INVESTIGATION OF THE PRESENCE OF IONIZED HYDROGEN IN THE OUTER ATMOSPHERE USING WHISTLER DISPERSIONS

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Abstract—A trial is made to detect the presence of ionized hydrogen in the outer atmosphere following a theory concerning the corrections of the dispersion law of whistlers given by Storey, which involves effects of light ion as well as electron on dispersion. Four short whistlers most suitable for this purpose are analysed, which were observed at Toyokawa and Wakkanai stations all on different dates. For they are pure and fairly well defined down to frequencies lower than 1,000 c/sec. and one of them is preceded by tweek type atmospheric click, which make the location of time origin easier. The electron density used by Storey is too small to produce the dispersions observed, so different distributions of electron density are assumed so as to satisfy the dispersions observed.

From these distributions two complete dispersion curves are calculated each for the case of Toyokawa and Wakkanai. The dispersions measured from whistler traces agreed fairly well with the theoretical curves.

From this result the presence of ionized hydrogen in the outer atmosphere is strongly supported. But it seems to be necessary to investigate the shape of the ray path as exactly as possible in order to solve the problem of distribution of ionization density and get further informations about the constituent of the ionization.

I. Introduction

At low latitudes and in the medium audio frequency range, the dispersions of whistlers satisfied the well known law that the inverse square root of the instantaneous frequency of a whistler is linearly proportional to the propagation time after the initiating lightning flash.

This law was theoretically derived under some simplifying assumptions by Eckersley¹⁾ and Storey.²⁾ But recently, the cases have been discussed by Helliwell *et al.*³⁾ and then by Sotrey,⁴⁾ that the following assumptions are broken down: firstly, the assumption that the critical frequency of the medium and the frequency of electron gyration caused by the magnetic field of the earth are much greater than the wave frequencies of whistler at all points along the path, secondly, the assumption that the influences of ions in the outer atmosphere on the wave propagation need not be considered. The former means that as the height of a whistler path rises the electron gyro-frequency decreases rapidly and becomes comparable with the high frequency end of the whistler.

This case is readily seen on whistlers observed at high latitudes where the apex of a whistler path reaches the height of several earth's radii from the center of the earth and the electron gyro-frequency falls down to the high audio frequency range. This produces conspicuous departures from the simple dispersion law, which are more marked for high latitudes and for the high frequencies of whistler, and its typical example is "nose" whistlers firstly observed by Helliwell *et al.*³⁾

The latter case is that the exersion of light ions on a propagation of a whistler becomes unnegligible at the low frequency end of the whistler, for the ionic gyro-frequency at the height of the path of a whistler is comparable to the low frequencies of the whistler. For instance, the gyro-frequency of ionized hydrogen at 300 km height takes an approximate value between 900 c/sec. at the geomagnetic poles and 450 c/sec. at the equator. This ionic effect is expected to cause low frequency departures of dispersion. So, if the low frequency departures are found in experiments, an information about the presence of light ions in the outer atmosphere will be obtained. But in order to measure the low frequency departures, a complete dispersion law must be known, including effects both of electron gyration and ionic gyration. For, the influence of electron gyration is indeed small at low audio frequencies, but it is still unnegligible at a frequency greater than a few kilo cycles per second even at low latitudes.

From this point of view, Storey presented two expressions of dispersion laws corrected respectively for high and low audio frequencies of a whistler. When put together they give a complete dispersion law. He calculated two complete dispersion curves applicable to geomagnetic latitudes 45° and 55° respectively, with a distribution of electron density given by Dungey,⁵⁾ and showed a possibility of detecting the existence of the ionized hydrogen as the main constituent of the ionization, which was enough to support the whistler propagation in the outer atmosphere.

The dispersion curve calculated for the case of geomagnetic latitude 45° has the minimum and maximum points occurring at frequencies of about 2,500 c/sec. and 300 c/sec. respectively. But at lower latitudes, these frequencies corresponding to the extremum points will rise up and the difference between the extrema will increase, which makes the detection of the ionic effect using this method easier for whistlers observed at low latitudes.

