RESEARCHES IN THE FREQUENCY ANALYSES OF WAVE-FORMS OF ATMOSPHERICS — I

KAZUO SAO

I. Introduction.

In order to illustrate the wave-forms of atmospherics with the knowledge of pulses radiated from the origin and theories of propagation of V. L. F. waves, investigations of the frequency spectra obtained with the aid of frequency analyses were proposed.

In this field F. W. Chapman and others observed the responses of a number of narrow-band receivers tuned to various frequencies. The results were based on the smooth daytime type of atmospherics originated at known distances up to about 4,000 km. From these results they reduced the frequency characteristics of attenuation factors of very long wave propagation.

In this paper the author analyzed a number of smooth type wave-forms of atmospherics and tried to investigate the characteristics of frequency spectra. In the consideration of wave-forms of received atmospherics there are two problems to be considered; first the discharge current wave-forms of lightning strokes, secondly attenuation characteristics of propagation paths. In dealing with these problems, first of all, current wave-forms of strokes observed by Norinder and Dahle were analyzed. Next the author analyzed the wave-forms of received atmospherics not only of smooth daytime type but of general complicated type. The author, however, only reported the frequency spectra of various types of wave-forms of atmospherics and of their sources, because the number of results were not enough to derive further detailed studies, i. e. the propagation characteristics of the waves. Besides, a brief description of the frequency analyzer used is given here.

II. Frequency Analysis of Wave-forms of Atmospherics.

It is well-known that the freqency spectrum of a transient nonperiodic signal is a continuous spectrum, and the relation between a wave-form and its frequency spectrum is represented by an expression of the form

$$G(t)=\frac{1}{\pi}\int_0^{\infty} S(\omega)\cos\left[\omega t+\phi(\omega)\right]d\omega \quad\cdots\cdots(1)$$

$$S(\omega)=\sqrt{\left[\int_{-\infty}^{+\infty} G(t)\cos\omega t dt\right]^2+\left[\int_{-\infty}^{+\infty} G(t)\sin\omega t dt\right]^2} \quad\cdots\cdots(2)$$

$$\tan\phi(\omega)=-\frac{\int_{-\infty}^{+\infty} G(t)\sin\omega t\,dt}{\int_{-\infty}^{+\infty} G(t)\cos\omega t\,dt} \quad\cdots\cdots(3)$$

where $G(t)$ is the wave-form, $S(\omega)$ is the amplitude-frequency spectrum and $\phi(\omega)$ is the phase-frequency spectrum.

According to the above expression, the frequency spectrum can be obtained from the wave-form $G(t)$ by computing expressions (2) and (3).

However, because of the difficulties of the computation of Fourier transform expressions, the author prepared an analog computer described below. An atmospheric wave-form enlarged to a suitable size is translated along the time axis by the automatic rotations of the drum in a constant speed. At the same time, by tracing this enlarged wave-form by hand the voltage proportional to the amplitude of wave-form is obtained with a potentiometer. Besides another voltage is induced with the sine (or cosine) potentiometer whose revolution is regulated in a speed corresponding to the analysing frequency. Both voltages are multiplied and integrated with time, then the value of the next expression can be obtained.

