MEASUREMENT OF POLARIZATION, INCIDENT ANGLE AND DIRECTION OF VLF EMISSIONS - (1)

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Abstract

Since 1967, at Showa Base (geomag. lat. 69° S) in the Antarctic, the polarization, incident angle and arriving direction of the VLF emissions in 5, 12 and 25 KHz bands, have been studied by means of two crossed loops and a vertical antenna, through the analysis of the Lissajous' figure. The sense of rotation of the polarization plane in 750 Hz, 12 and 25 KHz bands, has been studied by means of two crossed loop antennas and the polarimeter. In this paper, the principle and the system of observation are discussed.

Showa Base is constructed on a rocky ground, so that the earth conductivity will be very low. The error due to very low conductivity will occur only in the case of measurement of polarization ratio and phase difference between normal and abnormal waves. But, these errors are thought to be negligible in most cases.

Since 1963, the polarization and arriving direction of VLF emissions have been studied by means of two crossed loops with their planes in N-S and E-W directions, and the goniometer arrangement at Moshiri Observatory (geomag. lat. 34°). Except for few cases of high level emission events, the desirable data could not be obtained, because in the low latitude such as Moshiri, the emission levels are generally too low and the atmospherics are too strong to measure polarization and direction of the former.

1. Introduction

In Australia, Ellis (1960) studied the direction of arrival of VLF emissions by means of the goniometer arrangements. In the middle latitude, this method is reasonable, because the emissions are nearly vertically polarized and moderately strong, furthermore the duration is long enough and the variation of the amplitude is not much irregular.

In Sweden, Harang and Hauge (1965) studied the direction by switching alternately the crossed loop antennas to the preamplifier, and investigated the sense of rotation of the polarization plane by displaying the patterns on the cathode ray tube. They found that there were few occasions where the emissions had right-handed circular polarization for the case of almost vertical incidence and that the direction finding experiments with a simple goniometer arrangement were not suitable for the source area of high latitude.

2. Observation at Showa Base in the Antarctic

2. 1. Principle of the observation

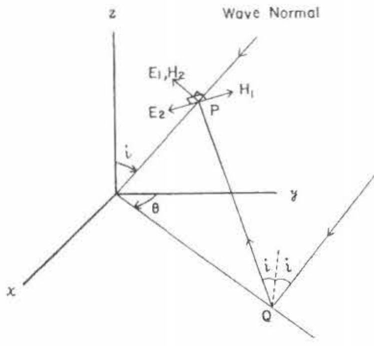

Fig. 1 Coordinate system

The principle of the observational method of atmospherics was once discussed by one of the authors (Iwai 1962) and this can be applied for Antarctic observation.

The symbols used in this paper are listed below.

x, y, z : cartesian coordinates: z is measured vertically upward and x-y plane is on the ground.

ω : angular wave frequency; a time factor $\exp(j\omega t)$ is used in this paper.

i : angle of incidence

θ : azimuthal angle

H_x, H_y, H_z, : x, y and z components of magnetic field at a point P, respectively.

E_x, E_y, E_z, : x, y and z components of electric field at a point P, respectively.

E_1, H_1, : electric and magnetic components of normal wave (E_1 exists in the plane of incidence)

E_2, H_2, : electric and magnetic components of abnormal wave (E_2 is perpendicular to the plane of incidence)

ρ_1, ρ_2, : complex reflection coefficients of normal and abnormal waves, respectively

λ : wave length

N, A, : amplitudes of magnetic field of normal and abnormal waves, respectively

ϕ : phase difference between normal and abnormal waves

σ : specific conductivity

κ : dielectric constant

h : height of a point P

Each component of magnetic and electric fields at P is

$$H_x = \left\{-1+\rho_2 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} H_2 \cos i \sin\theta - \left\{1+\rho_1 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} H_1 \cos\theta$$

$$H_y = \left\{-1+\rho_2 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} H_2 \cos i \cos\theta + \left\{1+\rho_1 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} H_1 \sin\theta$$

$$H_z = \left\{1+\rho_2 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} H_2 \sin i$$

$$E_x = \left\{-1+\rho_1 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} E_1 \cos i \sin\theta + \left\{1+\rho_2 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} E_2 \cos\theta$$

$$E_y = \left\{-1+\rho_1 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} E_1 \cos i \cos\theta - \left\{1+\rho_2 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} E_2 \sin\theta$$

$$E_z = \left\{1+\rho_1 \exp\left(-j\frac{4\pi h}{\lambda}\cos i\right)\right\} E_1 \sin i$$

Here, the authors assume as follows.

(a) The VLF emission is, in general, a downcoming plane wave and is elliptically polarized*.

(b) The ground is flat and perfectly conductive.

(c) The wave is a single frequency sine wave.

The assumption (a) is, perhaps, reasonable for the reception of the VLF emissions within source area. It is possible in most cases, to assume that the ground is perfectly conductive for VLF range. The assumption (c) will be accepted if the narrow band pass filters are used and the patterns for initial cycles are discussed.

