THE AMPLITUDE PROBABILITY DISTRIBUTION OF THE ATMOSPHERIC NOISE

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Abstract

On the basis of the observational results with the crossing rate distributions of the atmospheric noise envelope for the LF and MF bands, an adequate representation is proposed of the cumulative distribution of probability of the peak amplitude of the atmospheric impulses at the antenna. Then, on the basis of such a representation by a single power function or in particular by a composite of two different power functions, the expressions for the cumulative distributions of probability of amplitude of the atmospheric noise envelope are derived where particular emphasis is given on the effect of the change of band width. Furthermore, it is described that there are good agreements between the theoretically derived curves and measured ones of the amplitude probability distributions of the atmospheric noise envelope.

1. Introduction

The atmospheric radio noise generally gives rise to the interference on the wireless communication systems operating at various frequencies. In particular, it is considered to be the final noise sources of interference on the wireless communication systems for medium and long wave bands. As is well known, the characters of the atmospheric radio noise are very complex. Accordingly, in relation with the interference, the question has been introduced, "Which parameters of the atmospheric radio noise are the most closely related with the interference on the wireless communication systems of various types?". And it has attracted the profound interests of many experimenters of the atmospheric radio noise and inspired them to measure the various parameters of the atmospheric radio noise in detail. (1) While each of the parameters of the atmospheric radio noise is considered to be in particular ways related with the interference on the wireless communication system of some type, it should be noted that there is a remarkable fact as follows. With a narrow-band teletype system, a simple law has been found that the percentage of binary error is equal to one-half the probability that the noise envelope exceeds the signal carrier level. And this fact may have potentially promoted the study of amplitude characteristics of the atmospheric radio noise by the statistical method.

Since the I. G. Y., the cumulative distributions of the probability of amplitude (a. p. d.) of the instantaneous voltage of the atmospheric noise envelope have been measured at various frequencies on the VLF, LF, MF and HF bands by many experimenters over the world. From these measurements, a lot of informations on the amplitude characteristics of the atmospheric radio noise have been obtained, and many proposals have been made on the mathematical expressions as well as on the graphical derivation of the a. p. d. of the atmospheric noise envelope.

Clearly, these mathematical expressions and the graphical derivations (2)-(7) have potential meanings in pracitce. For example, some mathematical expressions represent many a. p. d. data of the atmospheric noise envelope in a compressive form, some is used for calculating the average and the r. m. s. of the amplitude of the atmospheric noise envelope, and some is used for deducing the effect of the band width change on the a. p. d. curve and so on. Crichlow and his co-operators proposed a graphical derivation method where they intended to draw the a. p. d. curve over a wide range of propability on a particular graph by using the average amplitude, the noise power and the average logarithm of the envelope voltage. (8) (9)

But, for these mathematical expressions and graphical derivations, a particular emphasis seems to have been placed on good agreement between the measured a. p. d. curves and mathematically or graphically derived a. p. d. curves. However, sufficiently satisfactory explanations, based upon the physical concept of the amplitude distribution, have not yet been established.

Furutsu and Ishida (10) have derived a theoretical expression of the a. p. d. of the atmospheric noise envelope for a given distribution of sources of atmospherics as a function of the distance from the receiver in a dimension, where they assume that the atmospheric impulses arriving at the antenna obey the Poisson law with respect to the time spacings of the occurrence of the impulses (such assumption is physically comprehensible and may be an appropriate basis for deducing the a. p. d. of the atmospheric noise envelope). Then they have proved that there are good agreements between the theoretically derived a. p. d. curves and the measured a. p. d. data of the atmospheric noise envelope on the HF band.

In spite of the success in the HF band, the assumption of one-dimensional distribution of sources of the atmospherics may not be acceptable within good accuracy basis of deducing the a. p. d. of the atmospheric noise envelope for the VLF, LF and MF bands. For these frequency bands, an alternative way is to estimate the distribution of peak amplitudes of the atmospheric impulses at the antenna from the analysis of the crossing rate distributions of the atmopheric noise envelope measured at the output of the receiver. When such estimation is adequately made, and at the same time the atmospheric impulses are assumed to arrive at the antenna at time spacings determined by the Poisson law, we can expect a good agreement between the theoretically derived a. p. d. curves and measured a. p. d. curves.

In this papar, the author assumes along the line of thought as described above that the distributions of peak amplitudes of the atmospheric impulses are represented by a single power function or a composite of two different power functions from the results of the analysis of the measured crossing rate distributions of the atmospheric noise envelope. On this basis, he will derive the expressions of the a. p. d. of the atmospheric noise envelope, and will show that a good agreement exists between the theortically derived curves and the measured data of the a. p. d. of the atmospheric noise envelope. Furthermore he will derive the effect of the change of band width on the a. p. d.

2. Distribution of peak amplitude of the atmospheric impulses.

The response of the receiver for an atmospheric impulse may well be considered to be identical with the response for an impulse with an uniform amplitude spectrum. Of course it is difficult to measure the distribution of peak amplitude of such the atmospheric impulses at the antenna directly, but it is easy to measure positive (or negative) crossings of a given voltage of the atmospheric noise envelope and accordingly the crossing rate distribution as a function of amplitude at the output of the receiver.

