

THE FREQUENCY SPECTRUM OF ATMOSPHERICS

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Summary: —

A method of frequency analysis is introduced on a basis of the Fourier integral, and is applied to certain types of atmospherics: (1) the atmospherics radiated from return streamer of lightning flash in the day time, (2) the quasi sinusoidal waveform at night.

The results obtained by this method show that: (1) the distance effects with relation to the spectrum of not only the complete waveform, but also each quasi one-cycle of the waveform, (2) the decreasing effects of the latter spectrum accompanied by the increasing order number of the quasi one-cycle in waveform.

In the latter part of this paper, one waveform, having two large maximum values in higher frequency region in the amplitude frequency spectrum, is given and investigated in relation to a feature of the waveform.

I. Introduction

The frequency spectrum of atmospherics has been previously investigated by several authors for some years.^{1), 2), n)}

In this paper a method of frequency analysis, after having been applied to certain types of waveforms of atmospherics, is introduced on a basis of the Fourier integral. These waveforms were photographically recorded at Kumamoto on a wide-band receiver, total duration 1 msec. in the summer of 1954. During these observations the direction finders were operated simultaneously at Toyokawa, Akita and Kumamoto for the accurate location of the origins of individual atmospherics by members of the Research Institute of Atmospheric of Nagoya University.

The results thus obtained have been investigated with particular reference to the effects of the distance not only for the complete waveform, but for each quasi one-cycle of the waveform. They also show the decreasing effect of the greatest component frequency in each quasi one-cycle of the waveform as its order number increases in the waveform. Further, they show that there is an atmospheric having two large maximum values in higher frequency region in the amplitude frequency spectrum, corresponding to some feature of the waveform.

II. The Method of Frequency Analysis

1. The Principle of Frequency Analysis

A waveform analyzed mathematically consists of an oscillatory portion composed of a sequence of half-cycles of sine wave having eigen amplitude and period respectively (see Fig. 1). Each of these half-cycles are numbered consecutively beginning with the waveform and a pair of half-cycles composed of the $(2n-1)$ th and $2n$ th half-cycle of the waveform is designated the n th quasi one-cycle of the waveform because the amplitude and period of the $(2n-1)$ th half-cycle are usually different from those of the $2n$ th.

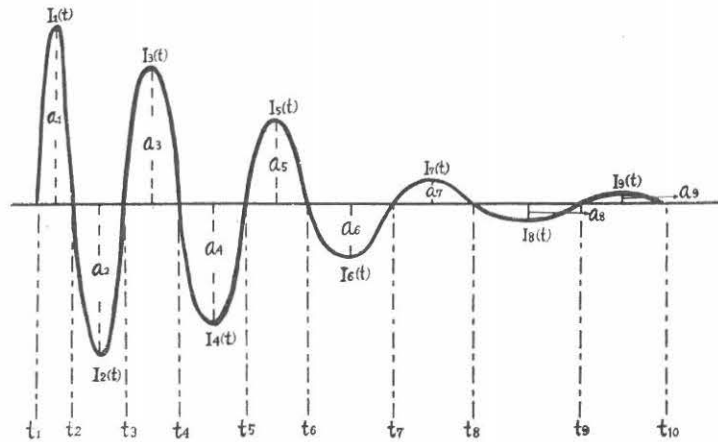


Fig .1. The Waveform Analyzed Mathematically

As is shown in Fig.1, the n th half-cycle of the waveform is expressed mathematically as a function of time by the expressions :

$$\left. \begin{aligned} I_n(t) &= (-1)^{n+1} a_n \sin \omega_n(t-t_n) & t_n \leq t \leq t_{n+1}, \\ &= 0 & t_n > t, \\ &= 0 & t > t_{n+1}, \end{aligned} \right\} \dots\dots\dots (1)$$

where

a_n is the amplitude of the n th half-cycle of sine wave

ω_n is the angular frequency of the n th half-cycle of sine wave and, equal to 2π times the reciprocal of its period T_n

t_n is the time at which the n th half-cycle of sine wave begins to appear in the waveform.

