

## OBSERVATION OF WHISTLER-LIKE ELF RADIO WAVES BETWEEN 3 AND 60 C/S

Michiko YAMASHITA

Observations of ELF radio waves at the Research Institute of Atmospheric have been carried on since 1964. Routine observation, however, was inaugurated in 1966 in the frequency range 3 to 60 c/s at Tottori Observatory. For the first six months of this observation, a solenoid antenna had been used. In September 1966, however, it was changed to a horizontal loop of 20 turns with a length of 300 meters per one turn, and has been in use to this data.

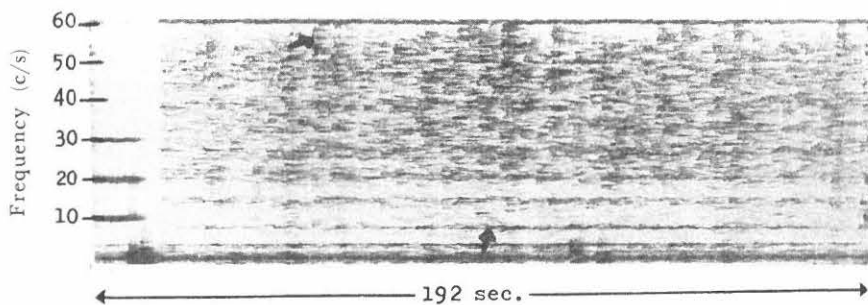


Fig. 1a. Sonagram (0<sup>h</sup> 30<sup>m</sup> J. S. T. November 8th, 1966)

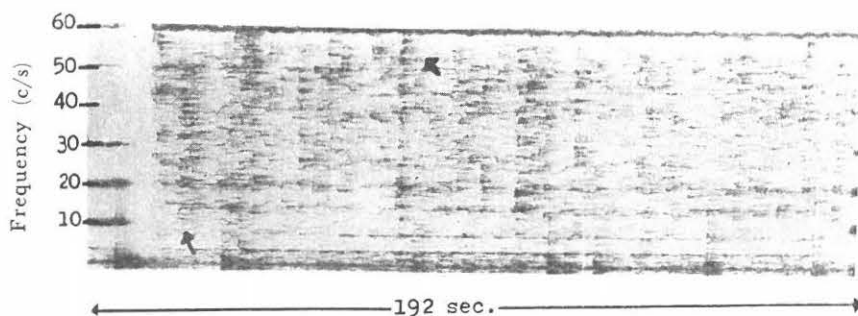


Fig. 1b. Sonagram (6<sup>h</sup> 20<sup>m</sup> J. S. T. November 8th, 1966)

In Fig. 1 are shown two samples of sonographic patterns analysed from the observation data. In these patterns, some distinct whistler-like traces could be recognized by the careful inspection between the lower and upper arrows. They are very weak in comparison with atmospherics, and they are especially faint and rare in summer.

As the first step of studying these phenomena, we made a very simple computations of group refractive index taking into consideration the O<sup>+</sup> and NO<sup>+</sup> ions that exist in the lower ionosphere. Thus, derived theoretical curves were compared with the patterns obtained by the observation.

Following Stix<sup>(1)</sup>, we assume a homogeneous and collisionless cold plasma on which an uniform and static magnetic field  $\vec{B}_0$  is superposed. Also all field quantities are assumed to vary as  $\exp j(\vec{K} \cdot \vec{r} - \omega t)$ , where  $\vec{K}$  is a propagation vector.

The condition that the following set of homogeneous field equations has a non-trivial solution yields the refractive index n.

$$\begin{pmatrix} S-n^2 \cdot \cos^2\theta & -jD & n^2 \cdot \cos\theta \cdot \sin\theta \\ jD & S-n^2 & 0 \\ n^2 \cdot \cos\theta \cdot \sin\theta & 0 & P-n^2 \cdot \sin^2\theta \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \dots\dots (1)$$

where  $\theta$  is the angle between  $\vec{B}_0 = z\vec{B}_0$  and  $\vec{n}$ , and  $\vec{n}$  is assumed to be in the x-z plane.