This is explained from the facts that for a propagation of a whistler from low latitudes the ionic gyro-frequency is kept at comparatively high values on a greater part of the whistler path, owing to its relatively low height; consequently the low frequency departures are more marked and the maximum point will shift towards higher frequencies, while the electron gyro-frequency at the top of the path, being most contributive to the high frequency departures, is comparatively high, owing also to its relatively low height, so the departures at high audio frequencies are less marked.

Further, whistlers observed at low latitudes have another character suitable for this purpose in that they are generally less widely spread and well defined, because they propagate within a relatively low height, so they encounter less the discontinuous layers of electron density which may be one of the main causes of setting up a multi-path for a whistler propagation and consequently producing a swish type whistler.

Since Storey's proposal, available whistlers have been carefully sought at Wakkanai (geomagnetic lat. 35.3°) and Toyokawa (geomagnetic lat. 24.5°), whose latitudes are among the lowest of the observatories of the world carrying out routine observations of whistlers, and the whistlers observed there are expected to have the advantages described above.

They are considerably pure and strong and are sometimes observed to have

frequencies near 600 c/sec., which may be the lowest limit in the present whistler observation tecknic.

II. Expressions

2.1. Expressions

Whistler waves were explained from the quasi-longitudinal propagation of the extra ordinary wave in the magneto-ionic theory under the conditions that the wave frequency f is small compared to the critical frequency f_0 and the electron gyro-frequency f_{ii} and large to the ionic gyro-frequency f_i at all the points on the ray path. And in a case of longitudinal propagation the whistler energy follows a magnetic line of force exactly and its travelling time is given in the next eq.,

that is
$$t \cdot f^{1/2} = 1/(2 c) \int f_0 / (f_H \cdot f)^{1/2} ds$$

that is $t \cdot f^{1/2} = 1/(2 c) \int f_0 / f_H^{1/2} ds = D$ (1)

where c is the velocity of light, ds is an element of the ray path and D is the dispersion of the whistler which is constant independently of f for any one path.

But in the case that the gyration effects of light ion and electron on the whistler propagation are not neglected, eq. (1) must be modified. According to storey, the correction term is generally small, and the complete dispersion law is expressed in the following form,

$$t \cdot f^{1/2} = 1/(2 c) \int f_0 / f_H^{1/2} (1 + \delta(f)) ds$$

= $D + 1/(2 c) \int f_0 / f_H^{1/2} \delta ds$ (2)

The correction factor δ is apporximately given as a sum of two expressions appropriate to low and high audio frequencies respectively.

The correction factor δ_l for the low audio frequencies is given in the next eq.,

$$\hat{o}_l = (1 + 2f_i/f) / (1 + f_i/f)^{3/2} - 1 \tag{3}$$

 δ_i varies from zero to -1 as f decreases from infinity to zero and is positive for frequencies greater than 0.618 f_i and negative for lower frequencies and takes its positive maximum value of 0.0887 at $f = 2 f_i$. This effect will be marked at frequencies less than $2 f_i$.

And the correction factor δ_h for high audio-frequencies is expressed by the next eq.,

$$\delta_h = 1/(1 - f/f_H)^{3/2} - 1$$

= 1.5(f/f_H) + 1.875(f/f_H)^2 (4)

 δ_h decreases nearly linearly as f decreases within audio frequency range for the value of f_{II} less than 200 kc/sec. This is more effective for higher frequencies.

The quantity $tf^{1/2}$ is not constant but varies with f owing to these correction terms.