On a basis of the Fourier transform, time functions and frequency functions now can be written for the half-cycle, the quasi one-cycle of the waveform, and the complete waveform. Each pair of time and frequency are known as mates. The mates are expressed for the n th half-cycle by the following two equations:

$$\left. \begin{aligned} \Phi_n(f) &= \int_{-\infty}^{+\infty} I_n(t) e^{-j\omega t} dt, \\ \text{and} \\ I_n(t) &= \int_{-\infty}^{+\infty} \Phi_n(f) e^{j\omega t} df, \end{aligned} \right\} \dots\dots\dots (2)$$

where

j is equal to $\sqrt{-1}$

ω is the angular frequency and equal to 2π times the frequency f

$I_n(t)$ is the time function for the n th half-cycle of the waveform

$\Phi_n(f)$ is the frequency function for the n th half-cycle of the waveform.

For the n th quasi one-cycle of the waveform, the mates are expressed by the following two equations:

$$\left. \begin{aligned} Q_n(f) &= \int_{-\infty}^{+\infty} q_n(t) e^{-j\omega t} dt, \\ \text{and} \\ q_n(t) &= \int_{-\infty}^{+\infty} Q_n(f) e^{j\omega t} df, \end{aligned} \right\} \dots\dots\dots (3)$$

where $q_n(t)$ and $Q_n(f)$ are the time function and the frequency function respectively for the n th quasi one-cycle of the waveform.

And for the complete waveform, the mates are given as

$$\left. \begin{aligned} \Phi(f) &= \int_{-\infty}^{+\infty} I(t) e^{-j\omega t} dt, \\ \text{and} \\ I(t) &= \int_{-\infty}^{+\infty} \Phi(f) e^{j\omega t} df, \end{aligned} \right\} \dots\dots\dots (4)$$

where $I(t)$ and $\Phi(f)$ are the time function and the frequency function respectively for the complete waveform.

Further, from the definition given above for the waveform, as shown in Fig. 1, the expressions (3) and (4) can be written as follows. For the quasi one-cycle of the waveform, that is, the expressions (3)

$$\left. \begin{aligned} Q_n(f) &= \Phi_{2n-1}(f) + \Phi_{2n}(f) \\ &= \int_{-\infty}^{+\infty} I_{2n-1}(t) e^{-j\omega t} dt + \int_{-\infty}^{+\infty} I_{2n}(t) e^{-j\omega t} dt, \\ \text{and} \\ q_n(t) &= I_{2n-1}(t) + I_{2n}(t) \\ &= \int_{-\infty}^{+\infty} \Phi_{2n-1}(f) e^{j\omega t} df + \int_{-\infty}^{+\infty} \Phi_{2n}(f) e^{j\omega t} df, \end{aligned} \right\} \dots\dots\dots (5)$$

and for the complete waveform, that is, the expressions (4)

$$\left. \begin{aligned} \Phi(f) &= \sum_{n=1}^m \Phi_n(f) = \int_{-\infty}^{+\infty} \left\{ \sum_{n=1}^m I_n(t) \right\} e^{-j\omega t} dt, \\ \text{and} \\ I(t) &= \sum_{n=1}^m I_n(t) = \int_{-\infty}^{+\infty} \left\{ \sum_{n=1}^m \Phi_n(f) \right\} e^{j\omega t} df, \end{aligned} \right\} \dots\dots\dots (6)$$

where m is the total number of the half-cycles of sine wave in the waveform.

Now let us convert each of these frequency spectra, $\Phi_n(f)$, $Q_n(f)$, and $\Phi(f)$, into a calculable form, for given values of a_n , t_n , and ω_i of sine wave in the waveform, that is, in the expressions (1). For this purpose, when $I_n(t)$ of expressions (1) is put into $\Phi_n(f)$ of expressions (5), $\Phi_n(f)$ becomes

$$\Phi_n(f) = \int_{-\infty}^{+\infty} I_n(t) e^{-j\omega t} dt = \int_{t_n}^{t_{n+1}} (-1)^{n+1} a_n \sin \omega_n(t - t_n) e^{-j\omega t} dt.$$

