CALCULATION OF INTERFERENCE ERROR ON A CROSSED-LOOP TYPE C. R. D. F.

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Summary— This paper describes the evaluation of interference error on a crossed-loop type C. R. D. F. First, an expression of interference error is obtained as a function of amplitude ratio, phase difference and relative direction between the interfering atmospherics and the observed atmospherics. Next, the mean-square error for a given amplitude ratio is computed assuming that the interfering atmospherics has random phase and bearing relative to the one under observation. And finally, the mean-square error and the probability of the mean-square error for the observed atmospherics are calculated either from the amplitude-occurrence distribution of atmospherics and the bandwidth of the amplifier or from the time rate of atmospheric noise and the bandwidth.

I. Introduction

In the observation of bearings of atmospherics, if an atmospherics is received during the time the effect of the previous one remains in the direction finder, the observed bearing of the atmospherics differs from the true one. The bearing error caused by this effect is the intererfence error.

As a result of recent developments in the radio direction finder, even a bearing error of a few degrees gives rise to discussion today. So it seems to be necessary to investigate the interference error as well as the site and polarization errors. In order to study the magnitude of the interference error, some calculation have been carried out along the same lines as some workers. [1) 2)]

Remark: The term "voltage" as used in the following means the voltage in the out put terminal of the amplifier of direction finder.

II. List of Symbols

ω = Tuned frequency of amplifier
T = Time-constant of amplifier
v = Amplitude of interfering atmospherics
E = Amplitude of observed atmospherics
ψ = Phase of interfering atmospherics relative to observed atmospherics
φ' = Bearing of interfering atmospherics.
φ = Bearing of observed atmospherics
$\alpha = \varphi' - \varphi$ Relative bearing of interfering atmospherics to observed one
Θ = Indicated bearing of observed atmospherics
$\Delta = \Theta - \varphi$ Error in indicated bearing
n = Number of atmospherics per second per unit range in voltage v
P = Time rate of atmospherics exceeding voltage v

III. Derivation of error formulae

1. Error by interference between two atmospherics

Since the selectivity of the direction finder is chiefly associated with single tuned circuit, the output from an amplifier due to an atmospherics is an oscillatory pulse rising almost instantaneously to its peak value v_0. The envelope of this pulse is defined by

$$v = v_0\ e^{-t/T}\ . \qquad (1)$$

If the interfering atmospherics rise to its peak value v_0 t' seconds before the atmospherics under observation rises to its peak value E, the amplitude of the interfering atmospherics at this moment becomes $v_0 e^{-t'/T}$.

Consequently, the output voltages from the N-S and E-W amplifiers in the direction finder are $e^{-t/T}\ \{E\cos\varphi\ \sin\omega t - v_0 e^{-t'/T}\cos\varphi'\ \sin\ (\omega t + \psi)\}$
and $e^{-t/T}\ \{E\sin\varphi\ \sin\omega t - v_0 e^{-t'/T}\sin\varphi'\ \sin\ (\omega t + \psi)\}$ respectively.

Then the indicated bearing is expressed as follows,

$$\tan 2\textcircled{H} = \frac{\sin 2\varphi + 2R\sin(\varphi+\varphi')\cos\psi + R^2\sin 2\varphi'}{\cos 2\varphi + 2R\cos(\varphi+\varphi')\cos\psi + R^2\cos 2\varphi'}\ ,$$

where R is used in place of $(v_0/E)e^{-t'/T}$ and is assumed to be less than unity in what follows. The indicated bearing is in error by a degree given by

$$\tan 2\Delta = \frac{2R\sin\alpha\ (\cos\psi + R\cos\alpha)}{1 + 2R\cos\alpha\cos\psi + R^2\cos 2\alpha}\ . \qquad (2)$$

From this eq. it can be seen that $\triangle$ vanishies when $\sin\alpha = 0$ for any values of ψ and R. This means that no interference error occurs between two atmospherics the same or counter direction. The maximum value of $|\Delta|$ for a given value of R occurs when $\cos\psi = 1$ and $\cos\alpha = -R$ or $\cos\psi = -1$ and $\cos\alpha = -R$, and the value is

$$\frac{1}{2}\tan^{-1}\frac{2R\sqrt{1-R^2}}{1-2R^2}\ . \qquad (3)$$

Figure 1 shows the values of Δ by eq. (2), and the curve A in Figure 2 shows (3).

