SOME CONSIDERATIONS OF THE WAVEFORM OF ATMOSPHERIC DUE TO IONOSPHERIC REFLECTION

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I. Introduction

Other authors have previously discussed the waveform of atmospherics received during the night hours, largely concerned with the waveform analysis, and have compared the distance from the source deduced by analysis with the distance located by C. R. D. F. Even for the most favourable peaked waveforms, considerable practical difficulties may be inherent in the estimation of the height of the reflecting layer and of the distance of origin, and perfect agreement between the two determinations is said to be obtained in only a small percent of estimation results. Perhaps a rather promising interpretation may be found by considering the finite conductivity of the reflecting layer. By assuming that the source of the atmospheric is a single rectangular pulse, and that the reflecting layer is a finite conductor sharply bounded at a height of 80 km and that the earth is a perfect conductor, the authors have shown by numerical computation how a pulse is transformed into a rather smooth half-wave, and how a kind of calculated waveform is consistent with that observed in practice.

It is concidered that the ground wave and the earth's magnetism are neglected in order to simplify the calculations.

II. Methods of calculation

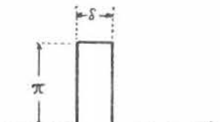

Fig. 1 Assumed source of atmospheric.

Let us consider a case in which a single rectangular pulse of short duration, polarized vertically, is radiated from a source.

In Fig. 1 δ is the width of pulse. The value of δ becomes more important in determining the computed waveform of atmospherics when we consider the time intervals between successive pulses. In this paper the authors have taken the values of δ to be 50 μ sec and 100 μ sec respectively.

The rectangular pulse can be expressed

$$H(t)=\int_0^\infty \left\{ \sin \omega t - \sin \omega(t-\delta) \right\} \frac{d\omega}{\omega}$$

In this case, the frequency spectrum is proportional to $\frac{2}{\pi\omega} \sin \frac{\omega\delta}{2}$ and here we may consider only the frequency range of $0<\omega<\frac{2\pi}{\delta}$. The reflection coefficient of the iono-

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sphere at very low frequency waves polarized vertically is given by

$$R=|R|e^{j\alpha}=\frac{\sqrt{\mu^2-\sin^2\theta}-\mu^2\cos\theta}{\sqrt{\mu^2-\sin^2\theta}+\mu^2\cos\theta}$$

where $\mu^2=1-j\frac{\omega_r}{\omega}$

$$\omega_r=\frac{4\pi Ne^2}{m\nu}$$

μ : refractive index of ionosphere,

θ : angle of incidence,

N : electron density in ionosphere,

ν : collision frequency of electrons.

and then the waveform of pulse reflected at the ionosphere becomes

$$h(t)=\int_0^\infty |R|\frac{\sin(\omega t+\alpha)}{\omega}d\omega-\int_0^\infty |R|\frac{\sin\{\omega(t-\delta)+\alpha\}}{\omega}d\omega$$

In calculating the above expression, we have assumed that both R and α are constant in the range of angular frequency $\omega_n\sim\omega_m$, and can thus obtain the next expression.

$$h(t)_{\omega_n\sim\omega_m}=|R|\int_{\omega_n}^{\omega_m}\frac{\sin(\omega t+\alpha)}{\omega}d\omega-|R|\int_{\omega_n}^{\omega_m}\frac{\sin\{\omega(t-\delta)+\alpha\}}{\omega}d\omega$$

It is evident that this leads to

$$\begin{aligned}h(t)_{\omega_n\sim\omega_m}=&|R|\cos\alpha\,[Si(\omega_m t)-Si\{\omega_m(t-\delta)\}]\\&-|R|\cos\alpha\,[Si(\omega_n t)-Si\{\omega_n(t-\delta)\}]\\&+|R|\sin\alpha\,[Ci(\omega_m t)-Ci\{\omega_m(t-\delta)\}]\\&-|R|\sin\alpha\,[Ci(\omega_n t)-Ci\{\omega_n(t-\delta)\}]\ \cdots\cdots\cdots\cdots\cdots(1)\end{aligned}$$

Where Si and Ci denote sine and cosine integral, respectively.

III. Results obtained

In numerical calculation we have used the values of ω_r as 6×10^6 sec^{-1} and 6×10^5 sec^{-1} because the calculated waveform can then be made to fit the waveform observed in practice.

