Calculated Statistical Characteristics of Atmospheric Radio Noise

T. Nakai

Summary

First the Poisson noise process is considered in which the pulses of rectangular form with a constant duration are arriving at the times determined by Poisson's distribution law, and the average number of the observed pulses per second, the average duration of the observed pulses per second and the probability of time length which is occupied by the existence of the observed pulses in the noise process are deduced respectively. Next, the deduced expressions of the parameters in Poisson's noise process are applied to the estimates of the same kind of parameters of a given voltage level of the envelope of the atmospheric noise by limiting their utilities in the direction of smaller amplitude probabilities at the high field strength levels, on an assumption of a crossing rate function of the amplitudes of impulses in the atmospheric noise field at the input of the receiver. And the expressions of the transformation factors (1) of the crossing rate (2) of the probability and (3) of the crossing rate and the probability are obtained. And the approximate range of the field strength or of the probability in which the expressions obtained may probably hold is estimated in a later section of the paper.

1. Introduction

Rice[1] has shown that the expected number of positive crossings of a given voltage level of the envelope of random noise or of random noise plus a carrier is equal to the bandwidth factor times the probability density of the envelope amplitudes. Watt and Maxwell[2] have shown that this relationship does not hold in the direction of smaller probability densities, in which the departure appears between the measured crossing rate curve and the derived envelope crossing rate curve by multiplying an appropriate constant by the measured probability density curve.

The auther has investigated the statistical properties of the small probability densities at the high field strength levels. A particular reference has been given to the relations (1) between two numbers of possitive crossings of a given voltage level of the envelope of the atmospheric noise for different values of the bandwidth, (2) between two probabilities for different values of the bandwidth when the envelope of the atmospheric noise exceeds a given voltage and (3) between the number of positive crossings and probability of a given voltage level of the envelope of the atmospheric noise for the same bandwidth. The three relations will be described in terms of the transformation factor

(1) of the crossing rate, (2) of the probability and (3) of the crossing rate and the probability, respectively.

At the first stage of the investigations, we have deduced the average number of the observed pulses per second, the average duration of the observed pulses per second and the probability of time length which is occupied by the existence of the observed pulses, in the noise process in which the pulses of rectangular form with a constant duration are arriving at the times spaced by Poisson's distribution law. The three parameters are respectively expressed by some functions of the average number of arrivals of the original pulses per second times a constant duration of the pulses respectively.

At the second stage of the investigations, we have applied the three expressions of the parameters deduced with respect to Poisson's noise process to estimate the same kind of parameters of a given voltage level of the envelope of the atmospheric noise, by limiting their utilities in the direction of smaller amplitude probabilities at the high field strength levels, where the interference between the IF response of the impulses at the input of the receiver may be negligibly small.

We have found from the results of the numerical calculations of the envelope crossing rate curves and the probability distributions of the amplitudes of the envelope of the armospheric noise that (1) the transformation factor of the crossing rate is equal to some power of a given bandwidth divided by a reference bandwidth, (2) the transformation factor of the probability is equal to some power of a given bandwidth divided by a reference bandwidth and (3) the transformation factor of the crossing rate and the probability is proportional to the reciprocal of a given bandwidth times a parameter of a given crossing rate function of the peak amplitudes of the impulses.

The details will be described in the paper.

2. Properties of Poisson's noise process

Let us assume a very long time interval T of the noise processes in which voltage or current pulses of rectangular form have arrived at the spaced times obeying Poisson's distribution law. Now we assign ν as the average number of the arrival of the pulses per second and x as the time spacing between two successively arrived pulses. x is a random variable and its probability density function is derived by differentiation of Poisson's distribution function as follows

$$q(x) = \nu e^{-\nu x} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

and Poisson's distribution function is

$$\int_x^\infty q(x)\,dx = e^{-\nu x} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

Here we assume that the initially arrived pulses have all the uniform duration τ. Then we can see that the following holds, that is, the probability that the spacing between two successively arrived pulses exceeds the time length τ is equal to the ratio of the average number of the observed pulses per second against the average number of the arrived
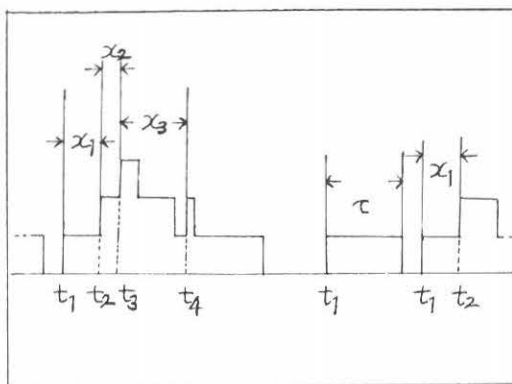
Fig. 1. Poisson noise process

