Ph.D. Thesis

Generalization of Higgs Effective Field Theory

（ヒッグス有効理論の拡張）

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Abstract

The Standard Model (SM) in particle physics is successful but still incomplete in the sense that the electroweak scale is unstable under quantum corrections. In order to solve this problem, we need to understand an origin of electroweak symmetry breaking and high energy physics that realize the mechanisms protecting the electroweak scale from the large quantum corrections. On the other hand, in experimental side, Higgs boson couplings are planned to be measured more precisely in the LHC. Because Higgs sector is deeply related to the symmetry breaking, deviations from SM predictions may enable us to extract information about the high energy physics. In such a situation, we have pursued the research to answer the following question: *how can we extract information about the new particles' properties such as spin, charge, helicity etc. from deviation patterns in Higgs precision measurements?*

For the purpose of obtaining model-independent information about the new heavy particles, it is well known that the Standard Model Effective Field Theory (SMEFT) is useful. In the SMEFT, it is, however, implicitly assumed that the observed Higgs boson is coming from the $SU(2)_L$ scalar doublet. In order to remove this assumption and consider the most general set up, we have focused on the Higgs Effective Field Theory (HEFT), which is constructed by adding the observed Higgs boson to the electroweak chiral perturbation theory. Furthermore, we have generalized the HEFT so that it includes additional scalar degrees of freedom. We also calculated one of the most precisely measured parameters: the Peskin Takeuchi S, T, U parameters. We saw theoretical correlations between these parameters and four-point scattering amplitudes, and found nontrivial relations.

Contents

Chapter 1

Introduction

In particle physics, there is one successful theory, the Standard Model (SM), which gives predictions reproducing various measurements correctly and precisely. In 2012, the last piece of the SM, the Higgs boson, was discovered at the Large Hadron Collider (LHC) [1,2], and all the particles predicted by the SM have been discovered. Now, the collider experiments in particle physics enter the new stage: the precision measurements of the observed particles' properties. In the near future, the Higgs couplings to the SM vectors (hVV) and fermions ($hf\bar{f}$) will be measured in $\mathcal{O}(1)\%$ accuracy in the LHC Run3, HL-LHC, and hopefully, ILC experiments.

Despite the brilliant successes on the experimental side, the SM has serious problems on the theoretical side called *hierarchy problem* or *naturalness problem*. This problem is deeply related to the Higgs boson mentioned above. In the SM, the Higgs boson causes the electroweak symmetry breaking at the energy scale of $\mathcal{O}(10^2)$ GeV. This energy scale is far below the cut-off scale of the SM, which is expected to be $\mathcal{O}(10^{16})$ GeV. In order to realize such a high hierarchy, we must expect unnaturally large cancellation between quantum corrections and the bare mass scale of the Higgs boson.

Considering these situations, we want to ask one simple question: *is the scalar boson discovered at the LHC really the Higgs boson predicted by the SM?* In many new physics models beyond the SM, the electroweak symmetry breaking sector is extended from that of the SM, and some of them predict new scalar bosons other than the observed Higgs boson. If the discovered scalar boson is not the Higgs boson predicted by the SM, the sign of the new physics may appear in the future Higgs precision measurements: Higgs couplings such as hVV and $hf\bar{f}$ may turn out to deviate from the SM predictions.

One important thing is that if hVV couplings turn out to deviate from the SM prediction, it may cause a serious problem involving unitarity: the scattering amplitude of longitudinal gauge boson mode, $V_L V_L \to V_L V_L$, shows energy growing behavior and it exceeds the upper bound coming from the unitarity arguments. In the SM, the energy growing behavior of $V_L V_L \to V_L V_L$ is completely canceled out thanks to the appropriately tuned hVV coupling, but if hVV deviates from the SM prediction, the observed scalar boson cannot restore the unitarity of $V_L V_L \to V_L V_L$ scattering completely. To cure this unitarity violation, new scalar fields should appear to cancel the remaining energy growing behavior. The unitarity conditions for $V_L V_L \to V_L V_L$ scattering are good probes for investigating new physics. In this thesis, we try to approach the new physics beyond the SM from the future precision measurements of the Higgs couplings, focusing on the unitarity conditions as one of the probes for new physics search.

The deviations from the SM predictions can be described by the effective theory approach. The Higgs Effective Field Theory (HEFT) is one of the most general effective field theory written in terms of the SM matter fields [3–19]. The HEFT is quite useful to parametrize the deviations from the SM predictions, but it also has disadvantages: in the framework of the HEFT, the heavy particle degrees of freedom are integrated out, so we cannot calculate the physical processes with these heavy particles appearing in the initial and the final state, such as production cross section and the decay rate of the heavy particles. In order to obtain concrete predictions on the properties of new particles, we must extend the HEFT so that it includes new particle degrees of freedom.

In this thesis, we extend the HEFT so that it includes the arbitrary number of neutral and charged scalar fields, and formulate "Generalized Higgs Effective Field Theory (GHEFT)." Using the GHEFT framework, we derive the unitarity conditions of $V_L V_L \to V_L V_L$ scattering amplitude. We also calculate one of the most precisely measured parameters, Peskin-Takeuchi S, T, U parameters in the GHEFT. Focusing on the geometry of the field space, we express these unitarity conditions and the S, T, U parameters in the covariant form. Furthermore, we derive the theoretical correlation between the unitarity conditions and oblique parameters in the model-independent manner.

This thesis is organized as follows: In Chap. 2, we will review the Standard Model (SM) in particle physics. In Chap. 3, we will review the Higgs Effective Field Theory (HEFT) and its key ingredient, the non-linearly realized symmetry. In Chap. 4, we will extend the HEFT so that it includes the arbitrary number of neutral and charged scalar fields, and formulate "Generalized Higgs Effective Field Theory (GHEFT)." In Chap. 5, we will point out difficulties in the analysis based on the effective field theory approach, and introduce a technique for overcoming the disadvantages.

In Chap. 6, we will derive a series of conditions for respecting the unitarity of $V_L V_L \to V_L V_L$ scattering amplitude in the GHEFT framework. In Chap. 7, we will calculate oblique parameters S, T, U in the GHEFT. In Chap. 8, we will relate the unitarity conditions derived in Chap. 6 and the expressions of oblique parameters derived in Chap. 7 by focusing on the geometry of the field space. We will summarize this thesis in Chap. 9.

Chapter 2

Standard Model in Particle Physics

The Standard Model in particle physics is a quite successful theory, established by S. L. Glashow, S. Weinberg, and A. Salam in the 1960s, constructed based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. In this chapter, we will briefly review the SM and its fundamental ingredients, spontaneous symmetry breaking.

2.1 Framework

Various phenomena observed in nature can be described by the behavior of elementary particles. The behavior of elementary particles are described by four fundamental interactions: gravitational interaction, strong interaction, weak interaction, and electromagnetic interaction. Among these four interactions, gravitational interaction is extremely weak. If we consider quite heavy particles, we must take the gravitational interaction into account, but because masses of elementary particles are quite small compared with the Planck scale, which is the mass scale where gravitational interaction become strong and non-negligible, we can ignore gravitational force as long as we focus on the interactions among the elementary particles. The remaining three interactions, strong interaction, weak interaction, and electromagnetic interaction can be successfully described by a framework of gauge symmetry.

Gauge symmetry of the SM is given by $SU(3)_C \times SU(2)_L \times U(1)_Y$. Associated with each gauge symmetry, a spin one vector boson called gauge boson is introduced, and these gauge bosons control the fundamental three interactions mentioned above. The gauge boson associated with $SU(3)_C$ gauge symmetry is called *gluon* and it rules the strong interaction. The gauge bosons associated with $SU(2)_L \times U(1)_Y$ symmetry are called *electroweak gauge bosons* altogether. As we will mention in Sec. 2.2, $SU(2)_L \times U(1)_Y$ is spontaneously broken to its subgroup $U(1)_{\rm em}$, generating one massless and three massive vector bosons. The massless gauge boson associated with unbroken $U(1)_{\rm em}$ symmetry is called *photon* and rules electromagnetic interaction. The three massive vector bosons are called *weak bosons*, controlling weak interaction.

Other than the gauge bosons controlling the fundamental interactions, the SM includes spin one-half fermions and spin-zero scalar boson as its matter contents. In Table 2.1, we listed all the matter fields comprising the SM, together with their representations under Lorentz symmetry and $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry.

	spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
G_μ	1	$\mathbf{8}$	$\mathbf{1}$	0
W_μ	1	$\mathbf{1}$	$\mathbf{3}$	0
B_μ	1	$\mathbf{1}$	$\mathbf{1}$	0
$q_L^i = (u_L^i, d_L^i)^T$	1/2	$\mathbf{3}$	$\mathbf{2}$	$+1/6$
u_R^i	1/2	$\mathbf{3}$	$\mathbf{1}$	$+2/3$
d_R^i	1/2	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
$l_L^i = (\nu_L^i, e_L^i)^T$	1/2	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_R^i	1/2	$\mathbf{1}$	$\mathbf{1}$	-1
H	0	$\mathbf{1}$	$\mathbf{2}$	$-1/2$

Table 2.1: Matter contents in the Standard Model

G_μ, W_μ, and B_μ in the first row of Table 2.1 denote gluon, $SU(2)_L$ electroweak gauge bosons, and $U(1)_Y$ electroweak gauge boson, respectively.

The spin one-half particles in the second row, q_L^i, u_R^i, and d_R^i, are fermions called *quarks*, and the spin one-half particles in the third row, l_L^i and e_R^i, are fermions called *leptons*. Contrary to the gauge bosons, these fermions comprise matters in nature. For a famous example, the electron in the third row of Table 2.1 comprises atoms together with the nucleus, and the nucleus is made of protons and neutrons, which are composed of quarks combined by the strong interaction.

The quarks and leptons have three kinds, which are labeled by index "i" running from 1 to 3. In particle physics, we call these particle sets generations and so the SM has three generations of quarks and leptons. In Table 2.2, we listed all the three generations of quarks and leptons.

