IMPEDANCE OF WHIP ANTENNA IN THE IONOSPHERE

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Abstract

A basic problem for the observation of radio noise in the ionosphere with sounding rocket is to know the antenna impedance in it. The purpose of this paper is to evaluate the impedance of an electrically short antenna in the ionosphere and to get the coefficient which converts the input voltage to a receiver into the electric field intensity in the ionosphere.

Theoretically to investigate the antenna impedance in the ionosphere, we have started from the equation of plasma dynamics and Maxwell's equation and derived a general expression for antenna impedance in the homogeneous ionosphere, which has enabled to obtain the conversion coefficient for this condition.

1. Introduction

The observation of radio noise spectrum in VLF bands in the ionosphere was carried out by us using a VLF noise spectrum receiver system aloft on the sounding rocket K-9M-19 flown at 21.00 on Aug. 10, 1966. The swept-frequency method was used to observe the amplitude-frequency spectrum. The observing frequency was sweeped in two ranges from 2 to 30 KHz and from 20 to 60 KHz. The sensor used was a umbrella rib type whip antenna which had one meter length and 4 elements. The radio noise in the ionosphere was successfully observed throughout the flight.

The intensity of the noise was described in terms of input voltage to the receiver in the previous report (Kamada, Kurahashi, 1966), but it is necessary to convert the input voltage obtained into the field intensity of radio waves to know more detailed and accurate information about them. To do this, the character of whip antenna in the ionosphere must be understood. The purpose of this paper is theoretically to investigate antenna impedance in a medium of ionospheric plasma which can be represented as a function of altitude and to obtain conversion coefficient which we need to convert the input voltage to receiver into the electric field intensity in the ionosphere.

2. Equivalent input circuit

The equivalent input circuit to receiver is shown in Fig. 1, where we get the following relation,

$$h\dot{E} = \left(\frac{\dot{Z}_a}{\dot{Z}_{in}} + 1\right) \dot{V}_{in} \qquad \text{—(1)—}$$

where $\dot{E}$ is the electric field intensity, h the effective height of antenna, $\dot{V}_{in}$ the input voltage to the receiver, $\dot{Z}_a$ the antenna impedance and $\dot{Z}_{in}$ the input impedance to the receiver.

The antenna impedance $\dot{Z}_a$ consists of sum of the plasma impedance $\dot{Z}_p$ and the sheath impedance $\dot{Z}_s$.

$$\dot{Z}_a = \dot{Z}_p + \dot{Z}_s \qquad \text{—(2)—}$$

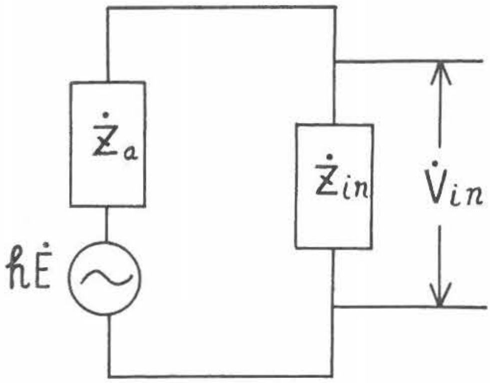

Fig. 1 The equivalent input circuit of the receiAer

Using Eq. 1 and 2, we find

$$\dot{E} = \left(\frac{\dot{Z}_p + \dot{Z}_s}{\dot{Z}_{in}} + 1\right) \frac{\dot{V}_{in}}{h} = \dot{k}_c \frac{\dot{V}_{in}}{h} \qquad \text{—(3)—}$$

where

$$\dot{k}_c = \left(\frac{\dot{Z}_p + \dot{Z}_s}{\dot{Z}_{in}} + 1\right)$$

If we can calculate $\dot{Z}_p$ and $\dot{Z}_s$, we can evaluate $\dot{k}_c$ in Eq. 3 and convert the input voltage to receiver into the electric field intensity from Eq. 3