pulses per second. And so, the average number R of the observed pulses per second in the time interval T is from the equation (1) as follows,

$$R = \nu e^{-\nu \tau} \quad \cdots\cdots\cdots\cdots\cdots \quad (3)$$

Now, we take a reference time at the instant of the beginning of an arbitrary pulse observed and take the beginngs of successively arrived pulses from early pulses to late ones as

$$t_1,\ t_2,\ t_3,\ \cdots\cdots\cdots\cdots$$

where x_1, x_2 and so on in Fig. 1 are the random variables satisfying the equation (1). When an observed pulse is the same one as an arrived pulse with the duration of τ, the condition

$$x_1 > \tau \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (4)$$

is required. Then the probability of the above specified case is

$$\int_{\tau}^{\infty} \nu e^{-\nu x_1}\,dx_1 = e^{-\nu \tau} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (5)$$

as is derived from the equation (1). Generally, the conditions where an observed pulse is one of the k-multiple pulses of which each is the pulse resulted from the overlappings of k successively arrived pulses, are expressed as

$$x_1 < \tau,\ x_2 < \tau,\ \cdots\cdots\cdots\cdots,\ x_{k-1} < \tau,\ \tau < x_k < \infty \cdots\cdots \quad (6)$$

and so the probability of the case in hand is given by the following expression

$$\int_{\tau}^{\infty}\int_{0}^{\tau} \cdots\cdot\int_{0}^{\tau} \nu^k \exp\{-\nu(x_1+x_2+\cdots\cdot+x_{k-1}+x_k)\}\,dx_1\,dx_2\cdots\cdot dx_{k-1}\,dx_k$$
$$= (1-e^{-\nu\tau})^{k-1}\cdot e^{-\nu\tau} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (7)$$

And summing up the probabilities, as is shown in the equation (7), ranging over all the positive integral numbers of k, the followiog equation holds,

$$\sum_{k=1}^{\infty} (1-e^{-\nu\tau})^{k-1}\cdot e^{-\nu\tau} = 1 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (8)$$

By multiplying R on each side of the equation (8), the following expression is obtained,

$$R = R\sum_{k=1}^{\infty} (1-e^{-\nu\tau})^{k-1}\cdot e^{-\nu\tau} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (9)$$

in which each term on the right side expresses the average number per second of the k-multiple pulses observed in the time interval T, which is expressed as follows

$$R_k = R(1-e^{-\nu\tau})^{k-1}\cdot e^{-\nu\tau} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (10)$$

Now, the duration of a k -multiple pulse is expressed by the following expression

$$\tau_k = x_1 + x_2 + \cdots + x_{k-1} + \tau \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where the following conditions must hold

$$0 \le x_s \le \tau : \quad s = 1, 2, \cdots, k-1 \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

and each of the $x_s's$ is an independent random variable for each other and satisfying the equation (1). Therefore, the average duration of all the k -multiple pulses is expressed as follows,

$$\bar{\tau}_k = \sum_{s=1}^{k-1} \frac{\int_0^\tau \nu \cdot \exp(-\nu x_s) \cdot x_s dx_s}{\int_0^\tau \nu \cdot \exp(-\nu x_s) dx_s} + \tau = \left\{ (k-1)\left(\frac{1}{\nu\tau} - \frac{e^{-\nu\tau}}{1-e^{-\nu\tau}}\right) + 1 \right\} \tau \cdots\cdots (13)$$

From the equations (10) and (13), the sum of all the durations of the k -multiple pulses is expressed as follows

$$R_k \bar{\tau}_k = R e^{-\nu\tau} (1-e^{-\nu\tau})^{k-1} \left\{ (k-1)\left(\frac{1}{\nu\tau} - \frac{e^{-\nu\tau}}{1-e^{-\nu\tau}}\right) + 1 \right\} \tau \cdots\cdots\cdots\cdots\cdots (14)$$

Furthermore, the sum of all the durations of the observed pulses per second in the time interval considered now, that is, the probability of time length occupied by the existence of the observed pulses P is as follows,

$$p = \sum_{k=1}^{\infty} R e^{-\nu\tau} (1-e^{-\nu\tau})^{k-1} \left\{ (k-1)\left(\frac{1}{\nu\tau} - \frac{e^{-\nu\tau}}{1-e^{-\nu\tau}}\right) + 1 \right\} \tau \cdots\cdots\cdots\cdots (15)$$