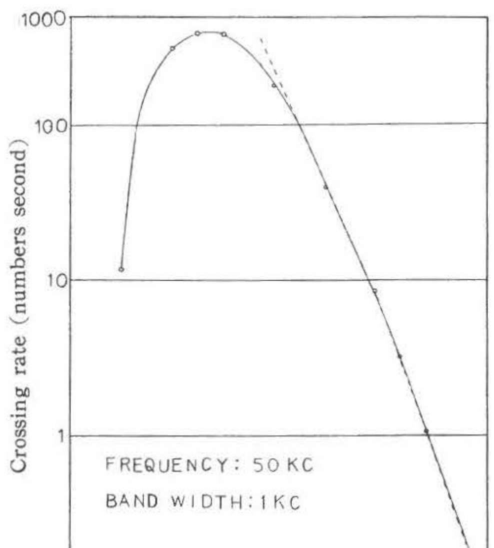

Fig. 1. Crossing rate distribution
• measured point,—measured curve,⋯⋯approximate straight line

Fig. 1 shows the typical distribution of the crossing rate distribution measured at Toyokawa with a 50 kc/s receiver of 3-dB band width 1 kc/s and a vertical antenna. As seen from Fig. 1, the crossing rate increases and gradually attains the maximum value as the voltage decreases, and thereafter, on the contrary, it decreases as the voltage decreases. And for the behaviors of the distributions not shown in Fig 1, a simple analysis shows that the crossing rate continues to decrease and finally approaches to zero as the voltage decreases. Such decrease of the crossing rate in the direction of low voltage is resulted from the effect of overlapping of successively arriving impulse responses. As the voltage increases, on the contrary, the effect of overlapping becomes smaller and at last becomes almost negligible. Accordingly, while the crossing rates correctly reflect the number of the atmospheric impulses received by the antenna at higher voltage range, such relation does not hold at lower voltage range.

Now the following observational results should be noted. (3)~(11) As is well known on the VLF, and LF bands (10kc, 23kc, 50kc, and 100kc and so on), when the measured crossing rate distributions are drawn on the graph with the same co-ordinate as shown in Fig. 1, the crossing rate distribution curves can frequently be approximated by a straight line within the range of crossing rate 1 to 80 numbers per second. And the slopes of the line are found to lie within the range 0.8 to 1.6 according to the various atmospheric conditions for all four seasons.

In addition to the well known observational results, we have got an information about the crossing rate distributions at the smaller crossing rate than about 1 number per second by the measurments on the LF band at Toyokawa. The analysis of the results of measurements shows that (1) the crossing rate distribution can be approximated by two different power functions over the voltage range after passing the peak of the crossing rate, (2) the value of the crossing rate at the connecting poing of the two power functions varies depending upon the season and time of measurement. Generally, this value is less than 20, and in most cases less than 10, (3) the slopes of the two linear parts of the distribution are different each other, one of which lies in the same range 0.8 to 1.6 as mentioned above, and the other lies in the variable range centering around 2 (the most probable value is 2).

Now we can represent the distribution of peak amplitude of the atmospheric impulses by a composite of specific two power functions on the basis of the observational results. Because the response of the receiver by the atmospheric impulses may well be considered to be discrete at higher voltage range where the crossing rate distribution can be approximated by a composite of the two power functions.

Then the average number n per second that the atmospheric impulses exceed a given voltage p at the antenna can be expressed as

$$n=\left(\frac{p}{A}\right)^{-r} \quad \cdots\cdots(1)$$

where A: the voltage level at the antenna where just one atmospheric impulse exeeds

r: the absolute value of the slope of the linear part of the crossing rate distribution

Of cource, eq. (1) holds for each of the two different linear parts of the distribution of peak amplitude of the atmospheric impulses by using pairs of (r_1, A_1) and (r_2, A_2). Here the suffixes 1 and 2 of (r, A) are for the lower linear part and the higher linear part respectively.

Now let us derive a few parameter in relation to eq. (1). Let p_0 be minimum measurable voltage at the antenna, then the average number ν of the atmospheric impulses that exceed the voltage p_0 per second is expressed by

$$\nu=\left(\frac{p_0}{A}\right)^{-r} \quad \cdots\cdots(2)$$

From eq. (1). we can derive the distribution $Q\ (p)$ of peak amplitude of the atmospheric impulses in a normalized form as

$$Q\ (p) = \left(\frac{p}{p_0}\right)^{-r} \qquad \cdots\cdots (3)$$

Also we can derive its probability density function $w\ (p)$ as

$$w\ (p) = r\frac{p_0{}^r}{p^{r+1}} \qquad \cdots\cdots (4)$$

3. Cumulative distribution of the probability of amplitude of the instantaneous voltage of the atmospheric noise envelope.

Now let us consider how the a. p. d. of the atmospheric noise envelope would be when the atmospheric impulses arrive at the antenna at time spacings determined by the Poisson law, and the amplitude variation of instantaneous envelope is statistically stationary over a very long time. Let us assume first that the distribution of peak amplitude of the atmospheric impulses is represented by a single power function i. e., eq. (1), over the entire voltage range.

Then, as is well known, [12] the probability density function $W(\vec{R})$ of the vector $\vec{R}$ of the amplitude of instantaneous envelope can be expressed as a double Fourier transformation of the characteristic function. That is we have

$$W(\vec{R}) = \frac{1}{(2\pi)^2}\int_0^\infty d\vec{K}\ \exp(i\vec{K}\vec{R})\ A(\vec{K}) \qquad \cdots\cdots (5)$$

where $\vec{R}$ represents the vector of the amplitude of envelope measured at the time t_0 in the cartesian co-ordinate, $\vec{K}$ is the vector of tranformation variable in the cartesian co-ordinate and $A\ (\vec{K})$ is the characteristic function as shown in eq. (App. 10) in the appendix. Then $A\ (\vec{K})$ can be expressed as

$$A(\vec{K}) = \exp(-\nu)\int_{t_0}^\infty dt\int_0^\infty dp w(p)\times[1-J_0\{KpF(t_0-t)\}] \cdots\cdots (6)$$

where K is the absolute value of vector $\vec{K}$, and J_0 is the Bessel function of zero order. ν is the average number per second of the atmospheric impulses given by eq. (2) and $w(p)$ is the probability density function expressed by eq. (4).

