ON THE TIME FUNCTIONS OF ATMOSPHERIC NOISE

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Abstract

The author once derived the probability distribution of pulse duration and that of time interval between the occurrence times of pulses in the noise process which resuls from a random series of rectangular pulses of equal amplitude and duration, their arrivals conforming to the exponential distribution. The distribution of duration and that of time interval between pulses were measured at 50 KHz with a receiver of 1 KHz 3-db-bandwidth using a vertical antenna at Toyokawa, 1959 and 1960. In this paper, the theoretical and experimental probability distributions are compared for duration and interval. The agreements found between them are well for the case of duration, except when the percentage the durations exceed a given length of time is very low. In the case of interval, the agreements are found to be poor for both intermediate and high thresholds.

For very low threshold, the experimental distribution approximate the exponential distribution, conforming to an analytical result. The unusual increase in probability density have been observed on interval distribution at very high thresholds.

1. Introduction

The type of informations studied to clarify the noise structure includes the four statistical parameters, amplitude probability distribution, the crossing rate distribution, the probability distribution of pulse duration and that of time interval between pulses. The study of the interfering effect of noise on radio communication systems involves not only the amplitude characteristics, but also the time sequence of the intensity variations (Horner 1967).

For the vertical noise field radiated form near origins, Aiya (1965), Aiya and Lakshiminarayan (1965) have recorded SF noise as a series of bursts which is the counterpart of a lightning flash and measured the duration of the burst and time interval between the bursts, where the two parameters have been found to be approximated by a log-normal distribution respectively. Clark (1960) measurd the duration of noise burst at 11 MHz, with a bandwidth 300 Hz, where the duration was taken as the longest time for which the level of atmospheric noise envelope was almost continuously kept above a given threshold.

Watt and Maxwell (1958) measured the probability distribution of interval between pulses at 22 KHz, with a bandwidth 1170 Hz, when the atmospheric noise envelope exceeds the given thresholds. Nakai and Suzuki (1961) measured the probability distribution of pulse duration and that of time interval between pulses for the atmospheric noise at 50 KHz, with a bandwidth 1000 Hz. Nakai (1964, 1965) derived the expression for the probability distribution of pulse duration and that of interval between occurrence times of pulses, in the noise process, which composed from a random series of rectangular pulses of equal amplitude and duration, their arrivals conforming to the exponential distribution. In this paper, the distributions measured at Toyokawa are compared with the theoretical distributions and their features are discussed.

2. Theory

Considering that the noise to be discussed can be composed from a random series of original pulses of rectangular shape of equal amplitude and equal duration, and their arrival can be determined by the exponential distribution, the overlapping of the original pulses results in longer composite pulses, the probability that the duration of composite pules exceeds a given length of time $(m+1)\ \tau$ is given as follows:

$$
\begin{aligned}
&\mathrm{P}\ (\mathrm{Y} \geq (m+1)\tau) = 1 - \mathrm{P}(\mathrm{Y} < (m+1)\tau) \\
&= 1 - \sum_{k=1}^{[m]+1} (1-e^{-\nu\tau})^{k-1} \cdot e^{-\nu\tau} \\
&+ m\nu\tau \cdot e^{-\nu\tau} - \sum_{s=0}^{[m]-1} e^{-\nu\tau} \left\{ 1 - e^{-m\nu\tau} \sum_{t=0}^{s} \frac{(m\nu\tau)^t}{t\,!} \right\} \\
&+ \sum_{n=1}^{[m]} (-1)^n \frac{e^{-\nu(n+1)\tau}}{n\,!} \left[\left\{ \nu\tau(m-n) \right\}^n + \frac{\{\nu\tau(m-n)\}^{n+1}}{n+1} \right] \\
&+ \sum_{n=1}^{[m]} \sum_{s=0}^{[m]-n} (-1)^{n+1} \cdot \frac{e^{-\nu\tau(n+1)\tau}}{n\,!} \cdot \frac{s+n}{s\,!} \cdot (s+n-1)\,! \left[1 - e^{-(m-n)\nu\tau} \right. \\
&\qquad \left. \sum_{t=0}^{s+n-1} \frac{\{(m-n)\nu\tau\}^t}{t\,!} \right] \cdots\cdots\cdots\cdots (1)
\end{aligned}
$$