The above equation is simplified by a short calculation as follows

$$p = R \cdot \frac{1-e^{-\nu\tau}}{\nu e^{-\nu\tau}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (16)$$

,where the multiplifying factor of R on the right side of the equation (16) expresses the average duration $\bar{\tau}$ of all the observed pulses per second in the noise processes considered now, that is,

$$\bar{\tau} = \frac{1-e^{-\nu\tau}}{\nu e^{-\nu\tau}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (17)$$

And the probability of time length occupied by the existence of the observed pulses in the noise processes considered now is as follows, from the equations (3) (16), and (17)

$$p = R\bar{\tau} = 1 - e^{-\nu\tau} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (18)$$

3. Statistical properties of the atmospheric noise

Here, we will assume a peak amplitude property of the impulses in the atmospheric noise field at the input of the receiver, and the characteristics of the bandpass filter of the receiver, in order to make use of the analitical results obtained in chapter 2 and to derive the statistical properties of the amplitudes of envelope of the atmospheric noise at the output of the receiver.

At first it may be commonly accepted that a disturbance in the atmospheric noise field at the input of the receiver has the same effect as an impulse on the response of a

narrow band receiver at the output. Then we will assume that the number n of possitive crossings of a given voltage level v of the amplitudes of impulses in the atmospheric noise field at the input is given as follows,

$$n = \left(\frac{v}{A} \right)^{-r} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (19)$$

where

    A : the voltage level exceeded just by one impulse per second

    r : the constant parameter that characterizes the crossing rate function

The above assumption is thought to be a reasonable deduction from the measured envelope crossing rate curves of the atmospheric noise in a very low and low frequencies.

Next, though the bandwidth of a receiver is usually determined by a resonance characteristic curve of the multi- tuned resonance circuits, it may be assumed without the loss of generality that the bandwidth used is determined by the frequency difference between 3-db points of the resonance characteristic curve of a single-tuned circuit. Then the waveform of the envelope voltage generated at the output of the receiver may be expressed as follows,

$$E(t) = 2\pi B G_o e^{-\pi B t} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (20)$$

when an impulse is applied at the input of the receiver, where

    B : a bandwidth between 3-db points of the receiver response curve

    G_0: a receiver gain at a reference frequency

Then, when the waveform of the envelope voltage generated at the output by an impulse of the amplitude v arrived at the input of the receiver, is sliced by a given voltage level v_0, the duration of the sliced part of the waveform of the envelope is given as follows, from the eqation (20)

$$\tau = -\frac{1}{\pi B} \cdot \log_e \left(\frac{v_0}{2\pi B G_o v} \right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (21)$$

Now, when the waveforms of the envelope voltage generated independent of each other at the output are sliced by a given voltage level v_0 for all the impulses in the field of atmospheric noise at the input of the receiver and a simple arithmetical sum of all the durations of the sliced parts of the envelope waveforms is obtained, the average duration of a given voltage v_0 may be expressed as follows, from the equations (19) and (21)

$$\bar{\tau}(v_0, B, \gamma) = \int_{v_0/2\pi B G_o}^{\infty} \frac{-1}{\pi B} \cdot \log_e \left(\frac{v_0}{2\pi B G_o v} \right) dp(v, n) \quad \cdots\cdots\cdots \quad (22)$$

$$dp(v, n) = \frac{1}{n_m} \cdot \frac{\gamma}{A} \left(\frac{v}{A} \right)^{-r-1} dv \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (23)$$

where

    n_m : an expected number per second that exceeds over the least measurable
        amplitude of the impulses

We are now at the stage of applying the analitical results obtained in chapter 2 to express some parameters in the statistical variations of the envelope amplitudes of the atmospheric noise by making use of the expressions described in this chapter. When

the average number of the arrived pulses per second ν and its duration τ in the equations (3), (16) and (17) are substituted by the average number of the impulses exceeding a given voltage level per second in the field of the atmospheric noise at the input of the receiver as is shown by the expression (19), and the average duration of the independently sliced parts by the given voltage level of all the impulse responses at the output of the receiver, as is shown in the equation (22), the following three expressions are respectively obtained,

the envelope crossing rate of a given voltage of the atmospheric noise

$$R(v_0, B, \gamma) = \left(\frac{v}{A}\right)^{-r} \exp\left\{-\left(\frac{v}{A}\right)^{-r} \bar{\tau}(v_0, B, r)\right\} \quad\cdots\cdots\cdots\cdots\cdots (24)$$

the probability of time length that the envelope amplitudes of the atmospheric noise exceed over a given voltage level