Next let the frequency characteristic of the receiver be represented by the resonance frequency characteristic curve of a single resonance circuit with the same 3-dB band width as the receiver. Then the envelope waveform $F\ (t)$ of the output response of the receiver for an atmospheric impulse can be expressed as follows:

$$F(t) = 2\pi BG_0\exp(-\pi Bt) \qquad \cdots\cdots (7)$$

where B: the 3-dB band width of the receiver

G_0: the receiver gain at the center bandwidth frequency

By substituting eq. (7) into $F\ (t-t_0)$ in eq. (6) and making the following transformations of the variables

$$\left.\begin{array}{l} p=p \\ u=2\pi BG_0Kp\ \exp(-\pi Bt) \end{array}\right\} \cdots\cdots(8)$$

eq. (6) becomes

$$A(\vec{K})=\exp\frac{-\nu}{\pi B}\int_0^\infty \frac{du}{u}[\{1-J_0(u)\}\times\int_{u/2\pi BG_0K}^\infty w(p)dp] \cdots(9)$$

From eq. (3) the integral with the integrand $w\ (p)$ on the right side of eq. (9) can be simplified as

$$\int_{u/2\pi BG_0K}^\infty w(p)dp=\left(\frac{p_0 2\pi BG_0K}{u}\right)^r \cdots\cdots(10)$$

By substituting eq. (10) into eq. (9), we obtain

$$A(\vec{K})=\exp\frac{-\nu}{\pi B}\int_0^\infty\frac{du}{u}\{1-J_0(u)\}\times\left(\frac{p_0 2\pi BG_0K}{u}\right)^r \cdots\cdots(11)$$

Eq. (11) holds for both $r\fallingdotseq 1$ and $r=1$, and furthermore in a particular case $r=1$, the integral has the simplest form

$$\int_0^\infty\frac{du}{u}\{1-J_0(u)\}\frac{1}{u}=1 \cdots\cdots(12)$$

so that from eq. (11), the characteristic function $A(\vec{K})$ reduces to

$$A(\vec{K})=\exp(-2\nu p_0G_0K) \cdots\cdots(13)$$

And in the case $r\fallingdotseq 1$, the integral has the form

$$\int_0^\infty\frac{du}{u}\ \{1-J_0(u)\}\frac{1}{u^r}=\frac{1}{r}\frac{1}{2^r}\frac{\Gamma\{(2-r)/2\}}{\Gamma\{(2+r)/2\}} \cdots\cdots(14)$$

so that from eq. (11) we get the following characteristic function as

$$A(\vec{K})=\exp\{-\frac{\nu}{\pi B}(2\pi BG_0p_0K)^r\xi\} \cdots\cdots(15)$$

where

$$\xi=\frac{1}{r}\frac{1}{2^r}\frac{\Gamma\{(2-r)/2\}}{\Gamma\{(2+r)/2\}} \cdots\cdots(16)$$

Then let us substitute the so far derived expressions of the characteristic function for that in eq. (5) which gives the propability density function of the vector $\vec{R}$. Firstly, in case $r=1$, substituting eq. (13) for $A(\vec{K})$ in eq. (5), we get

$$W(\vec{R})=\frac{1}{(2\pi)^2}\int_0^\infty d\vec{K}\exp\{(i\vec{K}\vec{R}-2\nu G_0p_0K)\} \cdots\cdots(17)$$

Here we replace each components of vectors $\vec{R}$ and $\vec{K}$ in the cartesian co-ordinate by the components in the polar co-ordinate as

$$R_1 = R\cos\phi, \quad R_2 = R\sin\phi \quad \cdots\cdots(18)$$

$$K_1 = K\cos w, \quad K_2 = K\sin w \quad \cdots\cdots(19)$$

Then we get

$$\begin{aligned} W(R, \phi) &= \frac{R}{4\pi^2}\int_0^\infty dKK\int_0^{2\pi} dw e^{iKR\cos(w-\phi) - 2\nu G_0 p_0 K} \\ &= \frac{R}{2\pi}\int_0^\infty dKKJ_0(KR)e^{-2\nu G_0 p_0 K} \\ &= \frac{R}{2\pi}\frac{2\nu G_0 p_0}{\{R^2 + (2\nu G_0 p_0)^2\}^{3/2}} \quad \cdots\cdots(20) \end{aligned}$$

By integrating eq. (20) from o up to 2π with respect to ϕ we get

$$W(R)\,dR = \frac{2\nu G_0 p_0 R dR}{\{R^2 + (2^\nu G_0 p_0)^2\}^{3/2}} \quad \cdots\cdots(21)$$

By integrating eq. (21) from p to infinity with respect to R, we can obtain probability P by which the instantaneous amplitude of the atmospheric noise envelope exceeds the given voltage ρ as

$$P(R > \rho) = \int_\rho^\infty W(R)dR = 1\Big/\left\{1 + \left(\frac{\rho}{2\nu G_0 p_0}\right)^2\right\}^{1/2} \cdots\cdots(22)$$

Next in the case $r \neq 1$, substituting eq. (15) for $A(\vec{K})$ in eq. (5), we get

$$W(\vec{R}) = \frac{1}{(2\pi)^2}\int_0^\infty d\vec{K}\exp\{i\vec{K}\vec{R} - \frac{\nu}{\pi B}(2\pi BG_0 p_0 K)^r \xi\} \quad \cdots\cdots(23)$$

By integrating eq. (23) with respect to w after making transformation of the variables by eqs. (18) and (19), we get

$$W(R, \phi) = \frac{R}{2\pi}\int_0^\infty dKKJ_0(KR)\exp\left\{-\frac{\nu}{\pi B}(2\pi BG_0 p_0 K)^r \xi\right\} \cdots\cdots(24)$$

By integrating eq. (24) from o up to 2π with respect to ϕ, we get

$$W(R)dR = RdR\int_0^\infty dKKJ_0(KR)\exp\left\{-\frac{\nu}{\pi B}(2\pi BG_0 p_0 K)^r \xi\right\} \cdots(25)$$

Then by denoting

$$\left.\begin{aligned} \gamma &= \frac{\nu}{\pi B}(2\pi BG_0 p_0)^r \xi \\ Z &= K(\gamma)^{1/r} \\ \rho &= \frac{R}{(\gamma)^{1/r}} \end{aligned}\right\} \quad \cdots\cdots(26)$$

we rewrite eq. (25) as follows:

$$W(R)dR = d\rho\,\{\rho\int_0^\infty dZZJ_0(Z\rho)e^{-Z^r}\} \quad \cdots\cdots(27)$$

By integrating the right side of eq. (27) from ρ to infinity, we obtain the probability by which the instantaneous amplitude R of the atmospheric noise envelope exceeds $(\gamma)^{1/r}\,\rho$ as

$$P\,(R>(\gamma)^{1/r}\rho) = \int_{(\gamma)^{1/r}\rho}^\infty W(R)\,dR = 1-\int_0^\infty dZ\rho J_1(Z\rho)\exp(-Z^r) \cdots\cdots(28)$$

4. Approximation of a composite of two power functions

In this section, we consider the case that the distribution of the probability of peak amplitude of the atmospheric impulses can be approximated by a composite of two different power functions connected at high voltage level. We obtain the expression of percentage of time by which the instantaneous amplitude of the atmospheric noise envelope exceeds a given voltage level for high voltage including the intersecting point of two power functions.