2.2. Distribution of Ionization

In order to calculate a whistler dispersion from the equations obtained in the last section, it is necessary to know the shape of ray path and the distribution of the electron density in the space the whistler propagates. The ray path will be represented by a magnetic line of force originating from a magnetic dipole at the center of the earth, for the ray path coincides with a magnetic line of force of the earth in a case of longitudinal propagation. The distribution of the electron density may be regarded as a function only of altitude in the outer atmosphere as well as in the ionosphere. In Storey's calculation it is expressed as N = 600 exp. $(2.5 r_0/r)$ cm⁻³ at the height above 300 km, where r is the distance from the center of the earth and r_0 is radius of the earth. This distribution is derived from an assumption given by Dungey⁵ and the other workers that the outer atmosphere consists of fully ionized hydrogen at a temperature of 1,500°k in thermal and hydrostatic equillibrium under gravity and in dynamical equilibrium with the ionization in the interplanetary space. This distribution gives the values of dispersion $43.5\sqrt{s}$ and $97\sqrt{s}$ for short whistlers occurring at geomagnetic latitudes 45° and 55° respectively. But this is too small to cause the dispersions of short whistlers observed at Toyokawa and Wakkanai stations, which amount to 40 to $50\sqrt{s}$. For instance, for a whistler propagating toward Wakkanai only the dispersions of magnitude $25\sqrt{s}$ is got from this distribution, about $7\sqrt{s}$ of which is caused in the ionosphere between 100 and 300 km height, an average density of 1.2×10^6 cm⁻³ assumed.

Consequently different distributions are adopted in the present calculation, which is obtained by the development of a convenient way reported previously.⁶) For this, there happened a favourable circumstance in which whistlers originating from one lightning flash were observed at both stations, Wakkanai and Toyokawa, at the same time during an observation period on 24 Dec. 1957, 1635 J.S.T. This is confirmed by a careful analysis with a sonagraph.

The dispersions of these two whistlers showed the same value of about $50\sqrt{s}$. The distributions of electron density are determined so as to reasonably satisfy this set of value of the dispersions according to the following procedures: The space in which whistlers propagate is divided into three regions as their altitudes and then a constant electron density is assigned to each region.

1. The region between 100 and 500 km high above the ground level. The constant electron density N_1 is the average value of a parabolic distribution assumed in this region and is expressed by next eq.,

$$N_1 = 1/400 \int_0^{400} N_m \cdot 2/z_m (z - z^2/2 z_m) dz$$

where N_m is the maximum electron density at the height of z_m measured from the base of the ionosphere which is taken at 100 kms' height.

Numerical values used here are $N_m = 1.61 \times 10^6$ cm⁻³ and $z_m = 300$ km, consequently $N_1 = 1.194 \times 10^6$.

Then the dispersion D_1 caused in this region is obtained from next eqs.,

$$D_{1} = 5.340 \times 10^{-2} N_{1}^{1/2} \int_{19^{\circ}5'}^{23^{\circ}30'} f(\theta) \cdot d\theta$$
$$f(\theta) = 2.7 + 1.737 \sin 2 \ \theta + 0.175 \sin 4 \ \theta - 0.0125 \sin 6 \ \theta$$

therefore, $D_1 = 24.08\sqrt{s}$.

2. The region between 500 km and 1,320 km which is the apex of the line of force going through Toyokawa.

In this region the constant density N_2 is determined so as to produce the remaining dispersion D_2 with the one caused in the region 1 subtracted from the total dispersion observed at Toyokawa station, that is, for the total dispersion $50\sqrt{s}$

$$D_2 = 5.340 \times 10^{-2} N_2^{1/2} \int_0^{19^{\circ}5'} f(\theta) \cdot d\theta = 50 - 24.08$$

therefore, $N_2 = 5.187 \times 10^4 \text{ cm}^{-3}$.

3. The region between 1,320 km and 3,200 kms' height which is the altitude of the top of the line of force coming from Wakkanai.

In this region the constant density N_3 is decided to cause the remaining dispersion D_3 the dispersions D_1 , and D_2 produced in the region 1 and 2 in the case of Wakkanai subtracted from the total dispersion observed at Wakkanai station.

Using the values of N_1 and N_2 obtained above, for the total dispersion of $50\sqrt{s}$, N_3 is given by

$$D_{1} = 9.218 \times 10^{-2} \sqrt{N_{1}} \int_{32^{0}5'}^{34^{0}42'} f(\theta) \cdot d\theta = 16.84$$
$$D_{2} = 9.218 \times 10^{-2} \sqrt{N_{2}} \int_{26^{0}18'}^{32^{0}5'} f(\theta) \cdot d\theta = 9.08$$
$$D_{3} = 9.218 \times 10^{-2} \sqrt{N_{3}} \int_{0}^{26^{0}18'} f(\theta) \cdot d\theta = 50 - (16.84 + 9.08)$$

that is, $N_3 = 8.810 \times 10^3 \text{ cm}^{-3}$.