By making the variable transformation $t - t_n = s$, because the period T_n is equal to $2(t_{n+1} - t_n)$, $\Phi_n(f)$ is equal to

$$\begin{aligned} \Phi_n(f) &= \int_0^{t_{n+1} - t_n} (-1)^{n+1} a_n \sin \omega s e^{-j\omega(s+t_n)} ds \\ &= (-1)^{n+1} a_n e^{-j\omega t_n} \int_0^{T_n/2} \sin \omega_n s e^{-j\omega s} ds \\ &= (-1)^{n+1} a_n e^{-j\omega t_n} \frac{2\omega_n}{(\omega_i - \omega)(\omega_n + \omega)} \cos \frac{\omega T_n}{4} e^{-j\omega \frac{T_n}{4}} \\ &= (-1)^{n+1} 2a_n \frac{\omega_n \cos \omega T_n/4}{(\omega_i - \omega)(\omega_n + \omega)} e^{-j\omega(t_n + \frac{T_n}{4})} \dots\dots\dots (7) \end{aligned}$$

It follows that the mate $I_n(t)$ for $\Phi_n(f)$ can be written:

$$I_n(t) = \int_{-\infty}^{+\infty} (-1)^{n+1} 2a_n \frac{\omega_n \cos \omega T_n/4}{(\omega_i - \omega)(\omega_n + \omega)} e^{j\omega(t - t_n - \frac{T_n}{4})} df.$$

$$= \int_0^{+\infty} (-1)^{n+1} 4a_n \frac{\omega_i \cos \omega T_n/4}{(\omega_i - \omega)(\omega_i + \omega)} \cos \omega(t - t_n - \frac{T_n}{4}) df \dots \dots \dots (8)$$

Thus making use of the expression (7), $\Phi_n(f)$, the following two expressions can easily be derived.

For the n th quasi one-cycle of the waveform, $q_n(t)$ in the expressions (5) is equal to

$$q_n(t) = \int_0^{+\infty} \left\{ \sum_{r=2n-1}^{2n} (-1)^{n+1} 4a_r \frac{\omega_r \cos \omega T_r/4}{(\omega_r + \omega)(\omega_r - \omega)} \cos \omega(t - t_r - \frac{T_r}{4}) \right\} df, \dots \dots \dots (9)$$

and for the complete waveform, $I(t)$ in the expressions (6) is equal to

$$I(t) = \int_0^{+\infty} \left\{ \sum_{n=1}^{\infty} (-1)^{n+1} 4a_n \frac{\omega_n \cos \omega T_n/4}{(\omega_n + \omega)(\omega_n - \omega)} \cos \omega(t - t_n - \frac{T_n}{4}) \right\} df \dots \dots \dots (10)$$

These three integrands of these three expressions (8), (9), and (10) are the final forms of the frequency spectra which forms are the aim of these derivations for the half-cycle, the quasi one-cycle of the waveform and the complete waveform.

2. The Method of Graphical Evaluation

If numerical values were calculated, depending on ω or $f = \frac{\omega}{2\pi}$, of each of the three integrands in the expressions (8), (9), and (10), the frequency spectrum of the complete waveform and also each half-cycle and each quasi one-cycle of the waveform would be gained. But such a method of evaluation is so tedious and cumbersome that it is faster and more convenient to make use of a method of the graphical evaluation described later. This method in outline is as follows:

For the parameter $f (= \frac{\omega}{2\pi})$ Fig. 2

gives the value $\frac{\omega_n \cos \omega T_n/4}{(\omega_n - \omega)(\omega_n + \omega)}$ versus the axis of the period $T_n (= \frac{2\pi}{\omega_i})$ of the n th half-cycle of the waveform according to the expression (8). For example, in the same figure the three curves shown correspond to the component frequencies 5, 10, and 15 kc/s respectively. For a given value of the period T_n through the use of such a curve, can easily be read the value of the relative amplitude $\frac{\omega_n \cos \omega T_n/4}{(\omega_n - \omega)(\omega_n + \omega)}$

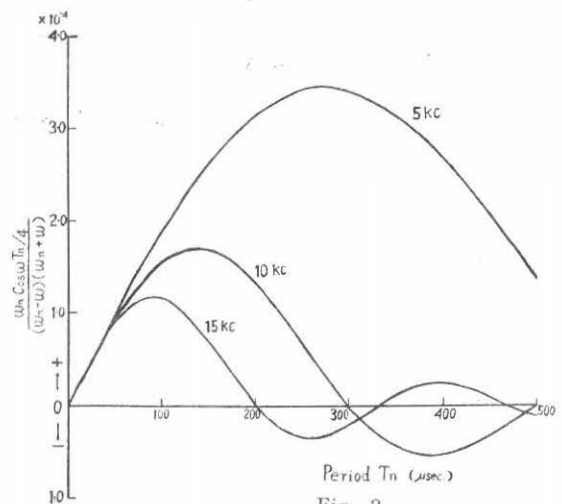


Fig. 2.

