# A STUDY OF ATMOSPHERIC NOISE AT 50 KC 

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#### Abstract

Summary-Measurements of the characteristics and level of atmospheric noise at 50 kc have been made since June 1957, that is, the cumulative amplitude probability distribution in the envelope of atmospheric noise and the field strength of integrated atmospherics have been studied. The relations between the statistical parameters of noise, i.e., the measured field strength of integrated atmospherics, the estimated average voltage and a series of noise parameters in the amplitude probability distribution have been obtained. The mean of the measured amplitude probability distributions is deduced with regard to the measured field strength of integrated atmospherics or the estimated average voltage, and in particular, for the latter parameter, is deduced an expression for the mean amplitude probability distribution. The impulsiveness of the noise is investigated with reference to the impulsive excursion of the trace of pen caused by the pulses of large peak appearing interruptedly. The contribution of the parts of noise envelope which exceeded over the threshold of very high voltage is investigated with regard to the average voltage or the mean square voltage of the noise envelope.


## I. Introduction

Measurements of the characteristics of atmospheric noise at very low frequencies have been made at the Research Institute of Atmospherics, Nagoya University since June 1957. In this paper, in particular, the summarizations are given of results derived from the observations at 50 kc . in the W.M.I. in June 1958, in which the cumulative amplitude probability distributions and the integrated strength of atmospheric noise were measured. The measurements of the amplitude probability distributions were made extending to the lower range of probability compared to that previously given ${ }^{11}$. An elaborate investigations were made of the characteristics of the noise parameters in the amplitude probability distributions. An attempt has been made to derive the mean of the measured amplitude probability distributions in terms of the measured strength of integrated atmospherics or the estimated average voltage. The latter has been deduced by integration of the measured amplitude probability distribution of the noise envelope. Another attempt has been made to describe the impulsiveness of the noise envelope by the deflections from the mean at the most impulsive excursions of the traces of the pen and with relation to the impulsiveness of the noise envelope has been derived the mean of the envelope of atmospheric noise. An investigation has been made of the average voltage or the r.m.s. voltage deduced by integration of the measured amplitude probability distribution respectively.

## II. Data

The measurements related to the envelope of the noise of atmospherics at 50 kc after it has been received on a 10 m rod antenna and passed through a narrow band receiver with the receiver bandwidth of about $1,000 \mathrm{c} / \mathrm{s}$ between 3 db points.

The strength of integrated atmospherics was recorded continuously for 10 days
in this period via an integrating circuit with a discharge time constant of about 80 sec . The recordings of parameters of the noise envelope in the amplitude probability distribution were made with an automatic equipment in the range of intermediate voltage and the percentage of time which the envelope exceeded over higher voltages was estimated visually by an operator with a micro-ammeter every 10 sec . Data were taken by estimating visually the mean of level of trace of the pen, or deducing the average of indications in the micro-ammeter over two minutes, just long enough to give statistically stationary data, for a series of threshold voltages. Then, the estimated mean of level or indications was plotted on a graph with the threshold voltage on a logarithmic-scale and with the probability on a normal-scale. In addition, a deflection from the mean at the most impulsive excursion of trace or the indications was estimated from every recordings of two minute measurement for a given threshold voltage.

The technique used in the observations of atmospheric noise in this period was approximately the same as was previously ${ }^{11}$ given except for the matters described in this section.

## III. The Method of Deduction of the Average Voltage or R.M.S. Voltage

The average voltage or r.m.s. voltage of the noise envelope could be deduced by integration of the measured amplitude probability distribution. But the process of integration is, in general, cumbersome and laborious in deriving each of two statistical parameters for a given amplitude probability distribution. When the characteristics of the measured amplitude probability distribution is more simplified, that is, is approximated by 2 or 3 logarithmic-normal distributions with different standard deviations in the whole range of probability, the deduction of each of two parameters can be made more easily by using a graph. The way of deduction of the graph is based upon a known formula ${ }^{2)}$ by the use of which the average voltage or r.m.s. voltage is deduced for a logarithmic-normal distribution with specified values of the standard deviation and the threshold voltage for $50 \%$ of time. In the following are shown the required equations in which some equations have been previously given ${ }^{1)}$, that is