Y : the duration of composite pulse

ν : the average number of original pulse per second

τ : the duration of original pulse

m : any real positive value

$[m]$: the integer value, where $m > [m] \geq m-1$

A series of theoretical distributions calculated from eq. (1) have already been demonstrated for various $\nu\tau$ values, elsewhere (Nakai, 1964).

Again considering the noise as described above, the probability that the interval between occurrence times of composite pulses exceeds a given length of time $(l-1)\tau$, is given as follows :

$$P(Z \geq (l+1)\tau) = 1 - \int_{\tau}^{l\tau} \sum_{n=0}^{[m]} \frac{(-1)^n \{(m-n)\nu\tau\}^n}{n\,!} \times \nu e^{-(n+1)\nu\tau} d(m\tau) \cdots\cdots\cdots (2)$$

Z : the interval between the occurrence times of composite pulses

m : a real number satisfying the equality $Z=(m+1)\tau$

A series of theoretical distributions calculated from eq. (2) have already been demonstrated for various $\nu\tau$ values, elsewhere (Nakai 1965). Here, the following statement will be necessary to be added; The theoretical distributions for large $\nu\tau$ values approach to the exponential distribution for the case of interval between the occurrence times of pulses, and they also conform to the exponential distribution for the case of the interval between pulses, for which the noise field disappears.

The theoretical treatment described above is believed to be one approach to the problem of time variation of intensity for the case of atmospheric noise. It is not always possible to secure a complete correspondence between the noise discussed above and the atmospheric noise envelope that exceeds various thresholds, however, we know still the existence of a certain simulation between the two. Therefore, it is interesting and instructive for the further study of time characteristics of atmospheric noise, to compare the theoretical and experimental distribution.

3. Measurement of time functions of the atmospheric noise

Nakai and Suzuki (1962) reported the experimental results of their measurements of pulse durations when the atmospheric noise envelope exceeds a seres of given thresholds in the intensity range from 31.5 db/ v/m to 81.5 db/ v/m. The measurements were made at 50 KHz with a receiver 1 KHz 3-db-bandwidth using a vertical antenna, for about 10 days in each period in March-April and September, 1959.

Some features derived from these data were describd there: The shape of distribution of pulse duration and the magnitude of duration are subject to a change depending on threshold value, time and day of measurement. But there exist a similarity in shape and magnitude among the distributions measured for different thresholds, time and day, where the threshold is expressed in a unit of the integrated field intensity of atmospherics measured with time constant 80 seconds, and the thresholds thus defined anew are proved approximately equal among themselves. Here, the integrated field intensity of atmospherics is estimated to be several db higher than average noise amplitude.

An additional measurements of the same parameters were made at Toyokawa for about 7 days in March, 1960. The system and method of measurement are similar to what was used in the previous measurements but the thresholds were lowered down to 1.5 db/μ v/m to investigate the time characteristics of atmospheric noise at such a low threshold.

4. Comparison between theory and experiment of distribution of the duration

In the following discuss, all the distributions of pulse duration measured for three periods as indicated above are investigated and compared with the theoretical distributions. A rough estimation of the magnitude of pulse duration and the dynamic range between 90 and 0.1 percent may be obtained from Fig. 1, where two distributions which have the smallest and the largest dynamic ranges, are selected among those distributions obtained at Toyokawa, and drawn on the Rayleigh-graph. The remaining