of the component frequency f on the one hand and the corresponding value of relative phase angle $\omega(-t_n - \frac{T_n}{4})$ can be easily evaluated on the other hand. Then, putting

the value of $(-1)^{n+1} a_n \frac{\omega_i \cos \omega T_n/4}{(\omega_n - \omega)(\omega_n + \omega)}$ along the radius of a circular section paper

and the corresponding value of $\omega(-t_n - \frac{T_n}{4})$ anticlockwise around the radius origin, only one point can be determined. This point represents two values i. e. the relative amplitude and relative phase angle of the component frequency f in the n th half-cycle of the waveform. Similarly, such a point can be determined on the same paper for each half-cycle of the waveform. According to the expressions (9) and (10) for the component frequency f , the point representing the two values, relative amplitude and relative phase angle, for each quasi one-cycle in (9) and for the complete waveform in (10) is determined on the paper as the vectorial superposition of the two required points, or all of the points determined for each half-cycle of the waveform. By repeating similar processes for a sufficient number of different component frequencies the frequency spectrum for each of the waveforms can be obtained, as defined above.

III. The Results of Frequency Analysis

1. The Type of Atmospherics

The present chapter gives the results of the frequency spectrum of the atmospheric waveforms, as analyzed by the method of graphical evaluation on a basis of the principle of frequency analysis described in detail in the preceding chapter. These atmospheric waveforms were recorded at Kumamoto during observations of waveforms on a waveform recorder having a wide-band amplifier. The observations on waveforms were made with a network of direction-finder stations at Toyokawa, Kumamoto, and Akita for the accurate location of the atmospheric to be recorded.

In this chapter, two types of atmospheric waveforms are analyzed: (a) atmospheric radiated from return streamer of lightning flash in the day time, and (b) quasi sinusoidal type atmospheric received at night because they are considered to be the most fit for the application of the method of frequency analysis.

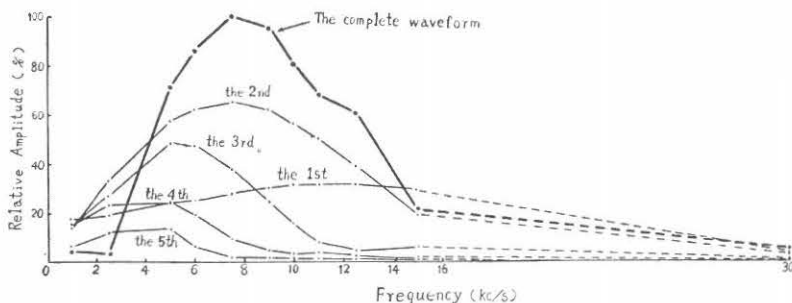


Fig. 3. The Amplitude frequency spectrum of not only the complete waveform, but each quasi one-cycle of the waveform. The mark "•" represents the relative amplitude of the component frequency having been evaluated.

Fig. 3 represents one result obtained for type (a) atmospheric waveforms received by day at Kumamoto; the amplitude frequency spectra are represented for both the complete waveform and each quasi one-cycle of the waveform. It is found in Fig. 3 that the spectrum shape changes and the maximum component frequency moves toward the lower frequency region for each quasi one-cycle of the waveform as its order number increases in the complete waveform.

2. The Frequency Spectrum of the Complete Waveform

Fig. 4 is the relative amplitude of the frequency components for atmospherics radiated from return streamer of day time lightning flash as received at different distances from the origin of the atmospherics. For the purpose of showing plainly the distance effect with relation to the spectrum, the maximum amplitude is equated at 100 % of the component frequency in each waveform of the atmospherics analyzed. As can be expected from the theory of wave guide,^{5) 6)} the frequency of the maximum component increases as the distance of propagation increases. (See Fig. 4)

