NUMERICAL STUDY OF TWEEKS BASED ON WAVEGUIDE MODE THEORY

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Bbstract

Assuming that the ionospheric reflecting layer is a sharp bounded and imperfect conductor attenuation factor and group velocities are calculated from the waveguide mode theory developed by Wait.

Results are obtained mainly for the first order mode for four frequencies between 1.8 kc and 2.2 kc with various values of the ionospheric parameterω_r. And in the cases of zero and second order modes calculations are performed only for ω_r of 5×10 sec^{-1} and for wave frequencies less that 2.0 kc and between 3.6 kc and 5.0 kc, respectively. The usually observed characteristics of propagation time vs. frequency of tweeks well explained by the attenuation and group velocity characteristics of first and second order modes And it is shown that for the determination of travelling distance and height of reflecting layer from the propagation characteristics of tweeks it is advantageous to make use of the part of a tweek corresponding to the first order mode, because of the easiness of seperating it from waves belonging to the other order modes and of its less attenuations. Differences which are caused by regarding an imperfectly conducting layer as a perfectly conducting one, in the determination of distance and height are discussed.

I. Introduction

Since the study by E. T Burton and E. M Boardman (1933)[1], the tweeks, which are now usually called long osillatory type atmospherics by workers of the waveforms, have heen interpreted to be produced from the propagation of a pulse radiated from a lightning discharge, by multiple reflections between the ground and ionospheric layers. And this propagation mechanism has been shown to afford imformations about the height of a reflection layer and the travelling distance of the pulse.

In the year 1958[2] we found that short whistlers observed at Toyokawa were frequently preceded by the tweek type atmospherics, and afterwards similar cases were observed also at Wakkanai. Further we could find such cases in the photographs of sonagrams of short whistlers which were observed in America*. So, very naturally we were given hints to make use of these tweeks measuring the distances from the whistler sources, for it is very important to know the locations of whistler source in order to investigate the problem of whistler propagation and the ionization distribution in the outer atmosphere. In this trial the ionospheric reflection layer has been assumed to be a sharp-bounded and perfectly conducting plane, but in reality it must be regarded as an imperfect conductor even for lower frequency waves in VLF range. The assumption of a perfect conductor for the reflection layer will more or less cause to give incorrect results. It needs to know the order of magnitude of the errors.

But the treatment of an imperfect conductor of a layer based on the multiple reflection theory, will lead to much trouble in obtaining the characteristics of propagation time vs. frequency, owing to a large number of reflection and to the reflection coefficient of the layer which is a function both of travelling distance and of the order of reflection of a pulse received, besides being a function of ionospheric layer properties.

Hence, in the present study we calculated from waveguide mode theory the attenuation factors and group velocities for some frequency ranges of tweeks which are thought to be necessary in the determination of height and distance. In addition, the same quantities are calculated for frequencies less than 2.0 kc in the zero order mode.

In §II natures of tweeks are briefly described, in §III the theory used in the calculations is explained, in §IV the results obtained from the calculations are shown and confirmed to explain the tweek characteristics, in §V a procedure to determine height, distance and ionospheric parameter ω_r is shown and an example of possible difference in height and distance obtained by assuming an imperfect layer as a perfect one are discussed.

*Fig. 14, p. 59, Fig. 28, p. 111 and Fig. 29, p. 113 in Low Frequency Propagation Studies Patr I : Whistlers and Related Phenomena.
R. A. Helliwell
Stanford Electronics Laboratories Stanford University May, 1958.

II. Tweeks

Examples of tweeks observed at Toyokawa appear in Figs. 1(a) and 1(b),

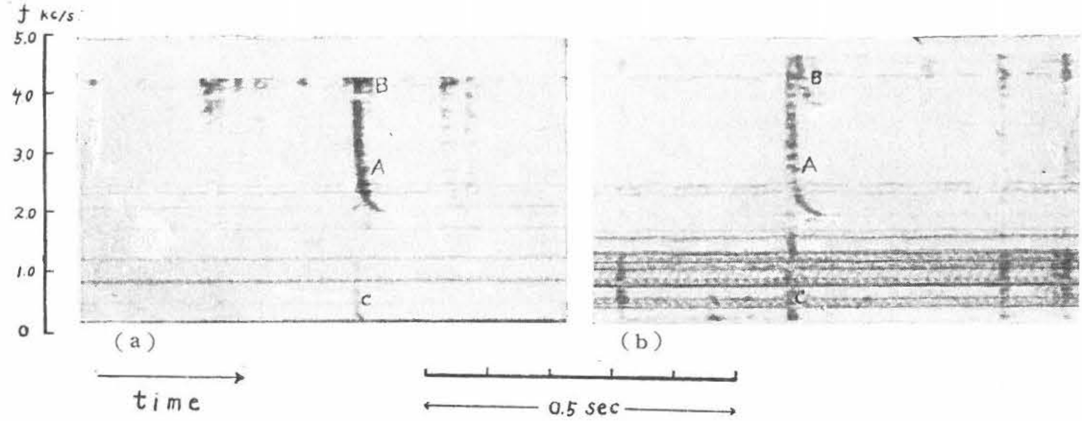


Fig. 1. Examples of tweek analysed by a sound spectrograph
A : fundamental tweek B : second harmonic tweek
C : part of a pulse corresponding to zero order mode
(a) 1840 5 Sep. 1956 J.S.T. Toyokawa The second harmonic tweek seems to exist above a frequency near 4 kc/s.
(b) 2110 24 Oct. 1956 J.S.T. Toyokawa The socond harmonic tweek can be seen to a frequency near the lower limit.
(note : Since in these analyses the output level and marking level of the analysing apparatus were manually changed, degrees of darkness do not correspond to the field intensity.)

which were obtained by an analysis using a sound spectrograph known as "Sona-graph". The frequency scale is marked at the left of the figures and the time elapses from left to right and the scale is shown at the bottom of the figures. As seen from part A's in these examples, the waves of tweeks travel slowly in degrees, as the frequency falls, and they go down exponentially to certain limiting frequencies which are usually observed at near 1.8 kc, although they vary as the travelling path conditions and the time of day change.