3. Plasma Impedance $\dot{Z}_p$

The problem of the plasma impedance in the ionosphere has been discussed and examined by many workers (Kane, et al, 1962; Kaiser, 1962; Whale, 1962; Balmain, 1964; Oya, et al, 1966) and, especially for the case of the cylindrical whip type antenna, it was calculated by Kaiser (1962) and Balmain (1964). But the effect of ion to the plasma impedance has not seemed to be sufficiently considered until to the present. So, we intend here to derive out the general expression of the plasma impedance which consider the effects above and, we try to calculate the plasma impedance in the ionosphere starting from the equation of plasma dynamics and from Maxwell's equation. The equations of plasma dynamics for electron and ion are respectively given by,

$$\left.\begin{aligned}&\frac{\partial \mathbf{J}_e}{\partial t}+\nu_e \mathbf{J}_e-\frac{n_e e^2}{m_e}(\mathbf{E}+\mathbf{V}_e\times\mathbf{B})+\frac{e}{m_e}\nabla \mathrm{P}_e=0\\&\frac{\partial \mathbf{J}_i}{\partial t}+\nu_i \mathbf{J}_i-\frac{n_i e^1}{m_i}(\mathbf{E}+\mathbf{V}_i\times\mathbf{B})-\frac{Ze}{m_i}\nabla \mathrm{P}_i=0\end{aligned}\right\} \qquad \text{—(4)—}$$

where subscript e and i express the quantity for electron and ion respectively. $\mathbf{J}$, ν, n, $\mathbf{V}$, P, m, are current density, collision frequency, density, velocity, pressure and mass of the particles to be considerd. e, $\mathbf{B}$, and Z are electric charge, magnetic flux density and average charge of positive ions. By adding two equations in (4), we find

$$\frac{\partial \mathbf{J}}{et}+\nu\mathbf{J}-\varepsilon_0\{\omega_{pe}^2(\mathbf{E}+\mathbf{V}_e\times\mathbf{B})+\omega_{pi}^2(\mathbf{E}+\mathbf{V}_i\times\mathbf{B})\}$$
$$+\left(\frac{\gamma_e \mathrm{k}\,\mathrm{T}_e e}{m_e}\nabla n_e-\frac{\gamma_i \mathrm{k}\,\mathrm{T}_i Ze}{m_i}\nabla n_i\right)=0 \qquad \text{—(5)—}$$

where $\mathbf{J}=\mathbf{J}_e+\mathbf{J}_i$, ω_p, γ, T are angular plasma frequency, ratio of the specific heat at constant presure to that at constant volume, and temperature, respectively, and ε_0, k are the dielectric constant in vacuum and Boltzmann constans; ν is defined as

$$\nu=\frac{\nu_e \mathbf{J}_e+\nu_i \mathbf{J}_i}{\mathbf{J}_e+\mathbf{J}_i} \qquad \text{—(6)—}$$

The current density which flows to the antenna is

$$\mathbf{I}=\varepsilon_0\frac{\partial \mathbf{E}}{\partial t}+\mathbf{J} \qquad \text{—(7)—}$$

and the plasma impedance $\mathbf{Z}_p$ is

$$\mathbf{Z}_p=\frac{\mathrm{V}}{\int_s \mathbf{I}ds}$$

where, V is the plasma potential.
As, the capacitance of antenna in the free space is

$$\frac{\varepsilon_0\int_s \mathbf{E}ds}{\mathrm{V}}=\mathrm{C}_{p0}$$

so that

$$\mathbf{Z}_p=\frac{\varepsilon_0\int_s \mathbf{E}ds}{\mathrm{C}_{p0}\int_s \mathbf{I}\,ds} \qquad \text{—(8)—}$$

Then, we can get the plasma impedance in the ionosphere from Eq. (5), (6), (7), and (8).