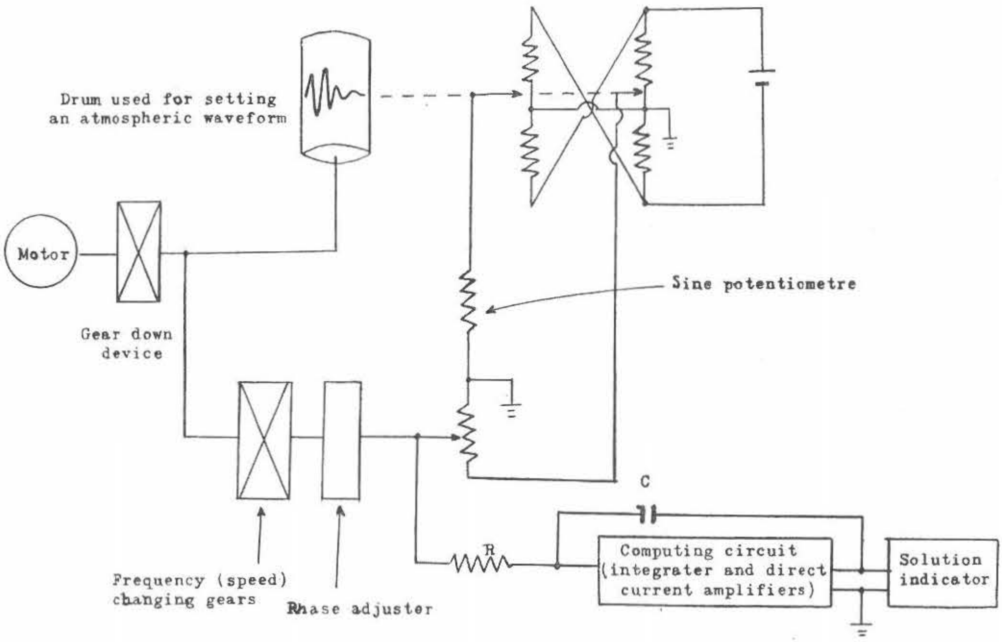

Fig. 1 Block diagram of frequency analyser for the use of wave-form of atmospheric.

$$\int_{-\infty}^{+\infty} G(t) \cdot \frac{\cos \omega t}{\sin \omega t} \cdot dt$$

From the expression now described $S(\omega)$ and $\phi(\omega)$ can be computed independently. The analysing frequency is easily regulated by changing the gear ratio of revolution of the sine potentiometer. The block diagram of this computer is shown in Fig. 1, and a photograph of the apparatus is shown in Photo. 1. It consists of four parts;

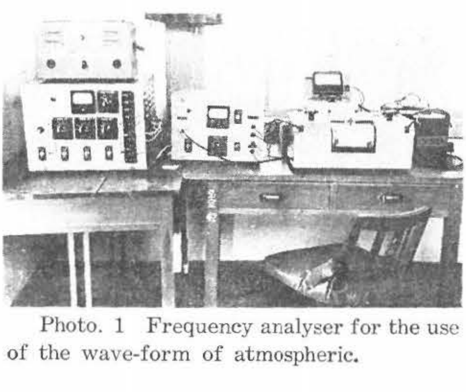

Photo. 1 Frequency analyser for the use of the wave-form of atmospheric.

tracer (right, front), electronic computer (centre), powor supply (left) and solution indicator (right, back). A frequency is adopted in the frequency range 2 kc/s to 30 kc/s, but it is to be noted that the frequency used must be corrected by the enlarged dimension of the wave-form.

III. Frequency Spectrum of the Origin of an Atmospheric.

3.1 Frequency spectrum derived from empirical formula reported by Morrison.

When we consider a lightning discharge as a source of an atmospheric, if we assume that a vertical cloud-to-ground lightning stroke is a vertical aerial, it is well known that the received electric field is proportional to the current of the aerial. Therefore it would be desirable to make an analysis of discharge procedure in a lightning, and we studied rather simple empirical formula reported by Morrison, although the actual current variations of lightning strokes represent complicated features. Fig. 2 shows the curve of discharge current variation reported by Morrison when $\alpha=0.7\times10^4\ \mathrm{sec}^{-1}$ and $\beta=4\times10^4\ \mathrm{sec}^{-1}$. The frequency spectrum derived from the formula is given by an expression of the form

$$-\int_0^\infty (e^{-\alpha t}-e^{-\beta t})\cdot e^{-j\omega t}\,dt$$
$$=\sqrt{\left(\frac{\alpha}{\alpha^2+\omega^2}-\frac{\beta}{\beta^2+\omega^2}\right)^2+\omega^2\left(\frac{1}{\alpha^2+\omega^2}-\frac{1}{\beta^2+\omega^2}\right)^2}\cdot e^{j\left\{\pi-\tan^{-1}\left[\frac{(\alpha+\beta)\omega}{\alpha\beta-\omega^2}\right]\right\}} \quad \cdots\cdots(4)$$

The amplitude- and the phase-frequency spectrum expressed in (4) are plotted in Fig. 3. In view of these results we find that in the amplitude-frequency spectrum of the origin the amount of the component at the lower frequencies, particularly in the region of 1 kc/s, is extremely large, although the pease-frequency spectrum cannot be varied more than π rad. These results now described are the most interesting results derived from the general knowledge of lightning strokes.