The significant feature of the SM fermions is that the left-handed fermions and the right-handed fermions are assigned different representations under electroweak gauge symmetry $SU(2)_L \times U(1)_Y$. For example, as you can find in the third row of Table 2.1, left-handed electron e_L and right handed electron e_R have different quantum numbers under $SU(2)_L \times U(1)_Y$: the former belongs to $SU(2)_L$ doublet and is assigned $-1/2$ as $U(1)_Y$ hypercharge, while the latter is $SU(2)_L$ singlet and its $U(1)_Y$ hypercharge is -1. Such fermions are called chiral fermions and, as we will see later, the chiral fermions cannot have mass terms. This means quarks and leptons are massless in the SM and they move around at the speed of light, which is far from our intuition. This description is drastically changed after the spontaneously breaking of electroweak symmetry occurs. We will explain the details of this topic in the next section.

The last particle denoted by H in the fourth row of Table 2.1 is a scalar field called Higgs doublet. As we will see in Sec. 2.2, this Higgs doublet plays a crucial role in spontaneously breaking of the electroweak symmetry.

1st generation ($i=1$)	2nd generation ($i=2$)	3rd generation ($i=3$)
$q_L^1=(u_L,d_L)^T$	$q_L^2=(c_L,s_L)^T$	$q_L^3=(t_L,b_L)^T$
u_R	c_R	t_R
d_R	s_R	b_R
$l_L^1=(\nu_{eL},e_L)^T$	$l_L^2=(\nu_{\mu L},\mu_L)^T$	$l_L^3=(\nu_{\tau L},\tau_L)^T$
e_R	μ_R	τ_R

Table 2.2: Generations of quarks and leptons in the Standard Model

The SM Lagrangian is given by writing down all the invariant operators under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry [1] ,

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermion}\,, \tag{2.1}$$

where $\mathcal{L}_{gauge}$, $\mathcal{L}_{scalar}$, and $\mathcal{L}_{fermion}$ each denotes the gauge sector, scalar sector, and fermion sector, respectively. In the remaining of this section, we will closely see each sector in turn.

Gauge sector

The Lagrangian of the gauge sector in the SM is given by the kinetic terms of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge fields with its explicit form given by

$$\mathcal{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,, \tag{2.2}$$

where $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$ are field strength tensors of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively. Their explicit form is written in terms of gauge fields listed in Table 2.1

[1] Other than operators listed in the SM Lagrangian, we can write down another $SU(3)\times SU(2)_L\times U(1)_Y$ invariant, CP violating operator so called theta term. This operator involves electromagnetic dipole moment (EDM). The EDM is a quite important topic in particle phenomenology, but is beyond the scope of this thesis.

as

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig_C[G_\mu, G_\nu]\,, \tag{2.3}$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig_W[W_\mu, W_\nu]\,, \tag{2.4}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \tag{2.5}$$

where g_C and g_W are gauge coupling of $SU(3)_C$ and $SU(2)_L$, respectively. G_μ and W_μ are given as

$$G_\mu := \sum_{A=1}^{8} G_\mu^A \frac{\lambda^A}{2}\,, \qquad W_\mu := \sum_{a=1}^{3} W_\mu^a \frac{\tau^a}{2}\,, \tag{2.6}$$

with λ^A being Gell-mann matrix and τ^a ($a = 1 \sim 3$) being Pauli matrix. The explicit form of Gell-mann matrices is given as

$$\begin{aligned}
\lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},
\end{aligned} \tag{2.7}$$

while that of Pauli matrices are given as

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.8}$$

The transformation law of each gauge field under each gauge symmetry is given by

$$SU(3)_C \;:\; G_\mu \to \mathfrak{g}_C G_\mu \mathfrak{g}_C^\dagger + \frac{i}{g_C} \mathfrak{g}_C \partial_\mu \mathfrak{g}_C^\dagger\,, \tag{2.9}$$

$$SU(2)_W \ : \ W_\mu \to \mathfrak{g}_W W_\mu \mathfrak{g}_W^\dagger + \frac{i}{g_W}\mathfrak{g}_W \partial_\mu \mathfrak{g}_W^\dagger \,, \tag{2.10}$$

$$U(1)_Y \ : \ B_\mu \to B_\mu + \frac{1}{g_Y}\partial_\mu \theta_Y \,, \tag{2.11}$$

where $\mathfrak{g}_C$ and $\mathfrak{g}_W$ are $SU(3)_C$ and $SU(2)_L$ transformation matrix, respectively, and their explicit form is given as

$$\mathfrak{g}_C = \exp\left(i\sum_{A=1}^{8}\theta_C^A \frac{\lambda^A}{2}\right), \tag{2.12}$$

$$\mathfrak{g}_W = \exp\left(i\sum_{a=1}^{3}\theta_W^a \frac{\tau^a}{2}\right). \tag{2.13}$$

θ_C^A, θ_W^a, and θ_Y are gauge parameters and all of them are the real numbers. We can easily find that Lagrangian of the gauge sector (2.2) is invariant under $SU(3)_C \times SU(2) \times U(1)_Y$ transformation with arbitrary θ_C^A, θ_W^a, and θ_Y.

Scalar sector

The Lagrangian of the scalar sector in the SM is composed of a kinetic term of Higgs doublet H and a scalar potential V:

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger D^\mu H - V(H) \,, \tag{2.14}$$

with

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4 \,. \tag{2.15}$$

The covariant derivative $D_\mu H$ is given as

$$D_\mu H = \left(\partial_\mu - i g_W W_\mu - i g_Y \frac{1}{2} B_\mu\right) H \,. \tag{2.16}$$

The transformation law of Higgs doublet H under $SU(2)_L \times U(1)_Y$ are given as

$$SU(2)_L \;:\; H \to \mathfrak{g}_W H\,, \tag{2.17}$$

$$U(1)_Y \;:\; H \to e^{i\frac{1}{2}\theta_Y} H\,, \tag{2.18}$$

with $\mathfrak{g}_W$ given by Eq.(2.13). We can easily check that the covariant derivative given by Eq.(2.16) transforms covariantly under $SU(2)_L \times U(1)_Y$.

In the SM, the mass of Higgs doublet H is taken to be negative, $\mu^2 < 0$. Due to this, the scalar potential V has an extreme minimum at the nonzero field value. This causes the electroweak symmetry breaking. We will explain its details in the next section.

Fermion sector

The Lagrangian of the fermion sector in the SM is composed of fermion kinetic terms and Yukawa terms, with its explicit form given as

$$\begin{aligned}\mathcal{L}_{fermion} = &\sum_{i=1}^{3} \bar{q}_L^i i\not{D} q_L^i + \sum_{i=1}^{3} \bar{l}_L^i i\not{D} l_L^i + \sum_{i=1}^{3} \bar{u}_R^i i\not{D} u_R^i + \sum_{i=1}^{3} \bar{d}_R^i i\not{D} d_R^i + \sum_{i=1}^{3} \bar{e}_R^i i\not{D} e_R^i \\ &+ \left(\sum_{i,j=1}^{3} y_{ij}^u \bar{q}_L^i \tilde{H} u_R^j + \sum_{i,j=1}^{3} y_{ij}^d \bar{q}_L^i H d_R^j + \sum_{i,j=1}^{3} y_{ij}^e \bar{l}_L^i H e_R^j + \text{h.c.} \right)\,,\end{aligned} \tag{2.19}$$

where $\tilde{H}$ in the second line is defines as

$$\tilde{H} := i\tau^2 H^*\,. \tag{2.20}$$

The covariant derivative for each fermion field is given as follows:

$$D_\mu q_L = \left(\partial_\mu - ig_C G_\mu - ig_W W_\mu - ig_Y \frac{1}{6} B_\mu\right) q_L\,, \tag{2.21}$$

$$D_\mu u_R = \left(\partial_\mu - ig_C G_\mu - ig_Y \frac{2}{3} B_\mu\right) u_R\,, \tag{2.22}$$

$$D_\mu d_R = \left(\partial_\mu - ig_C G_\mu - ig_Y \frac{-1}{3} B_\mu\right) d_R \,, \tag{2.23}$$

$$D_\mu l_L = \left(\partial_\mu - ig_W W_\mu - ig_Y \frac{-1}{2} B_\mu\right) l_L \,, \tag{2.24}$$

$$D_\mu e_R = \left(\partial_\mu - ig_Y (-1) B_\mu\right) e_R \,, \tag{2.25}$$

with flavor indices suppressed. y^u_{ij}, y^d_{ij}, and y^e_{ij} are three by three Yukawa matrices and their components can take arbitrary complex values.

Before closing this section, we will briefly mention the fermion mass terms in the SM. As you can see in the fermion sector given by Eq.(2.19), the SM fermions do not have mass terms. This is because the SM fermion is chiral, namely, left-handed fermions and right-handed fermions are assigned different quantum numbers under $SU(2)_L \times U(1)_Y$ symmetry. As shown in Table 2.1, all the fermions in the SM have nonzero $U(1)_Y$ hypercharge and so the mass terms of charged fermion are given by Dirac mass terms,

$$\Delta\mathcal{L}_{\text{mass}} = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \,, \tag{2.26}$$

with ψ_L and ψ_R denoting a left-handed and a right-handed Dirac fermion, respectively. A significant feature of the Dirac mass term is that it inevitably combines the left-handed fermion ψ_L and the right-handed fermion ψ_R. Therefore, if ψ_L and ψ_R have different quantum numbers under underlying symmetry, the mass terms (2.26) cannot be invariant under the symmetry. The same thing happens in the SM. Because the left- and right-handed SM fermions are assigned different quantum numbers under $SU(2)_L \times U(1)_Y$, their mass terns are forbidden by $SU(2)_L \times U(1)_Y$. In the real world, however, we can see that matters around us have nonzero masses. This puzzle is resolved by the symmetry breaking of the $SU(2)_L \times U(1)_Y$, which will be treated in the next section.

So far, we have explained the details of the SM framework. As we mentioned above, the SM gives descriptions that are far from our intuition: all the SM fermions are massless and they move around at the speed of light. In the next section, we will show how to fix this problem and introduce a mechanism generating masses of gauge bosons, quarks, and leptons.

2.2 Spontaneous symmetry breaking

In the previous section, we reviewed the foundation of the SM and found that the SM is the chiral theory: each SM fermion is assigned different quantum number according to its handedness under the $SU(2)_L \times U(1)_Y$ symmetry. Because the fermion mass terms inevitably combine the left-handed and right-handed fermions, we can not write down the mass terms in the chiral theory. In this section, we will focus on this topic.