Thus, we assume $\rho_1 = 1$, $\rho_2 = -1$ and $\lambda \gg h$

hence,

$$H_x = -2A \cos i \sin\theta \sin(\omega t+\phi) - 2N \cos\theta \sin\omega t$$
$$H_y = -2A \cos i \cos\theta \sin(\omega t+\phi) + 2N \sin\theta \sin\omega t$$
$$E_z = 2N \sin i \sin\omega t$$
$$H_z = E_x = E_y = 0$$

where,

$$H_1 = N \sin\omega t$$
$$H_2 = A \sin(\omega t+\phi)$$
$$E_1 = N \sin\omega t$$

It is, therefore, clear that only H_x, H_y and E_z have to be discussed for VLF range.

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* Unless the wave is elliptically polarized, the method that will be discussed in 2. 1. 1, 2. 1. 2 is not useful in general.

2. 1. 1 Observational method of rotating crossed loops

In practice, the goniometer is used in place of rotating crossed loops. If H_x and E_z are displayed on a rectangular coordinate, the pattern becomes an ellipse in general.* θ will have a moment where $\theta=0^\circ$ or 180° as the goniometer goes round. Then, we will get

$$H_x = \mp 2N \sin \omega t$$
$$H_y = \mp 2A \cos i \sin (\omega t+\phi)$$
$$E_z = 2N \sin i \sin \omega t$$

And $H_x/E_z = \mp (1/\sin i)$

Therefore, the pattern becomes a straight line as shown in Fig. 2.

If $\theta=0^\circ$, the pattern exists in the 2-4 quadrant.
If $\theta=180^\circ$, the pattern exists in the 1-3 quadrant.
And, $\psi=\tan^{-1}(\sin i)$

Thus, i is determined and θ is indicated by the rotation angle of the goniometer without 180° ambiguity. This method will be effectively used when the VLF emissions will come successively keeping the arrival direction constant throughout an observation period. If H_x and H_y are displayed on a rectangular coordinate, the pattern also becomes an ellipse in general.** As shown in Fig. 3, $A \cos i/N$ and $\sin \phi$ can be determined. Therefore, polarization ratio A/N is obtained. ϕ is determined by consideration of the quadrant where the long axis of the ellipse exists and of the sense of rotation of the point vector.*** Thus, all the physical properties of the emissions can be obtained.

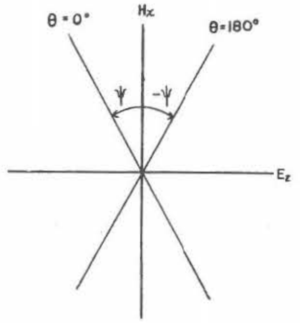

Fig. 2 H_x-E_z pattern when $\theta = 0^\circ$ or 180°

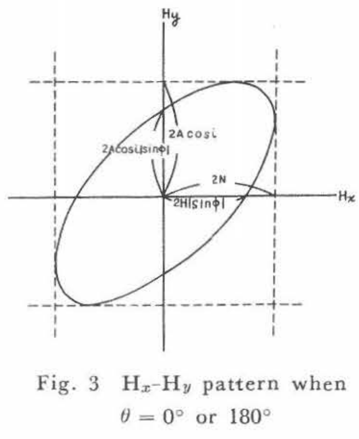

Fig. 3 H_x-H_y pattern when $\theta = 0^\circ$ or 180°

* When the wave is linearly polarized (i. e. $A=0$, $N=0$ or $\phi=0^\circ$ or 180°) H_x-H_z pattern becomes always a straight line.

** When the wave is linearly polarized, H_x-H_y pattern becomes always a straight line.

*** discussed in the appendixes

2. 1. 2 Observational method of fixed crossed loops

Within source area, the physical properties of the emission vary rapidly, so that, it is difficult, in general, to use the goniometer arrangement. Then, H_x-H_y figure whose brightness is modulated by E_z (Fig. 4) and H_x-E_z figure (Fig. 5) will be effective to find these quantities. If H_x-H_y figure is displayed only during plus half of few cycles of E_z, the symmetric line for the cutting axis of the half ellipse with respect to the straight line $H_x=H_y$ gives the direction.* Thus, the direction is briefly determined without 180° ambiguity.

If $E_z = 0$,

we have $H_x = \mp 2A \sin\phi \cos i \sin\theta$

$H_y = \mp 2A \sin\phi \cos i \cos\theta$

and $H_y/H_x = \cot\theta = \tan(\pi/2-\theta)$ for the cutting axis. If the coordinate axes (H_x, H_y) are rotated clockwise by angle θ and the new axes are written as $\bar{H}_x$, $\bar{H}_y$. $\bar{H}_x$, $\bar{H}_y$ can be given as follows:— $\bar{H}_x = -2N \sin\omega t$, $\bar{H}_y = -2A \sin(\omega t+\phi)\cos i$.