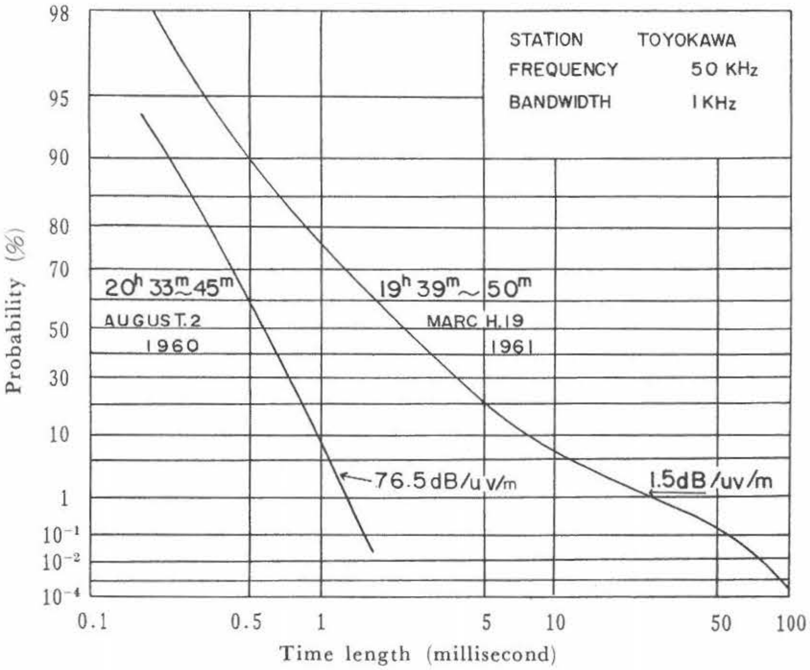

Fig. 1 The distributions of pulse duration with the largest and the smallest dynamic range between 90 and 0.1 percent for the atmospheric noise at 50 KHz.

measured distributions showed an intermediate characteristics with respect to the magnitudes of duration and dynamic range.

Fig. 2 (a) and (b) show the comparison between the theoretical and experimental distributions. The theoretical distributions of the probability that pulse duration exceeds the abscissa are depicted on the Rayleigh-graph with real curves for the two $\nu\tau$ values, 1.0 and 2.0, where the abscissa is expressed in $\log_{10} \tau$ unit, and the ordinate represents the probability. This graph can also be used in plotting the measured probability that the pulse duration exceeds a given length of time, where the abscissa is expressed in $\log_{10}$ (millisecond) unit and its absolute values can be taken arbitrarily. The plots on the two graphs are the measured points for the atmospheric noise envelope at the threshold 41.5 db/μ v/m.

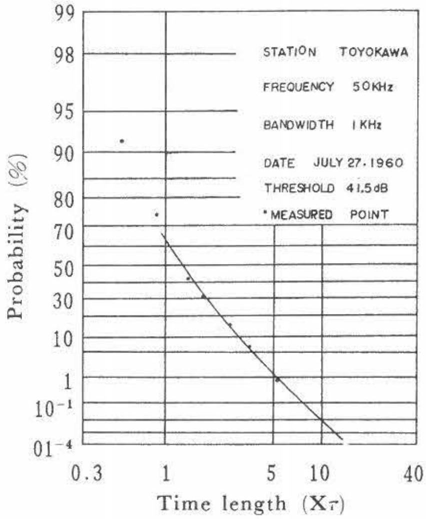

Fig. 2(a) Comparison between the theoretical and experimental distribution curves of pulse duraion (Rayleigh-graph).
• : measured points, real curve: theoretical curve ($\nu\tau = 1.0$).

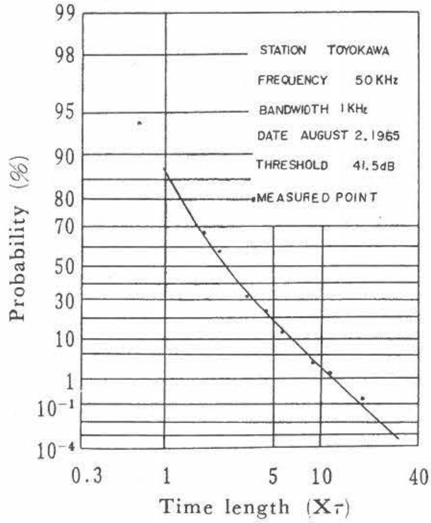

Fig. 2(b) comparison between the theoretical and experimental distribution curves of pulse duration (Rayleigh-graph).
• : measured points, real curve: theoretical curve ($\nu\tau = 2.0$).