Suppose that the term $\mathbf{V}\times\mathbf{B}$ in Eq. (5) is negligible because the whip antenna is non-sensitive to magnetic field, and further $\omega_{pe}^2 \gg \omega_{pi}^2$ because electron plasma frequency f_{pe} is larger than ion plasma frequency f_{pi} in the region of the altitude from 80 to 300 km, since we consider the ionospheric plasma to be cold for a simple treatment, temperture effect is also negligible. Then Eq. (5) may be written in a simplified from

$$\frac{\partial \mathbf{J}}{\partial t} + \nu \mathbf{J} - \varepsilon_0 \omega_{pe}^2 \mathbf{E} \simeq 0 \qquad \text{—(9)—}$$

Let us consider current density $\mathbf{J}$ and the electric field $\mathbf{E}$ to be a function of exp (jwt) and the plasma be homogeneous. Then, from Eq. (7), (8) and (9), we obtain

$$\dot{Z}_p \simeq \frac{1-j\left(\frac{\nu}{\omega}\right)}{C_{p0}\left[\nu + jw\left(1-\frac{\omega_{pe}^2}{\omega^2}\right)\right]} \qquad \text{—(10)—}$$

Now, we can express ν in Eq. (6) in terms of ν_e and ν_i by eliminating $\mathbf{J}_e$ and $\mathbf{J}_i$ from Eq. (4). Substituting them in Eq. (10) we obtain

$$\dot{Z}_p \simeq \frac{\omega\left[\frac{\nu_e+\nu_i}{\omega} + j\left(1-\frac{\nu_e\nu_i}{\omega^2}\right)\right]}{C_{p0}\,\omega_{pe}^2\left[\left(1+\frac{\nu_e\nu_i}{\omega_{pe}^2}\right)-j\left(\frac{\nu_i}{\omega}\right)\right]} \qquad \text{—(11)—}$$

where we assume

$$\omega_{pe}^2 \gg \omega^2$$

The collision frequency usully is smaller than the observing frequency in altitudes higher than 140 km, namely $\omega \gg \nu_e$, ν_i, thus we get from Eq. (11)

$$\dot{Z}_p \simeq \frac{\omega}{C_{po}\omega_{pe}^2} \qquad \text{—(12)—}$$

But the approximation cannot be adopted for the altitude between 80 and 140 km because the collision frequency is no longer small enough in this region, thus we must evaluate $\dot{Z}_p$ from Eq. (11).

In this case, the collision frequency is generally the sum of, the collision frequency of electrons (or ions) with neutral particles, and the collision frequency of electrons (or ions) with charged particles. But the collision with neutral particles is far more frequent than that with charged particles at the altitude from 80 to 140 km, so that we will consider only the collision of electrons (or ions) with neutral particles for ν_e (or ν_i). Then, ν_e anc ν_i are calculated from the following relation.

$$\nu_e \simeq \frac{4}{3} n_{nu} \pi \sigma_e^2 \left(\frac{8kT_e}{\pi m_e}\right)^{1/2} \qquad \text{—— (13) ——}$$

$$\nu_i \simeq \frac{16}{3} n_{nu} \pi \sigma_i^2 \left(\frac{k T_i}{\pi m_i}\right)^{1/2} \qquad \text{—— (14) ——}$$

The relations were given by M. Nicolet (1953) and σ_e, σ_i, n_{nu} are the collision distance for electron and ion and neutral particle density, respectively.

The capacitance C_{p0} in the free space can be measured it beforehand of firing of sounding rocket, and the angular plasma frequency ω_{pe} can be calculated from the exerimental electron density profile. Substituting these valuse into Eq. (11), and (12), we can get the plasma impedance $\dot{Z}_p$ in the ionosphere.

4. Sheath Impedance $\dot{Z}_s$

Sheath impedance may be treated similar to case of the plasma impedance. Hence we get the sheath impedance $\dot{Z}_s$ as

$$\dot{Z}_s \simeq \frac{1-j\left(\frac{\nu_{si}}{\omega}\right)}{C_{s0}\left[\nu_{si} + jw\left(1-\frac{\omega_{ps}^2}{\omega^2}\right)\right]} \qquad \text{—— (15) ——}$$

where C_{s0}, ν_{si}, ω_{ps} are capacitance, ion collision frequency and ion angular plasma frequency in the sheath, respectively.