Part B in Fig. 1(b) can clearly be seen to be the second harmonics of part A, though the full part of the harmonics is not shown. When R. K. Potter (1951)[3] analyzed tweeks recorded by Burton and Boardman with sound spectrograph, he found the seoncd and third harmonics. At that time he thought that they were possibly caused by the receiving and recording epuipment nonlinearities. We can not decide whether it was so or not in his case, but we are now certain of the possible existence of higher harmonic components, judging from the nature of tweeks, because they can be reasonably explained by the wave-guide mode theory. In the following, the parts A and B of the tweeks will be called the fundamental and the second harmonic tweek (component), respectively.

From this propagation vs. frequency characterisitics we can easily distinguish tweeks from other types of atmospherics, when signals from a vertical antenna are heard through a speaker after amplification with a audio-frequency amplifier. On the basis of the hearing method, it has been found at Toyokawa and Wakkanai that the tweeks very frequently occur in the night between near sunset and sunrise all the year round. This result is quite the same as has been reported by Burton and Boardman.

By analysis we have found the second harmonic tweeks to be a lesser event compared with the foundamental ones, and as for the third harmonic tweeks they have been hardly observed.

III. Theory

Idealizing the space between the ground and ionosphere as a waveguide with concentric spherical walls of finite conductivities and assuming the source is a vertical electric dipole, J. R. Wait (1957)[4] derived a formula for a vertical electric field at any reciving point. This formula is available for VLF radio waves and is expressed by a sum of terms which depend on mode numbers.

The main part which determines the propagation characteristics of the modes is expressed following exponential term,

$$\exp\ (-i2\pi S_n d/\lambda)$$

where d is a travelling distance along the great circle of the earth, λ is a wave length and $S_n=(1-C^2{}_n)^{1/2}$.

C_n is a cosine of a complex angle between wave normal and vertical, and is a solution of the next equation,

$$\frac{(L-i)\,C_n-\sqrt{C_n\,L^2-iL}}{(L-i)\,C_n+\sqrt{C^n\,L-iL}}=e^{i4\pi HC_n}\ e^{-i2\pi n} \qquad (1)$$

which was also given by Budden (1953).

This eq. is valid for perfectly conducting ground. But As seen from examples of tweeks in Figs. 1(a) and 1(b), frequencies lower than about 5 kc may be enough for the study of tweeks including the second harmonic one, and in this frequency range the ground may be approximated as a perfect conductor. So the eq. (1) will be valid for the present case.

Where

$L=\omega/\omega_r,\ \ H=h/\lambda,\ \ n=$ mode number $=0,\ 1,\ 2\cdots\cdots$

ω=angular velocity of wave$=2\pi f$, $\omega_r=(2\pi f_0)^2/\omega=3.182\times10^9\times N/\nu$
f_0=plasma frequency, N=electron number density/cm^3 and
h=height of ionospheric reflecting layer from the ground,
ν=collisional frequency/sec.

We write $C_n=x_n+iy_n$ and $S_n=X_n+iY_n$ then

$$\left.\begin{aligned} X_n&=\sqrt{(1-x_n^2+y_n^2)+\sqrt{(1-x_n^2+y_n^2)^2+4x_n^2y_n^2}}/\sqrt{2} \\ Y_n&=-x_ny_n\sqrt{2}/\sqrt{(1-x_n^2+y_n^2)+\sqrt{(1-x_n^2+y_n^2)^2+4x_n^2y_n^2}} \end{aligned}\right\}. \quad (2)$$

And the right side of eq. (1) becomes

$$\exp(-4\pi Hy_n)\cdot\exp i2\pi(2Hx_n-n), \quad (3)$$

and

$$\exp(-i2\pi S_nd/\lambda)=\exp(2\pi Y_nd/\lambda)\cdot\exp i(-2\pi X_nd/\lambda). \quad (4)$$

The first term of the right side of eq. (4) decides the attenuation of the mode n and $2\pi X_n/\lambda$ is the wave number of the mode.

According to Wait, the attenuation factor α_n is defined as the number of db per 1,000 km of path length; then

$$\alpha_n=1.819\times fY_n\times10^{-1}\text{db}. \quad (5)$$

The group velocity V_{gn} of the mode n is given from the wave theory by

$$V_{gn}=d(\text{wave number})/d\omega=c/(X_n+\omega dX_n/d\omega). \quad (6)$$

Here, X_n is seen to be a function of x_n and y_n from eq. (2) and then x_n and y_n are thought to be a function L from eq. (1) which is equal to ω/ω_r.
(c=light velocity$\fallingdotseq3\times10^5$km/s).
Hence,

$$\omega.\frac{dX_n}{d\omega}=\left(\frac{\partial X_n}{\partial x_n}\cdot\frac{\partial x_n}{\partial L}+\frac{\partial X_n}{\partial y_n}\cdot\frac{\partial y_n}{\partial L}\right)\cdot L$$

where, $\partial x_n/\partial L$ and $\partial y_n/\partial L$ can be determined from the simultaneous equations derived by the real and imaginary parts of eq. (1).