The agreement between the measured points and the theoretical distribution curves are very well. As described in the latter part of the second paragraph, the physical meanings of the quantities ν and τ are not clear in a strict sense, if we come to consider their magnitudes and to make a question, "Which parameters of the atmospheric noise should be attributed to." We know the fact that the magnitudes of ν and τ can be determined if the coincidence can be secured between the theoretical and experimental distributions, but we do not think that we have a physical means to measure ν directly.

Therefore, we will put the problem aside and place a particular attension only on the shape coincidence problem between the theoretical and experimental distributions.

All the measured distributions have been examined along the line of what has been described just above. The examination shows the evidence that the coincidence are poor when the thresholds are very high. The examples of such distribution are demonstrated on the Rayleigh-graph in Fig. 3, where they were measured at the threshold 61.5 db/μ v/m. The situation in this case is very similar to the case of higher thresholds. The reason of this is clear, because the atmospheric noises at such high thresholds are almost composed of discrete pulses which result from a sequence of responces after the higher level parts of the atmospheric noise field have passed through the narrow band receiver. Therefore, the theoretical treatment can not be applied appropriately in this case. The discrete character of a sequence of pulses should be taken into account in this case. An expression of the porbability distribution of pulse duration has been estabished for the noise that results from a sequence of discretely arriving pulses of idealized exponential shape with a distribution of peak amplitudes defined by a power law (Nakai 1964).

The figures 4 (a) through (b) show the comparison between the theoretical and experimental distribution curves of pulse durations for the atmospheric noise. Each of these graphs was obtained by overlaping, on the experimental graph, the transparent paper where the calculated distribution curves were depicted for three values $\nu\tau$ 1.0, 2.0 and 3.0. The measured points on experimental curves were transfered onto the transparent paper. Moreover, it

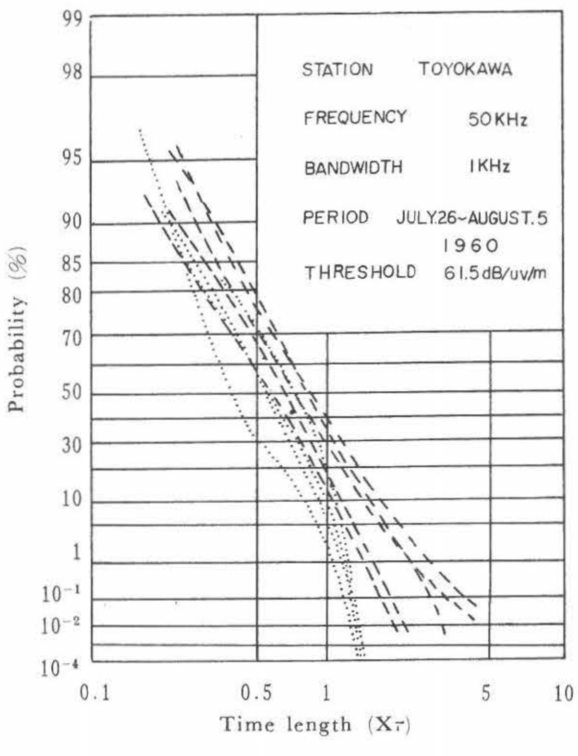

Fig. 3 The distribution curves of pulse duration when the atmospheric noise envelope exceeds high threshold 61.5 db/μ v/m (Rayleigh-graph).

The curves are the distributions measured during Sept. 13 to 23 and the dotted curves are those during July 24 to Aug. 5.