Sheath capacitance C_{s0} can not be measured directly, but we can estimate it considering as a coaxal codenser which consists of antenna and sheath. The antenna surface and the outer boundary of ion sheath being defined as the fransitional surface from ambient plasma to ion sheath, can be considered to form a pair of condenser electrodes, because the conductivity in the ambient plasma is very much larger than that in the ion sheath. Then, the calculation leads to

$$C_{s0} = \frac{2\pi \varepsilon_0}{l_n\left(\frac{R+r}{R}\right)} \cdot 4\,L \qquad \text{—— (16) ——}$$

where R is the antenna radius, r is the sheath's thickness and L is the antenna length respectively.

Rocket body too is coated with an ion sheath of the same nature as that of the antenna, thus it is necessary to consider the sheath impedance due to the rocket body. But the rocket radius is very larger than the antenna radius, and the sheath capacitance of rocket body must be very large. Therefore the sheath impedance due to rocket body can be neglected in comparison with that of the antenna. It is sufficient to cosider only the sheath impedacne of antenna.

To calculate C_{s0} given in Eq. (16), we must know the sheath's thichness r in the ionosphere. About this problem, there are the reports already published by several workers (Kane, et al., 1962; Whale, 1962;), but in our case, the idea of space charge limited-current in a cylindrical electrode in vacuum (Spangenberg, 1948) is adopted for the calcuation of the sheath's thickness. Therefore ion current in the sheath can be taken to be equal to the spac-charge limited current I_{si}, and it is written for a cylindrical antenna as follows,

$$I_{si} \simeq \frac{8\pi}{9}\sqrt{\frac{2e}{m_i}} \cdot \frac{\varepsilon_0 (4L)}{r\beta^2} (-V_0)^{3/2} \qquad \text{—— (17) ——}$$

where, β^2 is a function of $(R/R+r)$ and can be given theoretically (Spangenberg, 1948), $(-V_0)$ is the antenna surface potential with respect to the ambient plasma. I_{si} is equal to the random ion current $\int J_i ds$ in the plasma surrounding the sheath boundary surface, we have

$$\int_s J_i\, ds = \int_s n_i e V_i\, ds$$

$$= n_i e \sqrt{\frac{2kT_i}{m_i}} \int_s ds = n_i e \sqrt{\frac{2kT_i}{m_i}} \cdot 2\pi r (4L) \qquad \text{—— (18) ——}$$

From Eq. (17) and (18), we obtain

$$r = \left[\frac{4}{9} \cdot \frac{\varepsilon_0}{\beta^2 \sqrt{ke}} \cdot \frac{(-V_0)^{3/2}}{n_i \sqrt{T_i}}\right]^{1/2} \qquad \text{—— (19) ——}$$

For our antenna used, β^2 is nearly equal to 1. 1, so that substituting the numerical values for the constants including β^2 into (19), we have

$$r \simeq 4 \cdot 9 \times 10^4 \sqrt{\frac{(-V_0)^{3/2}}{n_i \sqrt{T_i}}} \qquad \text{—— (20) ——}$$

The valus of $(-V_0)$ is calulated theoretically by the following relation (Okada, et al., 1965).

$$(-V_0) \simeq 4 \cdot 3 \times 10^{-5}\, T_e\, l_n \left(\frac{m_i}{m_e}\right) \cdot \left(\frac{T_e}{T_i}\right) \qquad \text{—— (21) ——}$$

Then, If, we know electron and ion temperture, T_e, T_i, and ion density in the plasma, we can estimate C_{s0} from Eq. (16), (20) and (21).

To evaluate $\dot{Z}_s$ from Eq. (15), we must calculate the angular plasma frequency w_{ps} and the collision frequency ν_{si} in the sheath. But it is a troublesome problem, because these are a function of ion density in the sheath, what is more, ion density in the sheath is a function of position whose exact expression has not been well known at present. Thus, we shall assume that it nearly equals to ion density in the ambient plasma, therefore the angular plasma frequency in the sheath (ω_{ps}) also nearly equals to that in the ambient plsama (ω_{ps}).