Solutions of C_n in eq. (1) have been obtained by H. H. Howe and J. R. Wait (1959)[5] for each mode and for various values of L and H, and the attenuation factor and field intensity have been given by Wait (1957)[6] from the solutions of C_n. But they have not yet reported the group velocity for the mode. So we have tried to get it by a numerical calculation using eq. (6) mainly for the principal frequency range of tweeks, but we had to begin by solving eq. (1), because the values of C_n given by them are not directly usable. The first term of (4) gives the absolute value of the left side of eq. (1) and the second term determine the phase of it. The fact that the first term contains only y_n and the second only x_n makes it convenient for us to take a pair of starting values of x_i and y_n for successive approximation method adopted to solve the eq. (1) for the left side of eq. (1) is Fresnel's reflection coefficient and its absolute value must be nearly unit and its phase is to lie in a range between zero and $-\pi/2$ for sufficiently small values of L. The pair of starting value of zero mode has been refered to the results by How.

IV. Numerical Results

1. Group velocity for ionospheric reflecting layer of perfect conductor.

If the ionospheric reflecting layer can be regarded as a perfect conductor, then ω_n becomes infinite, so that L vanishes.

Accordingly, exp $(-4\pi Hy_n)\cdot$ exp $i2\pi(2Hx_n-n)=1$ i. e. $y_n=0$ and $x_n=n/2H=nc/2hf$. From eq. (2) $X_n=(1-x_n^2)^{1/2}$ and $y_n=0$ are obtained.

Hence, $X_n+\omega(dX_n/d\omega)=\{1-(nc/2\mathrm{hf})^2\}^{-1/2}$ and group velocity of the mode n is given by

$$V_{gn}=c\{1-(nc/2\mathrm{hf})^2\}^{1/2}. \tag{7}$$

From this expression it is seen that the group velocity decreases by slow degrees as the wave frequency falls and when it reaches the value $nc/2$h, the velocity retards infinitely, so the wave below this limiting frequency is unable to propagate. This limiting frequency is called the cut-off frequency of a waveguide. (In the following the term "cut-off frequency" used in the case of imperfectly conducting layer means the one defined by eq. (7).)

If the height of reflecting layer is 90 km, the cut-off frequencies are 1.67 kc and 3.33 kc for $n=1$ and $n=2$, respectively.

These cut-off frequencies are seen to be nearly the same as the limiting frequencies of tweeks shown in Figs. 1 (a) and 1 (b). And it is also clear from eq. (7) that the velocity of wave of any frequency of the first order mode equals that of the twice large frequency of the second order mode.

These facts lead to a conculusion that the propagation time vs. frequency characteristics of fundamental and second harmonic tweeks can possibly be explained by the first and second order modes, respectively of the waveguide mode theory.

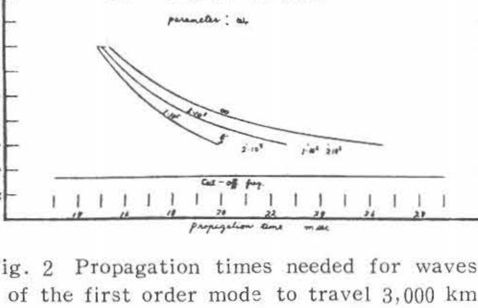


Fig. 2 Propagation times needed for waves of the first order mode to travel 3,000 km when h=90 km for various values of the ionospheric parameter ω_r

$\omega_r=\infty$ corresponds to the perfectly conducting reflection layer.

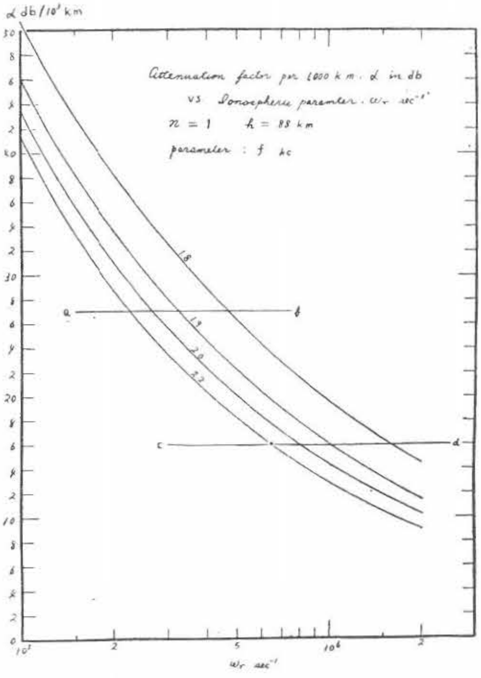
Cut—off freq. is obtained by calculation based on the waveguide mode theory for height of 90 km of the perfectly conducting reflection layer.

The propagation time vs. frequency characteristics of first mode obtained by eq. (7) for 3,000 km propagation with $h=90$ km is shownin Fig. 2 by a curve marked with ∞.