In the parabolic distribution, $z_m = 300$ km will be rather greater, so the value of N_1 may be excessive, but this may not be so influential upon the present problem to detect the effect of proton.

Another set of distribution is determined in the similar way to fit a dispersion $40\sqrt{s}$ of whistlers observed at Toyokawa station. The results are that $N_1 = 1.194 \times 10^6$ cm⁻⁵ and $N_2 = 1.954 \times 10^4$ cm⁻³.

2.3. Calculated Dispersion Curves

Since the distributions of electron density are got, the complete dispersion curves can be calculated. Before the calculation we rewrite eq. (2) as

$$tf^{1/2} = D + 1/2 \ c \int f_0 / f_H^{1/2} \delta ds$$

= $D + 1/2 \ c \int f_0 / f_H^{1/2} \delta_h ds + 1/2 \ c \int f_0 / f_H^{1/2} \delta_l ds$
= $D + 4D_h + 4D_l$,

The quantities ΔD_h and ΔD_l give the departures of dispersion from the simple law for high audio frequencies and low audio frequencies, respectively.

For the dipole field, the electron gyro-frequency f_H at the top of the line of force passing through Wakkanai is about 240 kc/sec. and the value of $f/f_H^{1/2}$ is small even for the high audio frequencies, so for the right hand of the eq. (4) only the first term is enough.

Assuming the existence of proton in the outer atmosphere higher than 300 km, the integrations are numerically performed at following intervals of magnetic latitude, the independent variable; from $1^{\circ}42'$ to $2^{\circ}36'$ in the case of Toyokawa and from 39' to $5^{\circ}54'$ in the case of Wakkanai. The calculated results are shown by curves of ΔD_h , ΔD_l and $t \cdot f^{1/2}$ against f in Figs. 1 a and 1 b for the cases of Wakkanai and Toyokawa, respectively.

In each Fig. the lines $D = 49.8\sqrt{s}$ and $D = 38.2\sqrt{s}$ show constant dispersions obtained from eq. (1). The complete dispersion curves have a maximum point at a frequency between 650 and 700 c/sec. in the both cases, and the minimum point occurrs at a frequency near 5,000 c/sec. in the Wakkanai case, while in the Toyokawa case it does not occur in the frequency range treated here owing to the slow increment of D_{1} curve. The amount of the variation of the curves within the observable frequency range is $1.5 - 1.6\sqrt{s}$ for both cases, therefore the facilities to detect the ionic effect from the complete dispersion curve are almost equal in both cases, except for the question at which station the whistlers are





b: in the case of Toyokawa (geomag. lat. 24.5°) constant dispersion $38.2\sqrt{s}$

Di: low frequency correction term woing to the effect of proton gyration.

 D_h : high frequency correction term owing to the effect of electron gyration.

purer and stronger in wide frequency range. As for this point, they were also equally qualified.

III. Analysis

3.1. Experimental Method

Whistlers recorded on magnetic tapes are analysed with a sonagraph, a voice spectrum analyser. The result, known as a sonagram, is portrayed on a dry facsimile paper by a recording stylus. The frequency axis is nearly linearly scaled from 85 to 8,000 c/sec., which is checked with a 1,000 c/sec. signal and its higher harmonics divided down from a 100 kc crystal standard oscillator giving an accuracy of 10^{-7} , and the average distance per 1,000 c/sec. along the frequency axis is 12.1 mm. The distance equivalent to one second along the time axis is about 132.0 mm and a phenomenon durating about 2.35 seconds is covered on the sonagram. The bandwidth of the band-pass filter is about 30 c/sec. This is operated by a stabilized ac power source.