$$p(v_0, B, \gamma) = 1 - \exp\left\{-\left(\frac{v}{A}\right)^{-r} \bar{\tau}(v_0, B, \gamma)\right\} \quad\cdots\cdots\cdots\cdots\cdots (25)$$

the average duration of the parts of which the envelope amplitudes of the atmospheric noise exceed over a given level

$$\bar{w}(v_0, B, \gamma) = \frac{1 - \exp\left\{-\left(\frac{v}{A}\right)^{-r} \bar{\tau}(v_0, B, \gamma)\right\}}{\left(\frac{v}{A}\right)^{-r} \exp\left\{-\left(\frac{v}{A}\right)^{-r} \bar{\tau}(v_0, B, \gamma)\right\}} \quad\cdots\cdots\cdots\cdots\cdots (26)$$

Now, the three expressions may well be thought to be the accurate expressions of the statistical parameters considered respectively, when limiting the usabilities of the expressions to the small probabilities of the envelope amplitudes at the high field strength levels where the interference between the IF responses of the impulses may negligibly be small.

4. Results of numerical calculations

4. 1 Envelope crossing rate curves

Figs. 2 and 3 were obtained from the results of the numerical calculations of the expressions (19) and (24). The upper curve in Fig. 2 shows the crossing rate curve of the impulse amplitudes in the field of the atmospheric noise at the input of the receiver. And the lower curve shows the envelope crossing rate curve of the atmospheric noise. The reference level of the field strength in the abscissa is taken at the voltage exceeded by just an impulse at the input per second, and the receiver gain is assumed so that the two curves fit at the higher field strength levels. Fig. 3 shows the calculated envelope crossing rate curves of the atmospheric noise for a few values of the bandwidth for a constant value of the parameter r. Here we will pay much attention to the properties of the calculated envelope crossing rate curves at the higher field strength levels.

Fig. 4 shows the way of the voltage dependance of the ratio of the positively crossing rate of a given voltage level of the envelope for a few values of the bandwidth, to the positively crossing rate of the same level of the envelope for a 1000 c/s bandwidth. Now, we will estimate the mean of the values of the ratio of two crossing rates,
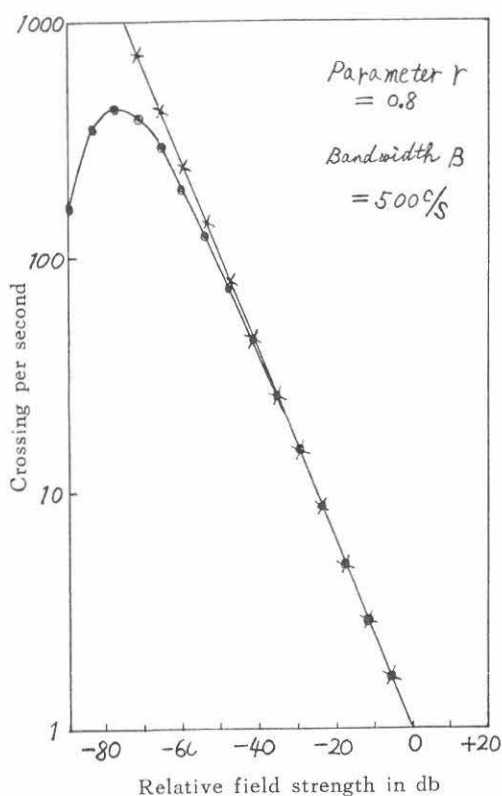
Fig. 2. Crossing rate curves of amplitudes
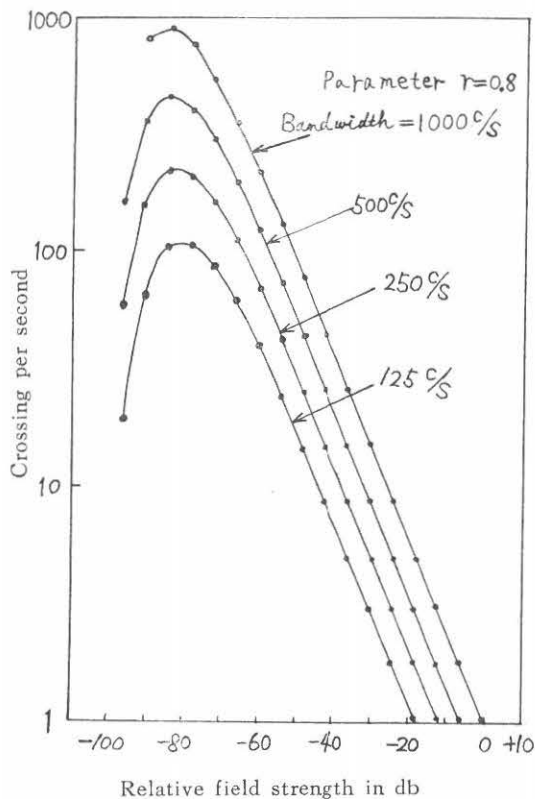of impulse and envelope