Frequency ranges used in calculations are as follows:

$\frac{\omega_r}{\omega}$: 20-30, 30-60, 60-100, 100-200, 200-600, 600-2000, and 2000-∞

Absolute reflection coefficient $|R|$ and its phase α with the function of $\frac{\omega_r}{\omega}$ are

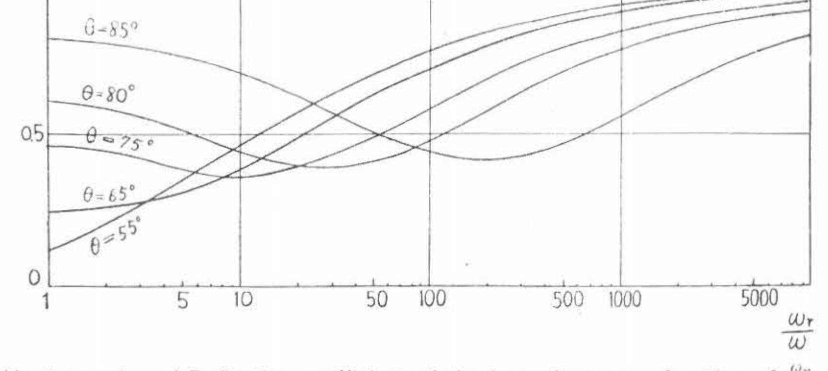

Fig. 2 Absolute value of Reflection coefficient of the ionosphere as a function of $\frac{\omega_r}{\omega}$

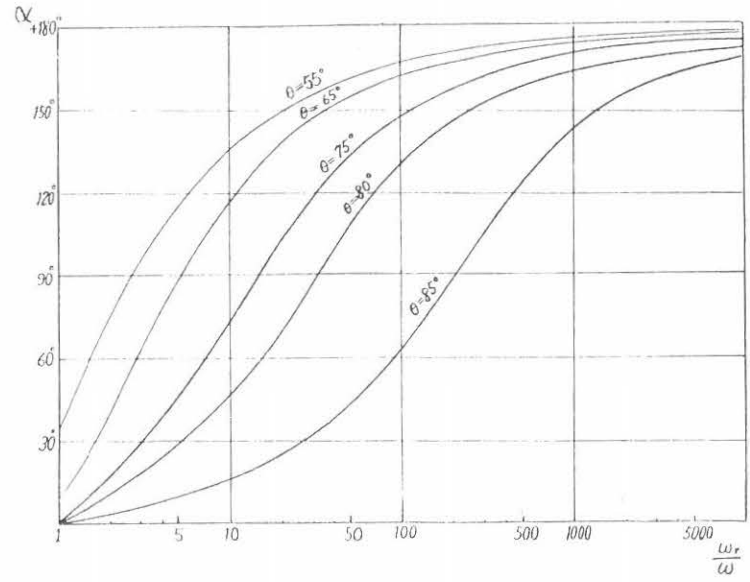

Fig. 3 Phase angle of Reflection coefficient of the ionosphere as a function of $\frac{\omega_r}{\omega}$

shown in Figs. 2 and 3, respectively.

Now we shall consider a case where $\delta=100\mu$ sec, and calculate Expression (1) over the range of frequency 0-32 kc/s. Within this frequency range, the original rectangular pulse given by Expression (1) when $|R|=\alpha=0$ is fairly well represented by illustration in Fig. 4.

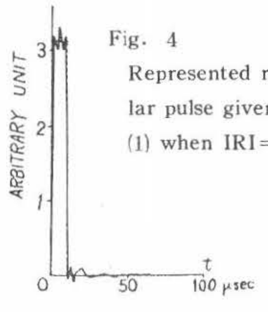

Fig. 4 Represented rectangular pulse given by eqn. (1) when $|R|=\alpha=0$

Assuming that the distance travelled d is 1000 km or 2000 km and that the orders of reflection at the ionosphere n are 1, 2, 3 and 4, the reflected pulses deduced from Expression (1) ara as shown in Figs. 5, 6, 7 and 8. By these figures it can be seen that the time at which each waveform shows its peak value grows later as distance d and the order of reflection n increase, and as the value of ω_r decreases. This tendency is shown in Table 1. In other words, the higher the order of reflection and the lower the value of ω_r, the pulses are more transformed. Figs. 9, 10 and 11 show the ionospheric reflection type waveforms obtained by superimposing these pulses with successive n but withidentical d and ω_r.