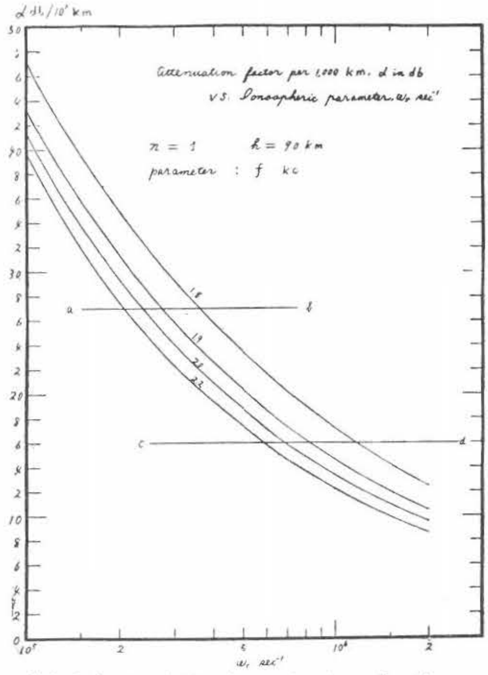
This curve compared with tweeks shown in Figs. 1 (a) and 1 (b), both propagation time characteristics may be seen to resemble on the whole, though the comparison is not enough since the frequency range shown is limited and the time scales differ so.

2. Ionospheric reflecting layer of imperfect conductor

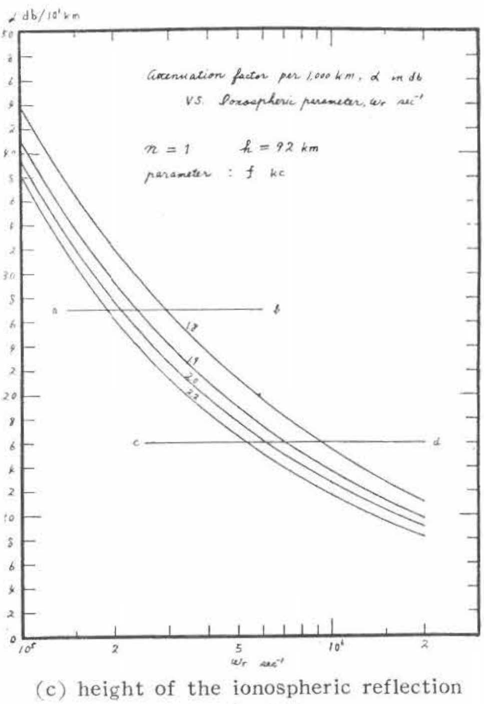
If the ionospehric reflecting layer is really a perfect cond-



(a) height of the ionospheric reflection layer h=88 km



(b) height of the ionospheric reflection layer h=90 km



(c) height of the ionospheric reflection layer h=92 km

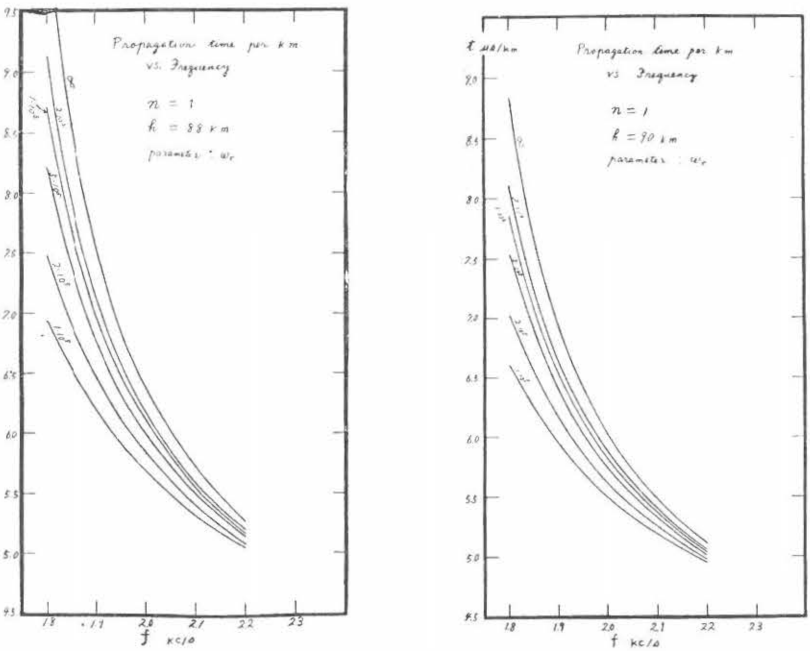
Fig. 3. Attenuation factor α_n of the first order mode
The lines ab and cd are drawn assuming that the limiting attenuation for waves to be observable at a receiving point is 80 db and the lower parts of these lines show possible regions of 3,000 km propagation and 5,000 km propagation, respectively.

uctor the higher harmonic tweeks should be observable as well as the fundamental ones, for the attenuation factors of each mode are vanished then. But in practice the harmonic tweeks higher than the second have been hardly detected. This may surely means that the ionospheric layer should not be treated as a perfect conductor.

In Figs. 3 (a), 3 (b) and 3 (c) the attenuation factors are plotted against the ionospheric parameter ω_r for layer height h=88, 90 and 92 km, respectively.