The condition that Eq. (1) has a nontrivial solution is that the determinant of the matrix is zero. This condition gives the refractive index n.

$$An^4 - Bn^2 + RLP = 0 \dots\dots\dots (2)$$

where

$$A = S \cdot \sin^2\theta + P \cdot \cos^2\theta, \dots\dots\dots (3)$$

$$B = RL \cdot \sin^2\theta + PS(1 + \cos^2\theta), \dots\dots\dots (4)$$

$$R = 1 - \sum_k \frac{\Pi_k^2}{\omega(\omega + \varepsilon_k \cdot \Omega_k)}, \dots\dots\dots (5)$$

$$L = 1 - \sum_k \frac{\Pi_k^2}{\omega(\omega - \varepsilon_k \cdot \Omega_k)}, \dots\dots\dots (6)$$

$$P = 1 - \sum_k \frac{\Pi_k^2}{\omega^2}, \dots\dots\dots (7)$$

$$S = \frac{1}{2}(R+L), D = \frac{1}{2}(R-L), \dots\dots\dots (8)$$

$$\Pi_k^2 = \frac{4\pi \cdot n_k \cdot q_k^2}{m_k}, \dots\dots\dots (9)$$

$$\Omega_k = \left| \frac{q_k \cdot B_0}{m_k \cdot c} \right|, \dots\dots\dots (10)$$

$$\varepsilon_k = \frac{q_k}{|q_k|}, \dots\dots\dots (11)$$

The subscript k refers to the particles of type k with mass  $m_k$ , charge  $q_k$  and number density  $n_k$ .  $\Pi_k$  and  $\Omega_k$  are the plasma- and gyro-frequency of the k-th constituent, respectively.

The solution of Eq. (2) is

$$n^2 = \frac{B \pm F}{2A} \dots\dots\dots (12)$$

where  $F^2 = (RL - PS)^2 \cdot \sin^4\theta + 4P^2D^2 \cdot \cos^2\theta \dots\dots\dots (13)$

The squared quantities of the refractive index for propagation at  $\theta = 0$  and  $\theta = \frac{\pi}{2}$  are

$$\begin{aligned} n_R^2 = R & & n_L^2 = L & & \text{for } \theta = 0, \\ n_X^2 = \frac{2RL}{R+L} & & n_O^2 = P & & \text{for } \theta = \frac{\pi}{2}. \end{aligned} \dots\dots\dots (14)$$

Here, R, L, X and O represent right-handed, left-handed, extraordinary and ordinary waves, respectively.

In order to simplify the computation of the squared quantities of the refractive index, we introduce a normalized frequency parameter  $\Lambda = \frac{\omega}{\Omega_1}$  in which  $\Omega_1$  denotes the gyro-frequency of  $O^+$  ion and concentration parameters  $\alpha = \frac{n(O^+)}{n_e}$  and  $\beta = \frac{n(NO^+)}{n_e}$ , following Gurnett et al<sup>(2)</sup>.

When  $\omega \ll \Pi_e$ , R, L and P become as follows :

$$R = 1 + \left( \frac{\Pi_e}{\Omega_e} \right)^2 + \frac{\Pi_e^2}{\Omega_1 \Omega_e} \cdot \left[ \frac{1}{\Lambda} - \frac{\alpha}{\Lambda(1+\Lambda)} - \frac{\beta}{\Lambda(1+\frac{15}{8}\Lambda)} \right], \dots\dots\dots (15)$$

$$L = 1 + \left( \frac{\Pi_e}{\Omega_e} \right)^2 - \frac{\Pi_e^2}{\Omega_1 \Omega_e} \cdot \left[ \frac{1}{\Lambda} - \frac{\alpha}{\Lambda(1-\Lambda)} - \frac{\beta}{\Lambda(1-\frac{15}{8}\Lambda)} \right], \dots\dots\dots (16)$$

$$P = 1 - \frac{1}{\Lambda^2} \left( \frac{\Pi_e}{\Omega_1} \right)^2, \dots\dots\dots (17)$$

where  $\alpha + \beta = 1$ .

As is well known, group refractive index  $n_g$  and group propagation time  $\tau_g$  are obtained from

$$n_g = n + \omega \frac{dn}{d\omega}, \dots\dots\dots (18)$$

$$\tau_g(\omega) = \frac{1}{c} \int_{\text{path}} n_g \cdot ds \dots\dots\dots (19)$$

For the case of  $\alpha=1$ , with assumption  $\frac{\Pi_e}{\Omega_e} \cong 1$ ,  $n_g$  and  $\tau_g$  for longitudinal propagation are calculated approximately,

$$(n_g)_R \cong \sqrt{\frac{\Pi_e^2}{\Omega_1 \Omega_e}} \cdot \frac{1 + \frac{1}{2} \Lambda}{(1 + \Lambda)^{3/2}},$$

$$(n_g)_L \cong \sqrt{\frac{\Pi_e^2}{\Omega_1 \Omega_e}} \cdot \frac{1 - \frac{1}{2} \Lambda}{(1 - \Lambda)^{3/2}}. \dots\dots\dots (18)^*$$