Fig. 3. Crossing rate curves of the amplitudes
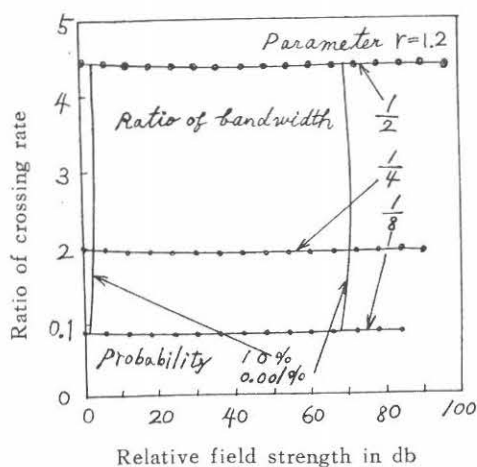of envelope

Fig. 4. Ratio of crossing rate against
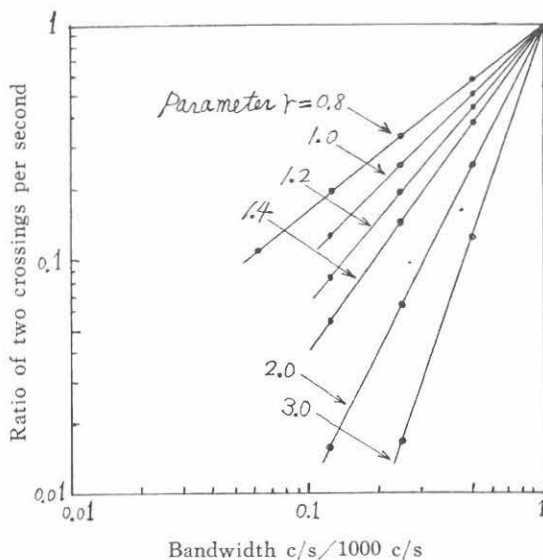the field strength

Fig. 5. Ratio of two crossings per second
between two different values of the
bandwidth

as is shown by the flat portion of the curve for each value of the bandwidth ratio in Fig. 4. The means of the ratios considered now are plotted against the ratio of the value of the bandwidth to a 1000 c/s bandwidth in Fig. 5. It will easily be seen that the plots are on a straight line of which the slope is equal to the value of the parameter r. Accordingly, a simple power law holds for the mean of the values of ratio of the two crossing rates between two different values of the bandwidth, or plainly speaking, the transformation factor of the crossing rate, that is,

$$\text{the transformation factor of the crossing rate}$$
$$= (\text{bandwidth}/1000)^r \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (27)$$

4. 2 Amplitude probability distribution curves

Fig. 6 was obtained from the results of numerical calculations of the equation (25). Attention is again paied to the high field strength levels. The reference level of the field strength in the abscissa is taken at the voltage level exceeded by just an impulse in the atmospheric noise field.

Fig. 7 shows the way of the voltage dependence of the ratio of the amplitude probability of a given voltage level of the atmospheric noise envelope for a few values of the bandwidth ratio, to the amplitude probability of the same level of the atmospheric noise envelope for a 1000 c/s bandwidth. Now, we will estimate the mean of the values of the ratios of the two probabilities, as is shown by the flat portion of the curves in Fig. 7 for each value of the bandwidth
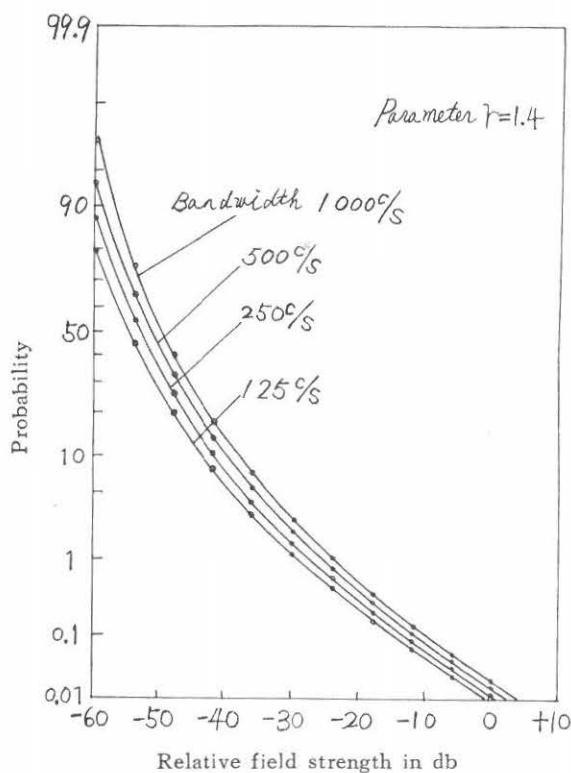