This is, therefore, the same expression as 2. 1. 1, where $\theta = 0°$. Thus, A cos i/N and ϕ are determined by the same way as 2. 1. 1, and H_x-E_z figure is an ellipse in general (Fig. 5). The symbols given in Fig. 5 have the meaning as follows:—

$$p = 2kN\,|\sin\theta \sin\phi|, \quad m = 2N \sin i$$

$$q = \frac{2kN \sin i\,|\sin\theta \sin\phi|}{(k^2 \sin^2\theta + 2k\sin\theta\cos\theta\cos\phi + \cos^2\theta)^{1/2}}$$

$$L = 2N\,(k^2\sin^2\theta + 2k\sin\theta\cos\theta\cos\phi + \cos^2\theta)^{1/2}$$

$$k = A\cos i/N$$

Therefore, we get $m/p = \sin i/k\,|\sin\theta\sin\phi|$, from which we can determine i value.

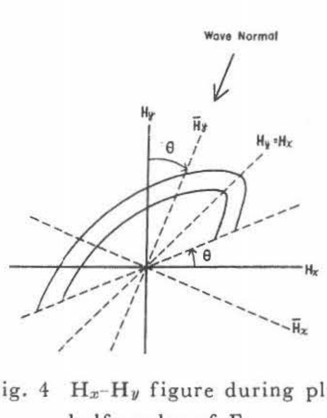

Fig. 4 H_x-H_y figure during plus half cycles of E_z

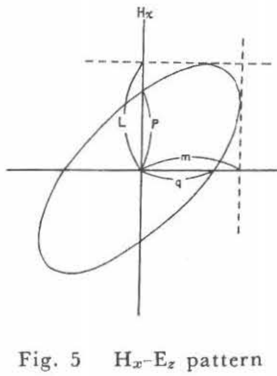

Fig. 5 H_x-E_z pattern

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* discussed in the appendixes

Thus, all the quantities can be obtained without any ambiguity. But, if the wave is linearly polarized, the quantities can not be determined in general.*

2. 1. 3 Observation of the sense of rotation of the polarization plane

Through the phase shifting circuit, two H_x's are led or lagged at right angles and added to two H_y's, respectively.

Accordingly,

$$\text{Amplitude of } (H_x^+ + H_y) = 2\{(A^2 \cos^2 i + N^2) + 2AN \cos i \sin \phi\}^{1/2}$$

$$\text{Amplitude of } (H_x^- + H_y) = 2\{(A^2 \cos^2 i + N^2) - 2AN \cos i \sin \phi\}^{1/2}$$

And,

$$\frac{\text{Amplitude of } (H_x^+ + H_y)}{\text{Amplitude of } (H_x^- + H_y)} = \left\{\frac{k^2 + 1 + 2k \sin \phi}{k^2 + 1 - 2k \sin \phi}\right\}^{1/2} = C$$

where, H_x^+ : H_x led by 90°

H_x^- : H_x lagged by 90°

therefore, if $C > 1$: right-handed elliptically polarized

if $C < 1$: left-handed elliptically polarized

if $C = 1$: linearly polarized.

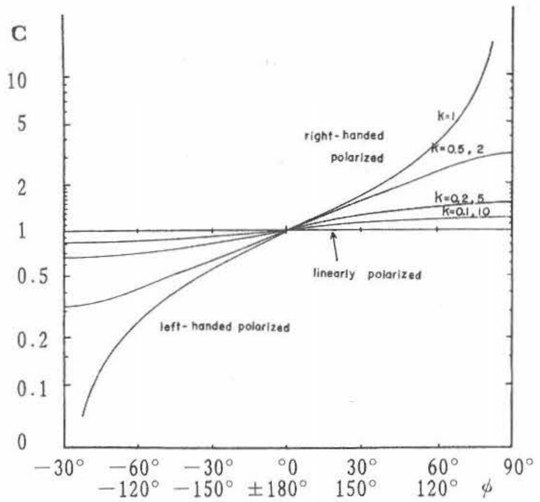

Fig. 6 C represents the ratio of right-handed polarized wave to left-handed polarized wave.

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* If the wave is vertically polarized ($A=0$) or horizontally polarized ($N=0$), the quantities can be obtained. But, if $\phi=0°$ or $180°$ ($N \neq 0$ and $A \neq 0$), the quantities can not be obtained.

Degree of the polarization can be roughly estimated from C (Fig. 6).

2. 2 Observing technique applied for Antarctic observation

The method of fixed goniometer corresponds to 2. 1. 2. In these observations, it is very important to keep fairly well the phase relation of the systems which contain antenna circuits. In the two cases (1) and (2), shown in Fig. 7, N-S and E-W systems must have strictly equal gain and phase while in the case (3), it is necessary not only to keep fairly well the phase relation but also to check the gain difference between N-S system and vertical system.

Next, the method of rotating goniometer corresponds to 2. 1. 1. For example, for the investigation of the relation between the movement of the source, (which means the transmission point through the ionosphere) and the auroral motion, the goniometer has only to be rotated for H_x-E_z pattern to assume a straight line. And then, it is able to detect the direction and incident angle successively, even if N and ϕ make a variation. On the one hand, right-handed and left-handed polarized waves are continuously recorded on the chart paper, so that it can be discussed how it polarizes, if we investigate the ratio of both levels.