$$
\begin{gather*}
X=\int_{0}^{F x} E p(E) d E \fallingdotseq \sum_{0}^{E x} E_{s}\left\{\phi\left(t_{q}\right)-\phi\left(t_{r}\right)\right\}=\sum_{0}^{F_{x}} E_{s} \Delta \tau  \tag{1}\\
Y^{2}=\int_{0}^{\infty} E^{2} p(E) d E \fallingdotseq \sum_{0}^{F_{x}} E_{s}\left\{\phi\left(t_{q}\right)-\phi\left(t_{r}\right)\right\}=\sum_{0}^{F_{x}} E_{S}^{2} \Delta \tau  \tag{1}\\
\bar{X}-X=\int_{0}^{\infty} E p(E) d E-\int_{0}^{F_{x}} E p(E) d E=\int_{F x}^{\infty} E p(E) d E  \tag{2}\\
\bar{X}^{2}-Y^{2}=\int_{0}^{\infty} E^{2} p(E) d E-\int_{0}^{F_{0}} E^{2} p(E) d E=\int_{F x}^{\infty} E^{2} p(E) d E  \tag{2}\\
\bar{X}=E_{50} e^{2.65 \sigma^{\prime 2}}  \tag{3}\\
\sqrt{X^{2}}=E_{50} e^{6.3 \sigma^{\prime 2}}  \tag{3}\\
\sigma^{\prime}=\log \frac{E_{16}}{E_{50}} \tag{4}
\end{gather*}
$$

where
$\bar{X}$ : the average voltage of the noise envelope
$\sqrt{\bar{X}^{2}}$ : the r.m.s. voltage of the noise envelope
$E_{16}, E_{50}, E_{x}$ : the threshold voltage exceeded by the noise envelope for 16,50 and $x$ percent of time

One pair of equations (1), (2), (3) and the other pair of equations (1)', (2)', (3)' are necessary in deriving the graph respectively for the average voltage or r.m.s. voltage of the noise envelope. Numerical calculations of these equations were made for different logarithmic-normal distributions with standard deviations 5, 7.5, $10,12.5,15,17.5$ and 20 in db .

Final results deduced from the numerical calculations with particular reference to the average voltage are shown in Fig. 1, the abscissa take the threshold voltage on a logarithmic-scale and the ordinate take the value of ( $\bar{X}-X$ ) on a logarithmic scale. The $E_{x}$ and the $(\bar{X}-X)$ are values corresponding to a unit of the threshold voltage for 50 percent of time. A series of curves are drawn with a parameter of


Fig. 1. The graph for estimating the average voltage.
standard deviation and another series of curves are passed through the points of equal probability on the calculated curves. The latter series of curves are with a parameter of probability. The curves of equal probability are drawn more densely in actually estimating a parameter such as the average voltage or r.m.s. voltage.

The way of deduction of the average voltage is described by integration of a given amplitude probability distribution when the deduced graph can be used. Let it be the amplitude probability distribution which consists of two logarithmic-normal distributions with different standard deviations on the whole range of probability, in which a departure occurs from a logarithmic-normal distribution at the probability of $p \%$, and $E_{x}$ is the threshold voltage exceeded by the noise envelope for $p \%$ of time. Then the ratio between $E_{x}$ and the threshold voltage exceeded by the noise envelope for $50 \%$ of time is estimated for each of the two logarithmic-normal distributions, that is, $E_{1 x}$ and $E_{2 x}$ are the estimated ratios for the logarithmic-normal distribution in higher range or in lower range of the threshold voltage. Here, by using the graph are read readily the two following quantities for a set of values of $E_{1 x}, p$ and other set of values of $E_{2 x}, p$ respectively, that is,

$$
\begin{align*}
& \bar{X}_{1}-X_{1}=\int_{E 1 x}^{\infty} E_{1} P_{1}\left(E_{1}\right) d E_{1}  \tag{5}\\
& \bar{X}_{2}-X_{2}=\int_{E 2 x}^{\infty} E_{2} P_{2}\left(E_{2}\right) d E_{2} \tag{6}
\end{align*}
$$