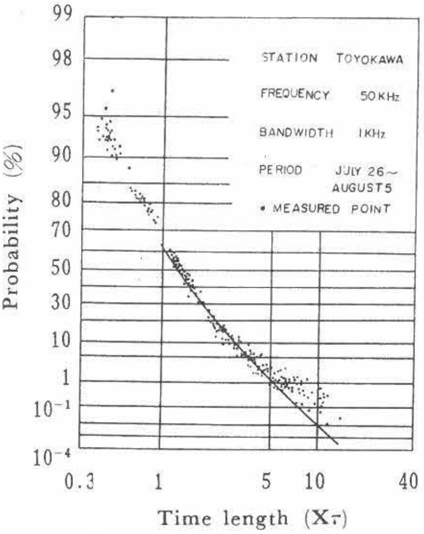

Fig. 4 (a) Scatter plots showing comparison between theoretical and experimental distribution curves of pulse duration (Rayleigh-graph).
• : measured (thresholds are 21.5 to 51.5 db/μ v/m), real curve : theoretical curve ($\nu\tau=1.0$).

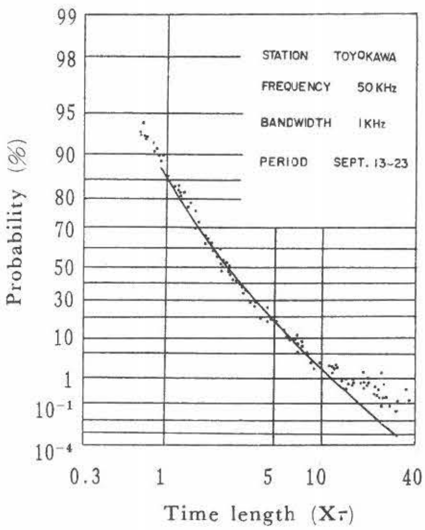

Fig. 4 (b) Scatter plots showing comparison between theoretical and experimental distribution curves of pulse duration (Rayleigh-graph).
• : measured points (thresholds are 21.5 to 51.5 db/μ v/m), real curve: theoretical curve ($\nu\tau=2.0$)

must be added that the coincidences similar to the case shown Fig. 4 (a) to 4 (b) have been found ın most cases of the remaining distributions.

The results of the comparisons in Fig. 4 (a) to 4 (b) are fairly well, except for the discrepancies in the ranges less than a few percent. The distributions compared here include that measured at a threshold from 21.5 to 51.5 db/μ v/m. In addition, it has been found that such agreements as shown in Fig. 4 (a) and 4 (b) have been obtained with the distributions measured at different thresholds for a given value of $\nu\tau$. Therefore, another conclusion of this comparision is that there is no great difference in the shapes among the distributions measured at different thresholds in the range as specified above, if an emphasis is placed only the conformity of shapes of the distributions. Fig. 4 (c) is an example of large discrepancies and they occur in the probability range less than about 10 percent, where the thresholds are more than 1.5 db/μ v/m.

In ending the description about the results of comparison, the reason why these discrepancies have frequently occurred in the ranges of low probability and of long duration, must be shown. They can be caused by large amplitude atmospherics occurred infrequently during time duration of measurement of a distribution. Because