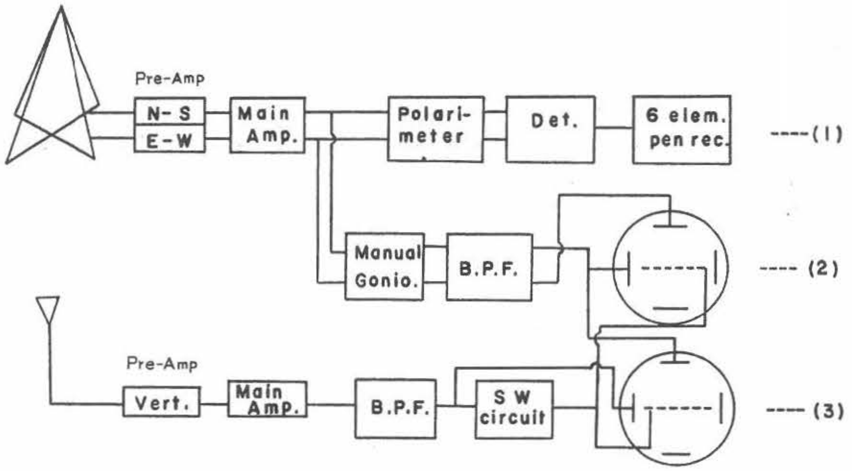

Fig. 7 Block diagram of the apparatus at Showa Base
loop antenna:
triangle type, 20m (height), 40m (base), 2 turns
vertical antenna: 10m (height)
(1): for right-handed and left-handed polarized wave at 750 Hz, 12 KHz and 25 KHz
(2): H_x-H_y figure at 5, 12 or 25 KHz
(3): H_x-E_z figure at 5, 12 or 25 KHz

2. 3 Errors due to the very low conductivity

At Showa Base, the ground consists of huge block of rocks and the conductivity is thought to be very low. Therefore, the ground can not be thought to be perfectly conductive, even in VLF range. The reflection coefficients ρ_1 and ρ_2 are complex in general and written as follows:—

$$\rho_1 = |\rho_1| \exp(-j\Theta_1)$$

$$\rho_2 = |\rho_2| \exp(-j\Theta_2)$$

For 2. 1. 1,

if $\theta=0°$ or $180°$, $H_x/E_z = \mp(1/\sin i)$

Thus, i and θ contain no error.

For 2. 1. 2,

if $E_z=0$, $H_y/H_x=\cot\theta$ for the cutting axis of half ellipse.

Thus, θ contains no error.

When H_x and H_y are rotated clockwise by angle θ, the new axes are written as $\overline{H}_x$ and $\overline{H}_y$.

$$\overline{H}_x = -(1+\rho_1)H_1$$

$$\overline{H}_y = (-1+\rho_2)H_2 \cos i$$

These are equivalent to H_x, H_y, when $\theta=0°$.

The errors of A/N and ϕ can be discussed on $\overline{H}_x$-$\overline{H}_y$ figure given by Fig. 8-1, if we investigate the K value which is given by

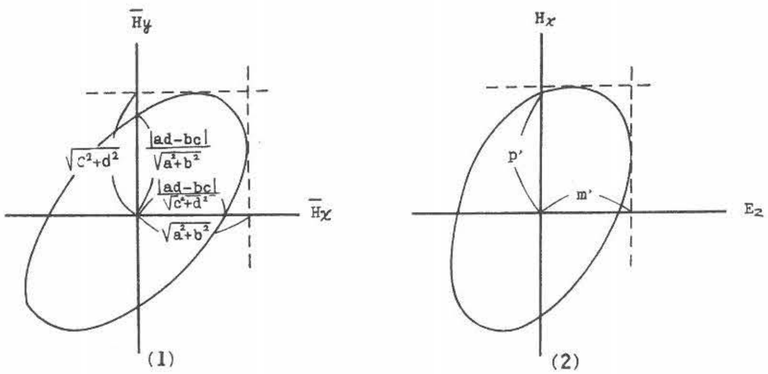

Fig. 8 (1): $\overline{H}_x$-$\overline{H}_y$ figure (i.e. H_x-H_y figure when $\theta=0°$ or $180°$) when the conductivity is low

(2): H_x-E_z figure when the conductivity is low

$$K=\frac{(c^2+d^2)^{1/2}}{(a^2+b^2)^{1/2}}=\frac{A\ \cos\ i}{N}\left\{\frac{1-2\,|\rho_2|\cos\ \Theta_2+|\rho_2|^2}{1+2\,|\rho_1|\cos\ \Theta_1+|\rho_1|^2}\right\}^{1/2}$$

If we define Δk as $\Delta k=K-k$, we get

$$\Delta\,k/K=1-\left\{\frac{1+2\,|\rho_1|\cos\ \Theta_1+|\rho_1|^2}{1-2\,|\rho_2|\cos\ \Theta_2+|\rho_2|^2}\right\}^{1/2}$$

If ω, i, σ, κ are given, ρ_1, ρ_2, Θ_1, Θ_2 are calculated and we can evaluate $\Delta\,k/K$,

where, $a=-N(1+|\rho_1|\cos\ \Theta_1)$

$b=N\,|\rho_1|\ \sin\Theta_1$

$c=A\ \cos\ i\{-\cos\ \phi+|\rho_2|\cos(\phi-\Theta_2)\}$

$d=A\ \cos\ i\{-\sin\ \phi+|\rho_2|\sin(\phi-\Theta_2)\}$

And

$$\sin(\phi+\Delta\phi)=\frac{\dfrac{|ad-bc|}{(c^2+d^2)^{1/2}}}{(a^2+b^2)^{1/2}}$$