2. The mean-square error

In section 1. the interference error is expressed in individual values of α, ψ and R, but in practice it is impossible to find these values. So in this section the mean-square error is treated as a practical expression for the interference error.

The mean-square error over all phases of interfering atmospherics is defined by

$\sigma^2 = \frac{1}{2\pi}\int_0^{2\pi}\Delta^2\ d\psi$, where Δ is taken from (2).

This integration has not been possible to carry out, but on replacing Δ^2 with $\frac{1}{4}\sin^2 2\Delta$ the integration can be accomplished. So, define $\sigma_\psi{}^2$ by the relation

$$\sigma_\psi{}^2 = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{4}\sin^2 2\Delta\ d\psi\ . \qquad (4)$$

The value of $\sigma_\psi{}^2$ will be good approximation to σ^2 if Δ is not too large for any value of ψ. Though this condition becomes unsatisfactory as R increases to unity, $\Delta^2 \geqq \frac{1}{4}\sin^2 2\Delta$ is always true, accordingly $\sigma^2 \geqq \sigma_\psi{}^2$ is also always true.

So the calculation on this replacement leads to the underestimation of the mean-

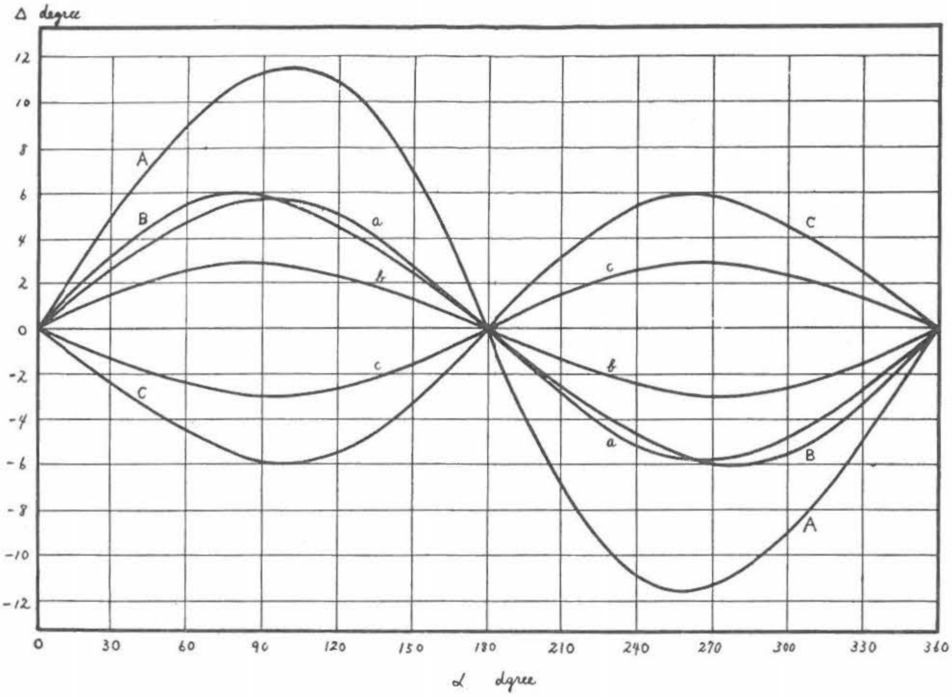

FIG. 1 $\Delta = \frac{1}{2} \tan^{-1} \frac{2R \sin \alpha \cdot \cos \psi + R^2 \sin 2\alpha}{1 + 2R \cos\alpha \cdot \cos \psi + R^2 \cos 2\alpha}$

A :	$R = 0.2$	$\psi = 0°$	a :	$R = 0.1$	$\psi = 0°$
B :	〃	$\psi = 60°$	b :	〃	$\psi = 60°$
C :	〃	$\psi = 120°$	c :	〃	$\psi = 120°$

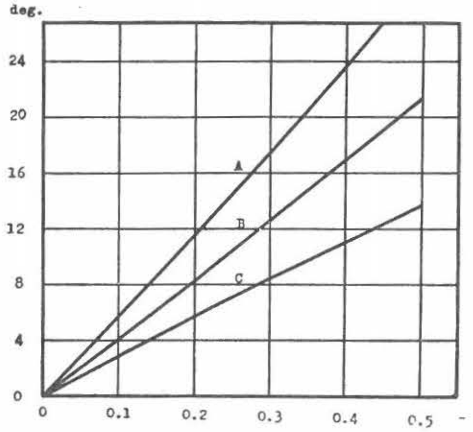

FIG. 2

A. $\frac{1}{2} \tan^{-1} \frac{2R\sqrt{1-R^2}}{1-2R^2}$ (value of $|\Delta|$ when $\cos \psi = 1$, $\cos \alpha = -R$)

B: $\frac{R\sqrt{1+R^2}}{\sqrt{2(1+R^4)}}$ (value of σ_{ψ} when $\alpha = 90°$)