In order to measure the quantity of $t \cdot f^{1/2}$ from a whistler trace on a sonagram, the position of the time origin of the whistler must be known, but this can not be done directly in cases of short whistlers except the one described later. Consequently following procedures are made. At first the magnitude of dispersion D is determined assuming that the trace apporximately satisfys the relation of $t \cdot f^{1/2} =$ constant, then taking a point on the trace corresponding to a given frequency f the distance given by the equation $t = Df^{-1/2}$ is measured on the left side from the point along the time axis. This position will lie near the time origin. Next, values of $t \cdot f^{1/2}$ are plotted for certain frequencies after measuring their traveling times with respect to the origin decided, and comparing the curve with the theoretical one, the time origin is slightly shifted till an appropriate curve is produced.

A shifting of the time origin Δt causes a change of $\Delta t \cdot f^{1/2}$ in the quantity of $t \cdot f^{1/2}$, which is independent of the magnitude of $t \cdot f^{1/2}$ but varies in proportion to $f^{1/2}$. Thus, though the dispersion of a whistler is constant, the curve of $t \cdot f^{1/2}$ against f measured in this way may deviate from the straight line expressing the constant value, if the time origin adopted departs from the true one. This circumstance is shown in Fig. 2, in which the curves corresponding to negative values of Δt are seen to have a similar curvature to the theoretical $t \cdot f^{1/2}$ curve. Accordingly, this effect should not be overlooked.

3.2. Errors

The errors produced in measure-



FIG. 2. Shifting effect of time origin on the quantity of $t \cdot f^{1/2}$.

ment of $t \cdot f^{1/2}$ are expressed by the following equation,

$$\begin{aligned} |\varDelta(tf^{1/2})| &= f^{1/2} \cdot |\varDelta t| + 1/2 \cdot tf^{-1/2} |\varDelta f| \\ &= f^{1/2} \cdot |\varDelta t| + 1/2 \cdot D/f \cdot |\varDelta f| \end{aligned}$$

where Δt and Δf are errors in time and frequency. Δt depends chiefly on the bandwidth of analyser and on the intensity of signal received, so that this must be determined for individual whistlers from the spreadness of their traces along the time axis on sonagrams, but it is somewhat troublesome to carry out in each case, so the values averaged with many data shown in Table 1 are used here.

TABLE 1. Average Errors Produced in the Measurement of Time and Frequency

| f | c/s | 800 | 1,000 | 1,500 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 |
|-----------------------------|-------------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Δf | \mathbf{c}/\mathbf{s} | 30 | 30 | 30 | 30 | 30 | 30 | 35 | 35 | 40 | 40 |
| $\Delta t \times 10^{-3}$ s | | 10.0 | 7.6 | 5.6 | 4.2 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 |

While, Δf is a function of the bandwidth only, because the time axis is scaled with the standard 1,000 c/sec. signal and with their higher harmonics.

 Δf measured at each frequency is also given in Table 1.

Using these numerical values of $|\Delta t|$ and $|\Delta f|$ the curve of $|\Delta (tf^{1/2})|$ is plotted against f for a case of $D = 45\sqrt{s}$ in Fig. 3. This curve rises up rapidly at frequencies lower than 2,000 c/sec., but this is still smaller than the calculated departures of whistler dispersion, so



FIG. 3. Probable error produced in the calculation of $t*f^{1/2}$ in a case of $D = 45\sqrt{s}$, using the values given in the Table 1.

it will be possible to detect the effect of proton, if existing, by the method adopted.