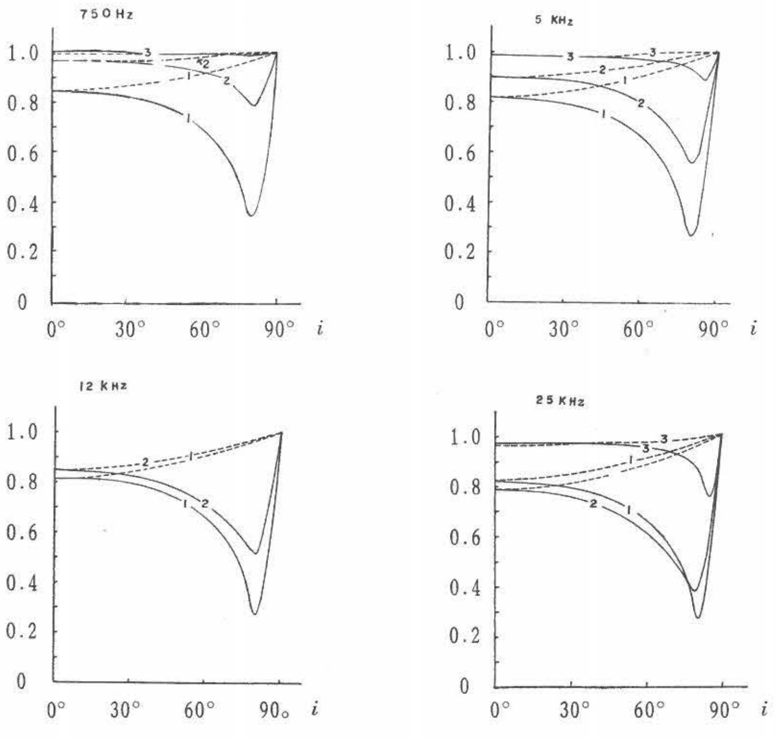

Fig. 9 Absolute value of reflection coefficient, Solid curves show the coefficients vertically polarized waves — i. e. $|\rho_1|$, Broken curves show the coefficients of horizontally polarized waves — i.e. $|\rho_2|$

1: $\kappa=100$, $\sigma=2.78\times10^{-6}$ ℧/m

2: $\kappa=4$, $\sigma=5\times10^{-5}$ ℧/m

3: $\kappa=8$, $\sigma=5\times10^{-3}$ ℧/m where

$$\cos\,\Delta\phi=\frac{1-|\rho_2|\cos\Theta_2+|\rho_1|\cos\Theta_1-|\rho_1|\cdot|\rho_2|\cos(\Theta_2-\Theta_1)}{(1+2|\rho_1|\cos\Theta_1+|\rho_1|^2)^{1/2}\ (1-2|\rho_2|\cos\Theta_2+|\rho_2|^2)^{1/2}}$$

$$\sin\,\Delta\phi=\frac{|\rho_2|\sin\Theta_2+|\rho_1|\sin\Theta_1+|\rho_1|\cdot|\rho_2|\sin(\Theta_2-\Theta_1)}{(1+2|\rho_1|\cos\Theta_1+|\rho_1|^2)^{1/2}\ (1-2|\rho_2|\cos\Theta_2+|\rho_2|^2)^{1/2}}$$

Assuming that the reflecting surface is dry earth ($\kappa=4$, $\sigma=5\times10^{-5}\mho$/m) or ice ($\kappa=100$, $\sigma=2.78\times^{-6}\mho$/m), ρ_1, ρ_2, Θ_1, Θ_2 for 750 Hz, 5, 12 and 25 KHz can be obtained as shown in Figs. 9, 10 and then $\Delta k/K$ and $\Delta\phi$ are calculated as shown in Fig. 11. And i is determined on H_x-E_z figure.

If $S'=m'/p'$ (see Fig. 8-2), $\sin i=S'\ |\sin\theta|\cdot K\cdot|\sin(\phi+\Delta\phi)|$.

Therefore, i has no error.

Thus, for 2. 1. 1 or 2. 1. 2, i, θ have no error and the errors in A/N and ϕ are roughly negligible if the frequency is in VLF range and incident angle is moderate.