Therefore, very approximately,

$$(\tau_g)_R \cong \frac{1}{\sqrt{1 + \frac{\omega}{\Omega_1}}} \cdot \left( \frac{1}{c} \cdot \int_{\text{path}} \sqrt{\frac{\Pi_e^2}{\Omega_1 \Omega_e}} \cdot ds \right),$$

$$(\tau_g)_L \cong \frac{1}{\sqrt{1 - \frac{\omega}{\Omega_1}}} \cdot \left( \frac{1}{c} \cdot \int_{\text{path}} \sqrt{\frac{\Pi_e^2}{\Omega_1 \Omega_e}} \cdot ds \right). \dots\dots\dots (19)^*$$

Computations by the use of Eq.(18)\* give the frequency-time relation as in Fig. (2), in which the propagation time  $\tau_g$  is shown in an arbitrary unit, since the propagation distance is still unknown in the present situation.

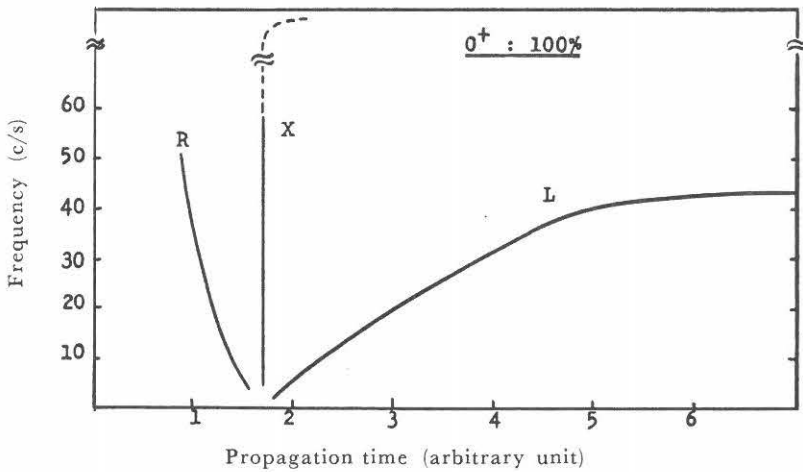


Fig. 2 Frequency-time relation.

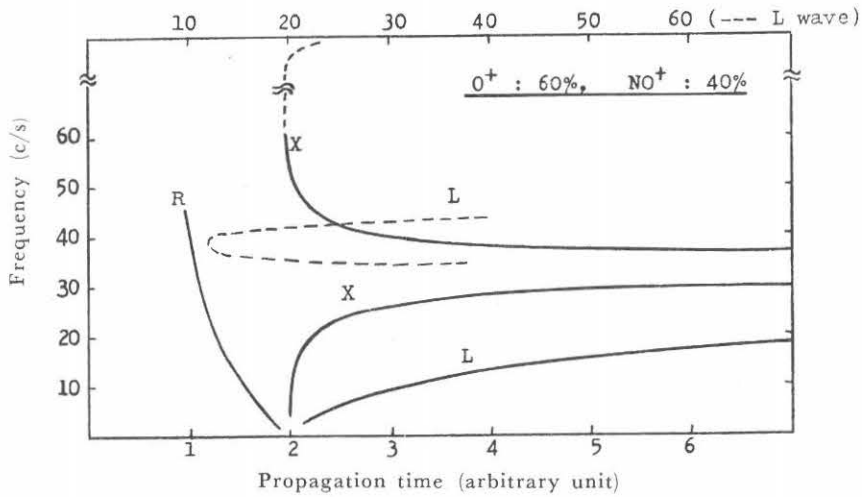


Fig. 3 Frequency-time relation.

For the case of two kinds of ions as  $\alpha=0.6$  and  $\beta=0.4$ , we can not derive the simple expression in the case of  $\alpha=1$  as in Eq.(19)\*. After quite tedious computation, only the result of the computation can be obtained as is shown in Fig.(3). The computed frequency-time patterns shown in Figs.(2) and (3) seem to be quite similar to those obtained in observations.

As for the future works, we expect to continue those observations to acquire observational data with which we may find the correlations between ELF phenomena and whistlers, solar activities, geomagnetic disturbances and etc..... These may help us to study the physics of the source of those phenomena, the location of the source and the propagation.

### References

- (1) Stix, T. H. : The Theory of Plasma Waves, McGraw-Hill Book Company, New York (1962)
- (2) Gurnett, D. A., Shawhan, S. D., Brice, N. M. and Smith, R. L.: Ion Cyclotron Whistlers, J. Geophys. Res., **70**, 1665-1688 (1965)