Now in measuring the percentage of time, by amplifying the part of the atmospheric noise envelope that exceeds the given voltage level, and limiting its amplitude, we can get a time series of pulses of rectangular form with definite amplitude. Here the probability at which the pulses exist per second is of course identical to the amplitude probability by which atmospheric noise envelope exceeds the given voltage level.

Here, as a model for the time series of such sliced pulses as described above, we take a mathematical time series of pulses resulted from the overlapping of the original pulses, where the original pulses are of definite amplitude and definite duration τ. When they occur at the rate of average number ν per second and at the time spacings determined by the Poisson law, as is already known, [13] the total sum P of pulse widths (measured with respect to the zero-level voltage) per second can be expressed as follows:

$$P = 1-\exp(-\nu\tau) \cdots\cdots(29)$$

Here it is assumed that the distribution of peak amplitude of the atmospheric impulses is represented by a single power function, and the characteristic of the receiver is represented by eq. (7). We can obtain the percentage of time by which the atmospheric noise envelope exceeds a given voltage level for high voltage. The procedure is as follows.

We assume that, due to the single impulse with amplitude p, the envelope waveform of response of the receiver is sliced at a given height v_0 in voltage. Then the duration τ of the part of the waveform just on the voltage level v_0, from eq. (7), can be expressed as follows:

$$\tau=-\frac{1}{\pi B}\log_e\left(\frac{v_0}{2\pi BG_0 p}\right) \cdots\cdots(30)$$

And we assume that the response of the receiver for each impulse is independent each other, i. e., there are no effects of overlapping among successively appearing responses. Then we can calculate the average duration $\bar{\tau}$ at the voltage level v_0 for the envelope waveform of the individual response of the receiver. Thus from eqs. (4) and (30), we get $\bar{\tau}$ as

$$\bar{\tau}(v_0, B, r)=\int_{v_0/(2\pi BG_0)}^{\infty}\frac{-1}{\pi B}\times\log_e\left(\frac{v_0}{2\pi BG_0 p}\right)w(p)dp \cdots\cdots(31)$$

where $w(p)$ represents the probability density function of amplitude of the atmospheric impulses as shown in eq. (4), and the integrand represents the contribution per unit amplitude interval for impulses in the amplitude interval p up to $p+dp$. Also $\bar{\tau}$ is a function of the envelope voltage v_0, the band width B and the parameter r.

Next, as seen from eq. (7), the lower limit $v_0/(2\pi BG_0)$ of p for the integral on the right side of eq. (31), represents the voltage level at the antenna and corresponds to the envelope voltage v_0. Accordingly, when $v_0/(2\pi BG_0)$ is substituted into in eq. (1), and N denotes the average number of the atmospheric impulses per second as shown by eq. (1), N should also represent the average number of response of the receiver per second exceeding the envelope voltage v_0 and contributing to $\bar{\tau}$ in eq. (31). Thus we can express

$$N=\left\{\frac{v_0/(2\pi BG_0)}{A}\right\}^{-r} \cdots\cdots(32)$$

Substituting N and $\bar{\tau}$ for ν and τ in eq. (29), we can express the percentage of time P as

$$P(v_0, B, r)=1-\exp\left\{\frac{v_0/(2\pi BG_0)}{A}\right\}^{-r}\times\int_{v_0/(2\pi BG_0)}^{\infty}\frac{1}{\pi B}\log_e\left(\frac{v_0}{2\pi BG_0}\right)w(p)dp\cdots(33)$$

where the left-hand side of the above equation is a function of the envelope voltage v_0, band width B and parameter r.

Now let us compare eq. (33) with eq. (22). Fig. 2 shows the comparison between the two curves of the percentage of time, which are calculated from eqs. (33) and (22) for the parameter $r=1$, the band width=1 kc/s. As seen from Fig. 2, some difference is observed between two curves at low voltage level, but they agree completely in the ranges of medium and high voltages. However, it has been found that the range of the agreement beetween eqs. (33) and (22) is signifcantly dependent on the magnitude of the parameter r. Summarizing the effect of r by comparing the case $r=1$ as shown in Fig. 2, the range of agreement for the case $r<1$ has a trend to extend toward the lower voltage level, and on the contrary, the range of agreement in the case $r>1$ has a trend to shrink toward the higher voltage level. Accordingly, it is concluded that two a. p. d. curves calculated from eq. (33) and eq.

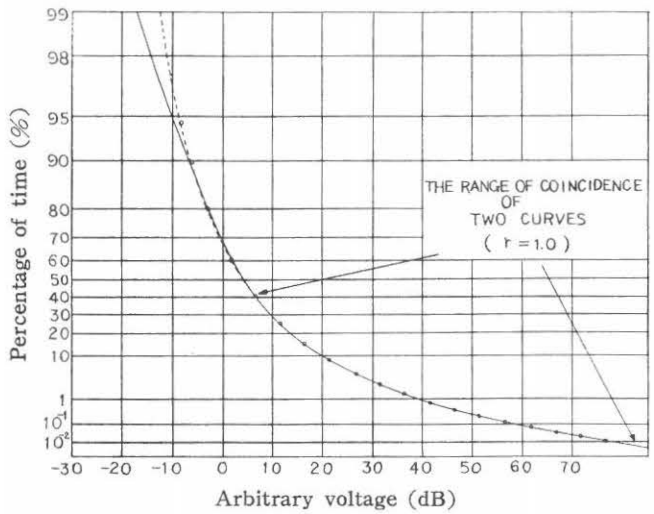

Fig. 2. Comparison between two calculated distribution curves (band width : 1kc)—
—Rayleigh graph.—: curve calculated from eq(22),······: curve calculated from eq.(33)

(22) or eq. (28) agree regardless of the value of the parameter r in a higher voltage range. In other words, it is sure that eq. (33) expresses the percentage of time with the same accuracy as eqs. (22) and (28) in the higher voltage range.