These curves indicate that the attenuation increases greatly as the frequencies approach the cut-off frequencies and ω_r decreases, and they also increase when the layer height falls. This attenuation vs. frequency characteristics well explains the existnece of a lower limit of frequency of waves. The lines ab and cd in each of Figs. 3 (a), 3 (b) and 3 (c) are drawn assuming that the limiting attenuation for waves to be observable is 80db, apart from the question of the radiated field intensity at the source. The line ab in each figure corresponds to 3,000 km propagation and cd corresponds to 5,000 km propagation. Conditions for a wave of 1.8 kc to be detectable after 3,000 km and 5,000 km propagations are that values of ω_r must be greater than 5×10^5 and 1.6×10^6, respectively for the reflecting layer height of 88 km. The vaule of ω_r depends on the electron density N and collision frequeny ν, but the distribution of both of these quantites against altitude differ with different observers. If we adopt the value of ν given by M. Nicolet (1953) and that of N given by A. P. Mitra (1957), $\omega_r=5\times10^5$ sec^{-1} will be produced for h=88 km and eight hours after sunset.

Instead of the group velocities the propagation times are plotted in Figs. 4(a),

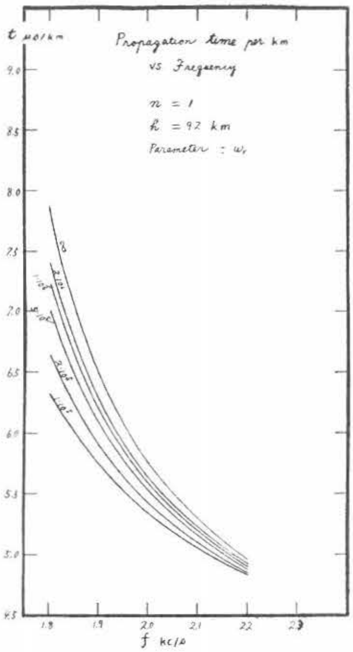


(a) h=88 km (b) h=90 km

Fig. 4. propagation time needed for waves of the first order mode to travel a kilometer

4 (b) and 4 (c) against wave frequency with the parametric variable of w_r for h= 88, 90 and 92 km, respectively, which is a time needed for wave to travel a distance of one kilometer.

These curves indicate that the propagation time increases markedly as



(c) $h=92$ km

frequency approaches the cut-off frequency, and ionospheric parameter ω_r decreases and the height of reflection layer lowers. And the difference of propagation time between any two frequencies also increases as the ionospheric reflective layer becomes more conductive and its height falls. The propagation time vs. frequency curves for 3,000 km travelling when $h=90$ km are shown in Fig. 2 for $\omega_r=5\times10^5$ and 1×10^6, respectively. It is evident that this propagation time vs. frequency characteristics makes it possible to account for the fundamental tweeks as in prefectly conductive layer model.

In Fig. 5 the attenuation factors of first and second order modes are shown against frequencies ranging from 1.8 kc to 2.2 kc and from 3.6 kc to 5.0 kc, respectively, for $\omega_r=5\times10^5$ and $h=90$ km.

The attenuation factor of wave of any frequency of second mode is seen to be far greater compared with that of the wave of its half frequency in the first mode. When $\omega_r=5\times10^5$ and $h=90$ km, the difference of the attenuation factor between 3.6 kc of second mode and 1.8 kc of first mode amounts to about 21db and that between 4.4kc of second mode and 2.2 kc of first mode is about 17db. It is clear that the waves of second mode attenuate more quickly than the waves of first mode. In Fig. 6 curves of propagation time per 1 km are plotted against frequency. It is seen that the propagation time of second mode is slightly greater than that of first mode if we compare any frequency wave of first mode with the twice larger frequency wave of second mode. This differs from the case of perfectly conducting layer. This attenuation vs. frequency and propagation time vs. frequency characteristics of first and second order modes reasonably account for the existence of the fundamental and second

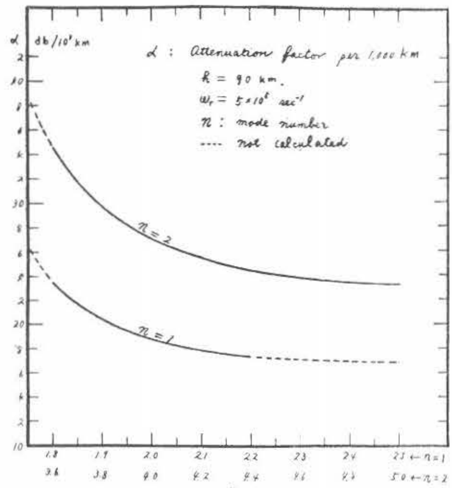


Fig. 5. Comparision of the attenuation factors between the first and the second order modes when $h=90$ km and $\omega_r=5\times10^5$ sec^{-1}

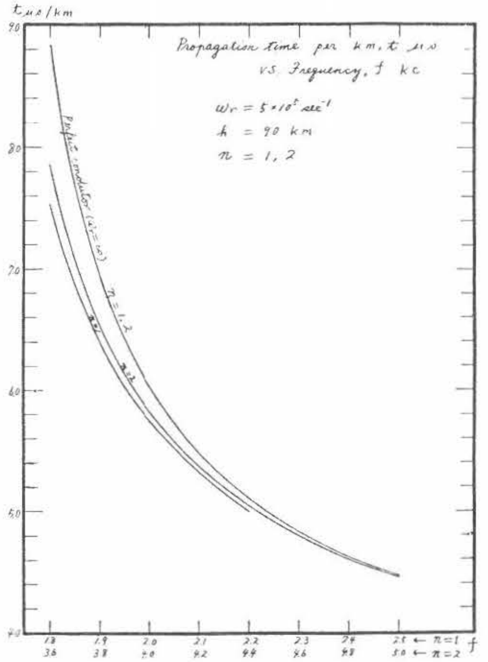


Fig. 6. Comparision of the propagation times between the first and the second order modes when $h=90$ km and $\omega_r=\infty$ and $5\times10^5\text{sec}^{-1}$

harmonic tweeks and the reason why occurrences of the second harmonic tweeks are reduced. And also harmonic tweeks higher than the second are supposed to be explained by the corresponding number of mode, and their more rapid attenuations can be expected, for Wait has shown that the higher the mode the greater the attenuation in the ranges of ω_r and f interested here.