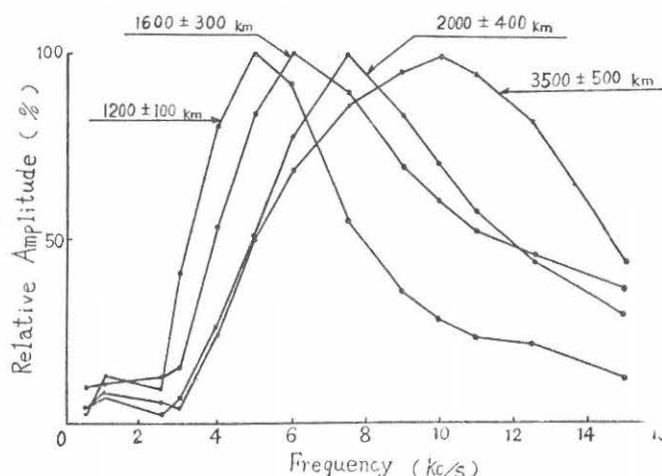


Fig. 4. Relative amplitude of the frequency components in the waveform of the atmospheric resulting from 'return streamers' of lightning flashes observed by day at different distances from the source. The components have been evaluated for frequency of 0.5, 1, 2.5, 3, 4, 5, 6, 7.5, 9, 10, 11, 12.5, 15 kc/s.

Other features of these curves are that there is the minimum value at frequencies in the regions from 2.5 to 3 kc/s and as the frequency decreases, one more maximum value appears below the marked drop in the curve. Now, the ratio of the new maximum value to the maximum amplitude described above is quite small, 8 to 15 %, and is not of sufficient percentage to be expected in the atmospherics of close origin. However, it may be considered the effect of the slow tail in the waveform. Further, the minimum value indicates the selective attenuation of the component frequencies near the drop in the curve.

For the quasi sinusoidal type atmospherics at night, several waveforms were analyzed in the same way. Although the analysis was insufficient for the aim of deriving conclusive results, owing to the small number of the waveforms analyzed, the results obtained did show the distance effect in the greatest component frequency for the complete waveform, and also for each quasi one-cycle of the waveform, described in the next paragraph, the small effect of the slow tail, and the selective attenuation in the amplitude frequency spectrum.

3. The Frequency Spectrum of the Quasi One-cycle of the Waveform

The atmospheric waveform of the first type treated above generally consists of an

oscillatory portion composed of a sequence of quasi half-cycles of increasing duration. Correspondingly, as described in Chapter II, a frequency component in the atmospheric waveform of this type can well be considered the vectorial superposition of all the vectors representing the corresponding component in each quasi one-cycle of the complete waveform.

Points in Fig. 5 represent the maximum component frequency for each quasi one-cycle against its order number in the complete waveform for the greatest component frequency 7.5 kc/s in the frequency spectrum of the complete waveform. Among these points having some degree of diversity of character, it is found that the maximum component frequency generally decreases as the corresponding order number increases in the complete waveform. In the same figure, similarly, the frequency of $\frac{1}{2}$ times amplitude of the greatest component, higher or lower than the maximum component frequency, is also shown. Similar diversities of character and similar trends can be seen among these points for both higher and lower frequencies considered. Fig. 6 represents the same kind of curve, on an average, for frequency of 5 or 6 kc/s respectively, with comparison of curves for frequency of 7.5 kc/s described above.

In Fig. 7 also is shown the maximum component frequency for each order number of quasi one-cycle against the maximum one in the complete waveform. The upward trend shows that the former frequency for any order number of quasi one-cycle in the waveform, almost always increases with the latter, according to the distance from the origin of the atmospherics, as may be inferred from Fig. 4.

4. A Particular Type of Frequency Spectrum

Some characteristics of the frequency spectrum of the waveform, have been discussed. Fig. 3 shows one large maximum value in higher frequency region above the marked drop in the curve representing the amplitude frequency spectrum. Among those waveforms which have already been described, we did not include one exception, which we will now describe. This particular type has two large maximum value in higher frequency region. Fig. 8 shows one maximum value at 10 kc/s and one at 5 kc/s.