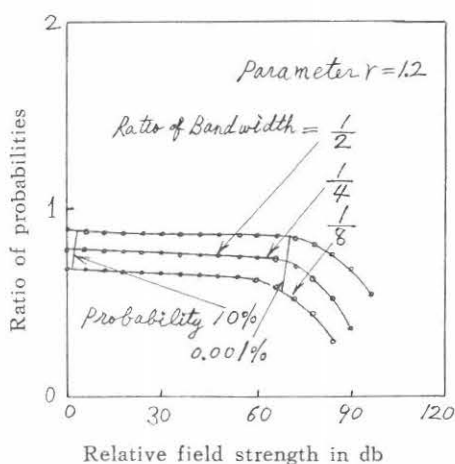Fig. 6. Cumulative amplitude probability distribution of atmospheric noise

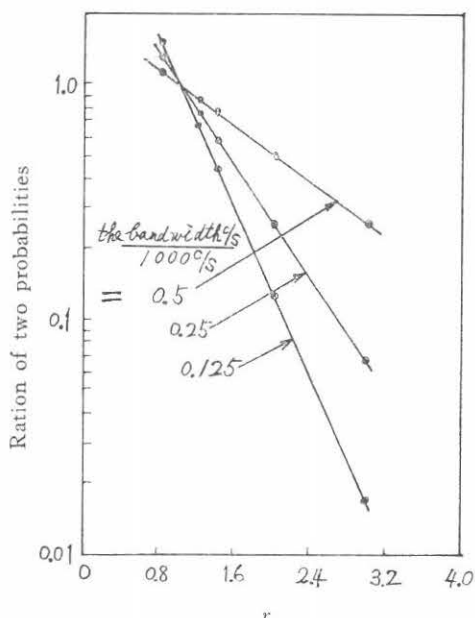Fig. 7. Ratio of probabilities against the field strength

Fig. 8. Ratio of two probabilities for
different bandwidth against the
parameter r

ratio. The means of the ratio considered now are plotted against the ratio of the value of the bandwidth to a 1000 c/s bandwidth in Fig. 8. It will easily be seen that the plots are on a straight line of which the slope is equal to the value of the parameter r minus one. Accordingly, a simple power law holds for the mean of the values of the ratio of the two probabilities of the atmospheric noise envelope between two different values of the bandwidths, or speaking simply the transformaiton factor of the probability, that is,

the transformation factor of the probability
$$= (\text{bandwidth}/1000)^{r-1} \cdots\cdots\cdots\cdots (28)$$

4. 3 Crossing rate curves and amplitude probability distribution curves

Due to the results of numerical calculations of the expression (26), Fig. 9 shows the way of the voltage dependence of the average duration of which the atmospheric noise envelope exceeds a given voltage, which is equal to the probability divided by the number of the positive crossings of the voltage level considered now of the envelope. On the other hand, Fig. 10 shows the amplitude probability distribution curve and an appropriate constant times the calculated envelope crossing rate curve. It can readily be seen from the fits of the two curves that the probability is equal to the constant times the expected number of the positive crossings of the envelope in the majority parts of the voltage levels now considered and the departure appears in the directions of very small probabilities at the very high field strength levels. It may easily be inferred from Figs. 9 and 10 that the multiplying constant is equal to the mean of the average durations in the flat portion of the average duration curve, as is shown in Fig. 9 for the values of the bandwidth 1000 c/s.