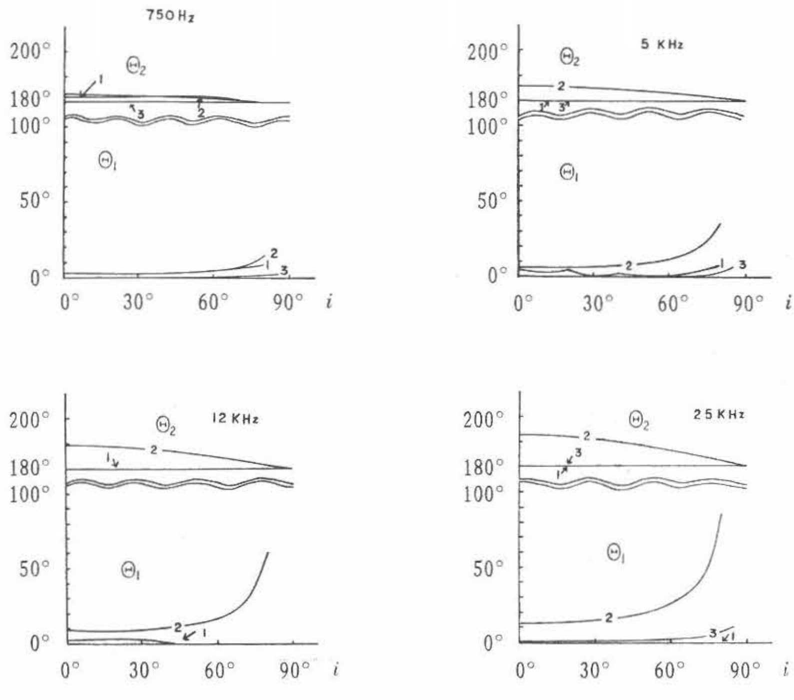

Fig. 10 Phase angle of reflection coefficient

Θ_1 corresponds to vertically polarized wave.

Θ_2 corresponds to horizontally polarized wave.

1: $\kappa=100$, $\sigma=2.78\times10^{-6}\mho$/m

2: $\kappa=4$, $\sigma=5\times10^{-5}\mho$/m

3: $\kappa=8$, $\sigma=5\times10^{-3}\mho$/m

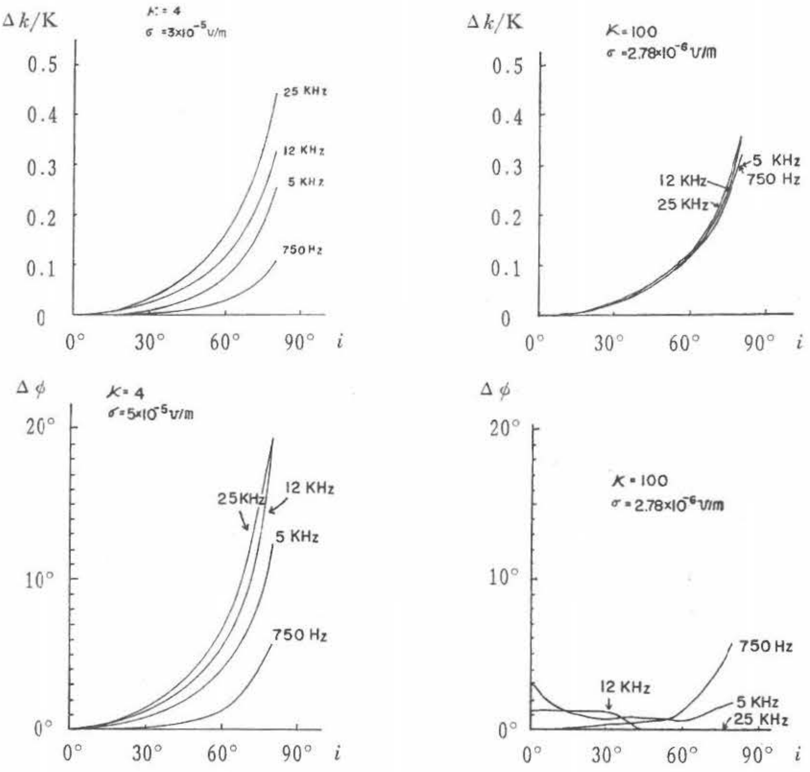

Fig. 11

3. Observation at Moshiri

In the low or middle latitude, the levels of the VLF emissions are much lower than those of atmospherics. So that, only the continuous type emissions can be observed by using the minimum level reading circuit developed by Ellis (1959) and Iwai (1964). In order to distinguish the VLF emissions from the atmospherics, the strength of the atmospherics is recorded on the chart paper. One of the purposes of the observation of the physical properties described above is, of course, to investigate nature of VLF emissions, however, another purpose is to find VLF emissions burying in atmospherics.

3. 1 Observing technique at Moshiri

Two channel outputs from the goniometer that automatically turns round with a period of 30 minutes, are divided into two branches, respectively and the signal on the first channel is supplied to the phase shifting circuit, in which the signal is led and lagged by right angle and addec to the signal on the second channel. Then, the incoming signals are divided into right-handed and left-handed polarized waves. Five signals — two channel outputs from the goniometer, vertical antenna output, right-handed and left-handed polarized waves —, are supplied separately to five narrow band amplifiers and each is rectifies to d-c and averaged for 1 milli-second and then fed to a resistance-capacitance circuit with a charging time constant of 1 minute and a discharging time constant of 2 milli-second and then recorded on a running chart paper. (Iwai et al. 1964)

3. 2 Principle of the observation

The authors assume as follows:—

(a) VLF emission is, in general, a downcoming plane and single frequency sine wave and elliptically polarized.

(b) The ground is flat and perfectly conductive.

(c) Physical properties of the emissions do not vary quickly.