The waveform of the quasi sinusoidal type has been recorded at night at a distance of about 4,000 km from the origin of the atmospherics. The remarkable difference between this waveform and the others under consideration is that the first half of the waveform shows a trend similar to the others, but subsequently in the last half smooth oscillations of comparatively larger amplitude and of increasingly longer periods

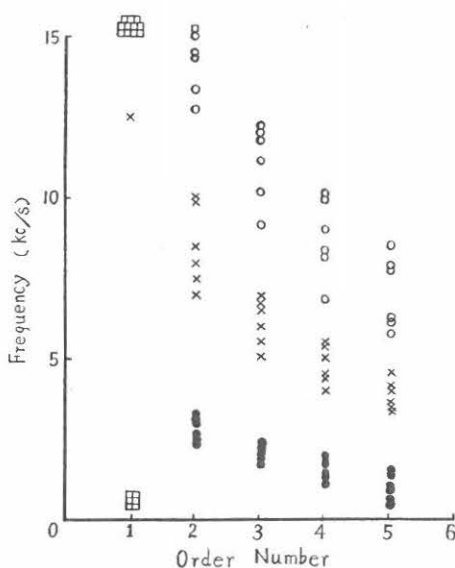


Fig. 5. The mark \times represents the maximum component frequency, and the marks \circ and \bullet represent the frequency of $\frac{1}{2}$ times of the maximum component, higher or lower than the maximum component frequency respectively. And, the mark \square shows that these frequencies are higher than 15 kc/s or lower than 0.5 kc/s.

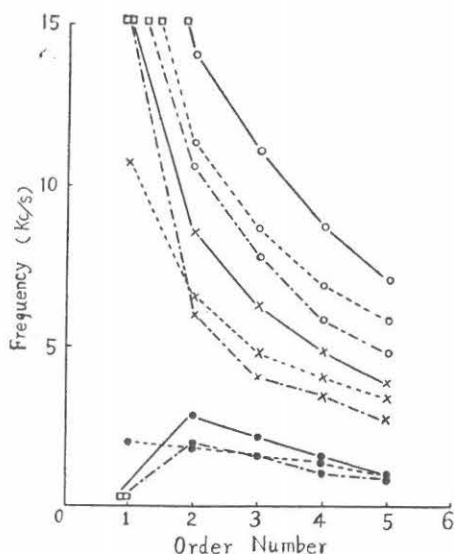


Fig. 6.

— is designated for the frequency of 7.5 kc/s.
 ---- is for the frequency of 6 kc/s.
 - · - is for the frequency of 5 kc/s.
 The marks \circ , \times and \bullet are used in the same sense, as in Fig. 5.

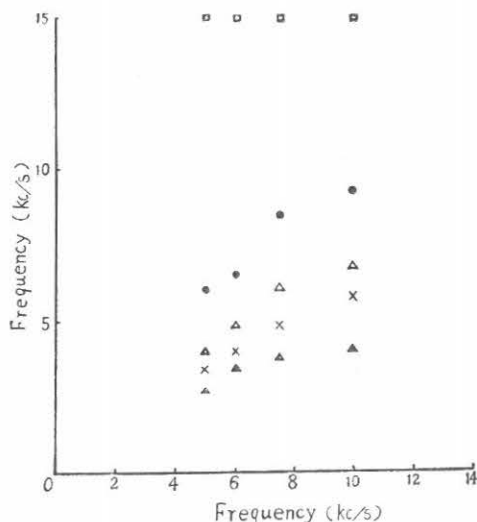


Fig. 7. The maximum component frequency of each quasi one-cycle of the waveform of the greatest component frequency of 5, 6, 7.5 and 10 kc/s; Δ , the 1st quasi one-cycle; \times , the 2nd; \triangle , the 3rd; \bullet , the 4th; \square , the 5th. The mark \square show, at the same time, that the corresponding frequencies are higher than 15 kc/s.

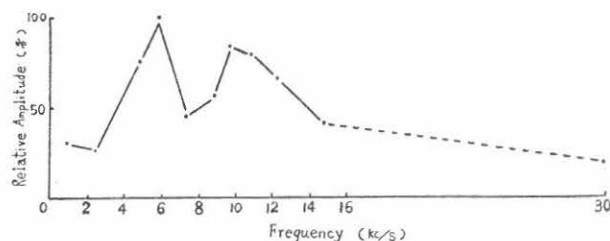


Fig. 8.

become re-established and, thereafter, the amplitude decreases by several half-cycles.

The characteristics of the several oscillations of slightly larger amplitude and of longer periods in the last half of the waveform produce the new maximum value of 5 kc/s different from the value at 10 kc/s in the amplitude frequency spectrum. The effect has been proved by checking the total sum of the vectors representing the relative amplitude and relative phase angle of the component frequency for each quasi one-cycle in the complete waveform under consideration.

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