FREQUENCY SPECTRUM OF THE DAYTIME WAVE-FORM OF ATMOSPHERICS RADIATED FROM THE RETURN STREAMER OF LIGHTNING FLASH.

KAZUO SAO

I. Introduction.

Researches into the theoretical treatment of the waveforms of atmospherics have been published by Budden and others. Since it is clearly of interest to deduce the frequency spectrum of atmospherics using the theory recently proposed by Budden, the author has determined the frequency spectrum of atmospherics received in the daytime according to the wave guide mode theory and with present information on the lightning discharge reported by Norinder.

II. Long radio wave field intensity on the surface of the earth by Budden's wave guide mode theory.

Budden's theory is successful for the following reasons; first, because frequencies lower than about 7.5 kc/s are greatly attenuated in the daytime, and, second, it is found that the oscillatory portion moves ahead of the slower disturbance as the distance of propagation increases. He obtained an expression for the horizontal magnetic field produced at a great distance in a parallel plane wave guide by a vertical electric dipole situated on the surface of the earth. In this theory, it is assumed that the atmospherics travel in a wave guide formed between the earth as a perfect conductor and a sharply bounded ionosphere as an inperfect conductor. Furthermore, it is possible to prove the waveforms of atmospherics in either daytime or night time. However, I shall deal with the daytime type of waveforms only.

The horizontal magnetic field of continuous wave received at a distance ρ from the transmitter, is approximately proportional to

$$\sin^{\frac{1}{2}} i \cdot p^{3/2} \cdot e^{\frac{p\rho}{c} I_m(\sin i)} \cdot e^{j\left\{pt - \frac{p\rho}{c} Re(\sin i)\right\}} \quad \cdots\cdots\cdots\cdots (1)$$

where i is the angle of incidence (complex number), p is the angular frequency of a continuous wave, and c is the light velocity. But it is necessary to point out that the effect of the earth's magnetic field is neglected here.

III. Frequency spectrum of received atmospherics.

Budden deduced his expression for atmospheric waveform from Expression (1) by assuming the instantaneous destruction of a vertical electric dipole situated on the surface of the earth as the source of atmospherics. In a continuous frequency spectrum of this source, the spectrum is proportional to $\frac{1}{p}$, thus the spectrum of waveform is given by

$$|\sin^{\frac{1}{2}} i| \cdot p^{\frac{1}{2}} \cdot e^{\frac{p\rho}{c} I_m (\sin i)}$$

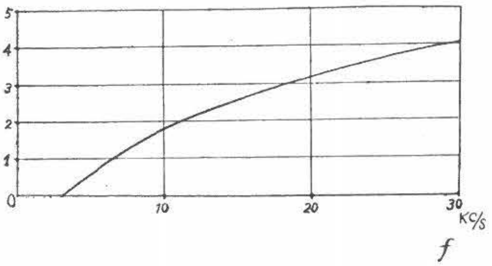

Fig. 1

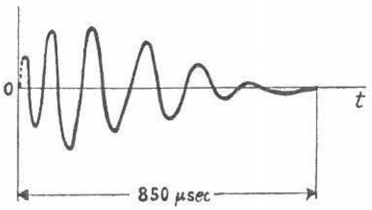

Fig. 2

However, it is difficult to explain the frequency spectrum of atmospherics by the curve of the spectrum as shown in Fig. 1.

Fig. 2 shows a typical atmospheric waveform $\Psi(t)$ observed in usual deytime conditions. The result of applying the Fourier transformation to this waveform to derive the frequency spectrum of the received atmospherics is :