As we briefly mentioned in the previous section, the scalar sector of the SM, given by Eq.(2.14) plays a crucial rule in electroweak symmetry breaking. The scalar potential V in the SM scalar sector is given by

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4 \,. \tag{2.27}$$

Due to the negative mass squared, $\mu^2 < 0$, the scalar potential V has extreme minimum at the nonzero field value. The field value at the extreme minimum is called vacuum expectation value and in this case, its explicit form is given by

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \,, \tag{2.28}$$

with

$$v = \sqrt{\frac{-\mu^2}{\lambda}} \,. \tag{2.29}$$

The nonzero vacuum expectation value of the Higgs doublet cause the electroweak symmetry breaking. This can be easily seen by acting $SU(2)_L \times U(1)_Y$ transformation matrix on the vacuum expectation value given by Eq.(2.28). The transformation law of Higgs doublet H under $SU(2)_L$ and $U(1)_Y$ symmetry is given by Eq.(2.17) and Eq.(2.18), respectively. If we take the transformation parameters θ_W^a and θ_Y quite small and consider infinitesimal transformation under $SU(2)_L \times U(1)_Y$, H transforms under each symmetry as

$$SU(2)_L \ : \ H \to H + \delta_W H \,, \tag{2.30}$$

$$U(1)_Y \ : \ H \to H + \delta_Y H \,, \tag{2.31}$$

with

$$\delta_W H := i \left(\sum_{a=1}^{3} \theta_W^a \frac{\tau^a}{2} \right) H \,, \qquad \delta_Y H := i\theta_Y \frac{1}{2} H \,. \tag{2.32}$$

Replacing Higgs doublet H in Eq.(2.32) with its vacuum expectation value $\langle H \rangle$ given by Eq.(2.28), we can easily see that vacuum expectation value $\langle H \rangle$ breaks $SU(2)_L \times U(1)_Y$ symmetry because it changes its value under $SU(2)_L$ and $U(1)_Y$ transformation:

$$\left(\sum_{a=1}^{3} \theta_W^a \frac{\tau^a}{2} \right) \langle H \rangle \neq 0 \,, \tag{2.33}$$

$$\theta_Y \frac{1}{2} \langle H \rangle \neq 0 \,. \tag{2.34}$$

Considering the special transformation with $\theta_W^1 = \theta_W^2 = 0$ and $\theta_W^3 = \theta_Y$, however, we can keep $\langle H \rangle$ invariant:

$$\theta_{\rm em} \left(\frac{\tau^3}{2} + \frac{\mathbf{1}_2}{2} \right) \langle H \rangle = 0 \,, \tag{2.35}$$

with

$$\theta_{\rm em} := \theta_W^3 = \theta_Y \,. \tag{2.36}$$

This means that $\langle H \rangle$ breaks $SU(2)_L \times U(1)_Y$ to the diagonal combination of $U(1)_L$ and $U(1)_Y$, where $U(1)_L$ denotes the subgroup of $SU(2)_L$ with its generator given by $\tau^3/2$. As notation suggests, this diagonal combination is the symmetry associated with electromagnetic interaction, $U(1)_{\rm em}$. Therefore, the nonzero vacuum expectation value of Higgs doublet $\langle H \rangle$ breaks $SU(2)_L \times U(1)_Y$ symmetry to its subgroup

$U(1)_{\rm em}$.

It is worth pointing out that the field value of H giving an extreme minimum of the scalar potential V is not only the value given by Eq.(2.28), but there are a series of equivalent vacuum. When Higgs doublet H takes nonzero vacuum expectation value, only one vacuum is chosen from the series of equivalent vacuum and this cause symmetry breaking of $SU(2)_L \times U(1)_Y$. Such symmetry breaking is called *spontaneous symmetry breaking*. As we will see in the remainder of this section, spontaneous symmetry breaking plays a crucial role in giving masses to the SM gauge bosons and fermions.

Now, let's consider the consequence of spontaneously breaking of the electroweak symmetry. Rewriting Higgs doublet H in terms of its vacuum expectation value given by Eq.(2.28) and the fluctuations from the vacuum,

$$H = \begin{pmatrix} w^+ \\ \\ (v + h + iz)/\sqrt{2} \end{pmatrix} , \tag{2.37}$$

and substituting Eq.(2.37) into the scalar sector (2.14), we get Lagrangian in the broken phase. From the potential term of Eq.(2.14), we get

$$\begin{aligned} V(H) = \frac{1}{2} m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 + \lambda (w^+ w^-) h^2 + \frac{\lambda}{2} h^2 z^2 \\ + 2\lambda v (w^+ w^-) h + \lambda v z h + \lambda (w^+ w^-)^2 + \lambda (w^+ w^-) z^2 + \frac{\lambda}{4} z^4 \, , \end{aligned} \tag{2.38}$$

with the mass of h given by

$$m_h^2 = \sqrt{2\lambda} v \, . \tag{2.39}$$

The scalar matter field h is called Higgs boson. In 2012, the scalar boson with its properties consistent with the SM Higgs boson was finally discovered in the LHC and all the particles predicted by the SM are observed. The mass of the Higgs boson is measured and turns out to be

$$m_h^2 = (125\text{GeV})^2 \,. \tag{2.40}$$

As you can see in Eq.(2.38), scalar fields $w^\pm$ and z have no mass terms. These massless scalar bosons correspond to the fluctuations along with the flat directions of the scalar potential: as we mentioned previously, in the spontaneous symmetry breaking, one vacuum is chosen from the series of equivalent vacuum, so there always exist the flat direction starting from chosen vacuum to the other equivalent vacuum. The massless scalar bosons resulting from spontaneous symmetry breaking are called *Nambu-Goldstone bosons* (*NG bosons*). From Goldstone theorem, the number of NG bosons are equal to the number of broken generators. In the case of electroweak symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$, there are three broken generators and so the three NG bosons should exist, which are denoted by $w^\pm$ and z in Eq.(2.38).

From the kinetic term of the Higgs doublet in Eq.(2.14), we get the mass terms of gauge bosons

$$(D_\mu H)^\dagger (D^\mu H) \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \,, \tag{2.41}$$

where weak bosons $W^\pm$ and Z are the linear combinations of $SU(2)_L \times U(1)_Y$ electroweak gauge bosons:

$$W_\mu^\pm := \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}} \,, \qquad Z_\mu := \frac{g_W W_\mu^3 - g_Y B_\mu}{\sqrt{g_W^2 + g_Y^2}} \,. \tag{2.42}$$

Note that the masses of weak bosons $W^\pm$, Z are expressed as

$$m_W^2 := \frac{1}{4} g_W^2 v^2 \,, \qquad m_Z^2 := \frac{1}{4} \sqrt{g_W^2 + g_Y^2} v^2 \,. \tag{2.43}$$

It is worth mentioning degrees of freedom before and after the electroweak symmetry breaking. A massless vector boson has two physical degrees of freedom called transverse modes, while a massive vector boson has three physical degrees of freedom, a longitudinal mode and two transverse modes. Therefore, in order for a massless vector boson to acquire its mass, they need to absorb one degree of freedom. In the case of the SM, three gauge bosons, $W^{\pm}$ and Z, acquire their masses after electroweak symmetry breaking, so three degrees of freedom should be provided to these three gauge bosons. Where do these additional degrees of freedom come from? Actually, they are provided by three NG bosons arising in the process of spontaneous symmetry breaking: $W^{\pm}$ and Z absorb massless NG bosons $w^{\pm}$ and z, respectively, and obtain the longitudinal mode to become massive. The mechanism for massless gauge bosons acquiring their masses through the absorption of would-be NG bosons is called the *Higgs mechanism*, and plays a crucial role in generating the gauge bosons' masses in the SM. As we will see in the next section, the Higgs mechanism may cause a serious problem involving unitarity, however. The unitarity violation caused by the longitudinal gauge bosons is one of the main topics of this thesis.

Before closing this section, we will mention the masses of the SM fermions after the electroweak symmetry breaking. Replacing the Higgs doublet H in the fermion sector (2.19) with its vacuum expectation value $\langle H \rangle$, we can easily see that the Yukawa terms in the second line of Eq.(2.19) generate the mass terms of the SM fermions. For a concreat example, from the Yukawa term of lepton fields, we get

$$\sum_{i,j=1}^{3} y^e_{ij} \bar{l}^i_L \langle H \rangle e^j_R = \sum_{i,j=1}^{3} \frac{y^e_{ij}}{\sqrt{2}} v \bar{e}^i_L e^j_R \,. \tag{2.44}$$

Diagonalizing Yukawa matrix y^e_{ij} by appropriate field redefinition, we get the mass terms of the SM leptons.

2.3 Perturbative unitarity

As we mentioned in the previous section, after electroweak symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken to its subgroup $U(1)_{\text{em}}$, weak bosons acquire the third degree of freedom, longitudinal mode, and become massive. The longitudinal mode of massive gauge bosons may cause a serious problem involving unitarity, however: the scattering amplitude of the longitudinal mode may show an energy growing behavior and exceed the upper bound coming from the unitarity argument. In this section, we will see the details of this topic.

Before the electroweak symmetry breaking, all the gauge bosons are massless and each of them has two transverse modes denoted by $\epsilon^\mu_{T_1}$ and $\epsilon^\mu_{T_2}$. The important features of these transverse modes is that they are orthogonal to the momentum vector k^μ:

$$k_\mu \epsilon^\mu_{T_1}(k) = k_\mu \epsilon^\mu_{T_2}(k) = 0\,. \tag{2.45}$$

After the electroweak symmetry breaking, weak gauge bosons acquire the third degree of freedom, longitudinal mode, by absorbing NG boson (Higgs mechanism). The explicit form of the polarization vector of the longitudinal mode ϵ^μ_L with its momentum being $k^\mu = (E_{\mathbf{k}}, 0, 0, k)$ is given as

$$\epsilon^\mu_L(k) = (\frac{k}{m}, 0, 0, \frac{E_{\mathbf{k}}}{m})\,. \tag{2.46}$$

The significant feature of the longitudinal mode, which is absent in the transverse modes, is that as the energy of the gauge boson increases, the polarization vector ϵ^μ_L become gradually parallel to its momentum k^μ:

$$\epsilon^\mu_L(k) = \frac{k^\mu}{m} + \mathcal{O}(m/E_{\mathbf{k}})\,. \tag{2.47}$$

This means that the scattering amplitude of longitudinal modes picks up a momentum dependence through the polarization vectors of the external gauge bosons, and shows energy growing behavior. This is not good from the view of unitarity arguments: in order for the scattering amplitude to be calculated perturbatively, they should not exceed upper bound coming from the unitarity of the scattering amplitude. We will explain the details of perturbative unitarity in the remainder of this section.

