THE COMPARISON OF FULL- AND HALF-WAVE SYNCHRONOUS DETECTION IN THE DICKE RADIOMETER FROM THE VIEWPOINT OF S/N RATIO

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In Vol. 1 of these Proceedings (the correction in Vol. 2),[1] we calculated the minimum detectable temperature of the Dicke radiometer and made the comparison of S/N ratios between two cases, where a selective amplifier of the modulating frequency is used and where a broad-band l-f amplifier is used. Our expression for the minimum detectable temperature was as follows,

$$\Delta T = \frac{4}{\pi} \frac{F T_0}{\sqrt{\alpha B}} \quad \cdots\cdots (1)$$

where T_0 : the room temperature (the temperature of the reference resistive load)
F : the noise figure of the receiver
B : band-width of i-f amplifier
α : a constant determined by the transfer characteristics of l-f amplifier and the low pass filter.

If a selective amplifier and the single RC low pass filter are used and the band-width of the latter is much narrower than the former, Eq. (1) is reduced to

$$\Delta T = \frac{\pi}{2} \frac{F T_0}{\sqrt{(RC) B}} \quad \cdots\cdots (2)$$

When the band-width of l-f amplifier is wide enough to amplify the square wave of the modulating frequency, S/N ratio is slightly improved and hence we do not use the selective circuit in our radiometers. Eq. (2) agrees with the expressions given by other authors.[2]

When Eq. (1) was derived, the synchronous detector (rectifier) was assumed to be of half-wave type. Selove[3] has described that if a half-wave detector is used, S/N voltage ratio will be cut by 1.5 db, compared with that in the case of a full wave detector (the output noise voltage will be cut $\sqrt{2}$, the output signal voltage (d. c.) a factor of 2). However, our measurements showed no difference between the two cases.

In Vol. I, the power spectrum of the output noise of the synchronous detector was calculated for the case of half-wave rectification, assuming that the noise passed through l-f amplifier is Gaussian. The audio noise is not Gaussian, but as the band-width of l-f amplifier is much narrower than that of i-f amplifier, the noise at the input of the synchronous detector may be considered to be Gaussian.[4] The power spectrum G(f) is represented as follows,

$$G(f)=\frac{W}{4}S(f)+\frac{W}{\pi^2}S(f\sim f_0)+\frac{W}{\pi^2}S(f+f_0)+\frac{W}{9\pi^2}S(f\sim 3f_0)$$
$$+\frac{W}{9\pi^2}S(f+3f_0)+\frac{W}{25\pi^2}S(f\sim 5f_0)+\cdots\cdots\cdots \qquad \cdots\cdots (3)$$

where W is the total noise power at the input, f_0 is the frequency of the Dicke switch and S(f) is the normalized power spectrum of the input noise. If the power transfer characteristic of I-f amplifier $A(f)=1$ for $f_1 < f < f_2$ and $=0$ for $f < f_1$, $f_2 < f$, and $f_2-f_1 \equiv \Delta \ll B$,

$$S(f)=1/(f_2-f_1)\equiv 1/\Delta \qquad f_1 < f < f_2$$
$$=0 \qquad f < f_1,\ f_2 < f$$

As $f_1 < f_0 < f_2$,

$$S(f\sim f_0)=1/\Delta \qquad 0 < f < f_0-f_1$$
$$=1/\Delta \qquad f_1+f_0 < f < f_0+f_2$$
$$=0 \qquad f_0-f_1 < f < f_1+f_0,\ f_0+f_2 < f$$
$$S(f+f_0)=1/\Delta \qquad 0 < f < f_2-f_0$$
$$=0 \qquad f_2-f_0 < f$$

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The power spectrum of the output noise for a full-wave detector can be calculated in the same way as the case of a half-wave detector. In this case, the output noise voltage y(t) can be represented as $y(t)=f(t)N(t)$,
where

$$f(t)=1 \qquad 0 < t < T/2$$
$$=-1 \qquad T/2 < t < T$$

$T=1/f_0$: the period of f(t).
N(t) : the input noise voltage.

The auto-correlation function of y(t)

$$R(\tau)=\frac{8}{\pi^2}\cos 2\pi f_0\tau+\frac{8}{3^2\pi^2}\cos 2\pi(3f_0)\tau+\frac{8}{5^2\pi^2}\cos 2\pi(5f_0)\tau+\cdots\cdots\cdots$$

Accordingly, the power spectrum of y(t)

$$G(f)=\frac{4W}{\pi^2}S(f\sim f_0)+\frac{4W}{\pi^2}S(f+f_0)+\frac{4W}{9\pi^2}S(f\sim 3f_0)+\frac{4W}{9\pi^2}S(f+3f_0)+\cdots\cdots \quad \cdots\cdot (4)$$

As the output noise of the synchronous detector goes through a low pass filter of very narrow band, only the components near d. c. appear at the output of radiometer and hence the first term of Eq. (3) does not generally contribute to the output noise. Therefore, from Eqs. (3) and (4), we can see that if a full-wave detector is used, the output noise power of the radiometer increase by 4 times. While the signal power also increases by 4 times. Accordingly, the signal to noise ratio is equal to that of the case of a half-wave detector.

If the frequency band of I-f amplifier extends down to d.c., the first term of

Eq. (3) contributes to the output noise of the radiometer. In this case, a full-wave detector is preferable, as described by Selove. But such I-f amplifier is not used in practice.

Colvin[2] has calculated the minimum detectable temperature of the Dicke radiometer and showed that S/N ratio depends on f(t) for the case of no video filter, but not for the case of a narrow-band video filter that passes only the fundamental component of the modulation waveform. According to his formula, when a half-wave detector is used, S/N ratio is cut by 1.5 db for the case of no video filter. But "no video filter" means that the frequency band of I-f amplifier extends down to d. c.

In practice, S/N ratio does not depend on the type of the synchronous detector (half-wave or full-wave). In order to obtain the maximum sensitivity, the band-width of I-f amplifier should be wide enough to amplify the square wave of the modulating frequency and, even in this case, we may use a half-wave detector.

References

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