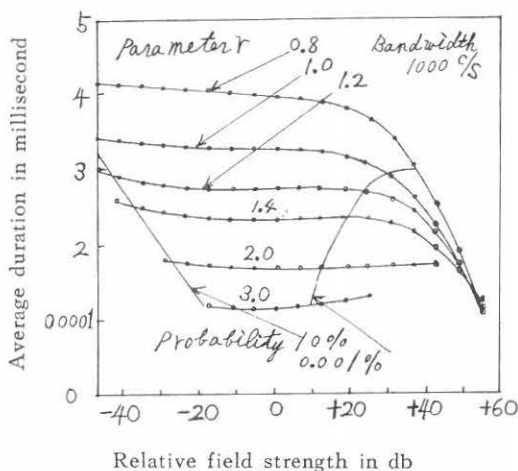The means of the average durations now considered are plotted in Fig. 11

Fig. 9. Average duration of parts of which
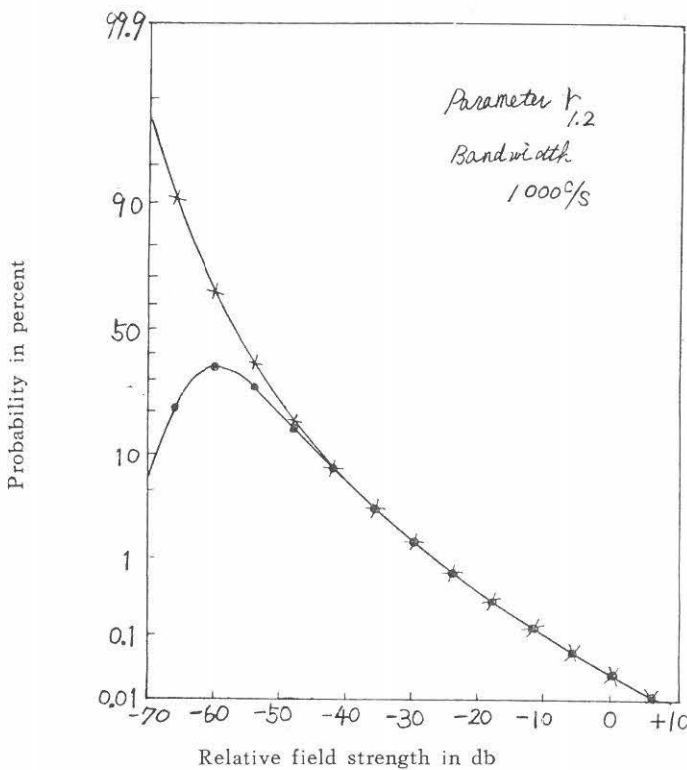the envelope exceeds over the abscissa

Fig. 10.　Amplitude probability distribution and modified envelope crossing rate curve

against the ratio of the bandwidth to a 1000 c/s band width. And the same quantities are plotted in Fig. 12 against the parameter r.

The plottings for a specified value of the parameter r in Fig. 11 are on a straight line of which the slope is approximately equal to minus one. On the other hand, the plottings for a specified value of the band width in Fig. 12 are on a straight line of which the slope is approximately equal to minus one. After all, the mean of the probability to the expected number of the positive crossings of the envelope, or the transformation factor of the probability and the crossing rate obeys a simple power law, that is,

the transformation factor between the probability and the crossing rate
$= (K/\text{bandwidth times parameter } r)$ ················· (29)
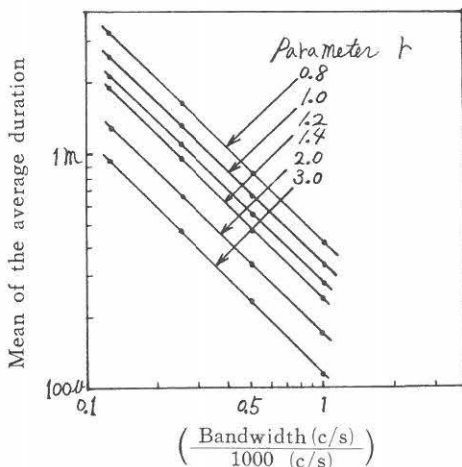
where K is a constant.

Fig. 11.　Mean of the average duration against $\left(\dfrac{\text{Bandwidth (c/s)}}{1000 \ (\text{c/s})} \right)$
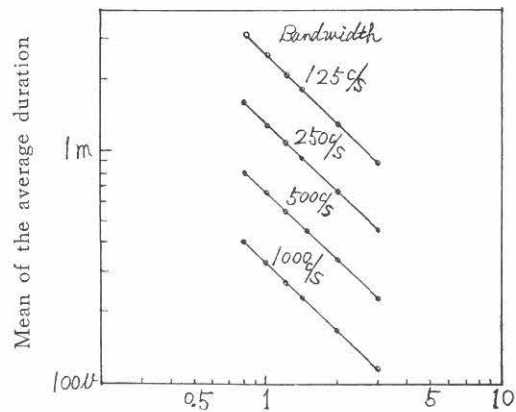