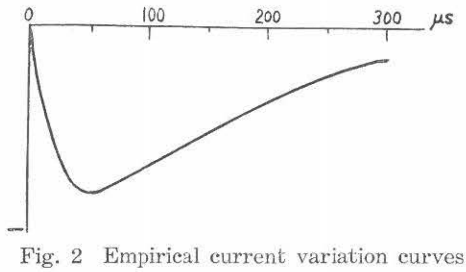

Fig. 2 Empirical current variation curves caused by lightning flashes by Morrison.

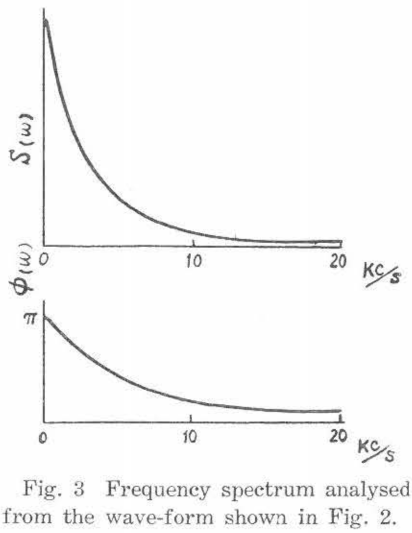

Fig. 3 Frequency spectrum analysed from the wave-form shown in Fig. 2.

3.2 Frequency spectrum derived from the discharge current observed by Norinder and Dahle.

Some curves of various types of discharge currents described in their paper were selected, and their frequency spectra were obtained. When we reflect upon the actual current variations of lightning discharges, in many cases superimposed quasi-periodic variations are found. These are mainly caused by influx of charges to the ionized path from adjacent volumes, and to some extent small fluctuations in the resistance in the lightning channel may be responsible for such superimposed variations.