The suffix 1 and 2 indicate the parameter related to the logarithmic-normal distribution in higher range or in lower range of the threshold voltage. On the other hand, the relationship between the $\bar{X}$ and the $\sigma$ has been deduced as a curve from a known formula (3), which is drawn on a graph with the $\bar{X}$ on a logarithmicscale and with the standard deviation on a logarithmic-scale. And the values of the $X_{2}$ can be deduced in the following subtraction, that is,

$$
\begin{equation*}
X_{2}=\bar{X}_{2}-\left(\bar{X}_{2}-X_{2}\right)=\int_{0}^{E 2 x} E_{2} P_{2}\left(E_{2}\right) d E_{2} \tag{7}
\end{equation*}
$$

Final result, namely, the average voltage of the noise envelope in the given amplitude probability distribution is the following:

$$
\begin{equation*}
\frac{E_{x}}{E_{1 x}} \cdot\left(\bar{X}_{1}-X\right)+\frac{E_{x}}{E_{2 x}} \cdot X_{2} \tag{8}
\end{equation*}
$$

because the three quantities expressed in equations (5), (6) and (7) are deduced in a unit of the threshold voltage for $50 \%$ of time.

A graph has been prepared for deducing the r.m.s. voltage of the noise envelope after the process of deduction has been carried out as has been described in this section for the r.m.s. voltage.

## IV. The Duration of the Envelope

An attempt is discussed to express the impulsiveness of the noise envelope in terms of the deflections of the trace of pen of the recorder used in measuring the noise parameter in the amplitude probability distribution. The behavior of the trace of pen gives an information of the sliced parts of the noise envelope after
it has been passed over a series of threshold voltages. As has been given previously ${ }^{11}$, a time series of the parts of the noise envelope sliced by a series of threshold voltages can be interpreted in the following way, that is, the time series consists mainly of periodical series of pulse current with equal height with much shorter period than the time constant of the integrating circuit, but with pulses of long duration appearing interruptedly. Particular reference is now directed to the impulsive excursions of the trace of pen caused by the noise peak appearing interruptedly. When it is supposed that the appearing of a pulse of a large peak is statistically at random with regard to the phase of the periodical series of pulses, the deflection from the mean at the impulsive excursion of the trace of pen may be given as follows,

$$
\underset{R}{E}\left\{1-\left(1-\frac{p}{100}\right) \varepsilon^{-\alpha \tau}-p\right\}
$$

where
$p$ : the mean of trace of pen, i.e, percentage of time
$\alpha$ : the time constant of the integrating circuit
$\tau$ : the duration of the parts of the noise envelope considered which excceded over a given threshold voltage
$E / R$ : the current indicating the full scale of the ammeter
And, the duration of the part of the noise envelope which excceded over a given threshold voltage could be deduced from the recordings of the noise parameters in the amplitude probability distributions.

## V. Results

5.1. The Field Strength of Integrated Atmospherics, and the Average Voltage

The recordings of integrated strength of atmospheric noise were made continuously with an automatic equipment over ten days. The trace of pen has shown the fluctuations corresponding to the incidence of atmospheric noise and hourly values were estimated at the mean of the fluctuations over the period of measurement of the amplitude probability distribution. Fig. 2 indicates the diurnal change of the median of hourly values and dashed curves show upper deciles and lower deciles respectively. The field strength on the ordinate is expressed in a scale of equivalent field strength at the aerial as will be made


Fig. 2. Diurnal changes of the integrated field strength and the average voltage. in this paper. Fig. 2 shows also the median of hourly values of the average vol-
tages which have been deduced by integration of the measured amplitude probability distributions. Two dotted curves indicate upper deciles and lower deciles in this period. The two levels display the similar trend through a day for the measured field strength and the estimated average voltage. The ratios between the night level and the noon level are approximately $7 \sim 8$ times and $5 \sim 6$ times for the integrated strength and the average voltage respectively. A high afternoon level is approximately comparable with the night level.