$$\psi_{(f)} = \int_{-\infty}^{\infty} \Psi(t) \cdot e^{-jpt} \cdot dt$$

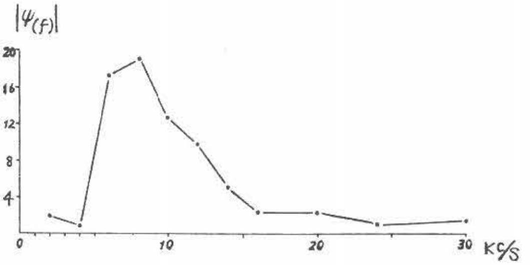

Fig. 3

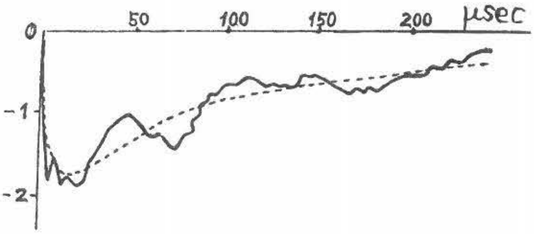

Fig. 4

and when the integral is evaluated numerically at 6.25 μsec intervals, $|\psi_{(f)}|$ is obtained as shown in Fig. 3.

The author tried to deduce the frequency spectrum on the basis of the information on the lightning discharge observed by Norinder in order to explain such a spectrum. Norinder proposes that the time rate of the change of discharge current can be shown in Fig. 4, and that it may be written

$$e^{-\alpha t} - e^{-\beta t}$$

neglecting the higher frequency deviation, where the values of α and β are 0.7×10^4 sec^{-1}, 4×10^4 sec^{-1}, respectively.

Thus the frequency spectrum of the discharge current is given by

$$\psi_0(f) = \int_0^{\infty} (e^{-\alpha t} - e^{-\beta t}) e^{-jpt} dt = \frac{1}{\alpha + jp} - \frac{1}{\beta + jp}$$

Then, from the above expression we obtain the next expression for the time function

$$\Psi_0(t) = \int_{-\infty}^{\infty} \left(\frac{1}{\alpha + jp} - \frac{1}{\beta + jp}\right) e^{jpt} dp$$

$$= \int_{-\infty}^{\infty} \sqrt{\left(\frac{\alpha}{\alpha^2 + p^2} - \frac{\beta}{\beta^2 + p^2}\right)^2 + p^2\left(\frac{1}{\alpha^2 + p^2} - \frac{1}{\beta^2 + p^2}\right)^2}\, e^{j\left\{pt - \mathrm{Tan}^{-1}\frac{p(\alpha+\beta)}{\alpha\beta - p^2}\right\}} dp \qquad \cdots\cdots\cdots\cdots (2)$$

Let us consider the radiation from the current of the lightning flash as radiation from the vertical antenna current, then we must multiply $\frac{1}{p}$ by Expression (2) .

Hence, from Expressions (1) and (2), frequency spectrum of received atmospherics is

$$|\sin^{\frac{1}{2}} i| \cdot e^{\frac{p\rho}{c} I_m(\sin i)} \cdot p^{\frac{1}{2}} \sqrt{\left(\frac{\alpha}{\alpha^2+p^2} - \frac{\beta}{\beta^2+p^2}\right)^2 + p^2\left(\frac{1}{\alpha^2+p^2} - \frac{1}{\beta^2+p^2}\right)^2}$$

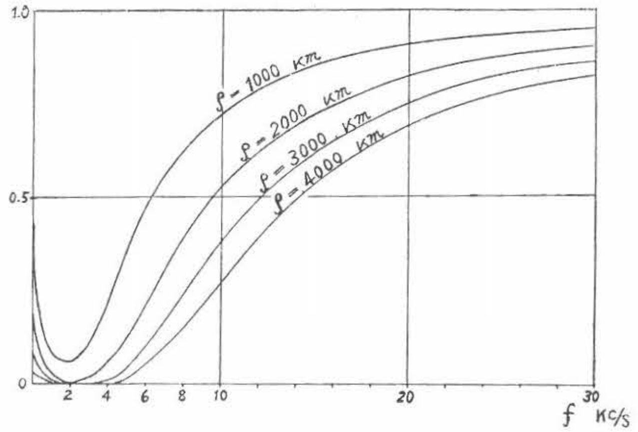

Fig. 5

This calculation has been made using the mode of order 0, and for the values of $h=70$ km, $p_1=2\times10^4\,\mathrm{sec}^{-1}$.