These figures resemble to some extent the waveforms observed in practice. (see Figs. 12, 13). It is a matter of fact that we must take into account the time interval between each reflected pulse and sin θ, since the amplitude of pulse received with vertical antenna is proportional to sin θ, where θ is the angle of incidence.

Finally, a calculation of the analysed distances of the waveforms resulting from a series of transformed pulses are given in Table 2, where it is found that these distances from sources are shorter than those assumed in calculating and where p and q are the orders of reflections.

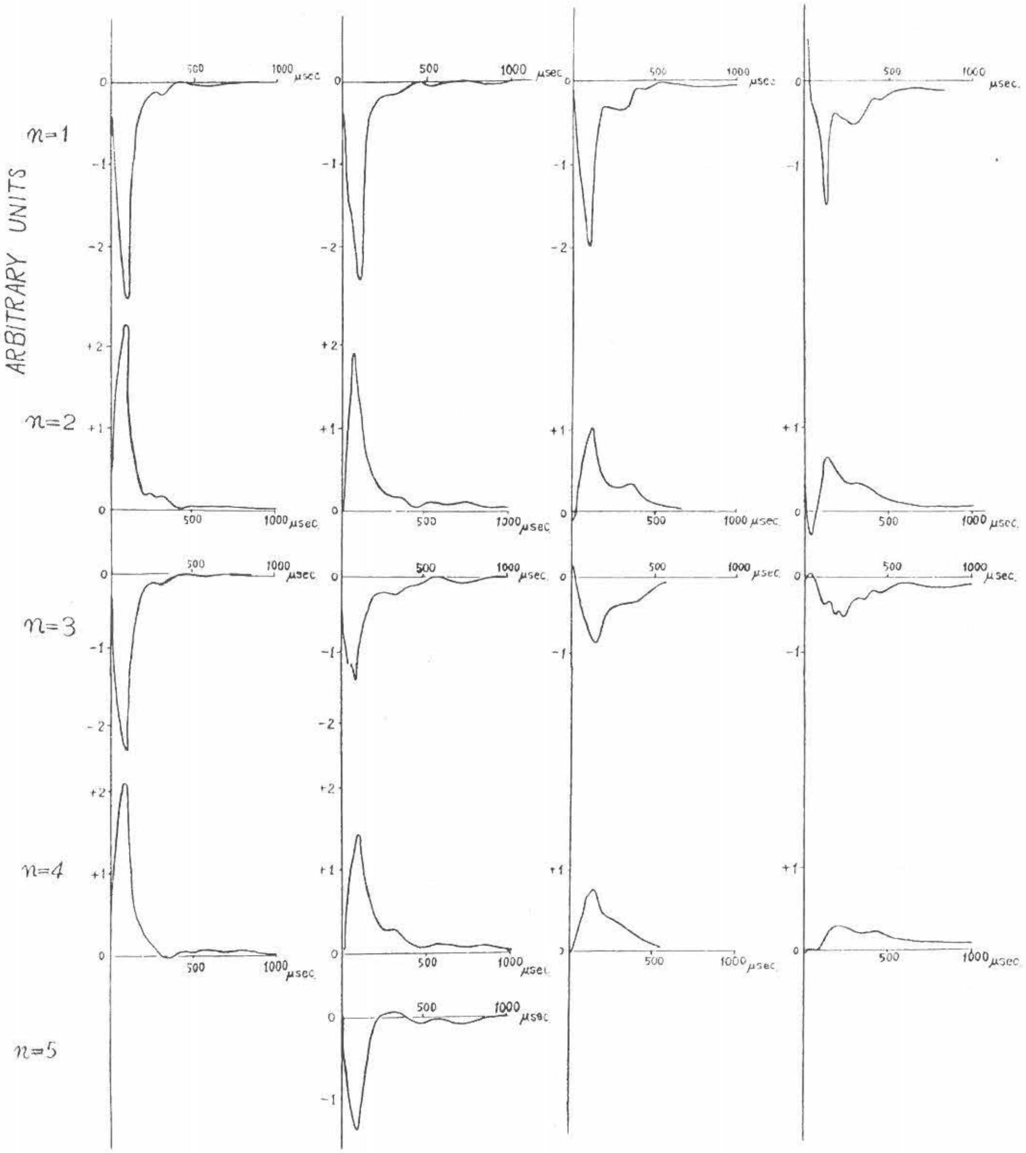

Fig. 5
Reflected pulses from the ionosphere.
$\delta = 100\mu\text{sec}$,
$\omega_r = 6 \times 10^6 \text{sec}^{-1}$,
d = 1000km