Table 2 shows the attenuation factor and propagation time of zero mode for frequencies lower than 2.0 kc with $\omega_r=5\times10^5$ and $h=90$ km. In this case the attenuation factors increase with frequency, but their magnitudes are very small compared with the first and second order modes and the propagation time decreases with frequency, but they are near that of the light. From these characteristics of zero mode it is clear that in the lower part of VLF range waves

TABEL 1

f c/s	10	30	300	600	1,000	1,500	2,000
α db	0.32	0.59	2.06	3.04	4.10	5.59	7.13
t μs	3.709	3.546			3.366	3.357	3.347

of zero order mode are the most dominant ignoring the dependence of radiated field intensity on frequency at source and they travel fastest, so they reach a receiving point more quickly than any waves of tweeks. Examples of the waves of zero mode can be seen in Figs 1 (a) and 1 (b) by the parts marked with C.

If waves belonging to any single mode is selectively observed, the wavefrom received can be thought to suffer no distortion which is possibly caused by superposition of waves of other modes. Then the time intervals of successive wave crests will change exactly in accordance with the propagatlon time vs. frequency characteristics of the mode. If we record the waveform of tweeks through a low pass filter of cut-off frequency about 3.0 kc, the result shows that the waveform observed consists of waves corresponding to the zero and first order modes. But the waves of the zero order mode arrives so much earlier than the first order mode, that the latter can be easily seperated from the former. In order to determine the propagation distance and height of reflection layer from the propagation time characteristics of tweeks, it seems best to make use of the waves corresponding to both modes, but then the duration time will be too long to measure with an electronic counting aparatus. Hence, it is advantageous to utilize the wave of frequency between about 1.8 kc and 2.2 kc in which the variation of propagation time vs. frequency is marked.

V. A graphical procedure to determine the propagation distance

When the reflecting layer is treated as a perfect conductor, the determination of the travelling distance and layer height from the propagation time characteristics of tweeks is performed by comparing the observed propagation time vs. frequency curve with curves previously calculated with eq. (7) for various values of distance and height. But this fit-method is supposed to become somewhat complicated in the case of imperfect conductor, because another parameter ω_r is introduced. So, we tookd the following procedure.

When the time needed for a wave of frequency f_i kc to travel a kilometor is expressed by $t(f_i)$, the propagation time $\tau\ (f_i)$ for travelling d km is shown as $\tau\ (f_i)=d \cdot t\ (f_r)$.

And for a pair of waves of frequencies f_i and f_j if we write $t(f_i)-t(f_j)=\Delta t(f_i-f_j)$ and $\tau(f_i)-\tau(f_j)=\Delta\tau(f_i-f_j)$, then it will be

$$\Delta\tau(f_i-f_j)=d \cdot\ \Delta t(f_i-f_j.). \tag{8}$$

Taking another pair of waves f_l and f_m we can derive the next equation,

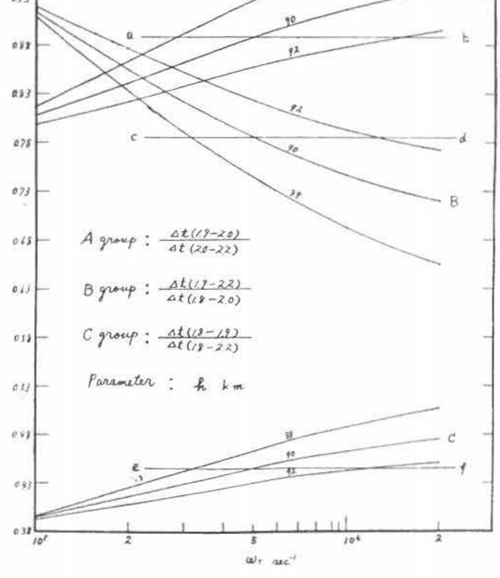


Fig. 7. Ratio of $\Delta t\,(f_i-f_j)$ to $\Delta t\,(f_l-f_m)$

$\Delta t\ (f_i-f_j)$ and $\Delta t\ (f_l-f_m)$ expres the propagation time difference between wave frequencies f_i and f_j, f_l and f_m, respectively of the first order mode. The lines ab, cd and ef show values of $\Delta t(1.9-2.0)/\Delta t(2.0-2.2)=0.888$, $\Delta t(1.9-2.2)/\Delta t(1.8-2.0)=0.785$ and $\Delta t(1.8-1.9)/\Delta t(1.8-2.2)=0.445$, respectively, which are obtained by calculation using the values of Δt given in Table 2.

$$\Delta\tau(f_i - f_j)/\Delta\tau(f_l - f_m) = \Delta t(f_i - f_j)/\Delta t(f_l - f_m). \tag{9}$$