3.3. Data

Four whistlers, all short type, are made use of for the present purpose, two of which were observed at Wakkanai station on 24 Dec. 1957, 1605 J.S.T. (Fig. 4 a) and on 9 Mar. 1958, 0435 J.S.T. (Fig. 4 b) and the remaining ones were observed at Toyokawa station on 23 Feb. 1956, 1715 J.S.T. (Fig. 5 a) and on 20 Feb. 1957, 1715 J.S.T. (Fig. 5 b). They are all considerablly pure, but the intensities of the signals at frequencies higher than 7,000 c/sec. are generally weak, especially so in the case of Toyokawa. In general, it has been found from a careful examination with sonagrams that a whistler, even of pure tone type, does not always show a uniformly defined trace of steadily decreasing tone, but shows a connection of several fragmental figures caused by the strength of its componet frequencies, which is more marked at Wakkanai station. This whistler characteristic also



a: 24 Dec. 1957, 1605 J.S.T., Wakkanai



Time origin Tweek b: 9 Mar. 1958, 0435 J.S.T., Wakkanai following an atmospheric click of tweek type One second FIG. 4. Sonagrams of short whistlers used for analyses.

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gives a difficulty to the present purpose. To avoid the difficulty as much as possible, the four whistlers, being comparably free from such a character, were selected from among the many whistlers analysed.

Though the whistler shown in Fig. 4 b is a short one, it is found to follow a preceding atmospheric click of tweek type (marked by an arrow in Fig. 4 b) as formerly reported in the case of Toyokawa.

This characteristic is very convenient in getting information about the position of time origin of the whistler.

3.4. Experimental Results and Discussions

The $t \cdot f^{1/2}$ curves obtained by analysis of the four whistlers of Wakkanai and Toyokawa are plotted in Figs. 6 *a*, 6 *b* and in Figs. 7 *a*, 7 *b*, respectively.

In each Figure, curves A and B indicate the experimental and theoretical $t \cdot f^{1/2}$ curves respectively. Curve A is determined to fit best in the theoretical



FIG. 5. Sonagrams of short whistlers used for analyses.

one by properly shifting its time origin as described in section 3.1. The shapes of the two sets of curves B are the same as the ones in Figs. 1 a and 1 b, but their levels are adequately changed to make comparisons with curves A easier. It is noted that the theoretical $t \cdot f^{1/2}$ curves will little change their shapes, even if the distribution of electron density may be altered to fit the individual cases; therefore it will be enough to use the B curves for the comparison.

From these figures, it will be seen that little difference exists between the result obtained by using time origin referred to in the preceding click and the results obtained in the way described in section 3.1.

Now, curve C is plotted in order to estimate whether the whistler disperses according to simple law or not. In each case, the curve is calculated to agree with curve A at high and low frequency ends by properly shortening the travelling time of a whistler assumed to have a constant dispersion appropriate in each case. Therefore, if the dispersion of the whistler analysed is constant in reality, though it shows an apparent departure from constant dispersion by its travelling





FIG. 7. Experimental $t \cdot f^{1/2}$ curves measured from whistler traces.

- A: experimental curve
- B: theoretical curve
- C: a curve calculated as a criterion to judge whether the whistler disperses according to the simple law or not

time being taken short, the curves A and C should agree in each case. But this is not the case, for in all cases curve A is clearly seen to agree better with curve B than with curve C.

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This fact strongly confirms the presence of ionized hydrogen in the outer atmosphere.

IV. Conclusions

Judging from what have been carefully examined, it is scarcely doubtful that all the dispersions of whistlers observed at two different locations on different dates satisfy the complete dispersion law including the effect of ionized hydrogen in the frequency range from 8,000 c/sec. to 700 c/sec. Therefore it may safelly be concluded that the ionized hydrogen is certain to exist in the outer atmosphere. But there remains a question about the distribution of electron density, and this necessarily relates to the problem of the constituent of the ionization in the outer atmosphere, because the distribution of ionization density presented by Storey is too small to produce the observed dispersions. In fact, the difference between the density adopted by Storey and the ones used in the present work is so great, that it seems to be somewhat doubtful that the ionization is composed of ionized hydrogen only. In order to get any informations about this problem, it is necessary to know the shape of the whistler path as exactly as possible, for the propagation of a whistler in low latitudes is theoretically predicted to deviate not a little from the magnetic line of force.⁷⁾

Though the results obtained so far give a sufficient evidence of the presence of ionized hydrogen in the outer atmosphere as mentioned, it will be a further proof, if dispersion curves are observed to have a maximum point at a frequency near 700 c/sec. as being presumed from the theoretical curves.

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