### 5.2. The Noise Parameters in the Amplitude Probability Distribution

Measurements of noise parameters in the amplitude probability distribution were made at each hour of a day with 5 measuring circuits. The measurement consists of 2 or 3 steps of measurements of noise parameters made successively in time for different series of threshold voltages, to measure the noise parameters in wide range of probability. Usually, two threshold voltages are given constantly through different steps of measurements to check a change of thunderstorm activity. When the variations of mean of trace is negligibly small for constant threshold voltages for the time elapsed for all steps of measurements, all the data are plotted on a graph with the probability on a normal-scale and with the threshold voltage on a logarithmic-scale.

About $90 \%$ of all the measured amplitude probability distributions were found to be approximated by 2 or 3 logarithmic-normal distributions with different standard deviations. The other distributions were approximated by 1 or more than 3 logarith-mic-normal distributions.

Here again the diurnal change is investigated in terms of the noise parameters in the amplitude probability distribution, that is, Fig. 3 indicates the diurnal change of the mean of the threshold voltage for 50,5 or $1 \%$ of time. Hourly values of the threshold voltage were estimated at


Fig. 3. Diurnal changes of the threshold voltages of $V_{50}, V_{5}$ and $V_{1}$. voltage intercepts with specified values of the percentage of time in the measured amplitude probability distributions. The $V_{50}, V_{5}, V_{1}$ indicates the threshold voltage for 50,5 or 1 percent of time respectively. Dashed curves and dotted curves show upper deciles and lower deciles for each of these noise parameters. Similar trends are seen in the diurnal changes for three statistical parameters of noise.

### 5.3. Relationship between a Series of Statistical Parameters of the Noise

Diurnal changes displayed similar trends for the mean level with respect to
statistical parameters, that is, the integrated strength, the average voltage, the threshold voltages for specified values of percent of time, as has been decribed in the preceeding section. Let us now investigate the relationship between the measured levels of the statistical parameters in a unit of measurement at each hour of a day.



Fig. 5. Relation between integrated field strength and threshold exceeded for $5 \%$ of time.

Fig. 4 indicates a relationship between the threshold voltage exceeded over for 5 percent of time and the integrated field strength of atmospherics. The abscissa takes the integrated field strength and the ordinate takes the threshold voltage exceeded over for 5 percent of time. Such a linear relation has been found between the integrated field strength and the threshold voltages for particular values of percent of time, i.e., voltage intercepts with $80,60,50,40,20,10,5,1$ and 0.1 percent in the measured amplitude probability distributions.

Another example of a linear relationship is shown in Fig. 5, in which the abscissa takes the estimated average voltage and the ordinate takes the threshold voltage exceeded over for 1 percent of time. Different marks indicate the plots for the periods of times specified in a day, in Figs. 4 and 5. Such a linear relation is also found for a series of time percentage, i.e., $80,60,50,40,20,10,5,1$ and $0.1 \%$ of time. These relations between the levels of two parameters have been deduced based on the data measured over the period from 08 h to 24 h of a day, because a reliable estimation of the average voltage can be made by integration of the measured amplitude probability distributions in this period of a day.

Now the mean of all the measured amplitude probability distributions in the specified period of a day can be described in terms of the integrated field strength or the average voltage of the noise envelope. It is shown in Fig. 6 in which the ordinate takes the percentage of time on a normal-scale and the abscissa takes the threshold voltage on a logarithmic-scale in a unit of the integrated field strength or the average voltage. The marks a and - refer to the plots determined


FIG. 6. Mean cumulative amplitude distribution.
for the integrated field strength or the average voltage respectively. Two dotted curves and two dashed curves show the limits between which four-fifth of the amplitude probability distributions lay for the integrated field strength and the average voltage, respectively. Similar shapes exist between the two mean amplitude probability distributions deduced independently with regard to the two different statistical parameters. On the other hand, the range of the two limits on each side of the mean amplitude probability distribution in Fig. 6 is larger for the integrated field strength than that for the average voltage. The difference may be attributed to the reason that the integrated field strength is influenced more by the incidence of pulses of large peaks apperring interruptedly than is the average voltage.