Fig. 6
Reflected pulses from the ionosphere.
$\delta = 100\mu\text{sec}$,
$\omega_r = 6 \times 10^6 \text{sec}^{-1}$,
d = 2000km

Fig. 7
Reflected pulses from the ionosphere.
$\delta = 100\mu\text{sec}$,
$\omega_r = 6 \times 10^5 \text{sec}^{-1}$,
d = 1000km

Fig. 8
Reflected pulses from the ionosphere.
$\delta = 100\mu\text{sec}$,
$\omega_r = 6 \times 10^5 \text{sec}^{-1}$,
d = 2000km

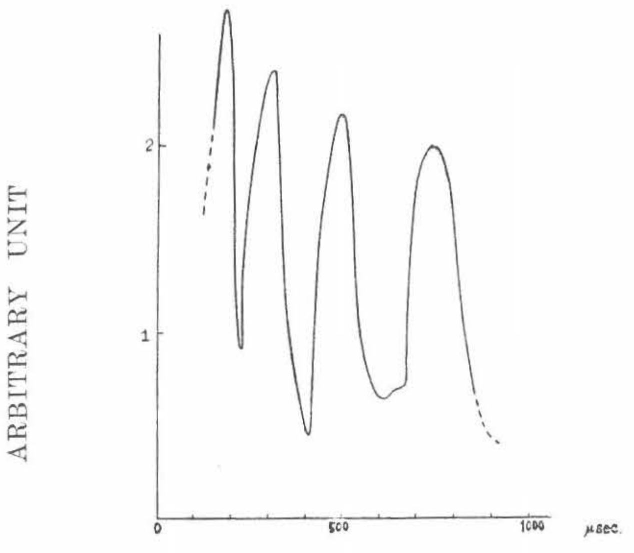

Fig.9 Waveform of atmospheric resulted from the pulses shown in Fig.5.
δ=100 μ sec, $\omega_r = 6 \times 10^6$ sec^{-1}, d=1000 km, h=80 km.

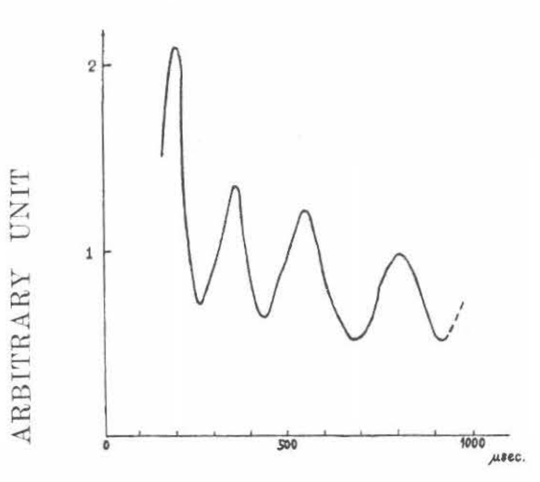

Fig.10 Waveforms of atmospheric resulted from the pulses shown in Fig.7.
δ=100 sec, $\omega_r = 6 \times 10^5$ sec^{-1}, d=1000km, h=80km.