Now expanding the above discussions, we can say that it is possible to calculate the a. p. d. curve in higher voltage range also for the other function than a single power function. In this case, if the function is specified, we can calculate N and $\bar{\tau}$ for the given function following the same procedure as described above for a single power function. Then substituting N and $\bar{\tau}$ for r and τ in eq. (29), we can calculate the percentage of time. Of course, this statement holds when the function is approximated by a composite of two different power functions. Therefore the percentage of time can accurately be calculated for the higher voltage range wherein the intersecting point of the two different power functions is included.

5. Effect of band width change on the distribution

As seen from eqs. (22) and (28), the parameter of the band width B is included in eq. (28) for the case $r \neq 1$. However it is not included in the eq. (22) for the case $r=1$. This means that the change of the bandwidth generally effects the characteristic of the a. p. d., but it has no effect in the case $r=1$. Now we investigate in detail the effect of the change of band width on the basis of eq. (28).

As seen from the second right side of eq. (28), the a. p. d. is a function of the parameter ρ only when the parameter r is given. Here we call the parameter ρ the general envelope voltage. Then we get the relation between the general envelope voltage ρ and the actual envelope voltage R as

$$R=2G_0p_0(\pi B)^{(r-1)/r}(\nu\xi)^{1/r}\rho \cdots\cdots (34)$$

Accordingly, using eq. (34), we can get the a. p. d. curve as a function of the actual envelope voltage from the a. p. d. curve calculated as the function of the general envelope voltage ρ from eq. (28)

Next let us assume that both parameters r and ν are given, and compare the two a. p. d. curves measured with two different receivers which are different for the band widths, but are the same in respect to the other characteristics. Designating as (ρ_1, R_1) and (ρ_2, R_2) pairs of the general and actual envelope voltages for two different band widths B_1 and B_2 respectively, we get the following relation for each pair of (ρ_1, R_1) and (ρ_2, R_2). That is from eq. (34), for the band width B_1

$$R_1=2G_0p_0(\pi B_1)^{(r-1)/r}(\nu\xi)^{1/r}\rho_1 \cdots\cdots (35)$$

and for the band width B_2

$$R_2=2G_0p_0(\pi B_2)^{(r-1)/r}(\nu\xi)^{1/r}\rho_2 \cdots\cdots (36)$$

As described at the beginning in this section, the a. p. d. curve can only be expressed by the general envelope voltage ρ, with a given parameter r. Accordingly, if the two general envelope voltages ρ_1 and ρ_2 are identical, the two percentages of time are also identical by which the atmospheric noise envelope exceeds the voltage ρ_1 (for the band width B_1) and ρ_2 (for the band width B_2). Then putting $\rho_1=\rho_2$ in a pair of eqs. (35) and (36), we have the effect of the change of band width for the same percentage of time on the a. p. d. curve as the difference between the two actual envelope voltage R_1 and R_2. From eqs. (35) and (36) we have

$$\log_{10}R_2=\log_{10}R_1+\frac{r-1}{r}\log_{10}\left(\frac{B_2}{B_1}\right) \cdots\cdots (37)$$

Eq. (37) holds for arbitrary value of the general envelope voltage, i. e., for the whole range of percentage of time. If the a. p. d. curves under consideration are drawn on the graph where the ordinate is the percentage of time and the abscissa is $\log_{10}$ of the actual envelope voltage, the effect of the change of the band width from B_1 to B_2 is represented by the parallel shift of a. p. d. curve, along the abscissa and the length of shift is expressed by the second term in eq. (37). But eq. (37) does not hold for the case of a composite of two power functions, while it accurately represents the effect of the change of band width on the distribution for the case of a single power function. For the former case, it is difficult to obtain exact analysis of the effect of change of band width.

In this case, however, we can analyze the effect of the change of band width on the a. p. d. curve by the following method. In this case, the effect of the change of band width can be divided into three partial effects over the entire voltage range, that is, (1) the effect in the lower voltage range calculated with the value of parameter r_1 in eq. (37)(r_1 determines the amplitude characteristics of the atmospheric impulses in the lower voltage range), (2) the effect in higher voltage range calculated with the value of parameter r_2 in eq. (37) (r_2 determines the amplitude characteristics of the atmospheric impulses in the higher voltage range), (3) the effect which is transient in the intermediate voltage range between the two voltage ranges mentioned in (1) and (2). Accordingly, if the parts of the a. p. d. curves for the two band widths are accurately calculated by the procedure as described in section 4 for the higher and intermediate voltage range, we can calculate the effect of the change of band width over the entire voltage range by combining the above results and the effect (1) which can be obtained from eq. (22) or (28).

6. Comparison of calculated and measured distribution curves

6.1. Results of numerical calculation

On the approximation of a single power function, Fig. 3 shows the a. p. d. curves calculated from eqs. (22), (28) and (33) for various values of the parameter r with 3-dB band width 1 kc/s. The curves in Fig. 3 are drawn on the graph, where the ordinate represents $\log_{10}$ -$\log_e$ of the percentage of time and the abscissa, 20 $\log_{10}$ of