Next, we must examine the collision frequency ν_{si} in the sheath. Unfortunately the expression for ν_{si} does not seems to be fully understood until to the present, thus we suppose here that $\nu_{si} \simeq \nu_i$ where ν_i is the collision frequency of ions with neutral particles in the ambient plasma which is given in Eq. (14).

In Eq. (15) ω is larger than 10^4 in the order of magnitude, since the observing frequency ranges from 3 to 60 KHz, So that, if ν_{si} does not reach the order of 10^4, we can neglect the term of ν_{si} in Eq. (15). In this case, Eq. (15) can be in the simplified as $\dot{Z}_{si} \simeq j\omega/C_{s0}\,\omega_{ps}^2$. If ν_{si} reaches the order of 10^4 or more in the lower ionosphere, the term can not be neglected no longer. And we must use Eq. (15) to calculate $\dot{Z}_{si}$. Substituting. into Eq. (15), the values of sheath capacitance C_{s0}, the angular plsama frequency ω_{ps}, and the collision frequency ν_{si}, we can get the sheath impedance $\dot{Z}_s$ in the ionosphere.

5. Numerical Calculation of Conversion Coeffcient

To calculate the conversion coefficient which converts the input voltage to receiver, into the field intensity in plasma, we must examine the variation with altitude in the ionosphere of density, temperature and collision frequency of electrons and ions to see it in the equations above mentioned. However, As the Radio noise measured with the sounding rocket K-9M-19 did not measure this kind of the quantity, we were obliged to use the standard values in the ionosphere at night which were measured or calculated by many workers.

Values adopted in our calculation is shown in following discussion. For electron and ion densities are estimated for the results obtained at night by S. Miyazaki in Japan (1966). That is

n_e : 1.0×10^2 ·········· 1.0×10^5 (cm^{-3})

n_i : 1.3×10^3 ·········· 1.0×10^5 (cm^{-3})

For electron and ion temperatues, we use the values obtained by J. V. Evans (1967). That is

T_e : 1.7×10^2 ·········· 2.2×10^3 (°K)

T_i : 1.7×10^2 ·········· 8.1×10^3 (°K)

For electron and ion collision frequencies, we use the values obtained by M. Nicolet (1953). That is,

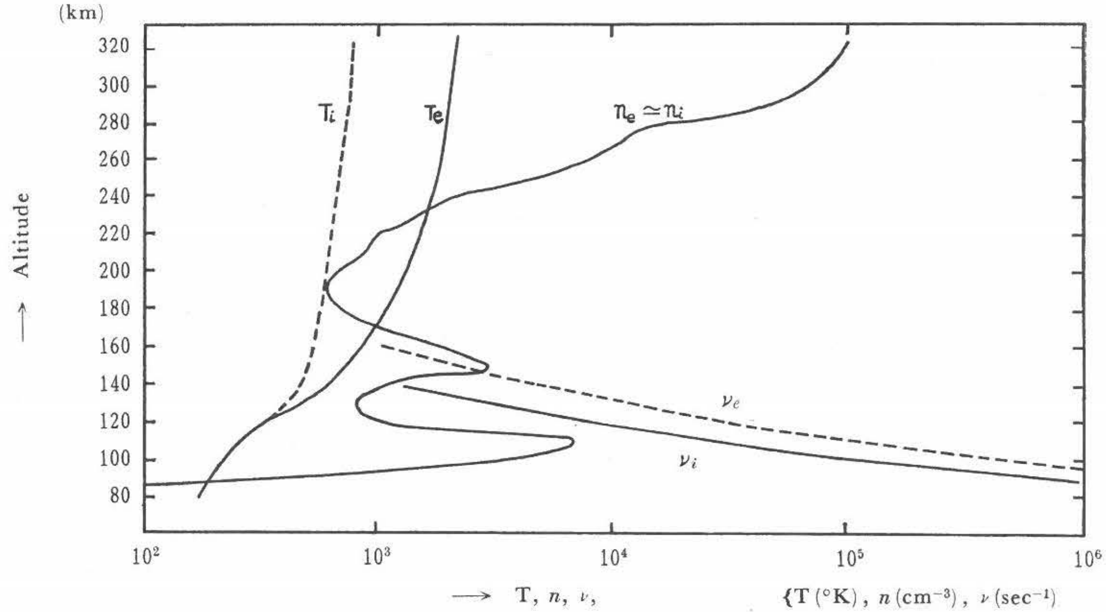

Fig. 2. Temperature (T), density (n), and Collision frequency (ν) of electron and ion in the night.