C: $\frac{R}{2}\sqrt{\left(1 - \frac{R^2}{2}\right)}$ (value of $\sigma_{\psi\alpha}$)

square error and the knowledge of the order of the mean-square error will be obtained by this calculation.

Introducing Δ in eq. (2), eq. (4) is

$$\sigma_{\psi}{}^2 = \frac{1}{8\pi}\int_0^{2\pi} \frac{(2R \sin \alpha \cdot \cos \psi + R^2 \sin 2\alpha)^2}{(1+2R\cos\alpha \cdot \cos\psi + R^2 \cos 2\alpha)^2 + (2R \sin\alpha \cdot \cos\psi + R^2 \sin 2\alpha)^2}\, d\psi.$$

Putting Z equal to $e^{i\psi}$, this integration is transformed into a contour integration of which the contour is the unit circle.

$$\sigma_{\psi}{}^2 = \frac{\sin^2\alpha}{8\pi}\oint \frac{dZ}{iZ}\, \frac{(Z+2R\ Z \cos \alpha+1)^2}{(Z+Re^{i\alpha})\ (Z+R^{-1}e^{-i\alpha})\ (Z+R^{-1}e^{i\alpha})\ (Z+Re^{-i\alpha})}\ .$$

As $R<1$, the poles of the integrand within the unit circle are zero, $-Re^{i\alpha}$ and $-Re^{-i\alpha}$.

Therefore, $\sigma_{\psi}{}^2 = \dfrac{R^2 \sin^2\alpha\ (1-R^2\ \cos 2\alpha)}{2\ (1-2R^2\ \cos\ ^2\alpha+R^4)}$ radian^2 . (5)

From this expression it can be seen that when $\alpha=90^\circ$, σ_{ψ}, takes the maximum value, $\dfrac{R\sqrt{R^2+1}}{\sqrt{2\,(1+R^4)}}$. So the effect of interference is most serious when the directions of two atmospheric sources to a receiving point cross each other at right angles. The curve B in Figure 3 expresses $\dfrac{R\sqrt{R^2+1}}{\sqrt{2\,(1+R^4)}}$.

Fathermore, when the bearing of the interfering atmospherics is random, $\sigma^2{}_{\psi\alpha}$ is defined by

$$\begin{aligned}\sigma^2{}_{\psi\alpha} &= \frac{1}{2\pi}\int_0^{2\pi} \sigma_{\psi\alpha}\, d\alpha \\ &= \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2\ \sin^2\ \alpha\ (1-R^2\ \cos 2\alpha)}{2\ (1-2R^2\ \cos 2\alpha+R^4)}\, d\alpha \\ &= \frac{1}{8}\ R^2\ (2-R^2) \qquad \text{radian}^2 \qquad (6)\end{aligned}$$

which is the mean-square error for a given value of R. The curve C in Figure 2 shows $\sigma^2{}_{\psi\alpha}$. Next, the mean-square error for an atmospherics giving pulse of amplitude E can be expressed with the amplitude-occurrence distribution of atmospherics and the band-width of amplifier or with the time rate of atmospheric noise measured by the C. R. D. F.

Consider an amplitude interval dv at level v, if a pulse has an amplitude greater than v, the decreasing voltage lies in the range dv at level v for a time

$$dt = \frac{T}{v}\, dv \qquad \text{taken from eq. (1).} \qquad (7)$$

If N is the number of pulses per second exceeding v and pulses do not overlap, the voltage lies in the range for a time $N \cdot dt$ per second.

$$N \cdot dt = \frac{T}{v} dv \int_v^{\infty} n\ (v) \cdot dv$$

The magintude of the mean-spuare error caused by interfering voltage v for the atmospherics of amplitude E is $\dfrac{1}{8}\ \dfrac{v^2}{E^2}\ (2-\dfrac{v^2}{E^2})$, therefore $\dfrac{1}{8}\ \dfrac{v^2}{E^2}(2-\dfrac{v^2}{E^2})\ N \cdot dt$ is the average of the mean-square error caused by interfering voltage interval dv at level v.