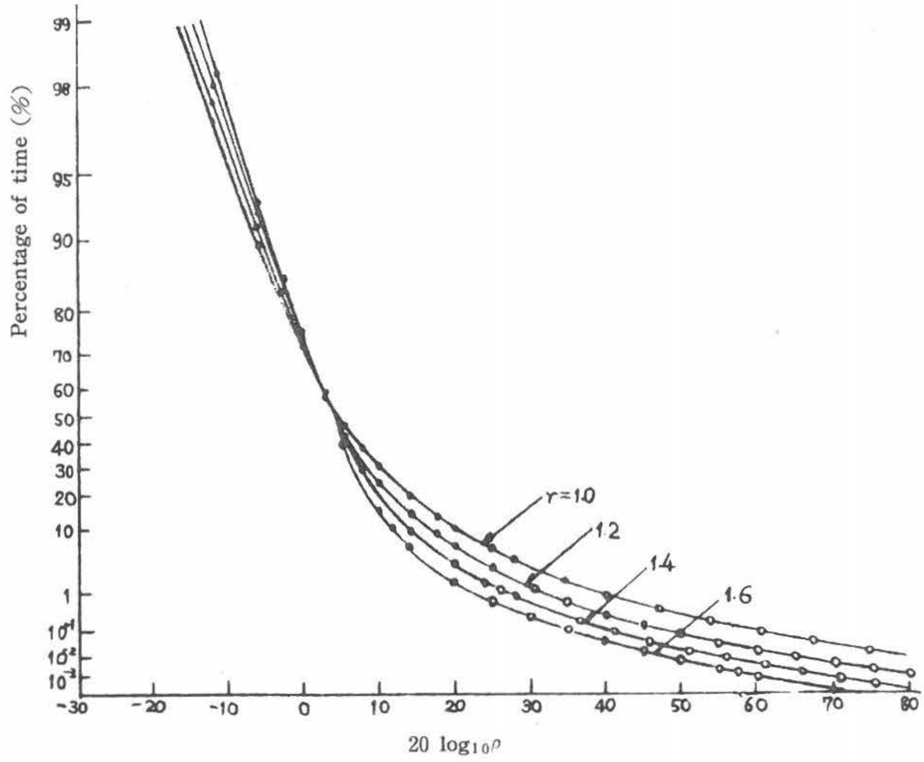

Fig. 3. Calculated distribution curves (band width 1kc)—Rayleigh graph

the general envelope voltage. The points • represent the values calculated from eqs. (22) and (28) and the points ∘, those from eq. (33).

For the approximation of two different power functions, it would be reasonable to consider the possible combinations for the parameters r_1, r_2 and the possible positions connecting the two power functions on the basis of the observational results. Therefore, let us suppose the following model for the distribution of peak amplitudes of the atmospheric impulses. That is, the parameter r_1 takes a value in the range of 0.8 to 1.6 and the parameter r_2 takes the probable value 2 and the two power functions are connencted at intervals of 12 dB in the unit of 20 $\log_{10}$ of the voltage at the antenna. Fig. 4 shows an example of the distribution of peak amplitude of the atmospheric impulses where $r_1=1$, $r_2=2$. And Fig. 5 shows the a. p. d. curves of the atmospheric noise envelope calculated for various peak amplitude distributions in Fig. 4.

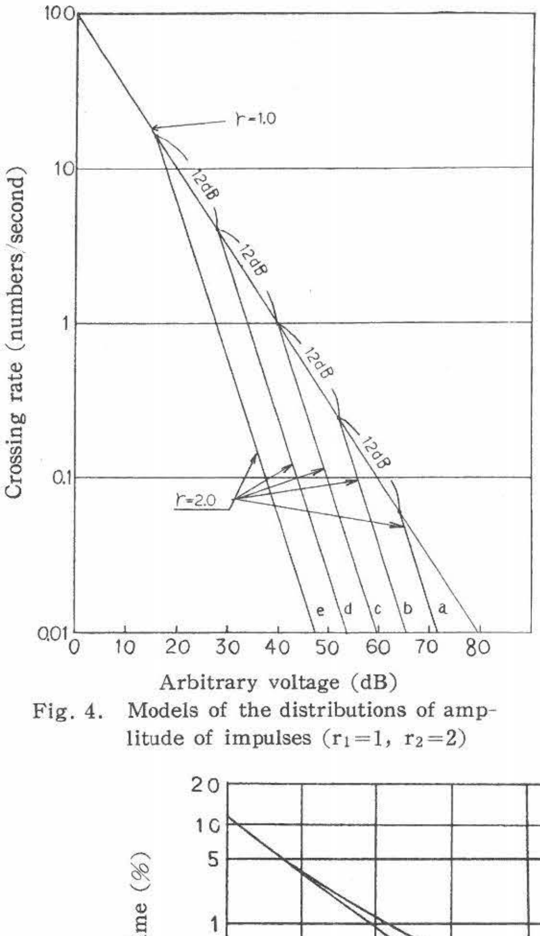

Fig. 4. Models of the distributions of amplitude of impulses ($r_1=1$, $r_2=2$)

Here, as seen from Figs. 4 and 5, it is worth while to note that the variation of the amplitude characteristics of the atmospheric impulses does not affect the a. p. d. in the range of the percentage of time higher than several percent. On the other hand, this variation does affect in the range of the percentage of time less than

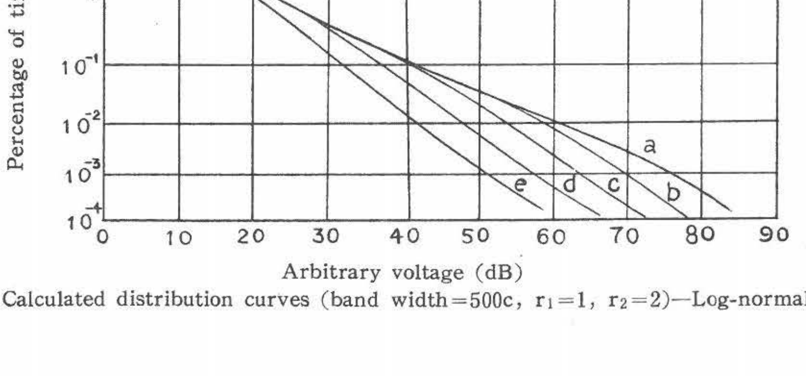

Fig. 5. Calculated distribution curves (band width=500c, $r_1=1$, $r_2=2$)—Log-normal graph.

several percent. The above relation holds similarily even if parameter r takes a value other than 1.