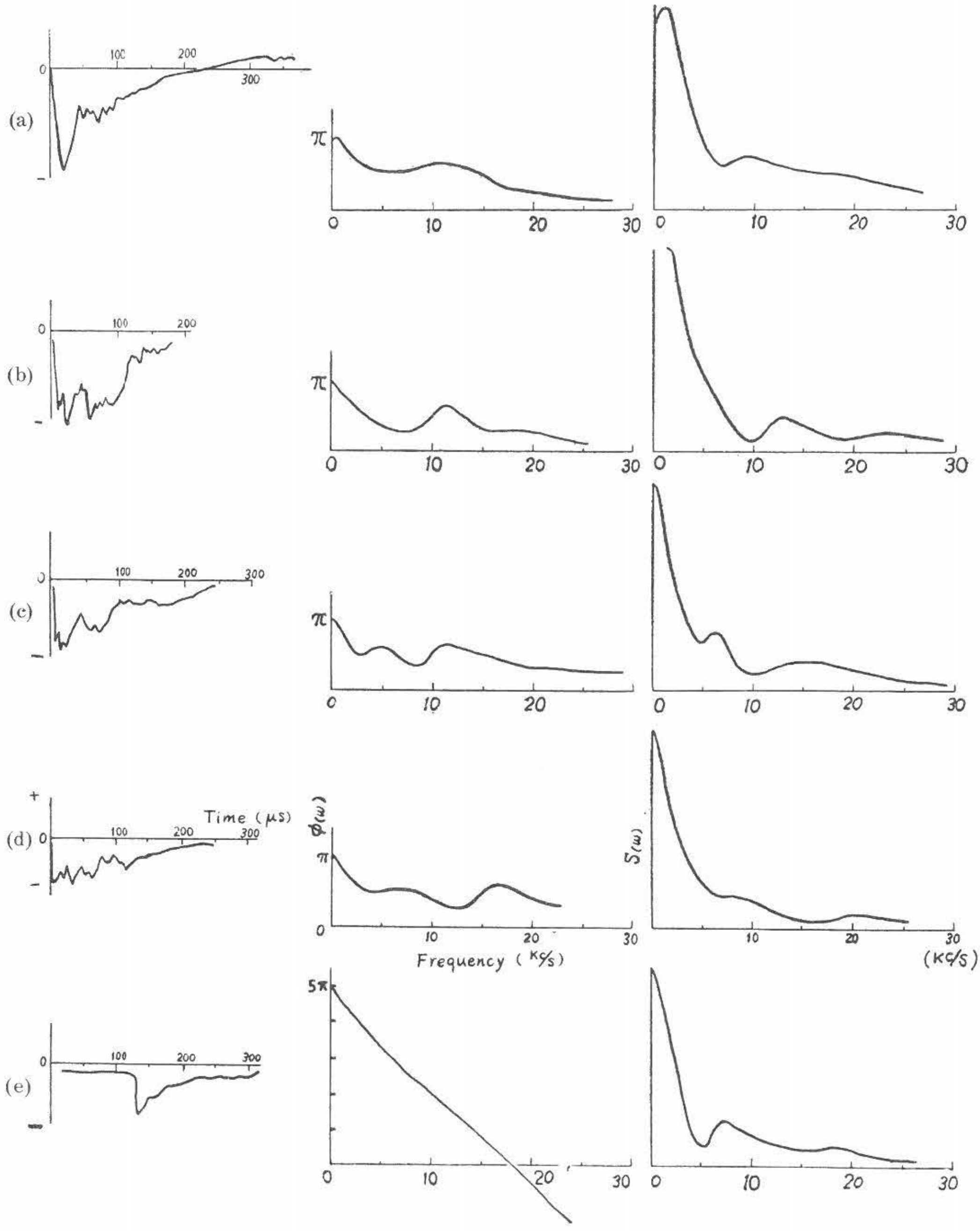

Fig. 4 Wave-forms of current variation caused by lightning flashes and their frequency spectra.

In Fig. 4 (a), (b), (c), (d) and (e) the original wave-forms and their spectra are represented. From these figures it is suggested that the frequency spectra at sources are variable, so it may be expected that the received wave-forms will be distorted in various types. These effects seem to justify some of the examples of spectra of received wave-forms discussed in 4. 2. Besides, we can find various maximum components in the amplitude-frequency spectrum at frequencies higher than about 5 kc/s; it is, therefore, conceivable that the amplitude-frequency specta of received wave-forms are affected by these maxima observed in the amplitude-frequency spectra at sources.

Furthermore, Fig. 4 (e) represents an example of the current of subsequent lightning discharge and its spectrum. From Fig. 4 (e) it follows that the phase-frequency spectrum shows remarkable change. It is interesting to know how the received wave-form is transformed when the the wave radiated from the source shown in Fig. 4 (e) is received at a distance. However, it is to be noted that the current variations described above were observed at Uppsala and the mechanism of lightning stroke originated at high latitude may differ from that originated at other places.

IV. Frequency Spectra of Received Wave-forms of Atmospherics.

4.1 Frequency spctra of the received wave-forms of atmospherics radiated from similar regions at about the same time.

As has been pointed out, it is conceivable that the frequency specta of received wave-forms are affected by those of discharge current at sources. In other words, even if we analyze the wave-forms radiated from a similar region at the same time, it may be expected that dissimilar types of amplitude-frequency spectrum can be obtained between them.

Two examples of these wave-forms and spectra are represented in Fig. 5 (a) and (b). It can be seen that each curve " i " in (a) and (b) shows one maximum and each curve "ii" shows two maxima in amplitude-frequency spectra in spite of their showing comparatively similar characteristics in phase-frequency spectra. It seems likely that the differences of the amplitude-frequency spectra between these two wave-forms depend only on the corresponding differences at the sources, since they travel the same path.