The scattering angle dependence of each scattering amplitude $\mathcal{A}$ can be parametrized by its expansion coefficients of Legendre polynomials,

$$\mathcal{A} = 16\pi \sum_l^\infty (2l+1) a_l P_l(\cos\theta)\,. \tag{2.48}$$

Each coefficient a_l has upper bound called *unitarity bound* coming from the dispersion relation. The absolute value of coefficient a_l must be equal or smaller than unity,

$$|a_l| \leq 1\,. \tag{2.49}$$

In Eq.(2.49), the coefficient a_l is interpreted to be calculated in all order of the perturbation. In the practical situation, however, we do not calculate scattering amplitude $\mathcal{A}$ in all order, but only consider the leading order denoted by $\mathcal{A}^{\text{tree}}$,

$$\mathcal{A}_{\text{tree}} = 16\pi \sum_{l}^{\infty} (2l+1) a_l^{\text{tree}} P_l(\cos\theta)\,. \tag{2.50}$$

If the perturbation works properly, the unitarity bound (2.49) should be respected by the leading order of the perturbation,

$$|a_l^{\text{tree}}| \leq 1\,, \tag{2.51}$$

otherwise, we must take into account the higher order contribution to restore the unitarity and this means the failure of the perturbation. Therefore, checking whether the unitarity bound (2.49) is respected by the leading order can be used to test the validity of the perturbation. If Eq.(2.50) is satisfied, we can say that the scattering amplitude respect the *tree level unitarity* or *perturbative unitarity*.

As we mentioned above, the scattering amplitude involving longitudinal mode of massive gauge bosons, $V_L V_L \to V_L V_L$, shows energy growing behavior. Therefore, it can easily exceed the unitarity bound at the high energy. In the case of the SM, however, these energy growing behavior are completely canceled out by the contributions coming from the Higgs exchange diagrams. In Fig. 2.1, we show a series of diagrams contributing to $V_L V_L \to V_L V_L$ scattering amplitude in the SM. The Higgs exchange diagrams which restore the unitarity are shown in the red color in Fig. 2.1. Thanks to appropriately tuned couplings among observed Higgs bosons and electroweak gauge fields, shortly the hVV couplings, the Higgs exchange diagrams completely cancel energy growing behavior and restore the perturbative unitarity.

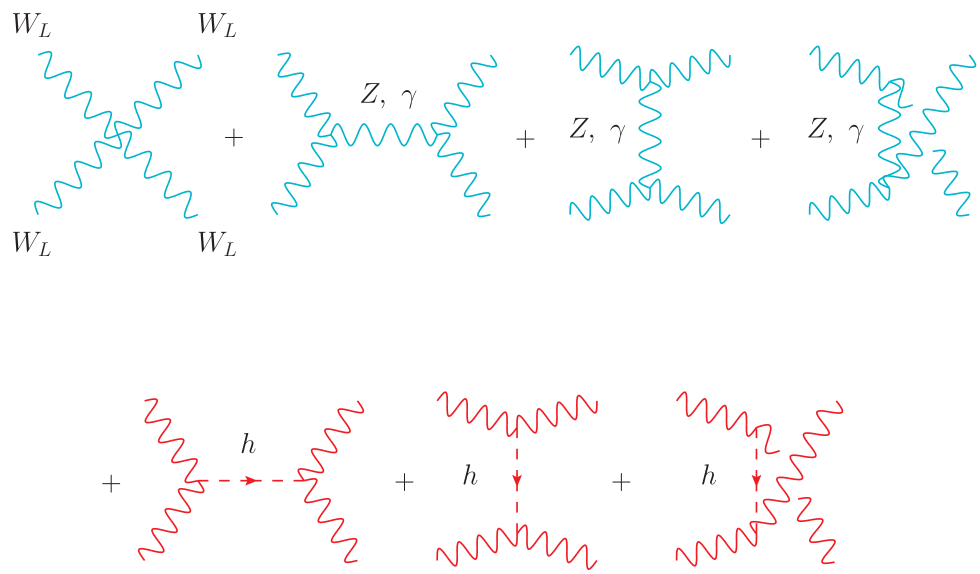


Figure 2.1: A series of diagrams contributing to $V_L V_L \to V_L V_L$ scattering amplitude in the SM

The important thing is that these delicate cancellations are spoiled if hVV couplings are turned out to deviate from the SM predictions. In that case, the observed Higgs boson unitalize $V_L V_L \to V_L V_L$ scattering only partially, and additional scalar fields are needed to restore the perturbative unitarity completely.

Chapter 3

Higgs Effective Field Theory

The SM in particle physics introduced in the previous chapter has some problems such as the hierarchy problem mentioned in Chap. 1. To solve these problems, many new physics models are suggested. Checking every new physics model one by one takes a lot of time. There is a good approach for extracting universal predictions about the new physics in a bottom-up manner, which is called the effective field theory approach. In the effective field theory approach, we write down all the invariant operators consistent with underlying symmetry and set the coefficient of each operator free parameter. These free parameters are used to parametrize the deviation from the SM predictions. Representative examples of the effective field theory approach in the context of the new physics beyond the SM are the Standard Model Effective Field Theory (SMEFT) and the Higgs Effective Field Theory (HEFT).

The SMEFT is the effective field theory written in terms of matter fields in the symmetric phase [20–22]. Its symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y$ and its matter contents are given by the SM matter contents listed in Table 2.1. The leading order of the SMEFT is given by the SM Lagrangian (2.1). The next leading order (NLO) Lagrangian is the dimension six operators consistent with $SU(3)_C \times SU(2)_L \times U(1)_Y$. For examples, the NLO Lagrangian involving Higgs doublet H is given as

$$\Delta\mathcal{L}_{\rm NLO} = \frac{c_H}{\Lambda^2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) + \frac{c_T}{\Lambda^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right), \tag{3.1}$$

where Λ denotes the cut-off scale of the SMEFT. In the SMEFT, it is, however, implicitly assumed that the observed Higgs boson is coming from the $SU(2)_L$ scalar doublet. To remove this assumption and consider the most general set up, we will focus on the Higgs Effective Field Theory (HEFT), which is constructed by adding the observed Higgs boson to the electroweak chiral perturbation theory.

3.1 Higgs Effective Field Theory

In this section, we will review the Higgs Effective Field Theory (HEFT) [3–19]. The HEFT is an effective field theory constructed by adding observed Higgs boson to the electroweak chiral perturbation theory [23–28]. In Table 3.1, we listed all the matter fields comprising the HEFT, together with their representations under Lorentz symmetry and $SU(3)_C \times U(1)_{\rm em}$ gauge symmetry. Note that the SM matter contents listed in Table 2.1 and the HEFT matter contents listed in Table 3.1 are a little bit different: the former is classified under $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry while the latter is classified under $SU(3)_C \times U(1)_Y$ symmetry. In this chapter, we will call the particles classified under $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetric phase particles and the particles classified under $SU(3)_C \times U(1)_{\rm em}$ broken phase particles.

Field	spin	$SU(3)_C$	$U(1)_{\rm em}$
G^A_μ	1	$\mathbf{8}$	0
$W^\pm_\mu$	1	$\mathbf{1}$	±1
Z_μ	1	$\mathbf{1}$	0
$u^i_{L,R}$	1/2	$\mathbf{3}$	$+2/3$
$d^i_{L,R}$	1/2	$\mathbf{3}$	$-1/3$
$e^i_{L,R}$	1/2	$\mathbf{1}$	-1
ν^i_L	1/2	$\mathbf{1}$	0
h	0	$\mathbf{1}$	0
$w^\pm$	0	$\mathbf{1}$	±1
z	0	$\mathbf{1}$	0

Table 3.1: Matter contents in Standard Model

Lagrangian of the HEFT in $\mathcal{O}(p^2)$ order is given as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermion}\,, \tag{3.2}$$

where $\mathcal{L}_{gauge}$, $\mathcal{L}_{scalar}$, and $\mathcal{L}_{fermion}$ each denotes the gauge sector, scalar sector, and fermion sector, respectively. In the remaining of this section, we will closely see each sector in turn.

Gauge sector

The Lagrangian of the gauge sector in the HEFT is given as

$$\mathcal{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,. \tag{3.3}$$

Note that the electroweak gauge fields in Eq.(3.3) are written in terms of symmetric phase fields W_μ, B_μ rather than broken phase fields $W^\pm_\mu$, Z_μ, A_μ. It is possible to rewrite the gauge sector in terms of broken phase fields $W^{\pm}_{\mu}$, Z_{μ}, A_{μ}, but for later convenience, we write it in terms of symmetric phase fields W_{μ} and B_{μ}.