The curves in Fig. 5 show how the quantity $e^{\frac{p\rho}{c} I_m\ (\sin i)}$ varies with frequency when distance is taken as 1000km, 2000km, and 3000km. In this way the frequency spectrum of the waveforms of atmospherics may be calculated for disfferent distances from the source, as shown in Fig. 6.

IV. Conclusion.

Using the expression of field intensity of very long radio wave by Budden's wave guide mode theory, as well as Norinder's records of the variation of discharge current of lightning flash with time, the author endeavoured to derive the frequency spectrum of the atmospherics radiated from the return streamer. However, his computed results are not in good agreement with the spectrum of atmospherics observed in practice, especially in a rather high frequency range.

In conclusion I should like to record my sincere appreciation to Prof. A. Kimpara, Director of our Institute for constant encouragement and advice. I am also indebted to Mr. Otsu for helpful discussions throughout the course of the work. It is a pleasure to thank Miss T. Maeda for her cooperation in making the calculations.

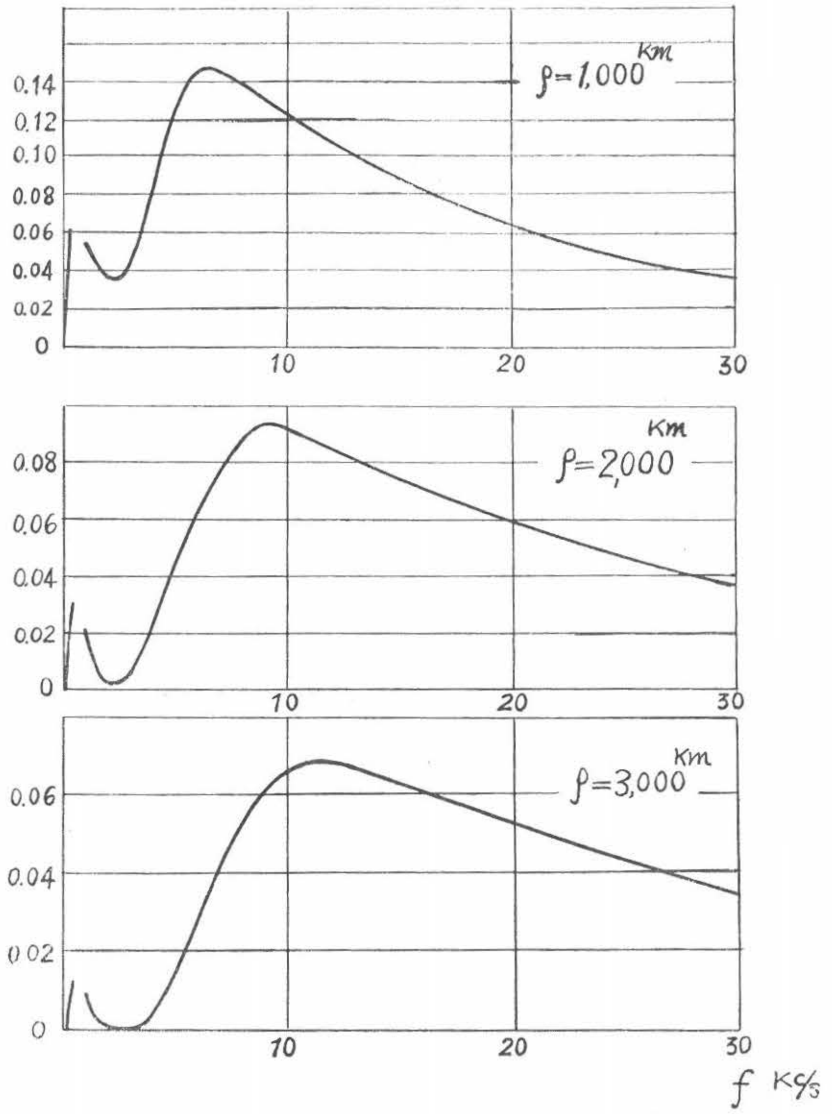
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
10
20
30
ρ=1,000 km
0.08
0.06
0.04
0.02
0
10
20
30
ρ=2,000 km
0.06
0.04
0.02
0
10
20
30
ρ=3,000 km
f Kc/s

Fig. 6