The relationship between the measured field strength of atmospherics and the estimated average voltage is shown in Fig. 7 over the period of a day considered. There is an approximate linear relation between the two statistical parameters.

Now, to express concisely the mean amplitude probability distribution, with particular reference to the term of the average voltage, the mean amplitude probability distribution considered was redrawn on a graph as is shown in Fig. 8, in which the ordinate takes the logarithm of $Q(v) / 1-Q(v)$ and the abscissa takes
the logarithm of threshold volt/average volt. Here, $Q(v)$ represents the percentage of time exceeded by the envelope over the threshold voltage $v$, and $1-$ $Q(v)$ represent the percent of time which was not exceeded by the envelope. The distribution redrawn in Fig. 8 is approximated by a straight line over the range of $60 \%$ to $0.1 \%$. When the expression ${ }^{4)}$

$$
Q(v)=\left[1+(a v / \bar{v})^{r}\right]^{-1}
$$

is applied to the approximated straight line for the distribution, $a$ and $r$ are deduced by the negative slope of line and the scale of the ordinate for thre-


Fig. 7. Relation between average voltage and integrated field strength. shold volt/average volt $=1$, that is, $a$ and $r$ are 2.7 and 1.4 respectively for the mean amplitude probability distribution deduced with regard to the average voltage.


FIG. 8. Mean cumulative amplitude distribution.

### 5.4. The Impulsiveness of the Noise Envelope

An attempt has been made to express the impulsiveness of the noise envelope and further to derive the mean waveform of the noise envelope which caused the largest impulsive excursion in the trace of pen, based upon the recordings of the noise parameters in the amplitude probability distribution. As has been described in section IV, particular attension is paied to the degree of deflection from the mean of trace at the largest of impulsive excursions in the trace of pen over two minutes' measurement. Fig. 9 shows the deflection plotted against the mean of trace, i.e. percent of time exceeded by the envelope over a given threshold voltage, where the abscissa takes the deflection in a unit scale of the percentage and the ordinate takes the percentage of time on a normal scale. Data were obtained from the recordings over the period from 19 h to 23 h for 5 days. With regard to data from all the recordings through this period of observations of atmospherics, the medians of degrees of deflection are shown against the probability as is shown in Fig. 10 on a normal-scale, in which four curves were deduced respectively for different periods of a day. Main feature obtained is that the degree of deflection is larger in the period from 08 h to 12 h than it in the other periods of a day, and further, the difference in the degree of deflection is approximately neglected over


Fig. 9, Relation between percent of time and the deflection.


FIG. 10. Relation between percent of time and the deflection.
the period of a day except in the morning. The feature is found to be in relation with the level of the integrated field strength or the average voltage by reference of Fig. 2, that is, the degree of deflection becomes more conspicuous when the level is low in the morning, and it is less conspicuous depending on the high level in the afternoon and at night. It may be considered that the largest peak of the noise envelope is less variable depending on the diurnal change of level of any of two parameters.

Let us describe a way of deriving the mean waveform of pulses of large peak which appeared interruptedly and caused the largest impulsive excursion of the trace of pen. At first, the mean degree of deflections as described above, is deduced against a series of percent of time from all the recordings of the noise parameters in the amplitude probability distributions for ten days. Next, by using the formula (8), the relationship between the deflection and the probability as derived above can be rewritten by the relationship between the duration of the part of the sliced envelope by a given threshold voltage and the probability. On the other hand, the relationship between the threshold voltage and the probability is given for a statistical mean by the mean amplitude probability distribution. as when the threshold voltage is expressed in a scale of a unit of the average vol-
tage. Therefore, the relationship between the duration and the probability can be rewritten by the relationship between the duration and the threshold voltage.