Scalar sector

The Lagrangian of the scalar sector in the HEFT is given as

$$\mathcal{L}_{scalar} = \frac{v^2}{4} F(h) \mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) \tag{3.4}$$

where v is the decay constant of NG bosons:

$$v = 246\,\mathrm{GeV}\,. \tag{3.5}$$

$F(h)$ and $V(h)$ are arbitrary functions of observed Higgs boson h,

$$F(h) = 1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)} \left(\frac{h}{v}\right)^2 + \cdots\,, \tag{3.6}$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + \lambda^{(3)} \left(\frac{h}{v}\right)^3 + \cdots\,, \tag{3.7}$$

with $\kappa_W,\ \kappa_W^{(2)},\ \cdots$ in Eq.(3.6) and $m_h,\ \lambda^{(3)},\ \cdots$ in Eq.(3.7) are free parameters. Here we assume $F(0) = 1$ so that the kinetic terms of NG fields are canonically normalized. The symbol U is two by two matrix and its explicit form is given as

$$U = \exp\left(\frac{i\sqrt{2}}{v}(w^+\tau_+ + w^-\tau_-) + \frac{i}{v} z\tau^3\right)\,, \tag{3.8}$$

where $\tau_\pm$ are given as

$$\tau_\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)\,. \tag{3.9}$$

The covariant derivative $D_\mu U$ is given as

$$D_\mu U = \partial_\mu U + i g_W W_\mu U - i g_Y U B_\mu\,. \tag{3.10}$$

Note that the transformation law of matrix U under $SU(2)_L \times U(1)_Y$ is

$$U \to U' = \mathfrak{g}_W U \mathfrak{g}_Y^\dagger \,. \tag{3.11}$$

Fermion sector

The Lagrangian of the fermion sector in the HEFT is composed of fermion kinetic terms and Yukawa-like terms, with its explicit form given as

$$\begin{aligned}
&\mathcal{L}_{fermion} \\
&= \sum_{i=1}^{3} \bar{q}_L^i i \not{D} q_L^i + \sum_{i=1}^{3} \bar{u}_R^i i \not{D} u_R^i + \sum_{i=1}^{3} \bar{d}_R^i i \not{D} d_R^i + \sum_{i=1}^{3} \bar{l}_L^i i \not{D} l_L^i + \sum_{i=1}^{3} \bar{e}_R^i i \not{D} e_R^i \\
&\quad + v \left(\sum_{i,j=1}^{3} \bar{q}_L^i Y_u^{ij}(h) U P_+ r^j + \sum_{i,j=1}^{3} \bar{q}_L^i Y_d^{ij}(h) U P_- r^j + \sum_{i,j=1}^{3} \bar{l}_L^i Y_e^{ij}(h) U P_- \eta^j + \text{h.c.} \right) .
\end{aligned} \tag{3.12}$$

Note that $Y_u^{ij}(h)$, $Y_d^{ij}(h)$, and $Y_e^{ij}(h)$ are three by three matrices and are arbitrary functions of observed Higgs boson h,

$$Y_u(h) = Y_u^{(0)} + \sum_{n=1}^{\infty} Y_u^{(n)} \left(\frac{h}{v} \right)^n \,, \tag{3.13}$$

$$Y_d(h) = Y_d^{(0)} + \sum_{n=1}^{\infty} Y_d^{(n)} \left(\frac{h}{v} \right)^n \,, \tag{3.14}$$

$$Y_e(h) = Y_e^{(0)} + \sum_{n=1}^{\infty} Y_e^{(n)} \left(\frac{h}{v} \right)^n \,, \tag{3.15}$$

with flavor indices suppressed. r and η are the two-component vectors composed of right-handed quarks and leptons:

$$r = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \,, \qquad \eta = \begin{pmatrix} e_R \\ 0 \end{pmatrix} \,. \tag{3.16}$$

$P_\pm$ is the projection operators projecting to the upper and lower components respectively,

$$P_\pm := \frac{1}{2}\mathbf{1}_2 \pm \frac{\tau_3}{2}\,, \tag{3.17}$$

with $\mathbf{1}_2$ being two times two identity matrix. Note that the transformation laws of the left-handed fermions q_L and l_L under $SU(2)_L$ symmetry are given as

$$q_L \to q'_L = \mathfrak{g}_W \cdot q_L\,, \tag{3.18}$$

$$l_L \to l'_L = \mathfrak{g}_W \cdot l_L\,, \tag{3.19}$$

while the right-handed fermions u_R, d_R, and e_R remain invariant.

The transformation law of all the fermions under $U(1)_Y$ symmetry are given as

$$q_L \to q'_L = \rho_q[\mathfrak{g}_Y] \cdot q_L\,, \tag{3.20}$$

$$r \to r' = \rho_r[\mathfrak{g}_Y] \cdot r\,, \tag{3.21}$$

$$l_L \to l'_L = \rho_l[\mathfrak{g}_Y] \cdot l_L\,, \tag{3.22}$$

$$\eta \to \eta' = \rho_\eta[\mathfrak{g}_Y] \cdot \eta\,, \tag{3.23}$$

with

$$\rho_q[\mathfrak{g}_Y] = \begin{pmatrix} e^{\frac{i}{6}\theta_Y} & \\ & e^{\frac{i}{6}\theta_Y} \end{pmatrix}\,, \tag{3.24}$$

$$\rho_r[\mathfrak{g}_Y] = \begin{pmatrix} e^{\frac{i2}{3}\theta_Y} & \\ & e^{-\frac{i}{3}\theta_Y} \end{pmatrix}\,, \tag{3.25}$$

$$\rho_l[\mathfrak{g}_Y] = \begin{pmatrix} e^{-\frac{i}{2}\theta_Y} & \\ & e^{-\frac{i}{2}\theta_Y} \end{pmatrix}\,, \tag{3.26}$$

$$\rho_\eta[\mathfrak{g}_Y] = \begin{pmatrix} e^{-i\theta_Y} & \\ & 1 \end{pmatrix}\,. \tag{3.27}$$

It is easy to see that the HEFT fermions sector (3.12) is invariant under the $SU(2)_L \times U(1)_Y$ gauge symmetry.

Note that, in order to pursue the perturbative calculation in the framework of the effective field theory, we need a power counting formula. In Appendix B, we give the power counting formula for the electroweak chiral perturbation theory (EWChPT). The EWChPT is the effective field theory written in terms of the SM fields other than the Higgs boson, and its leading order Lagrangian is obtained by setting $h \to 0$ in the leading order HEFT Lagrangian (3.2). Extending the power counting formula to incorporate observed Higgs boson is easy.

As we mentioned at the beginning of this section, the HEFT Lagrangian is written in terms of broken phase fields; all the fields in Table 3.1 are classified under the broken phase symmetry, $U(1)_{\rm em}$. In spite of this, the HEFT Lagrangian is invariant under not only $U(1)_{\rm em}$, but the whole set of $SU(2)_L \times U(1)_Y$ symmetry. This seems strange, but if we look closely at the interaction terms in the HEFT Lagrangian (3.2), we can find that the $SU(2)_L \times U(1)_Y$ symmetry actually exists, and forbid some parts of $U(1)_{\rm em}$-invariant interactions.

Assume that the HEFT respects only $U(1)_{\rm em}$ symmetry, not the whole set of $SU(2)_L \times U(1)_Y$ symmetry. In that case, the HEFT Lagrangian should include all the $U(1)_{\rm em}$-invariant and Lorentz-invariant interactions, because effective field theory should be composed of all the consistent terms with its symmetry. Then, in addition to interaction terms in Eq.(3.2), the HEFT Lagrangian should also include the mass terms for the NG bosons:

$$m_w w^+ w^- \,, \qquad \frac{1}{2} m_z^2 z^2 \,. \tag{3.28}$$

As we see in Eq.(3.2), however, the HEFT scalar potential $V(h)$ only depends on the observed Higgs boson h, and does not depend on the NG fields $w^\pm$ and z. The absence of the mass terms of NG fields is due to the remnant $SU(2)_L \times U(1)_Y$ symmetry. As we see later, the remnant $SU(2)_L \times U(1)_Y$ acts on NG fields as some kind of shift symmetry, and this shift symmetry forbids the potential of NG boson fields.

Generally speaking, if the symmetry group G is spontaneously broken to its subgroup H, the symmetry group G is not really broken, but it remains as non-linearly realized symmetry in the broken phase.

3.2 Non-linearly realized symmetry

As I mentioned at the end of the previous section, if global symmetry G is spontaneously broken to its subgroup H, G is not really broken but it remains as a non-linearly realized symmetry. The properties of the non-linearly realized symmetry play important roles in the HEFT, so we will treat its details and reveal what the non-linearly symmetry is, and how it can be useful for writing down EFT. In the remainder of this section, I will describe the concept of the non-linearly realized symmetry using a simple example: $O(N)$ linear sigma model with symmetry breaking pattern $O(N) \to O(N-1)$.

Firstly, we will briefly describe a setup of $O(N)$ linear sigma model. The Lagrangian of the $O(N)$ linear sigma model is given as follows:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - V(\phi)\,, \tag{3.29}$$

$$V(\phi) = \frac{1}{2}\mu^2(\vec{\phi})^2 + \frac{\lambda}{4}(\vec{\phi})^4\,, \tag{3.30}$$

where $\vec{\phi}$ is a N component scalar field in the fundamental representation of the global $O(N)$ symmetry. The transformation law of $\vec{\phi}$ under $O(N)$ is given as

$$O(N)\ :\ \vec{\phi} \to \vec{\phi}' = \mathfrak{g} \cdot \vec{\phi}\,, \tag{3.31}$$

where $\mathfrak{g}$ is a transformation matrix of $O(N)$ and its explicit form is written in terms of $O(N)$ generator T^a as

$$\mathfrak{g} = \exp\left(i\theta^a_{\mathfrak{g}} T^a\right). \tag{3.32}$$

Here we adopt the following normalization condition for the $O(N)$ generators,

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}\,. \tag{3.33}$$

If the scalar potential V has a negative mass term ($\mu^2 < 0$), then the potential V becomes a wine bottle type potential, just like the SM, and the scalar field takes a nonzero vacuum expectation value

$$\langle \phi \rangle = \sqrt{\frac{-\mu^2}{\lambda}} =: v\,. \tag{3.34}$$

As a result, $O(N)$ symmetry is spontaneously broken to its subgroup $O(N-1)$.