T_e, T_i : Temperature of electron and ion

n_e, n_i : Density of electron and ion

ν_e, ν_i : Collision frequency of electron and ion.

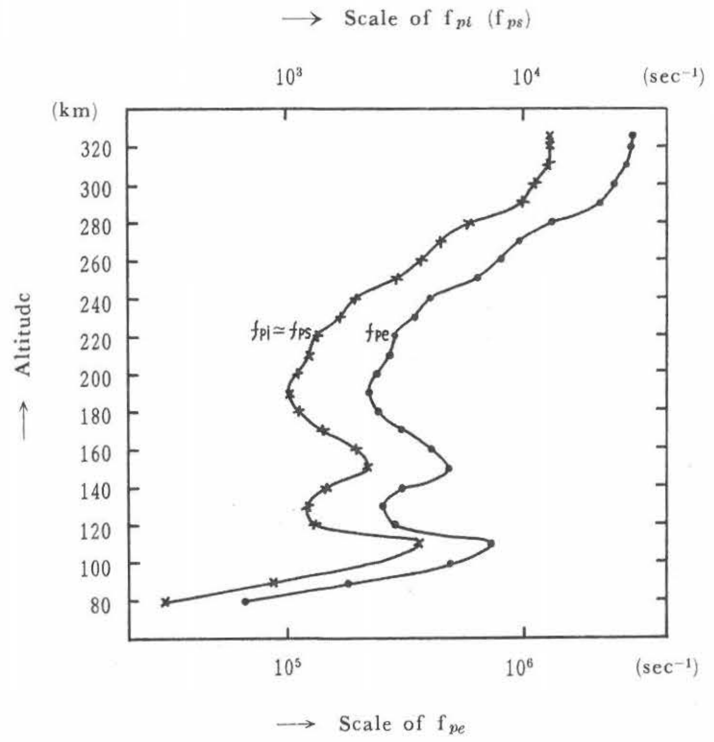

Fig. 3 The plasma frequency of ion (f_{pi}), electron (f_{pe}) and ion sheath (f_{ps}) in the night

ν_e : 10^3 ·········· 1.5×10^7 (sec^{-1})

ν_i : 10^3 ·········· 4.0×10^6 (sec^{-1})

The variations of these value with altiude are shown in Fig. 2.

Using these values, f_{pe}, f_{pi}, f_{ps} the plasma frequency of electrons, and of ions in and out of the sheath for altitude in the ionosphere are calculated and shown in Fig. 3.

By Using the values of T_e and T_i Fig. 2, the antenna surface potential ($-V_0$) referred to the ambient plasma is calcuated from Eq. 21 for the altitude in the ionosphere. Then the sheath's thichness r is also calculated from Eq. 20 by taking these calcualted values of ($-V_0$). Finally the capacitance C_{s0} is calculated from Eq. 16 using these values of r. These results are shown in Fig. 4.

Since the length of the whip antenna used in our experiment is very short compaired with the wave-lengh concerned, the antenna is considered to be the capacitive. So the antenna impedance in free space is defined by the capacity between the