There is another matter to be noted. Spectra shown in Fig. 5 (c) are examples analyzed in the same case as mentioned above, and remarkable differences between the phase-frequency spectra are observed while in the amplitude-frequency spectrum the curve " i " is roughly consistent with the curve " ii ". The differences between the phase-frequency spectra seem to be caused by the divergency of lightning stroke currents. In view of the results described in 3.2 it can be suggested that the phase-frequency spectrum mentioned above may be originated from the subsequent lightning discharge.

4.2 Differences between frequency spectra derived from wave-forms with long duration and those with short duration.

The wave-forms with long duration as well as short duration are observed in daytime. Results of analyses of the wave-forms with long duration are represented

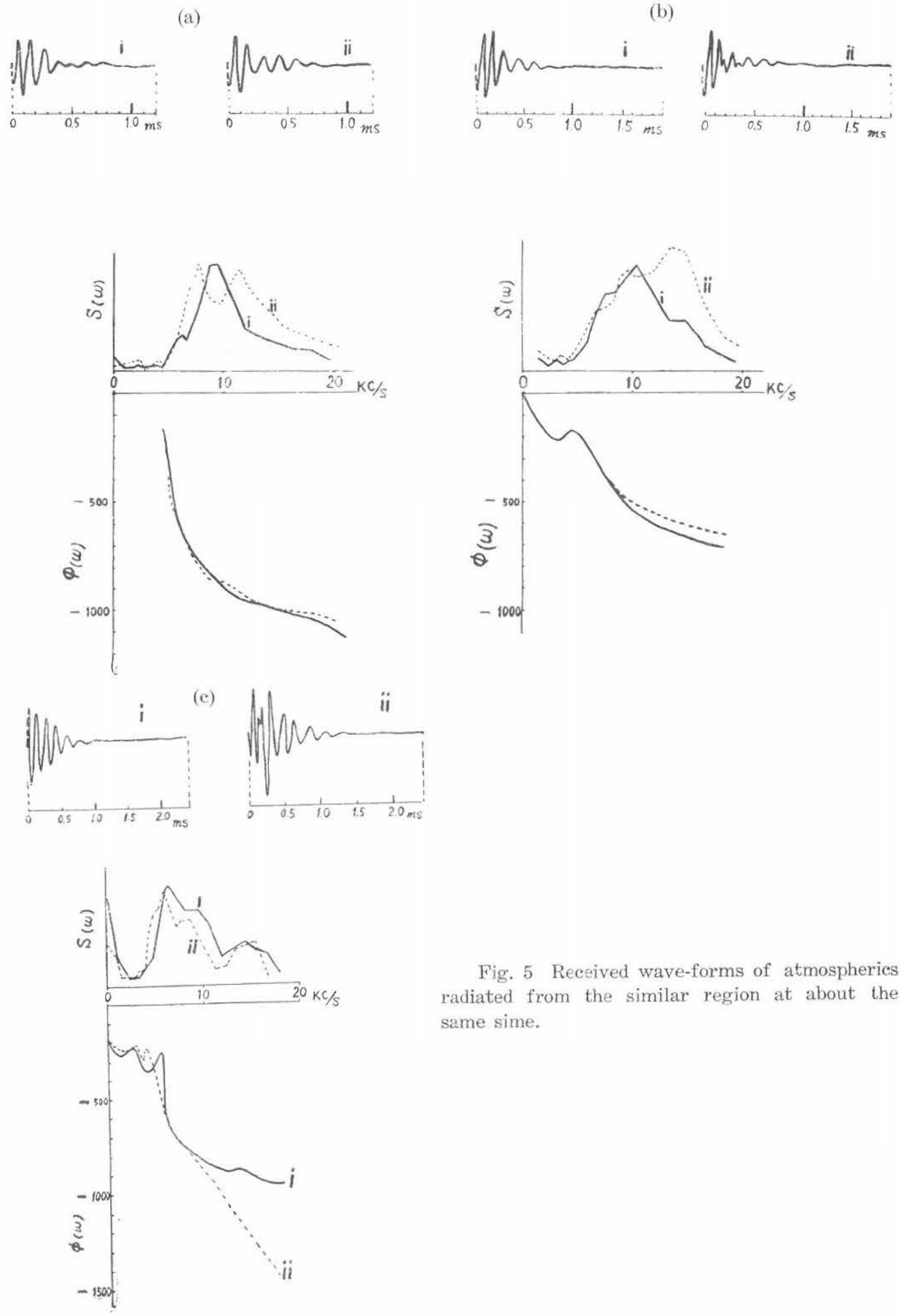

Fig. 5 Received wave-forms of atmospherics radiated from the similar region at about the same sime.