This is the set up of $O(N)$ linear sigma model, and in the remainder of this section, we will rewrite the symmetric phase Lagrangian (3.29) into the broken phase Lagrangian, and see how the $O(N)$ symmetry is realized in the broken phase. When $O(N)$ is spontaneously broken to its subgroup H, the generators of $O(N)$ symmetry, T^a, can be divided into the broken generator denoted by X^a and the unbroken generator denoted by S^a,

$$T^a = \{X^a, S^a\}\,. \tag{3.35}$$

Because the number of original $O(N)$ generators is $N(N-1)/2$ and the remaining unbroken $O(N-1)$ symmetry has $(N-1)(N-2)/2$ generators, the total number of broken generators is $N-1$. According to Goldstone's theorem, associated with these $N-1$ broken generators, the $N-1$ number of Nambu-Goldstone bosons (NG bosons) arise. Now the scalar field ϕ^i can be rewritten in terms of fluctuation fields from the new vacuum $\langle \phi \rangle$: ϕ^i can be expressed in terms of the $N-1$ NG bosons which are denoted by π^a ($a = 1 \sim N-1$) and one scalar matter field denoted by σ as

$$\vec{\phi} = (\pi^1, \pi^2, \cdots, \pi^{N-1}, v+\sigma)^T\,. \tag{3.36}$$

Substituting Eq.(3.36) into Eq.(3.29), and neglecting constant terms, we get the broken phase Lagrangian given as

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\lambda}{4}\sigma^4 - \lambda v \sigma^3 - \lambda v (\vec{\pi}\,)^2 \sigma - \frac{\lambda}{4}(\vec{\pi}\,)^4\,, \tag{3.37}$$

where the mass of the scalar field σ is given as

$$m_\sigma = \sqrt{2\lambda} v\,. \tag{3.38}$$

As we mentioned in the previous section, when the global symmetry G is spontaneously broken to its subgroup H, the broken phase Lagrangian respects not only the unbroken symmetry H, but also the whole set of G symmetry, which is realized as the remnant symmetry at the broken phase. Applying this statement to this $O(N)$ sigma model example, the broken phase Lagrangian (3.37) respects not only the unbroken $O(N-1)$ symmetry but also a remnant $O(N)$ symmetry.

Before we see the details of the remnant $O(N)$ symmetry, let us consider the choice of the coordinate system in the field space. When we write down the broken phase Lagrangian (3.37), we firstly express the original scalar field ϕ^i in terms of new fluctuation fields σ and π^a like Eq.(3.36). The parametrization Eq.(3.36) corresponds to the Cartesian coordinate system in the field space. For illustrating the nonlinearly realized symmetry, however, the other choice of the coordinate system, a polar coordinate system, turns out to be more convenient. In the remainder of this section, we will adopt the polar coordinate system and see how the remnant $O(N)$ symmetry is realized in the broken phase.

Firstly, let us consider the parametrization of the NG boson fields in the polar coordinate system. NG boson is a massless particle and therefore it parametrizes the flat direction in the field space. This means that NG boson is a fluctuation along with the super surface composed of a series of equivalent vacuum. Therefore, we can regard NG bosons as coordinate variables which transform one vacuum vector $\vec{F}$ to another equivalent vacuum:

$$\xi(\pi) \cdot \vec{F} = \vec{F}' \,. \tag{3.39}$$

On the other hand, a scalar matter field σ is a massive particle and needs nonzero energy when it is excited from the vacuum. Therefore it can be regarded as coordinate variable parametrizing the radial direction. As a result, $O(N)$ multiplet $\vec{\phi}$ can be decomposed into a fluctuation along with a flat direction and excitation along with the radial direction,

$$\vec{\phi} = (v + \sigma)\, \xi(\pi) \cdot \vec{F} \,, \tag{3.40}$$

where ξ is the $N \times N$ unitary matrix living in coset space G/H and $\vec{F}$ is a unit vector directing to the vacuum,

$$\xi(\pi) = \exp\left(\frac{i}{v}\pi^a X^a\right), \tag{3.41}$$

$$\vec{F} = (0, \cdots, 0, 1)^T. \tag{3.42}$$

with X denoting broken generator. The schematic picture of the decomposition (3.40) is shown in Fig. 3.1.

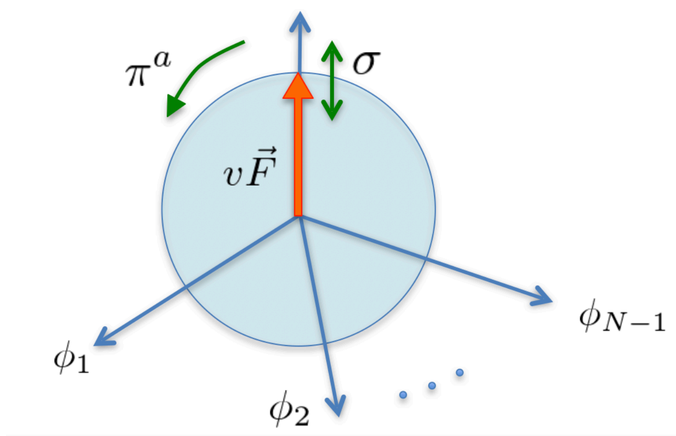


Figure 3.1: Polar decomposition of the $O(N)$ multiplet $\vec{\phi}$

Note that the vacuum vector $\vec{F}$ is invariant under the unbroken $O(N-1)$ transformation:

$$\mathfrak{h} \cdot \vec{F} = \vec{F}, \qquad \mathfrak{h} \in O(N-1), \tag{3.43}$$

where explicit form of $\mathfrak{h}$ is given as

$$\mathfrak{h} = \exp\left(i\theta_h^a S^a\right). \tag{3.44}$$

As we mentioned previously, $O(N)$ symmetry is not really broken in the vacuum but remained as the non-linearly realized symmetry. In the remainder of this section, we will see this using the polar coordinate system introduced above.

To understand the concept of non-linearly realized symmetry, let us consider the transformation law of σ and π^a under the $O(N)$ symmetry. We can easily anticipate the $O(N)$ transformation law of σ and π^a from Eq.(3.31) and Eq.(3.40). Substituting Eq.(3.40) into Eq.(3.31), we get

$$O(N) \ : \ \vec{\phi} = (v+\sigma)\, \xi(\pi) \cdot \vec{F} \to \vec{\phi}' = (v+\sigma)\, \mathfrak{g} \cdot \xi(\pi) \cdot \vec{F} \, . \tag{3.45}$$

As we see in (3.45), the transformation matrix $\mathfrak{g}$ pass through $(v+\sigma)$ and only acts on the $\xi(\pi) \cdot \vec{F}$. From Eq.(3.45), we can anticipate the $O(N)$ transformation law of σ and ξ as

$$O(N) \ : \ \sigma \to \sigma \, , \qquad \xi(\pi) \to \mathfrak{g} \cdot \xi(\pi) \, . \tag{3.46}$$

The fact that $\vec{F}$ is invariant under $O(N-1)$ (see Eq.(3.43)) allows $\mathfrak{h}$ to act on ξ from the right side. So the $O(N)$ transformation law of ξ is modified from Eq.(3.57) to

$$O(N) \ : \ \xi(\pi) \to \mathfrak{g} \cdot \xi(\pi) \cdot \mathfrak{h}^T \, . \tag{3.47}$$

At this point, the insertion of $\mathfrak{h}$ in Eq.(3.47) seems to be arbitrary, but in the following discussion, we will find that $\mathfrak{h}^T$ is necessary to give the appropriate transformation law of ξ.

From the explicit form of $\xi(\pi)$ given by Eq.(3.41), we anticipate that the $O(N)$ transformed matrix $\xi(\pi')$ should be expressed as

$$\xi(\pi') = \exp\left(\frac{i}{v} \pi'^a X^a \right) . \tag{3.48}$$

The $O(N)$ transformation law given by Eq.(3.57) is, however, contradict to Eq.(3.48) because $\mathfrak{g} \cdot \xi$ in Eq.(3.57) is no longer the element of G/H, but the element of G:

$$O(N) \ : \ \xi \to \mathfrak{g} \cdot \xi(\pi) \neq \xi(\pi') \, , \tag{3.49}$$

so we must conclude that Eq.(3.57) is not appropriate for the $O(N)$ transformation law of ξ.

This problem can be fixed by the following theorem: if G has subgroup H, the group element of G can be always expressed by the product of group element of H and that of G/H. According to this theorem, we can always divide $\mathfrak{g} \cdot \xi(\pi)$ into $\xi(\pi')$ living in G/H and $\mathfrak{h}$ living in H like

$$\mathfrak{g} \cdot \xi(\pi) = \xi(\pi') \cdot \mathfrak{h}(\pi, \theta_{\mathfrak{g}}) \,. \tag{3.50}$$

Here we write $\mathfrak{h}$ as $\mathfrak{h}(\pi, \theta_{\mathfrak{g}})$ to emphasize that $\mathfrak{h}$ in Eq.(3.50) generally depends on the NG fields π and $O(N)$ transformation parameter $\theta_{\mathfrak{g}}$ given in Eq.(3.32). Multiplying $\mathfrak{h}^T$ to both side of Eq.(3.50) from the right, we get

$$\xi(\pi') = \mathfrak{g} \cdot \xi(\pi) \cdot \mathfrak{h}^T(\pi . \theta_{\mathfrak{g}}) \,, \tag{3.51}$$

and we interpret Eq.(3.51) as the transformation law of ξ. As we mentioned, the insertion of $\mathfrak{h}$ is allowed from the discussion above Eq.(3.47).

Eventually, we get the following $O(N)$ transformation law of σ and ξ

$$O(N) \ : \ \sigma \to \sigma \,, \qquad \xi(\pi) \to \mathfrak{g} \cdot \xi(\pi) \cdot \mathfrak{h}^T(\pi, \theta_{\mathfrak{g}}) \,. \tag{3.52}$$

Now it is clear the meaning of "non-linearly realized symmetry." $O(N)$-transformed field π' is no longer the linear combination of the original field π but the non-linear function of π,

$$\pi' = \text{const. shift} + \pi + \mathcal{O}(\pi^2) \,, \tag{3.53}$$

and this is why π is called the non-linear representation of $O(N)$.

So far, we describe the concept of the non-linearly realized symmetry using $O(N)/O(N-1)$ as the example and see the associated NG bosons transform non-linearly under $O(N)$. In general, the non-linear property we described above is common in any symmetry breaking pattern $G \to H$: when G is spontaneously broken to its subgroup H, fields in the broken phase transforms non-linearly under G.

In any symmetry breaking pattern $G \to H$, the associated NG fields can be always regarded as the coordinate parametrizing the coset space G/H,

$$\xi(\pi) = \exp\left(\frac{i}{v}\pi^a X^a\right), \tag{3.54}$$

with X^a denoting the broken generator. The transformation law of ξ under G/H is given as

$$G \ : \ \xi(\pi) \to \xi(\pi') = \mathfrak{g} \cdot \xi(\pi) \cdot \mathfrak{h}^T(\pi, \theta_{\mathfrak{g}}) \,, \tag{3.55}$$

$$\mathfrak{g} \in G, \qquad \mathfrak{h} \in H \,. \tag{3.56}$$

This fact leads us interesting possibility: if our vacuum respects the symmetry H and if we somehow know that H is the remnant symmetry resulting from the spontaneous symmetry breaking $G \to H$, we can use not only the symmetry H but also the non-linearly realized symmetry G to restrict the allowed operator at the broken phase. How can we write down the non-linearly realized G invariant theory, then? The answer to this question will be given in the next section.