6.2. Results of comparison between calculated and measured distributions

In this section, let us examine the agreement between the calculated and measured a. p. d. curves. The a. p. d. of the atmospheric noise envelope were measured at Toyokawa in the day time conditions for almost ten days in July 1960.
The atmospheric noise arriving at the vertical antenna was received by the receiver of center frequency 50 kc and 3-dB band width 1 kc/s, and the percent of time was measured over the range of 90 % to 0.1 % by so-called Sullivan's type distribution meter. We measured as many as 35 distributions in the day time 08h to 17h. On the other hand, we prepared the calculated a. p. d. curves for the parameter $r_1=1$, 1.2, 1.4, $r_2=2$ and 3-dB band width 1 kc/s.

Fig. 6 shows the example of good agreement between the calculated ($r=1.0$) and measured a. p. d. curve, where the solid line represents the calculated curve and the points o represent the measured values. In addition to the above example, there were another four where $r_1=1$ held well. Fig. 7 shows another example of good agreement

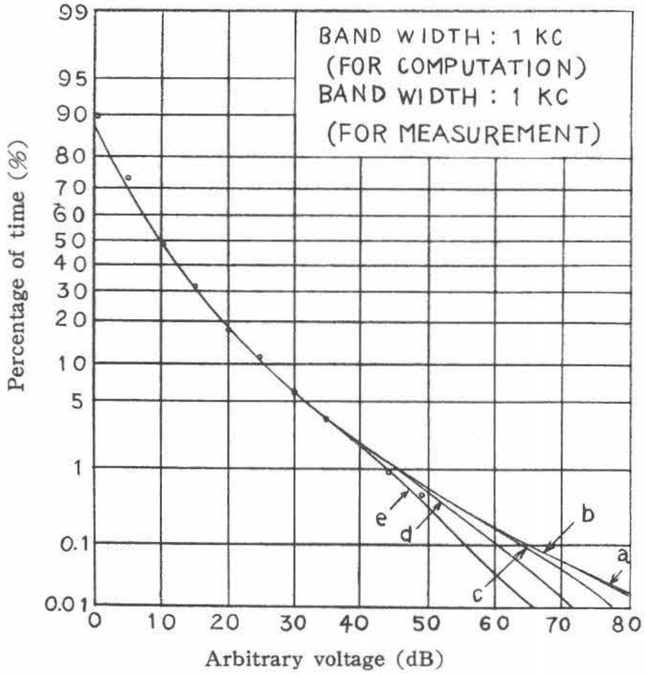

Fig. 6. Comparison between calculated ($r_1=1$, $r_2=2$) and measured time percent —Log-normal graph.
○ : measured value

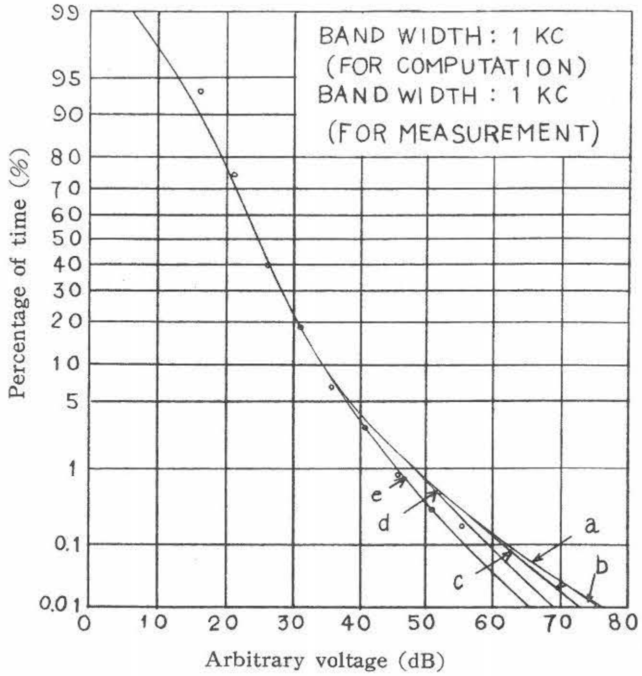

Fig. 7. Comparison between calculated ($r_1=1.4$, $r_2=2$) and measured time percent—Log-normal graph.

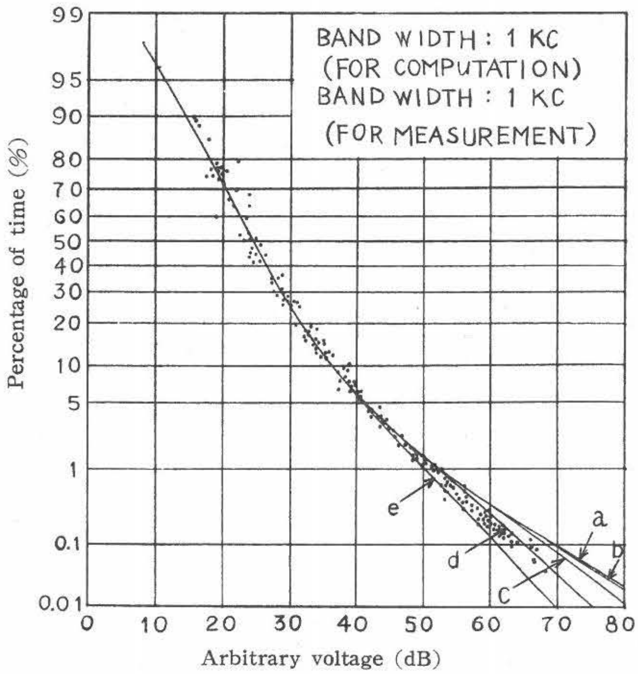

Fig. 8. Scatter plots showing comparison between calculated ($r_1=1.2$, $r_2=2$) and measured time percent—Log-normal graph.

between the calculated ($r=1.4$) and measured a. p. d. curve. We found two such cases where $r_1=1.4$ holds well. The rest of the measured distributions are in good agreement with the calculated curve for $r_1=1.2$, and shown in Fig. 8. This figure was obtained so that the transparent paper where the calculated a. p. d. curves for $r=1.2$ were drawn, was overlapped on the graph. Here the measured distribution curves were drawn on the graph and the measured points on the graph, plotted on the transparent paper.