Quantites given by a tweek observed are always the differences of the propagation time between any pair of frequency so the left sides of eqs. (8) and (9) are known. The right side of eq. (9) contains two unknown quantities h and ω_r but it can be numerically calculated for given values of both the quantities. These unknown quantities are determined graphically. The procedure will be explained with an illustration. In Fig. 7 the three curves of each group A, B and C show $\Delta t\,(1.9-2.0)/\Delta t\,(2.0-2.2)$, $\Delta t\,(1.9-2.2)/\Delta t\,(1.8-2.0)$ and $\Delta t\,(1.8-1.9)/\Delta t\,(1.8-2.2)$, respectively against ω_r for values of h=92, 90 and 88 km. If measured values of $\Delta\tau$ are those as shownt in Table 2, then the values of Δt (1.9−2.0) / Δt (2.0−2.2), Δt (1.9−2.2) / Δt (1.8−2.0), and Δt (1.8−1.9) / Δt (1.8−2.2) become 0.888, 0.785 and 0.445, respectively.

TABEL 2

$f_i - f_l$	1.8−2.2	1.8−2.0	1.8−1.9	1.9−2.2	1.9−2.0	2.0−2.2
$\Delta\tau(f_i - f_l)$ms	7.584	5.355	3.378	4.206	1.980	2.229
$\Delta t(f_i - f_l)\mu$s	2.528	1.785	1.126	1.402	0.660	0.743

Lines ab, cd and ef in Fig. 7 represent these values. Values of h and ω_r at the crossing points between A and ab, B and cd and C and ef are read and plotted in a $\omega_r - h$ plane and then connected by curves of ab, cd and ef as shown in Fig. 8. These three curves are

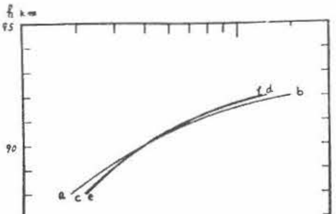


Fig. 8. $\omega_r - h$ plane
Values of (ω_r, h) at the crossing points between A and ab, B and cd and C and ef in Fig. 7. are plotted in this plane and then connected by curves ab, cd and ef, which are seen to cross at a point $\omega_r = 5 \times 10^5$ and $h = 90$.

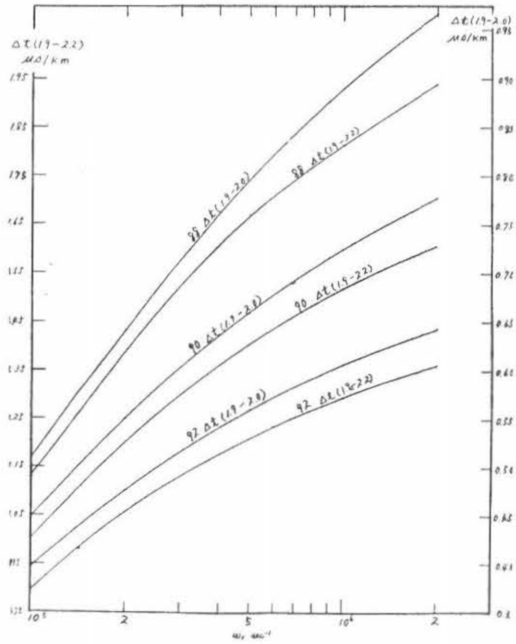


Fig. 9. $\Delta t(1.9-2.0)$ and $\Delta t(1.9-2.2)$ of the first order mode are plotted against ω_r for h=88, 90 and 92km.

seen to cross at a point $\omega_r=5\times10^5$ and h=90 km. But in general, these three curves do not always intersect at one point, so it is necessary to draw as many curves as possible and the most probable point will be determined from the group of crossing points.

Now, since the values of the pair h and ω_r are obtained, $\Delta t\ (f_i-f_j)$ becomes known. From curves $\Delta t\ (1.9-2.0)$ and $\Delta t\ (1.9-2.2)$ plotted in Fig 9 against ω_r for h=88, 90 and 92 km, the values of $\Delta t\ (1.9-2.0)$ and $\Delta t\ (1.9-2.2)$ at the point $h=90$ km and $\omega_r=5\times10^5$ are read as 9.660 and 1.402, respectively. From a pair of Δt and $\Delta\tau$ the propagation distance can be decided by eq. (8) but in general, different results may be obtained for different pairs. In the present case the results obatined from the two pairs of $\Delta t\ (1.9-2.0)$, $\Delta\tau\ (1.9-2.0)$ and $\Delta t\ (1.9-2.2)$, $\Delta\tau\ (1.9-2.2)$ are found to give the same value of about 3,000 km. The fact is that this illustration has been prepared by a calculation assuming $\omega_r=5\times10^5$, h=90 km and $d=3,000$ km.

Now, if we assume a perfectly conducting layer for values of $\Delta\tau$ in Table 2, the distance determined will differ. So, we rewrite the eq. (8) distinguishing by a dash as

$$\Delta\tau'\ (f_i-f_j)=d'\cdot\Delta t'\ (f_i-f_j). \tag{10}$$

In practice $\Delta\tau$ and $\Delta\tau'$ should be of the same value, for they are measured fom the same observational data. Accordingly, from eqs. (8) and (10)

$$d'/d=\Delta t\ (f_i-f_j)/\ \Delta t'\ (f_i-f_j) \tag{11}$$

can be derived.