3.3 CCWZ formalism

In the previous section, we describe the concept of the non-linearly realized symmetry and see that the NGB associated with G/H transforms non-linearly under G. In the last part of the previous section, we see that the non-linear realized symmetry may be useful to restrict the possible operator in the broken phase, but how can we write down effective field theory respecting non-linearly realized symmetry?

To capture the property of the non-linearly realized symmetry, we again focus on the transformation law of ξ matrix under the G symmetry,

$$O(N) \ : \ \xi(\pi) \to \xi(\pi') = \mathfrak{g} \cdot \xi(\pi) \cdot \mathfrak{h}^T(\pi, \theta_{\mathfrak{g}}) \,. \tag{3.57}$$

The key thing is that the transformation matrix $\mathfrak{h}(\pi, \theta_{\mathfrak{g}}) \in H$ depends on the space-time coordinate x^μ though the NB boson field $\pi(x)$, so even if we consider the *global* $O(N)$ transformation law of ξ, ξ matrix essentially experience the *local* transformation.

Now the important features of the non-linearly realized symmetry become clear:

- G is non-linearly realized at the broken phase in the sense that the NG bosons and matter fields transforms in a non-linear manner under G.
- Fields in the broken phase experience local transformations under non-linearly realized G symmetry even if G is introduced as the global symmetry.

In order to write down the effective field theory with symmetry breaking pattern $G \to H$, we must construct operators invariant under the local and non-linearly realized symmetry G. How can we construct such operators?

The answer to this question was given some decades ago by C. G. Callan, S. R. Coleman, J. Wess and B. Zumino [29–31]. They show how to construct the building blocks for writing down the non-linearly realized G invariant operators. The fundamental quantity is the Cartan one-form given by

$$\alpha_\mu(\pi) := \frac{1}{i}\xi(\pi)^{-1}\partial_\mu\xi(\pi)\,. \tag{3.58}$$

From Eq.(3.55), we can derive the transformation law of α_μ under the global symmetry G,

$$G : \alpha_\mu(\pi) \to \alpha_\mu(\pi') = \mathfrak{h}(\pi,\theta_\mathfrak{g})\alpha_\mu(\pi)\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g}) + \frac{1}{i}\mathfrak{h}(\pi,\theta_\mathfrak{g})\partial_\mu\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g})\,. \tag{3.59}$$

α_μ is $\mathcal{G}$-valued tensor and can be decomposed into broken and unbroken generator as

$$\alpha_\mu = \alpha_{\parallel\mu} + \alpha_{\perp\mu}\,, \tag{3.60}$$

with

$$\alpha_{\parallel\mu} := \mathrm{Tr}\left[\alpha_\mu S^a\right] S^a\,, \tag{3.61}$$
$$\alpha_{\perp\mu} := \mathrm{Tr}\left[\alpha_\mu X^a\right] X^a\,. \tag{3.62}$$

Note that the normalization conditions for unbroken and broken generators are given as

$$\mathrm{Tr}\left[S^a S^b\right] = \delta^{ab}\,, \tag{3.63}$$
$$\mathrm{Tr}\left[X^a X^b\right] = \delta^{ab}\,. \tag{3.64}$$

Substituting Eq.(3.60) into Eq.(3.59), we get

$$\begin{aligned}\alpha_{\parallel\mu}(\pi') + \alpha_{\perp\mu}(\pi') &= \mathfrak{h}(\pi,\theta_\mathfrak{g})\alpha_{\parallel\mu}(\pi)\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g}) + \frac{1}{i}\mathfrak{h}(\pi,\theta_\mathfrak{g})\partial_\mu\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g})\\ &\quad + \mathfrak{h}(\pi,\theta_\mathfrak{g})\alpha_{\perp\mu}(\pi)\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g})\,.\end{aligned} \tag{3.65}$$

We can easily show that the first line of Eq.(3.65) belongs to $\mathcal{H}$: by definition, $\alpha_{\parallel\mu} \in \mathcal{H}$ is the adjoint representation of H, so the first term is the element of $\mathcal{H}$. The second term also belongs to $\mathcal{H}$ for the same reason why α_μ given by Eq.(3.58) belongs to $\mathcal{G}$. As for the second line of Eq.(3.65), we can show that it belongs to $\mathcal{G} - \mathcal{H}$ by checking the orthogonality between $\mathfrak{h}\alpha_\perp\mathfrak{h}^{-1}$ and S^a:

$$\begin{aligned}\mathrm{Tr}\left[\mathfrak{h}\alpha_{\perp\mu}\mathfrak{h}^{-1}S^a\right] &= \mathrm{Tr}\left[\mathfrak{h}\alpha_{\perp\mu}\mathfrak{h}^{-1}S^a(\mathfrak{h}\mathfrak{h}^{-1})\right]\\ &= \mathrm{Tr}\left[\mathfrak{h}\alpha_{\perp\mu}S^b[\rho_\mathfrak{h}]_b{}^a\mathfrak{h}^{-1}\right]\\ &= \mathrm{Tr}\left[\alpha_{\perp\mu}S^b\right][\rho_\mathfrak{h}]_b{}^a\\ &= 0\,.\end{aligned} \tag{3.66}$$

Now we can extract the transformation law of $\alpha_{\perp\mu}$ and $\alpha_{\parallel\mu}$ under G by decomposing Eq.(3.65) into unbroken and broken generator part:

$$\alpha_{\parallel\mu}(\pi') = \mathfrak{h}(\pi,\theta_\mathfrak{g})\alpha_{\parallel\mu}(\pi)\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g}) + \frac{1}{i}\mathfrak{h}(\pi,\theta_\mathfrak{g})\partial_\mu\mathfrak{h}^{-1}(\pi,\theta_\mathfrak{g})\,, \tag{3.67}$$

$$\alpha_{\perp\mu}(\pi') = \mathfrak{h}(\pi,\theta_{\mathfrak{g}})\alpha_{\perp\mu}(\pi)\mathfrak{h}^{-1}(\pi,\theta_{\mathfrak{g}})\,. \tag{3.68}$$

As shown in Eq.(3.68), $\alpha_{\perp\mu}$ transforms homogeneously under G and we can build the non-linearly realized G invariant operator using $\alpha_{\perp\mu}$ as

$$\mathcal{L}_\pi = \frac{v^2}{2}\mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right]\,. \tag{3.69}$$

Rewriting Eq.(3.69) in terms of the NG boson π , we can see that Eq.(3.69) gives the kinetic term of NG bosons. The factor v^2 in front of the operator is for the canonical normalization of the NG bosons.

Note that a group of broken generators, $\{X^a\}$ comprise reducible representation under H in general: $\{X^a\}$ can be divided into some irreducible representation groups $\{X^\alpha\}$, $\{X^{\alpha'}\}$, $\cdots$, $\{X^{\alpha''}\}$ and they transform by $U(1)_{\rm em}$ matrix $\mathfrak{h}$ in particular representation, $\rho_X(\mathfrak{h})$,

$$\mathfrak{h}X^\alpha\mathfrak{h}^{-1} = \left[\rho_X(\mathfrak{h})\right]^\alpha{}_\beta X^\beta\,, \tag{3.70}$$
$$\mathfrak{h}X^{\alpha'}\mathfrak{h}^{-1} = \left[\rho_{X'}(\mathfrak{h})\right]^{\alpha'}{}_{\beta'} X^{\beta'}\,, \tag{3.71}$$
$$\vdots$$
$$\mathfrak{h}X^{\alpha''}\mathfrak{h}^{-1} = \left[\rho_{X''}(\mathfrak{h})\right]^{\alpha''}{}_{\beta''} X^{\beta''}\,. \tag{3.72}$$

In that case, associate NG boson fields $\{\pi^\alpha\}$, $\{\pi^{\alpha'}\}$, $\cdots$, $\{\pi^{\alpha''}\}$ also comprise different multiplets under $U(1)_{\rm em}$, and we can assign the different decay constants to these NG boson fields:

$$\xi(\pi^\alpha) = \exp\left(\frac{i}{v}\pi^\alpha X^\alpha\right)\,, \tag{3.73}$$
$$\xi(\pi^{\alpha'}) = \exp\left(\frac{i}{v'}\pi^{\alpha'} X^{\alpha'}\right)\,, \tag{3.74}$$
$$\vdots$$
$$\xi(\pi^{\alpha''}) = \exp\left(\frac{i}{v''}\pi^{\alpha''} X^{\alpha''}\right)\,. \tag{3.75}$$

Then, the operator Eq.(3.69) is modified to

$$\mathcal{L}_\pi = \frac{v^2}{2}\mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right] + \frac{v'^2}{2}\mathrm{Tr}\left[\alpha'_{\perp\mu}\alpha'^\mu_\perp\right] + \cdots + \frac{v''^2}{2}\mathrm{Tr}\left[\alpha''_{\perp\mu}\alpha''^\mu_\perp\right] \tag{3.76}$$

where $\alpha_{\perp\mu}$, $\alpha'_{\perp\mu}$, $\cdots$, $\alpha''_{\perp\mu}$ are defined according to Eq.(3.62) with broken generator X^a replaced to X^α, $X^{\alpha'}$, $\cdots$, $X^{\alpha''}$, respectively. The most familiar example of this case is the symmetry breaking pattern of SM: $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. We will give the explicit construction of the EFT with this symmetry breaking pattern in Sec. 4.

3.4 Perturbative unitarity in HEFT

Before closing this chapter, we will briefly mention the perturbative unitarity of the HEFT. Remember that the scattering amplitude among longitudinal gauge bosons $V_LV_L \to V_LV_L$ shows energy-growing behavior in general, and may violate the perturbative unitarity. In the case of the SM, this energy growing behavior are delicately canceled out by the contribution from the Higgs exchange diagrams. Because the cancellation in the SM is realized by appropriately-tuned hVV couplings, the deviation from the SM predictions in hVV coupling easily spoil the cancellation and reoccur the unitarity violation.