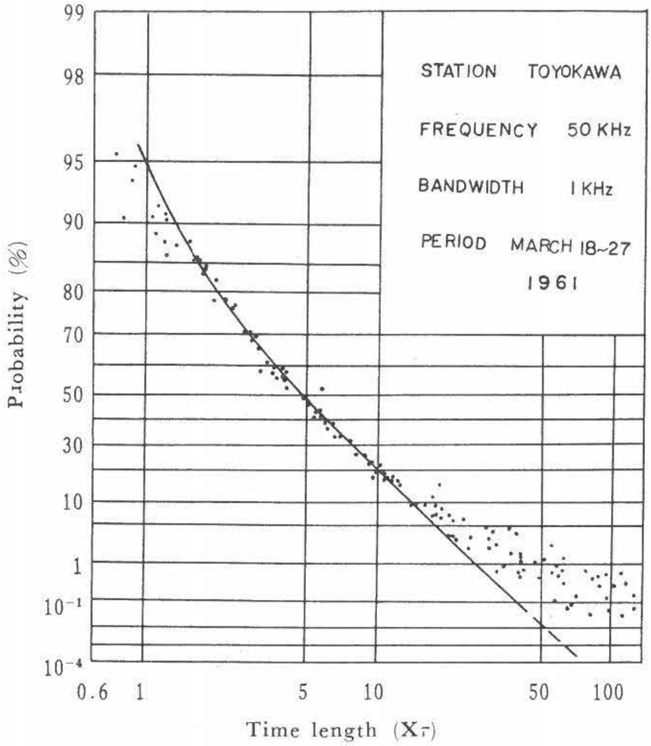

Fig. 4 (c) Scatter plots showing comparison between the theoretical and experimental distribution curves of pulse duration (Rayleigh-graph),
• : measured points (thresholds are 21.5 to 51.5 db/μ v/m), real curve: theoretical curve ($\nu\tau=3.0$)

the responce of large amplitude atmospherics after passing through the narrow band receiver should have a long duration at these given threshold levels, which are low compared with the peak amplitude of the responce. Whereas such long duration on only a single responce has not been taken into account for the case of the present theoretical treatment, which estimate the probability density and the cumulative probability distribution of duration of the composite pulse resulted from overlaping of the original pulses.

5. The measured distributions of pulse intervals

The measured distributions of pulse intervals of atmospheric noise envelope at various thresholds and the theoretical distributions of the interval between the occurrence times of pulses are compared. The difference between the interval between pulses and that between the occurrence times of pulses can be neglected in the following comparison, because the duration of pulse is small compared with each of these two intervals for intermediate and high threshold levels. But the above statement does not hold for very low threshold levels, at which the pulse duration is can

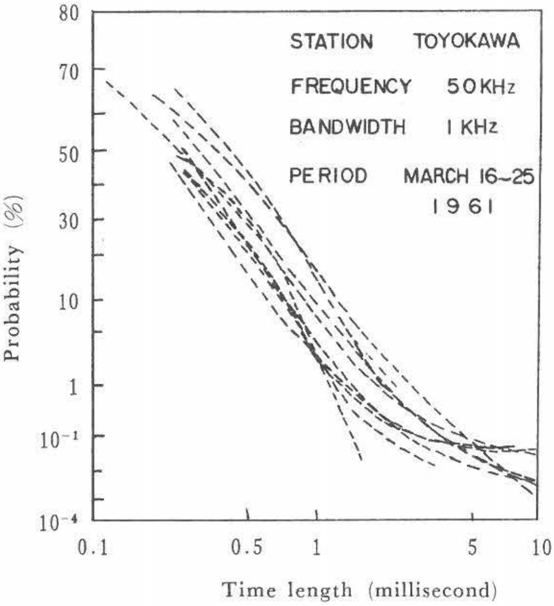

Fig. 5 The measured distribution curves of pulse interval when the atmospheric noise envelope exceeds low threshold 1.5 db/μ v/m (Rayleigh-graph).

be very large compared with these intervals.

Fig. 5 shows the measured distributions of pulse interval at the threshold 1.5 db/μ v/m drawn on the Reyleigh-graph. It is found that the slopes of the distribution curves are about (-1) in most cases, except for low probability section than several percent. Refering to the theoretical result as described in 2nd paragraph for the pulse interval, we can deduce that the times of arrival of impulses at the antenna approximate the exponential distribution.

For the intermediate and high thresholds, there are remarkable discrepancies between the theoretical and experimental distribution curves. Shortly speaking, there are some interaction between pulses occurring one after the other. Such situation was also observed for the atmospheric noise in VLF (watt and Maxwell 1959). Especially, it is remarkable that the distributions measured in the period of measurement in July-August, i. e., in mid-summer in Japan, showed an unusual increase in probability density of pulse interval. Such distributions are shown in Fig. 6 (a) on the Rayleigh-graph. The probability density of the time interval between pulses is shown in Fig. 6 (b), where Δ p is the differential of the probability with respect to the time length Δ t.