Fig. 12.　Mean of the average duration against the parameter r

5. Examination of the results

Now, we can easily see that the expression (27) of the transformation factor of the crossing rate holds in the levels of the field strength at which the interference between the IF responses of the impulses in the atmospheric noise field at the input is very small at the output of the receiver, where the envelope crossing rate curve is about the same as the crossing rate curve of the amplitudes of the impulses. Then, the envelope crossing rate curve for a given bandwidth B_1, may be expressed as follows,

$$n_1 \simeq \left(\frac{v_1}{2\pi B_1 G_0 A}\right)^{-r} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

And the envelope crossing rate curve for another given bandwidth B_2 may be expressed as follows

$$n_2 \simeq \left(\frac{v_2}{2\pi B_2 G_0 A}\right)^{-r} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (31)$$

where n_1 and n_2 are the numbers of the positive crossings of given voltage levels v_1 and v_2 of the envelope respectively.

When the numbers n_1 and n_2 express the number of the positive crossings of the same voltage of the envelope, it may easily be seen that the following relation holds in view of the equation (20).

$$\frac{v_1}{v_2} = \frac{B_2}{B_1} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (32)$$

Accordingly, the transformation factor of the crossing rate is expressed as follows,

$$\frac{n_2}{n_1} = \left(\frac{B_2}{B_1}\right)^r \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (33)$$

which is nothing but the expression (27).

Next let us assume that the expression (29) of the transformation factor between the probability and the crossing rate is true, then we can derive the expression (28) of the transformation factor of the probability. According to the assumption, for a given bandwidth B_1

$$\frac{n_1}{\gamma B_1} = K p_1 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (34)$$

and for another bandwidth B_2

$$\frac{n_2}{\gamma B_2} = K p_2 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (35)$$

By making use of the equations (29), (33), (34) and (35), the required expression is derived as follows

$$\frac{p_2}{p_1} = \left(\frac{B_2}{B_1}\right)^{r-1} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (36)$$

Lastly it is necessary to estimate the accuracies of the expressions (24), (25) and (26), respectively showing the crossing rate, the probability and the average duration of a given voltage level of the atmospheric noise envelope. It may be thought that the accuracies now considered commonly depend on the degree of the interference between the IF responses of the impulses in the atmospheric noise field at the input of the receiver. The degree of the interference may be larger unproportionately to the value of the ratio of the crossing rate of the envelope to the crossing rate of the impulses for a given voltage level, that is, from the equations (24) and (19)

$$\frac{R(v_0, B, \gamma)}{n(v)} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (37)$$

And the following equation is easily derived from the equations (19), (24) and (25)

$$p(v_0, B, \gamma) = 1 - \frac{R(v_0, B, \gamma)}{n(v)} \cdots\cdots\cdots\cdots\cdots\cdots (38)$$

The right side of the above equation (38) expresses the ratio of the decreased number of the observed pulses to the number of the arrived original pulses, which may well be said to define the degree of interference between the IF responses of the impulses. Accordingly we may be able to estimate the accuracies of the equations (24), (25) and (26) by the value of the left side of the equation (38), or the value of the amplitude probability of the envelope. But it may be inferred that the inaccuracies of three expressions considered now are less than the value of the probability of the amplitudes of the envelope for a given voltage level.

Next, it is necessary to consider the way of derivation of the equations (27), (28) and (29) and specify the approximate range of the field strength or of the probability in which the equations are true. For the three expressions the ranges considered now can be estimated from the range of the flat portions of such curves for given values of the bandwidth and the parameter r, as is shown in Fig. 4, 7 and 9. After all, it may be said that the range of the field strength levels now requested is from about 10 percent to 0.001 in terms of the probability of the envelope. Furthermore the range seems to exceed the lower limit of the probability with the increase of the value of the parameter r.

6. Conclusion

We have got the expressions for the three kinds of transformation factors respectively, and a rough estimation of the expected errors of the expressions. We think that the expressions of the transformation factors will remain to hold, even when exceeding over the range of the bandwidth considered in this paper, but we mean soon in future to define the usable range of the expressions of the transformation factors by numerical calculations of the amplitude probability distributions, the envelope crossing rate curves and the average duration curves in the questioned range of the bandwidth.

The many parts of the numerical calculations were made with the electronic computor NEAC-2203 at Nagoya university.

7. Acknowledgement

8. Reference

(1) S. O. Rice : Bell Sys. Tech. J., 27, 109 (1948).
(2) A. D. Watt and E.L. Maxwell : Proc. I. R. E., 55 (1956)