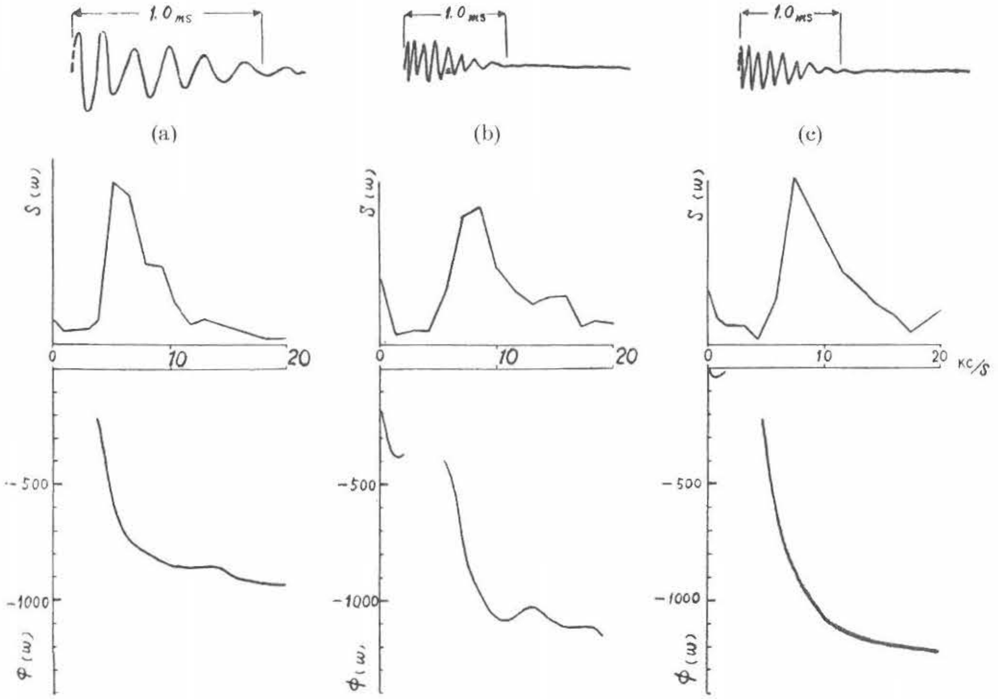

Fig. 6 Wave-forms with long duration and their frequency spectra.