For a group of distribution drawn with real curves, the numbers of pulses spaced by 10 to 40 milliseeonds were usually large, and for the other group drawn with dotted curves, the numbers of pulses spaced by 40 to 100 milliseconds were large in a less rate in terms of $\Delta p/\Delta t$. The increase in probability density is in good agreement with time interval between successive current peaks of a complete lightning

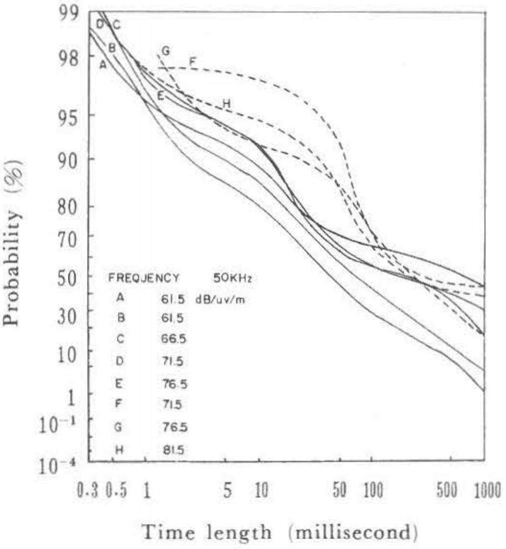

Fig. 6 (a) The measured distribution curves of pulse interval when the atmospheric noise envelope exceeds high thresholds (Rayleigh-graph).

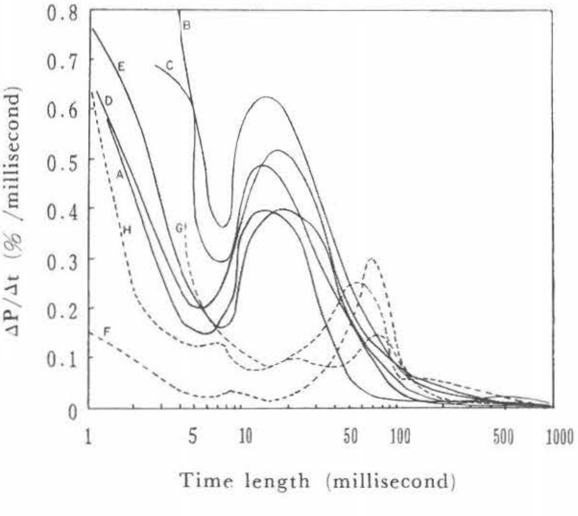

Fig. 6 (b) The derived probability density curves of pulse interval when the atmospheric noise envelope exceeds high thresholds (Rayleigh-graph).

discharge (Bruce and Golde 1941; Hagenguth 1947).

In addition to the above, it is interesting that the increase in the probability density in the time length range 40 to 100 milliseconds are predominant in the case of the distributions at higher thresholds, while the increase in the time length range from 10 to 40 milliseconds are predominant in the case of those at lower thresholds. For the other period of the observations, the distributions showing such increase in probability density were not found.

6. Conclusion

The time functions of the atmospheric noise and a kind of the Poission noise considered theroretically were compared each other. Good agreements were found for the distributions of pulse duration with respect to their shapes, but the agreement was poor for the distribution of time interval between pulses. It has been found that the observed pulse intervals approximate an exponential distribution only for very low threshold. For very high threshold, unusual increase in the probability density for the pulse interval in the range 10 to 100 milliseconds was found to agree with that observed in the source of atmospherics. This agreement stresses the importance of the time characteristics of the lightning discharge in the source of atmospherics with statistically measuring method. We have some information about the time variations of the atmospheric radio noise in VLF and LF waves, while only a scant information about it in SF wave. The time charcterisitics of the atmospheric noise in SF wave is considered to affect the behavior on the communication system working at this frequency. Therefore, further study of the time functions of the atmospheric noise in SF band is very important.

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