FIG. 11. The waveform of the noise envelope.
After the process of deriving has been fulfilled the result deduced is shown in Fig. 11 in which the ordinate takes the threshold voltage in the unit of average voltage and the abscissa takes the duration. Dashed curve shows the waveform of envelope of the noise after a short impulse applied at the aerial has been passed through a narrow band receiver 1 kc in width in which the intercept of two curves is made arbitrarily to fit, at the probability of $0.05 \%$, the threshold voltage of each of two different waveforms of the envelope. And the receiver is supposed to have one stage of tuned circuit at resonant frequency of 50 kc . The fit of two curves indicates that a comparable range seems to be limited approximately in the range of less than $0.1 \%$ or $0.5 \%$ and the difference becomes larger as the probability is higher.

## VI. Atmospheric Noise of Very High Voltage

Let us investigate the degree of influence which the noise envelope of large peak has on the average voltage or r.m.s. voltage of the noise envelope.

### 6.1. Average Voltage

Let us begin with a simple case, that is, a logarithmic-normal distribution. Fig. 12 shows the contribution to the average voltage from different ranges of probability in a logarithmic-normal distribution, where the abscissa takes the logarithm of the threshold voltage expressed in a scale of a unit of the threshold voltage for 50 percent of time, and the ordinate takes the contribution of the noise envelope every 3 db range of the threshold voltage. Different curves have been deduced for different standard deviations, $5,7.5,10,12.5,15,17.5,20$ in db .

Table 1 shows the limits where the contribution of the noise envelope can be neglected in terms of the probability or the ratio of the threshold voltage to it for 50 percent of time when the indicated admissible errors are assumed in estimation of the average voltage in the logarithmic-normal distributions with specified


Fig. 12

TAble 1

| Standard deviation $(\mathrm{db})$ | 10 |  | 15 |  | 20 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Admissible error | $(\mathrm{db})$ | 1 | 3 | 1 | 3 | 1 | 3 |
| Percent of time | $(\%)$ | 0.99 | 3.2 | 0.19 | 3.9 | 0.023 | 0.13 |
| Threshold volt. | $(\mathrm{db})$ | 23.3 | 18.5 | 43.5 | 36 | 70.2 | 30.2 |

values of standard deviation.
Next, the accuracy of the average voltage described in this paper is investigated, because the average voltage has been deduced by integration of the distribution which consists mainly of the measured amplitude probability distribution and in higher threshold voltage than the limits of measurement, the assumed distribution determined in the way of interpolation. The limit of measurement described in this paper is approximately 0.05 percent. And, the degree of influence on the deduced average voltage should be determined with regard to the contribution of the noise envelope in the range of probability assumed.

Now, let us consider a series of models of distributions as is shown in Fig. 13, which consist mainly of the mean amplitude probability distribution and the assumed logarithmic-normal distributions in the range of less than $0.05 \%$ with different standard deviations $7.5,10,15,17.5$ and 20 in db . The mean amplitude probability distribution is expressed in a scale of a unit of the average voltage and so the noise parameters in assumed logarithmic-normal distributions is expressed as well in the same scale. The average voltages deduced are shown in Table 2 by integration of each of models of amplitude probability distributions, in which the first line indicates the values of standard deviation for assumed logarithmic-normal distributions and the second line indicates the deduced average voltage. It is shown in the results that the deduced average voltages remain approximately the same


FIG. 13. A sevies of models of the amplitude probability distribution.
over the variable range of characteristics of the noise considered in the range of less than $0.05 \%$. Therefore, the average voltage deduced for the measured amplitude probability distribution at each hour of a day is considered to give a nearly correct value.

The other feature to be noted in Table 2 is that the average voltages are nearly 1. This means that the average voltage deduced by substantial integration of the amplitude probability distributions is well fit to the average voltage originally used as a unit of a scale.

TABLE 2

| Standard deviation (db) | 7.5 | 10 | 15 | 17.5 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Average voltage | 0.944 | 0.945 | 0.956 | 0.973 | 0.982 |

It is concluded that the contribution of the noise envelope of very high voltage is small for a statistical mean.