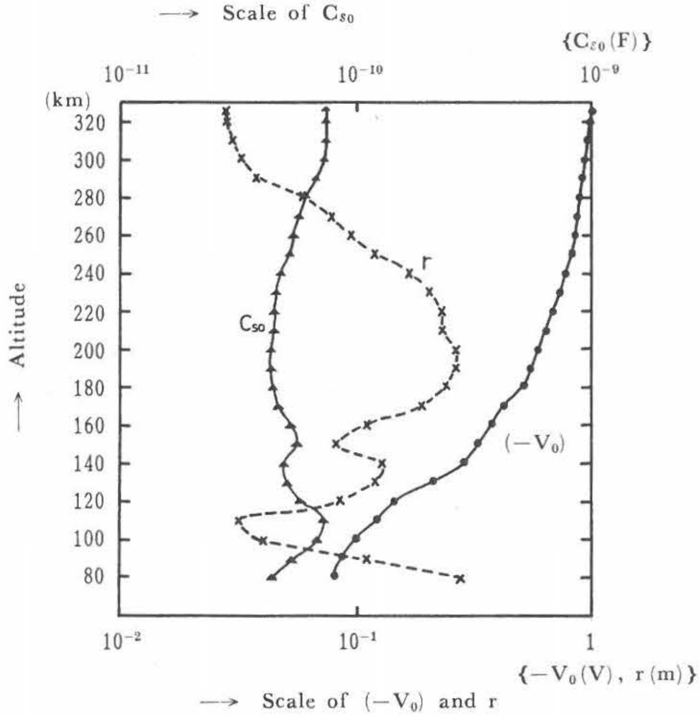

Fig. 4. The quantities in the sheath in the night; the antenna surface potential for the ambient plasma $(-V_0)$, the sheath thickness r. and the capacitance C_{s0}.

antenna and the rocket body and it can be measured easily. The capaciance C_{p0} in free space for the antenna used was measured to be 135 pF.

The antenna impedance in free space is easily converted into the antenna impedance in plasma by using Eq. 11 and 15.

Taking the values in Fig. 2, 3 and 4, the plasma and the sheath impedance are calculated from Eq. 11 and 15. Show in Fig. 5 the variation with altitude of the impedances for the frequencies 3, 10 and 21 KHz in the ionosphere.

In Fig. 5, we see that the plasma impedance varies in an inverse proportion to the electron density, while the sheath impedance does not follow the same trend. The latter is larger than the former in all altitudes, thus the antenna impedance is largely under the control of sheath impedance.

A remarkable peak being found in the sheath impedance profile at 250 km for 3 KHz and 290 km for 10 KHz indicate the point $\omega=\omega_{ps}$, which corresponds to the resonance of the sheath impedance.

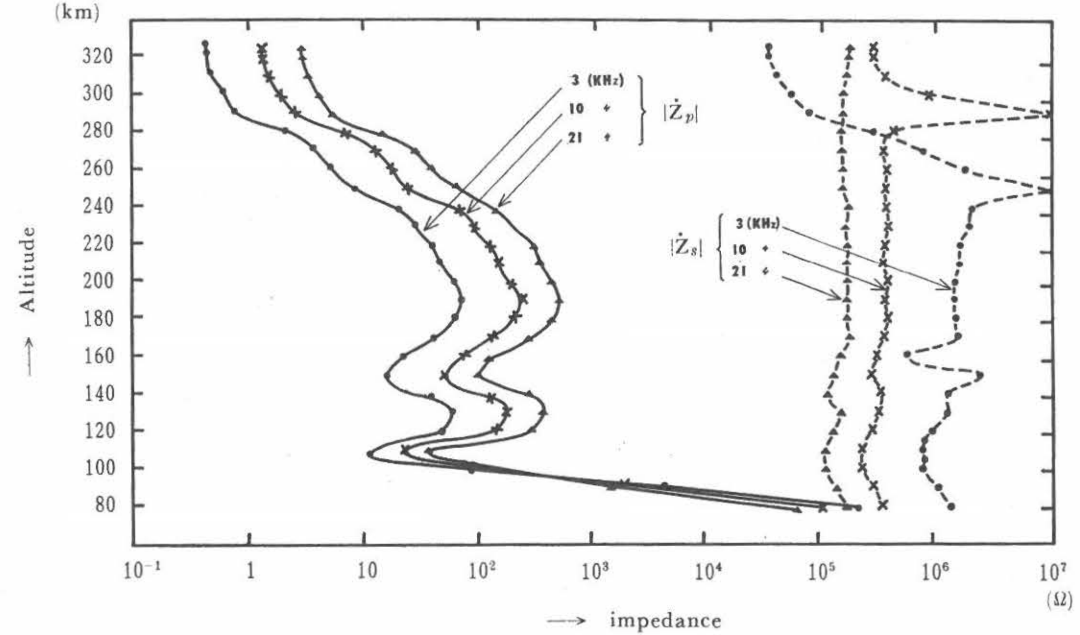

Fig. 5 Plasma impedance $|\dot{Z}_p|$ and ion sheath impedance $|\dot{Z}_s|$ in the night