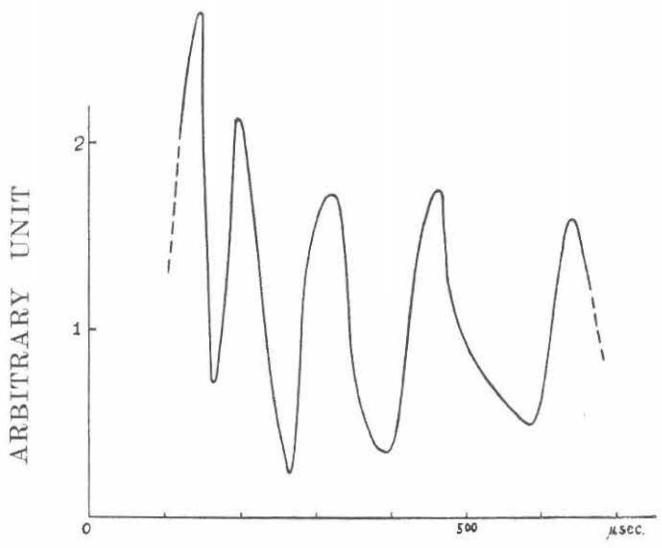

Fig.11 Computed waveform of atmospheric.
$\delta=50\ \mu sec,\ \omega_r=10^7\ sec^{-1},\ d=2000\ km,\ h=30\ km.$

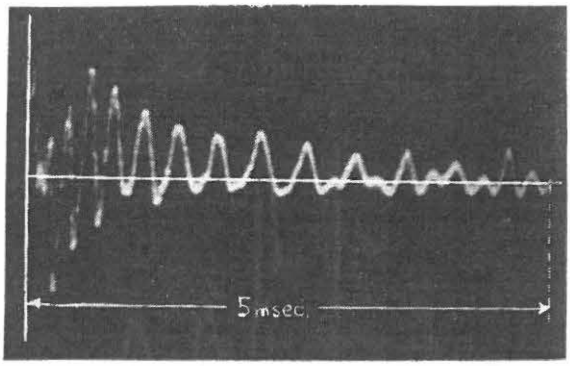

Fig. 12 An example of ionospheric reflection type waveform observed at 20 h, 15 Sep. 1952.

Distance analised from the waveform and that determined with Sferics fix are about 1200 km.

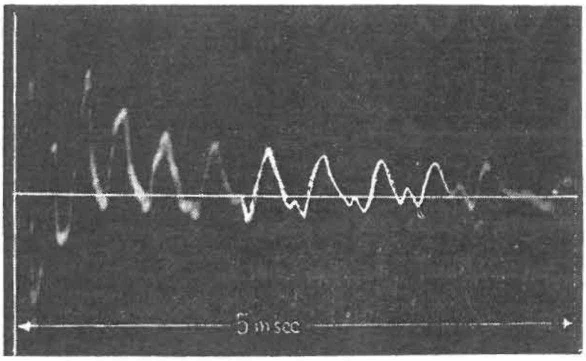

Fig. 13 An example of ionospheric reflection type waveform observed at 20 h, 15 Sep. 1952.
Distance analised from the waveform is about 800 km.

IV. Conclusion

(1) When Fresnel's reflection formula is used as in this paper, the attenuation becomes still smaller at lower frequency as the frequency decreases, that is, the value of reflection coefficient approaches unity as the value $\frac{\omega_r}{\omega}$ increases. (Fig. 2).

(2) As is to be expected, there are more transformations in the reflected pulse as the order of reflection and the distance from the source increase, and as the value of ω_r decreases.

(3) The deduced distance of source from the waveform is generally less than that assumed in calculations.

V. Acknowledgement

In conclusion the authors wish to thank Prof. A. Kimpara for his valuable suggestions in this investigation as well as for his constant guidance throughout the course of this work.

TABLE 1

ω_r	d(km) \ n	1 (μ sec)	2 (μ sec)	3 (μ sec)	4 (μ sec)	5 (μ sec)
6×10^6	1000	90	90–100	90–100	90–100	—
	2000	90–100	70	100–110	100–110	100
6×10^5	1000	110	130	130	140	—
	2000	110	120	230	200	—

TABLE 2

Figure	Distance from source employed in calculation (km)	Sign of pulse employed in analysis	Distance analysed from the waveform resulted from a series of pulses			
Fig. 9	1000	positive sign	p=1 q=3	990(km)	p=2 q=4	800(km)
		negative sign	p=2 q=4	930		
Fig. 10	1000	positive sign	p=1 q=3	900	p=2 q=4	980
		negative sign	p=2 q=4	880		
Fig. 11	2000	positive sign	p=1 q=3	1970	p=2 q=4	1250
		negative sign	p=2 q=4	1460	p=2 q=5	1300