This means that the ratio of distances determined under each assumption of a perfectly conducting layer and an imperfectly conducting one, equals the ratio of the differences of propagation time calculated theoretically for each case and it does not depend on the propagation distance, but it will more or less vary as the pair of reference frequencies are changed. The dependence of this variation on the pair of reference frequencies must be examined, but it has not yet been studied systematically. We will determine the ratio d'/d in the following procedure. Since $\Delta\tau'$ equals $\Delta\tau$, the next successive equations come into existence.

$$\frac{\Delta t\ (f_i-f_j)}{\Delta t\ (f_l-f_m)}=\frac{\Delta\tau\ (f_i-f_j)}{\Delta\tau\ (f_l-f_m)}=\frac{\Delta\tau'(f_i-f_j)}{\Delta\tau'(f_l-f_m)}=\frac{\Delta t'(f_i-f_i)}{\Delta t'(f_l-f_m)}$$

Accordingly, $\Delta t'(1.9-2.0)/\ \Delta t'(2.0-2.2)=0.888$ or
$\Delta t'(2.0-2.2)/\ \Delta t'(1.8-2.0)=0.416$ is derived from the given values $\Delta\tau$ in Table 2. Curves $\Delta t'(1.9-2.0)/\ \Delta t'(2.0-2.2)$ and $\Delta t'(2.0-2.2)\ /\ \Delta t'(1.8-2.0)$ are plotted in Fig. 10 against h.

Using these curves we read the values of h corresponding to values of the ratio 0.888 and 0.416 as 94.1 and 94.3 km, respectively. The determined values of h are seen to be nearly the same. From curves $\Delta t'$ plotted in the same figure $\Delta t'(1.8-2.0)=1.640$ and $\Delta t'(1.9-2.0)=0.602$ are obtained for $h=94.2$ km. Hence, from eq. (10)

$$d'/d=\Delta t\ (1.8-2.0)/\ \Delta t'(1.8-2.0)=1.088 \text{ or}$$
$$d'/d=\Delta t\ (1.9-2.0)/\ \Delta t'(1.9-2.0)=1.096$$

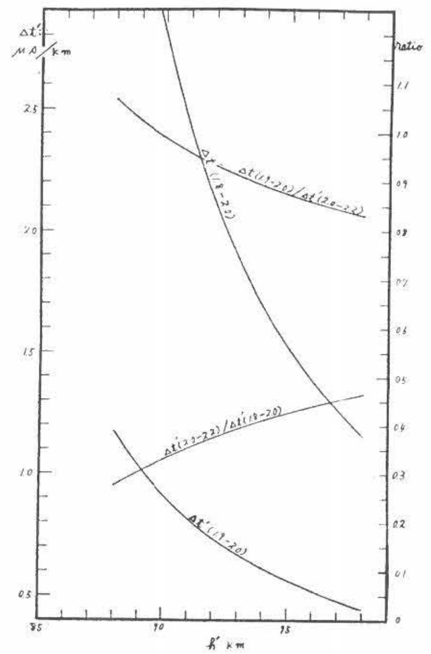


Fig. 10. For the case of perfectly conducting reflection layer $\Delta t'$ (1.8−2.0), $\Delta t'$(1.9−2.0) and Δt (1.9−2.0)/$\Delta t'$(2.0−2.2), $\Delta t'$(2.0 −2.2)/$\Delta t'$(1.8−2.0) of the first order mode are plotted against the layer height h'.

(the averaged value 1.092) can be decided.

These results signify that if we treat an imperfectly conducting layer characterized by $\omega_r=5\times10^5$ and h=90 km as a perfectly condcting layer, the travelling distance and reflecting layer height are determined to be about 9 % and 5 % larger, respectively.

If the value of ω_r is reduced more the results determined will become larger.

VI. Conclusions

It is shown that the existence of the lower limiting frequencies and the propagation time vs. frequency characteristics of tweeks can be well explained respectively by the attenuation factor and the group velocity calculated numerically based on waveguide mode theory.

And it is also shown that each order of harmonic tweeks corresponds to each order number of the mode and that the observational fact that the second harmonic tweeks occur less frequently than the fundamental ones is due to the difference in the magnitude of the attenuation factors between the two component tweeks. In order to determine the propagation distance and reflection layer height from the propagation time vs. frequency characteristics of tweeks it is advantageous to make use of the part of the fundamental tweek in frequency range between about 1.8 kc and 2.2 kc, because the waves in this frequency range can be easily seperated from the waves of other modes, consequently they are free from any possible distortions caused by an overlapping of waves of other modes. In the determination of distance and height, if the ionospheric reflecting layer is treated as a perfect conductor, errors are introduced. It is theoretically known that the ratio of distances determined under perfect and imperfect condctor assumptions equals the ratio of the differences in propagation time between any two frequencies calculated under each assumption.

For instance, when the ionospheric layer can be characterized by $\omega_r=5\times10^5$ and $h=90$ km, the ratio is about 1.09, and h becomes about 94.2 km. The ratio and h will be larger as the value of ω_r is reduced. But the actual values of ω_r and h are not yet finally known, so the errors can not be correctly estimated now. On the contrary, we expect that this analysis of tweeks will give some informations about these quantities and further expect that it will contribute to the study of the whistler propagation problem and to the fixing of atmospherics by one-station method.

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