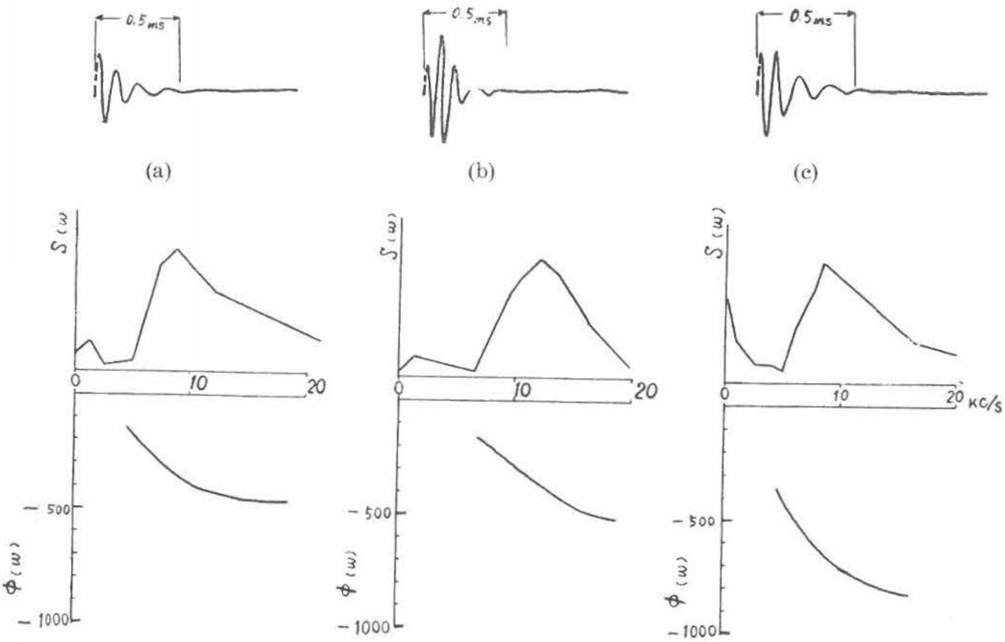

Fig. 7 Wave-forms with short duration and their frequency spectra.

in Fig. 6 (a), (b) and (c), while those with short duration are also represented in Fig. 7 (a), (b) and (c). The general conclusion to be drawn from these figures is that the mean value of tangent of any curve of phase-frequency spectrum analysed from each wave-form with long duration is large, and that the amplitude-frequency spectrum is distributed in a comparatively narrow frequency band. While in each wave-form with short duration both amplitude and phase-frequency spectra show the opposite behaviour.

The relationship of frequency analyses to the duration of wave-forms is thus considered.

4.3 Frequency spectra derived from the complicated wave-forms.

A rather complicated wave-form and the analysed frequency spectrum are represented in Fig. 8. As will be seen from Fig. 8, the amplitude-frequency spectrum is characterised by a distribution in a comparatively wide frequency range although the phase spectrum shows the usual tendency. As mentioned above it is possible that the relatively small declination of the curve of the phase-frequency spectrum shown in Fig. 8 maybe responsible for the wave-form with short duration.

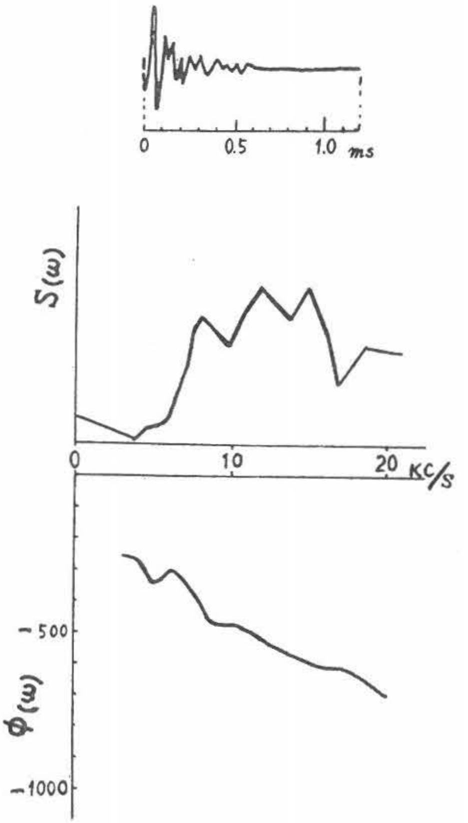

Fig. 8 Complicated wave-form and its frequency spectrum.

V. Conclusion.

The preliminary work reported here concerns with results of frequency spectra derived from the received wave-forms of atmospherics together with those of frequency spectra at the sources of atmospherics. But it is not our purpose to clarify here the characteristics of propagation paths employing the frequency spectra of received atmospherics with precise sferics fixes, the auther, therefore, has as yet no information as to what is involved in the propagation of V. L. F. waves. This work is continuing, and I shall have to discuss later the problems mentioned above as well as the significance of complicated wave-forms affected by the mechanism of lightning discharges on the bases of the results deduced from the frequency analyses.

VI. Acknowledgement.

I wish to express my thanks to Prof. Kimpara, director of our Institute, for his invaluable criticisms and encouragement. The author is indebted to Assistant

Prof. Iwai and others for their assistance in carrying out the observations and also to Miss T. Maeda and Miss M. Ito for their assistance in the course of the investigation.