Amplitude of H_x can be given as follows:—

$$\text{Amplitude of } H_x = [2\{(A^2\cos^2 i + N^2)^2 - 4A^2N^2\cos^2 i \sin^2\phi\}^{1/2}\sin(2\theta+\beta_1) + 2(A^2\cos^2 i + N^2)]^{1/2}$$

where

$$\sin\beta_1 = \frac{N^2 - A^2\cos^2 i}{\{(A^2\cos^2 i + N^2)^2 - 4A^2N^2\cos^2 i\sin^2\phi\}^{1/2}}$$

$$\cos\beta_1 = \frac{2AN\cos i\cos\phi}{\{(A^2\cos^2 i + N^2)^2 - 4A^2N^2\cos^2 i\sin^2\phi\}^{1/2}}$$

Thus. it is clear that the amplitude varies according as the goniometer is rotated.

And so,

Amplitude of H_x is max., if $\sin(2\theta+\beta_1) = 1$

Amplitude of H_x is min., if $\sin(2\theta+\beta_1) = -1$

Then, the direction is given by the next formula. — see Fig. 12

$$\theta = \theta_g + \theta_1(\beta_1) = \theta_g + \theta_1(A, N, i, \phi)$$

where, θ_g: rotation angle of the goniometer for the max. or min. of H_x.

Besides, the analysis for H_y is also identical.

As a result,

$$\text{Amplitude of } (H_x^{\pm} + H_y) = 2\{(A^2\cos^2 i + N^2) \pm 2AN\cos i\sin\phi\}^{1/2}$$

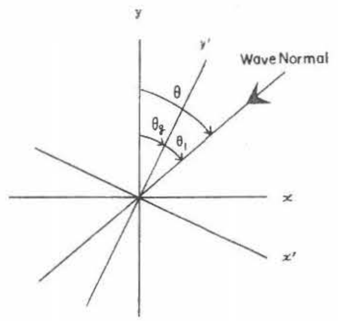

Fig. 12 y represents the direction toward north. N-S plane of the loop is, equivalently, directed along y' when the goniometer is rotated at angle θ_g. θ_g is the rotation angle of the goniometer when the amplitude becomes max. or min. and then θ_1 is the angle between N-S plane rotated (i. e. y') and the wave normal.

Amplitudes of $(H_x^{\pm}+H_y)$, H_x, H_y and E_z are calibrated by inserting a test signal. Then, $(A^2 \cos^2 i + N^2)$, $AN \cos i \sin \phi$, $N \sin i$ are given. Thus, there are only four equations for five unknown quantities (A, N, i, ϕ, θ) and it is necessary to give one quantity.

For example,

(1) If the wave is vertically polarized, $A=0$ and all the quantities are determined.

(2) If the wave propagates with the wave guide first mode between the ionosphere and the earth with perfect conductivity, $\cos i = c/2hf$

where c : light velocity
f : frequency
h : height of the ionosphere

Then, θ has 45° and 180° ambiguity in general. Thus, it is generally unable to detect the direction by using the goniometer arrangement, except vertically polarized wave.

3. 3 Observational result

At Moshiri, the ratio of the level of right-handed polarized wave to that of left-handed polarized wave is usually near 1 or slightly smaller than 1. Judging from this result, the wave seem to be linearly polarized or slightly left-handed polarized. Thus, it is assumed that the emissions received in the low latitude are vertically polarized. Therefore, in some few cases of high level emissions, the method of the goniometer is effective in order to detect the direction of the emissions even in the low latitude such as Moshiri. An example is shown in Fig. 13.

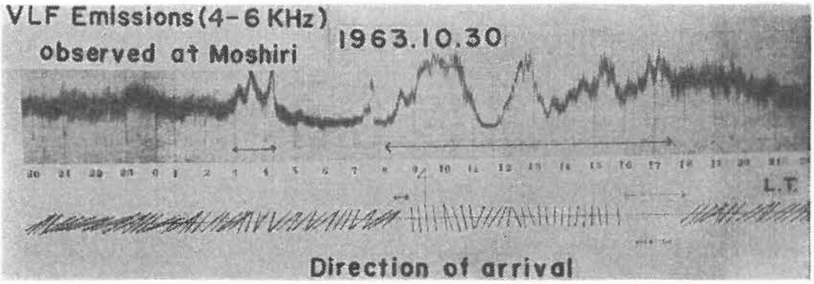

Fig. 13 A sample record showing the arrival direction of VLF emissions observed at Moshiri. When VLF emissions appear, the direction is seen to be roughly toward north (auroral zone). When VLF emissions do not appear, the direction is toward south-west (source region of atmospherics).

But, unfortunately, a trial to detect the incident angle by means of vertical antenna, ended in a failure, because the emissions were too weak and the error due to the ground noise was serious. VLF emissions observed at Moshiri seem to include local emissions (Tanaka et al. 1968).

If the local emissions come down through over-head ionosphere, they are expected to be elliptically polarized in general and so the observation of right-handed and left-handed polarized waves is interesting from this point of view.