As seen from the above comparisons, the agreement is good between the calculated and measured a. p. d. curves, and the author believes that better agreement will be obtained if the a. p. d. curves are prepared with smaller intervals of r.

7. Conclusion

It has been proved in this paper that there are good agreements between the calculated and measured a. p. d. curves of the atmospheric noise envelope at 50 kc/s. The author believes that similar good agreements can be obtained for the LF and MF bands. Because the characteristics of the crossing rate distributions which are the basis of the analysis in this paper are commonly found in the LF and MF bands. Furthermore, the author has profound interests in the comparison between the calculated and measured a. p. d. curves in the range of lower probability than that discussed in this paper. Finally the author believes that the discussions on the effect of the change of band width will give a powerful means for further complete investigation along this line.

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Appendix

The atmospheric impulses arriving at the antenna produce the responses determined by the frequency characteristic of the receiver at the output of the receiver. In this case it should be supposed that the atmospheric impulses arrive randomly at the antenna and the time spacings of the arrived impulses obey the Poisson law. Now designating the expected numbers of the atmospheric impulses as ν per second and the numbers of impulses which occur in the time interval T (duration T) as N, N is the random variable obeying the Poisson law of small number. In other words, we cen express the probability of N as

$$p(N)_T=\frac{(\nu T)^N}{N!}\exp(-\nu T) \quad \cdots\cdots\cdots\cdots (A1)$$

Now consider the noise processes in which exactly N impulses occur in the time interval T in the ensemble of the noise processes of the same characters as that under consideration. And we designate the end point of the time interval T as the instant t_0 of the measurement and the peak amplitude of the impulse occurring at the instant t_i as p_i, when the time of occurrence is measured in the reverse direction from the instant t_0. Then the voltage contribution $s_i\,(t_i)$ at the instant t_0 due to the response when the impulse p_i has passed through the receiver, can be expressed by

$$s_i(t)=p_iF(t_0-t_i)\sin\{\omega(t_0-t_i)+\varphi_i\} \quad \cdots\cdots\cdots\cdots (A2)$$

The voltage contribution is rewritten in a vector from as

$$\vec{s}=\{p_iF(t_0-t_i)\cos\varphi_i, p_iF(t_0-t_i)\sin\varphi_i\} \cdots\cdots\cdots\cdots (A3)$$

where $F\,(t)$ is the envelope waveform of the response of the receiver for the impulse with unit amplitude, and accordingly, $F\,(t_0-t_i)$ is the envelope voltage at the instant t_0 when (t_0-t_i) elapses after the occurrence time of the impulse with unit amplitude. ω is the angular frequency, given by 2π times the intermediate frequency of the receiver. φ_i is the phase at the intermediate frequency.

Then, because the vector $\vec{R}$ of the envelope voltage at the instant t_o can be obtained by the sum of the vectors of the voltage contributions $\vec{s_i}$'s, we have

$$\vec{R}=\sum_{i=1}^{N}\vec{s_i} \quad \cdots\cdots(\mathrm{A4})$$

when the voltage contributions are produced by each of N impulses occurring in the time interval T. Of course, $\vec{R}$ and $\vec{s_i}$ are the vector random variables as functions of random variables t_i, p_i and φ_i. By using the characteristic function method, we get the characteristic function of the vector $\vec{R}$ as

$$A_T'(\vec{K})=\prod_{N=1}^{N}\int_{t_0}^{T}dt_i\int_0^{\infty}dp_i\int_0^{2\pi}d\varphi_i\omega(p_i,t_i,\varphi_i)\times\exp i\vec{K}\vec{p_i}F(t_0-t_i) \quad \cdots\cdots(\mathrm{A5})$$

where $\vec{K}$ is the vector of the transformation variable. Also we have

$$\vec{p_i}=(p_i\cos\varphi_i, p_i\sin\varphi_i) \quad \cdots\cdots(\mathrm{A6})$$

Next we can obtain the ensemble average of the characteristic function $A_T(\vec{K})$ for the noise processes of duration T as

$$A_T(\vec{K})=\sum_{N=0}^{\infty}\frac{(\nu T)^N}{N!}\exp(-\nu T)\times\Big[\int_{t_0}^{T}dt\int_0^{\infty}dp\int_0^{2\pi}\omega(p,t,\varphi)$$
$$\times\exp i\vec{K}\vec{P}F(t_0-t)\Big]^N \quad \cdots\cdots(\mathrm{A7})$$

where three random variables p_i, t_i and φ_i are independent of the suffix i. Here the connected probability density function $\omega\ (p,\ t,\ \varphi)$ is expressed as

$$\omega(p,t,\varphi)=\frac{1}{T}\frac{1}{2\pi}\omega(p) \quad \cdots\cdots(\mathrm{A8})$$

where $\omega\ (p)$ is the probability density function of the peak amplitude of the impulses in a normalyzed form, and $1/T$ and $1/2\pi$ are the propability density functions of t and φ in a normalyzed form respectively.

After substituting eq. (App. 8) into eq. (App. 7), the length of the time interval T is taken to infinity, and we have

$$A(\vec{K})=\exp\Big\{-\nu\int_0^{\infty}dt\int_0^{\infty}dp\int_0^{2\pi}d\varphi\frac{1}{2\pi}\omega(p)\Big[1-\exp i\vec{K}\vec{p}F(t_0-t)\Big]\Big\} \quad \cdots\cdots(\mathrm{A9})$$

Intergrating the right side of eq. (App. 9) with respect to φ, $A\ (\vec{K})$ becomes

$$A(\vec{K})=\exp\Big[-\nu\int_0^{\infty}dt\int_0^{\infty}dp\omega(p)\times\Big[1-J_0\{KpF(t_0-t)\}\Big]\Big] \quad \cdots(\mathrm{A10})$$

where J_0 is the Bessel function of zero order, and K is the absolute value of the vector $\vec{K}$.