### 6.2. The Root-Mean-Square Voltage

Let us investigate once more the models of the amplitude probability distribution with regard to the contribution of the noise envelope to the r.m.s. voltage. The r.m.s. voltage has been deduced by the integration of each of the models. The results are shown in Table 3, in which the first line indicates the values of the standard deviation for the asssumed logarithmic-normal distributions in each of the models of distributions, and the second line indicates the deduced r.m.s. voltage. The 17.6 db is the degree of the standard deviation for the logarithmicnormal distribution determined in interpolating method in the mean amplitude probability distribution. The figures in Table 3 show that the error of approximately $\pm 6 \mathrm{db}$ could occur in the estimated values of the r.m.s. voltage in the variable range of characteristics of the noise assumed, when the estimation of the r.m.s. voltage is made for the measured amplitude probability distribution in the same way as has been made in the estimation of the average voltage.

TABLE 3

| Standard deviation (db) | 7.5 | 10 | 15 | 17.5 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R.M.S. voltage | 9.6 | 9.8 | 12.1 | 18.7 | 38.7 |

Next, the contribution of the envelope to the mean square voltage is investigated from different parts of the amplitude probability distribution, that is, two parts of the amplitude distribution divided at the probability of $0.05 \%$ for each of the models of distridutions. The first line indicates, as is shown in Table 4, the degree of standard deviation of the assumed logarithmic-normal distribution in the range of less than $0.05 \%$ for each model of the distribution, and the second and third lines indicate the contribution of envelope to the mean square voltage for the range of more than $0.05 \%$ or the range of less than $0.05 \%$.

TABLE 4

| Standard deviation (db) | 7.5 | 10 | 15 | 17.5 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Contribution in the range of <br> more than 0.05\% | 92.3 | 92.3 | 92.3 | 92.3 | 92.3 |
| Contribution in the range of <br> less than 0.05\% | 1.01 | 3.70 | 55.6 | 258 | 1404 |

It is found in Table 4 that a reliable estimation of the r.m.s. voltage cannot be made when the amplitude probability distribution is not experimentally determined in the range of less than about $0.05 \%$, and the relative importance of the noise envelope in this range is remarkably variable with regard to the contribution of the r.m.s. voltage depending on the characteristics of the noise.

### 6.3. The Ratio Between the R.M.S. Voltage and the Average Voltage

The ratio between the r.m.s. voltage and the average voltage can be estimated
by assuming the amplitude probability distribution in the range of less than 0.05 $\%$. The ratio is found readily in Table 3 where small error is admissible. The ratio is 10,19 or 34 for the standard deviation $10,17.5$ or 20 in db which is allotted to the assumed logarithmic-normal distribution. Any of these values of ratio is clearly divided, compared to it for thermal noise.

## VII. Conclusion

The results of the measurement have been described so far of the envelope after the atmospheric noise has been passed through a narrow band receiver with the band width of about 1 kc . Main features are summarised in the following.

1. There exists a close relationship between the measured field strength of integrated atmospherics, the average voltage and the statistical parameters of the noise in the amplitude probability distribution. Any of these parameters represents some statistical quantity associated with atmospheric noise, but there are different levels for different parameters.
2. The mean amplitude probability distribution has been deduced with regard to the integrated field strength or the average voltage. In particular, an expression has been obtained in terms of the average voltage for the mean amplitude probability distribution, that is, $a=2.7$ and $r=1.4$.
3. The average voltage can be estimated within a negligible error, but the r.m.s. voltage cannot be estimated with reliable accuracy, by integration of the measured amplitude probability distribution described in this paper.
4. The waveform has been deduced of the noise envelope which has appeared interruptedly and caused the largest impulsive excursion of the trace of pen over two minutes' measurement. It shows the largest limits of duration of time for which a series of threshold voltages are exceeded by the envelope. Compared to the duration of the envelope at the output of the receiver, when a short impulse is applied to the input of the receiver, a comparable order of durations seems to be limited in the range of less than about 0.1 or $0.5 \%$. The difference between the two cases is remarkable in other ranges of the probability.

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