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Appendixes

*** In H_x-H_y figure for $\theta=0°$ or $180°$ (i.e. $\overline{H}_x$-$\overline{H}_y$ figure), the equation of the ellipse is

$$F\,(H_x,\,H_y)=a\;H_x^2+2h\;H_xH_y+b\;H_y^2+c=0$$

where, $a=4A^2\cos^2 i,\; h=-4AN\cos i\cos\phi$

$$b=4N^2,\qquad c=-16A^2N^2\cos^2 i\sin^2\phi$$

If the axes $(H_x,\,H_y)$ are rotated by angle α and the new axes are represented by $(H_x',\,H_y')$,

$$F(H_x',\,H_y')=a'H_x'^2+2h'H_x'H_y'+b'H_y'^2+c=0$$

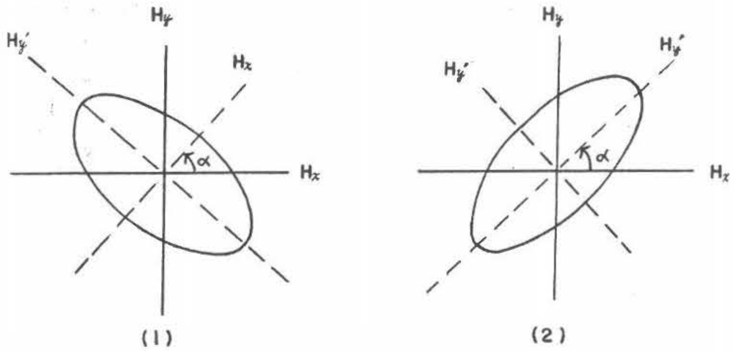

Fig. 14 H_x-H_y figures for the two different ranges of $\cos \phi$
(1): for long axis in 2-4 quadrant, $\cos \phi < 0$
(2): for long axis in 1-3 quadrant, $\cos \phi > 0$

where, $a' = a \cos^2 \alpha + h \sin 2\alpha + b \sin^2 \alpha$, $2h' = 2h \cos 2\alpha - (a-b) \sin 2\alpha$
$b' = a \sin^2 \alpha - h \sin 2\alpha + b \cos^2 \alpha$

If $h' = 0$ (i. e. $\tan 2\alpha = 2h/(a-b)$, provided $0 < \alpha < \pi/2$),

the equation of the ellipse is written on the new coordinate axis as

$$\frac{H_x'^2}{\dfrac{(16A^2N^2 \cos^2 i \sin^2 \phi)}{a'}} + \frac{H_y'^2}{\dfrac{(16A^2N^2 \cos^2 i \sin^2 \phi)}{b'}} = 1$$

And $a' - b' = (1/2h) \sin 2\alpha \{4h^2 + (a-b)^2\}$

Therefore,

if $h > 0$ (i. e $\cos \phi < 0$), $a' > b'$ (i. e. the long axis exists in 2-4 quadrant)

if $h < 0$ (i. e. $\cos \phi > 0$), $a' < b'$ (i. e. the long axis exists in 1-3 quadrant)

and vice versa.

Applying the brightness control method to H_x-H_y figure on a CRT, the tail of the half ellipse can be made to vanish gradually, so that the sense of rotation of the point vector can easily be seen on it.

If right-handed polarization, we get $\sin \phi > 0$,

if left-handed polarization, we get $\sin \phi < 0$.

And $|\sin \phi|$ is given in H_x-H_y figure. Thus, ϕ is determined.

* If H_x-H_y figure is displayed during plus half cycles, H_x and H_y at the starting point of the half ellipse (i. e. $\otimes$ shown in Fig. 15) are given by

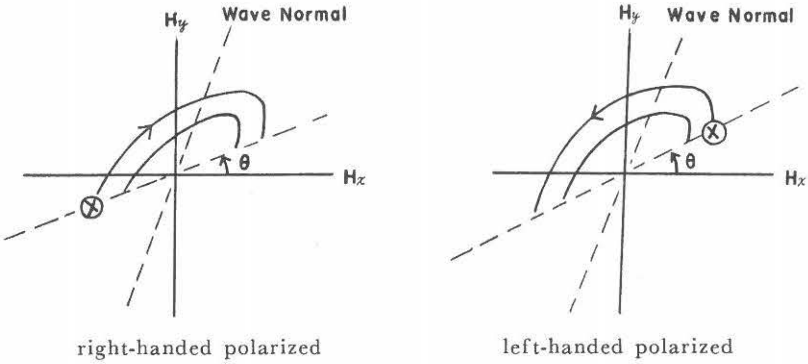

Fig. 15 H_x-H_y figure during plus half cycles of E_z
⊗: starting point of the locus, The wave comes from the direction of arrow.

$$H_x = -2A \cos i \sin \phi \sin \theta$$

$$H_y = -2A \cos i \sin \phi \cos \theta, \text{ for } \sin \omega t = 0, \cos \omega t = 1$$

Then, for right-handed polarized wave, starting point angle θ takes a value in first quadrant.
And for left-handed polarized wave, starting point angle θ takes a value in third quadrant.
Therefore, θ is an angle in the quadrant, where the locus of the half ellipse exists. And, for the figure during minus cycles, θ is an angle in the quadrant, in which the locus does not exist.