Consequently the mean-square error caused by interfering voltages of all amplitudes up to kE is given by,

$$\frac{1}{8}\int_0^{kE}\frac{v^2}{E^2}\left(2-\frac{v^2}{E^2}\right)N\cdot dt$$

$$=\frac{1}{8}\int_0^{kE}\frac{v^2}{E^2}\left(2-\frac{v^2}{E^2}\right)\frac{T}{v}dv\int_v^{\infty}n\,(v')\,dv' \qquad \text{radian}^2 \qquad (8)$$

where k is any value smaller than unity.

In case the main sources of atmospherics are limited to two locations where relative direction from receiving point is α, the mean-square error is obtained, using (5) by

$$\frac{\sin^2\alpha}{2}\int_0^{kE}\frac{\frac{v^2}{E^2}\left(1-\frac{v^2}{E^2}\cos 2\alpha\right)}{1-2\frac{v^2}{E^2}\cos 2\alpha+\left(\frac{v^2}{E^2}\right)^2}\frac{T}{v}dv\int_v^{\infty}n\,(v')\,dv' \ \text{radian}^2 \quad . \qquad (9)$$

Since the pulse in the amplifier is oscillatory one, it is evident that $\frac{1}{2}N\cdot dt$ expresses a time rate of atmospheric noise having voltage between v and $v+dv$. Therefore, (8) and (9) can be written also as follows,

$$\frac{1}{8}\int_0^{kE}2\,\frac{v^2}{E^2}\left(2-\frac{v^2}{E^2}\right)\left(\frac{dP}{dv}\right)dv \qquad \text{radian}^2 \qquad (10)$$

$$\frac{\sin^2\alpha}{2}\int_0^{kE}2\,\frac{\frac{v^2}{E^2}\left(1-\frac{v^2}{E^2}\cos 2\alpha\right)}{1-2\frac{v^2}{E^2}\cos 2\alpha+\frac{v^4}{E^4}}\cdot\left(\frac{dP}{dv}\right)\cdot dv \qquad \text{radian}^2 \qquad (11)$$

where P is the time rate of atmospheric noise over the level v.

3. Probability of the mean-square error

The probability that one atmospherics is received during the voltage in the amplifier lies over the level v, is equal to the total time duration when the voltage lies over the level v during one second.

Since the pulse of peak value v_0 decreases as $v_0e^{-t/T}$, the time duration when the pulse lies over the level v is $T\log_e\frac{v_0}{v}$. Thus the total time duration per second is obtained by

$$\int_v^{\infty}n\,(v_0)\ T\log_e\frac{v_0}{v}\ dv_0 \qquad (12)$$

if the pulses do not overlap.

So the probability is given by eq. (12) for any value of v, on the other hand the mean-square error is also given from eq. (6) for any value of v, if the amplitude of received atmospherics is known. Therefore it can be seen that the mean-square error greater than $\frac{1}{8}\frac{v^2}{E^2}\left(2-\frac{v^2}{E^2}\right)$ occurs with the probability $T\int_v^{\infty}n\,(v_0)\,\log_e\frac{v_0}{v}dv_0$ for atmospherics of amplitude E. The same probability can be very easily expressed if the time rate is used, because the time rate is the probability itself. So it can be said similarly that the mean-square error greater than $\frac{1}{8}\frac{v^2}{E^2}\left(2-\frac{v^2}{E^2}\right)$ occurs with the probability P for atmospherics amplitude E.

IV. Conclusion

If the amplitude-occurrence distribibition or the time rate of atmospheric noise is Observed the magnitude of interference error in practice can be known from the results obtained, but it is possible to say that atmospherics of weak intensity have fairly good chance to suffer errors of several degrees. It has been shown that the effect of interference increases when two directions from atmospheric sources to a receiving point meet at right angles.

To reduce the errors, it is effective to make the time constant of the amplifier of direction finder as small as possible and to neglect bearings of atmospherics with intensities so small that they are exceeded by great number of atmospherics.

V. Aknowledgement

The authors wish to express their appreciation to Professor A. Kimpara, Director of their Reserarch Institute, for his constant guidance and Mr. Iwai for his advices and assistance during the course of this work and in the preparation of this paper.

VI. References

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