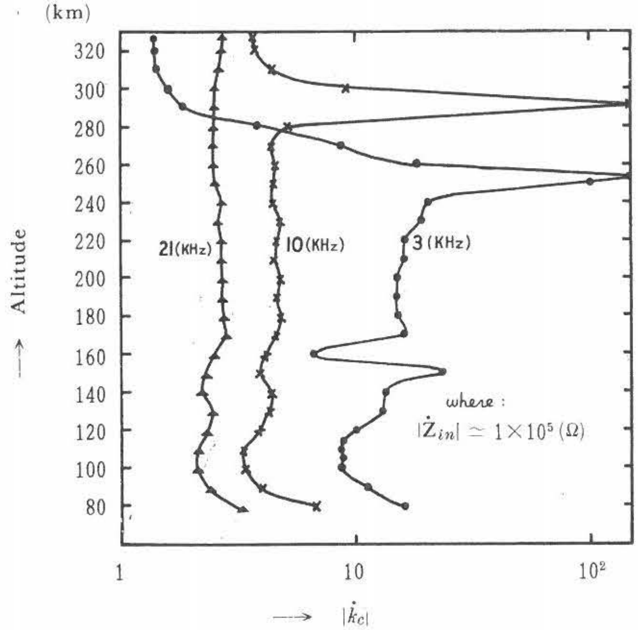

Fig. 6 Characteristics of the factor $|\dot{k}_c|$ $\left(=\left|\frac{\dot{Z}_p+\dot{Z}_s}{\dot{Z}_{in}}+1\right|\right)$ in the night for frequency 3, 10 and 21 (KHz)

Finally, taking the impedance values shown in Fig. 5, we can get the conversion coefficient $|\dot{k}_c|$ from Eq. 3. The variation with altitude of $|\dot{k}_c|$ in night time in the ionosphere, for the frequencies 3, 10 and 21 KHz are shown in Fig. 6.

We see that $|\dot{k}_c|$ varies in proportion to the sheath impedance $|\dot{Z}_{s}|$ and accordingly becomes larger as the frequency decreases with in limit altitudes below about 270 (km). But, in the higher alitude, the reverse tendency shows up with decreare in the frequency. The peaks appearing at 250 km for 3 KHz and 290 km for 10 KHz are caused by the ion sheath resonance.

6. Conclusion

In this paper, we derived on expression for the antenna impedance in the ionosphere starting from the equation of plasma dynamics and Maxwell's equation and obtained the conversion coefficient which converts the input voltage to receiver into the electric field intensity in the ionosphere. The characteristics of antanna impedance for the whip type in the ionosphere has been demonstrated for a whip type antenna system.

From the results obtained, we found that it is desirable to bring the conversion factor k_c as closely as possible to unity in all the altitudes concerned so as to minimize the influence of plasma, as well as of ion sheath in the ionosphere. To realize it, the input impedance to reeciver must be so determined as it has the largest possible value within the limit of antenna design.

On way of the calcuation, we supposed the influence of pressure in the ionospheric plasma to be very small, so as we get in Eq. 5, the following inequality

$$E_0 \omega_{pe}^2 \mathbf{E} \gg \left(\frac{\gamma_e k T_e e}{m_e} \nabla n_e - \frac{\gamma_i k T_i Ze}{m_i} \nabla n_i \right)$$

If we could confirm the validity of the assumption for frequency range 3-60 KHz, we would be able to see whether the ionospheric plasma being cold or warm in reference to the above inequality. To confirm this, we must have an exact measurement of the electric field component of electromagnetic waves in the ionosphere, which seems to be a very imoprtant problem remained in future.

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