The purpose of the HEFT is describing the effects of the new physics and the HEFT has many free parameters for describing the deviations from the SM predictions. The hVV coupling is also set to be a free parameter in the HEFT. As we mentioned above, the deviation from the SM predictions in hVV coupling spoils the cancelation of the energy growing behavior in $V_LV_L \to V_LV_L$ scattering amplitude. We expect that this energy growing behavior may be cancelled by the new particles' contribution. Because the HEFT include only the SM particles, however, the energy growing behavior in $V_LV_L \to V_LV_L$ scattering amplitude is never cancelled out once hVV coupling deviate from the SM predictions. Eventually, $V_LV_L \to V_LV_L$ scattering amplitude violate the perturbative unitarity at a certain energy scale and this sets the upper limit of the energy scale where the HEFT is applicable. The energy scale of the unitarity violation can be expressed in terms of the deviation in the hVV

coupling as

$$\Lambda \sim \frac{4\pi v}{\sqrt{|1-\kappa_W^2|}} \tag{3.77}$$

with κ_W given in Eq.(3.6). At the energy scale above Λ, the perturbative calculation in the HEFT becomes non-reliable.

If the new physics weakly interact with the SM particles, we expect that the new particles appear below the energy scale Λ, namely, $M_{\rm new} < \Lambda$ with $M_{\rm new}$ denoting new particle's mass. In order to directly produce the new physics particles, we should know which scattering channel is more efficient for producing the new particles. This cannot be done in the existing effective field theory framework because the new particles are integrated out and we cannot treat the production process of the new particles. We must extend the HEFT to include the new particles degrees of freedom. In the next section, we will extend the HEFT to incorporate the arbitrary number of the scalar fields with arbitrary electromagnetic charges.

Chapter 4

Generalization of Higgs Effective Field Theory

As we mentioned in the previous chapter, the HEFT introduced in Sec. 3 is the most general effective field theory composed of the observed particles. The HEFT includes all the operators consistent with $SU(3)_C \times SU(2)_L \times U(2)_Y$ gauge symmetry, and a coefficient of each effective operator is set to be a free parameter, which can parametrize the deviation from the SM prediction. Because the HEFT has so many free parameters, it can deal with any pattern of deviations from the SM predictions. Therefore, the HEFT can take into account the various types of new physics effects to low-energy physics.

The important thing is that the HEFT is quite useful for parametrizing deviations from the SM predictions, but because it is written in terms of the observed particles only, it does not suit for constraining the new particles' properties. For a concrete example, within the HEFT framework, we cannot calculate the production cross section of the new particles. This is because, in the framework of effective field theory, all the heavy particle degrees of freedom are removed by integrating out. This means we cannot treat physical processes with the heavy particles appearing in the initial or final state, such as decay or production process. Of course, the information about the new particles does not vanish completely in the low-energy effective Lagrangian, but is parametrized in the coefficient of each effective operator.

Because the value of each operator coefficient reflects various effects of new particles, however, it is difficult to extract the information about each new particle's properties from these coefficients. Therefore, in order to obtain the concrete predictions about the new particles' properties, we must extend the HEFT so that it includes new particle degrees of freedom.

Considering the possible UV models realized above the electroweak scale, there are various kinds of candidate of new particles: they can be scalar fields, fermion fields, and vector fields. In this chapter, we will focus on new scalar particles, because we are interested in the mechanism realizing electroweak symmetry breaking: as we mentioned previously, we believe that understanding the way to break electroweak symmetry is the key to solve the hierarchy problem, and the scalar particles may play an important role in the symmetry breaking. For a concrete example, if we try to solve the hierarchy problem by generating the electroweak scale dynamically, the observed Higgs boson can be a composite particle and various interactions involving the Higgs boson are modified from the SM predictions. This model is a so-called composite Higgs model. In the framework of the composite Higgs models, the additional global symmetry breaking is assumed at the energy scale above the electroweak symmetry breaking scale, and the observed Higgs boson is interpreted as a pseudo NG boson getting its nonzero mass through the explicit breaking of the underlying global symmetry. The minimal realization of the composite Higgs models consistent with the electroweak precision measurements is well-known as the minimal composite Higgs model with its global symmetry breaking pattern given by $SO(5)/SO(4)$ [49]. Besides, if we also try to incorporate dark matter into the composite Higgs' framework, its coset space is enlarged from the minimal one and the additional scalar fields are predicted. This model is called the non-minimal composite Higgs models. For an explicit example, the non-minimal composite Higgs model with its symmetry breaking pattern given by $SO(6)/SO(5)$ predicts the additional light scalar particle, which plays the role of dark matter.

In this chapter, we extend the HEFT so that it includes the arbitrary number of neutral and charged scalar fields. The authors of [50] already constructed the extended HEFT including the arbitrary number of neutral scalar fields. The work of this thesis is the further extension of [50].

4.1 Generalized Higgs Effective Field Theory

In this section, we will construct generalized Higgs Effective Field Theory (GHEFT) so that it includes the arbitrary number of neutral and charged scalar fields. Note that, just like the HEFT introduced in Sec. 3, the GHEFT is written in terms of broken phase fields. To add new scalar fields to the HEFT keeping the invariance under non-linearly realized $SU(2)_L \times U(1)_Y$, we must rely on the CCWZ formalism introduced in Sec. 3.3.

Before entering the details of GHEFT, let us introduce the main feature of the GHEFT. The symmetry of the GHEFT is $SU(3)_C \times SU(2)_L \times U(1)_Y$, and its matter contents include new charged and neutral scalar matter fields as well as SM fields. The list of matter contents of the GHEFT is given in Table 4.1.

Field	spin	$SU(3)_C$	$U(1)_{\text{em}}$
G^A_μ	1	$\mathbf{8}$	0
$W^\pm_\mu$	1	$\mathbf{1}$	± 1
Z_μ	1	$\mathbf{1}$	0
$u^i_{L,R}$	1/2	$\mathbf{3}$	$+2/3$
$d^i_{L,R}$	1/2	$\mathbf{3}$	$-1/3$
$e^i_{L,R}$	1/2	$\mathbf{1}$	-1
ν^i_L	1/2	$\mathbf{1}$	0
h	0	$\mathbf{1}$	0
$w^\pm$	0	$\mathbf{1}$	± 1
z	0	$\mathbf{1}$	0
ϕ^I	0	$\boldsymbol{n}_I$	q_I

Table 4.1: Matter contents in the Generalized Higgs Effective Field Theory

Note that we collectively express observed Higgs boson as well as new scalar matter fields by ϕ^I, which will be explained in detail later. In the next section, we focus on the NG boson sectors and the matter sectors in the GHEFT in turn.

4.2 Construction of GHEFT

In this section, we will construct the GHEFT introduced in the previous section relying on the CCWZ method.

4.2.1 NG bosons

The generator of unbroken symmetry $U(1)_{\text{em}}$ can be written as

$$S = \frac{1}{2} \begin{pmatrix} \tau^3 & \\ & \tau^3 \end{pmatrix} . \tag{4.1}$$

Note that, in (4.1), upper-left two by two matrix denotes $SU(2)_L$ subspace, and lower-right two by two denotes that of $U(1)_Y$. The form of unbroken $U(1)_{\text{em}}$ generator is reasonable because unbroken $U(1)_{\text{em}}$ is the symmetry which rotate $U(1)_Y$ and $U(1)_L$ subgroup of $SU(2)_L$ in the same angle. In the symmetry breaking pattern $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$, there are three would-be NG bosons, $w^\pm$ and z, which are going to be eaten by gauge fields $W^\pm$ and Z. In this chapter, we choose the three broken generator as

$$X^\alpha = \frac{1}{2} \begin{pmatrix} \tau^\alpha & \\ & 0 \end{pmatrix} \quad (\alpha = 1, 2)\,, \qquad X^3 = \frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix} . \tag{4.2}$$

Using the unbroken generator introduced above, we can express transformation matrix of the unbroken symmetry $U(1)_{\text{em}}$ as

$$\mathfrak{h} = \exp\left(i\theta_h S\right) = \begin{pmatrix} e^{i\theta_h \frac{\tau^3}{2}} & \\ & e^{-i\theta_h \frac{\tau^3}{2}} \end{pmatrix} . \tag{4.3}$$

The important thing is that the broken generators $X^{1,2}$ and X^3 transform independently under $U(1)_{\text{em}}$ symmetry, namely, $\{X^1, X^2\}$ and X^3 comprise irreducible representation [1] :

$$\mathfrak{h}\,(X^1, X^2, X^3)\,\mathfrak{h}^\dagger = (X^1, X^2, X^3) \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{4.7}$$

We can see that a transformation matrix on the RHS is block diagonalized. This means that broken generators $\{X^1, X^2\}$ and X^3 transform independently under $U(1)_{\rm em}$, and so each comprise irreducible representation. In that case, as we mentioned in the end of Sec. 3.3, we can assign different decay constants to NG bosons associated with $\{X^1, X^2\}$ and X^3. Here we assign a decay constant v to NG boson fields $w^{1,2}$, which are associated with broken generators $X^{1,2}$, and assign a decay constant v_Z to NG boson field z, which is associated to the broken generator X^3.

In this case, ξ can be written as

$$\xi = \exp\left(\frac{i}{v}\sum_{\alpha=1,2} w^\alpha X^\alpha\right)\exp\left(\frac{i}{v_Z} z X^3\right) . \tag{4.8}$$

The transformation law of matrix ξ under $SU(2)_L \times U(1)_Y$ are given as

$$\xi \to \xi' = \mathfrak{g}\cdot\xi\cdot\mathfrak{h}^\dagger(\pi, \theta_{\mathfrak{g}})\,, \tag{4.9}$$

$$\mathfrak{g} \in SU(2)_L \times U(1)_Y\,, \tag{4.10}$$

$$\mathfrak{h} \in U(1)_{\rm em}\,. \tag{4.11}$$

Cartan one-form defined by Eq.(3.58) is written in terms of ξ given by Eq.(4.8) as

$$\alpha_\mu = \frac{1}{i}\xi^\dagger\partial_\mu\xi\,. \tag{4.12}$$

[1] To get Eq.(4.7), we used the following formulae:

$$e^{i\theta\frac{\tau_3}{2}}\tau_1 e^{-i\theta\frac{\tau_3}{2}} = \tau_1\cos\theta - \tau_2\sin\theta \tag{4.4}$$

$$e^{i\theta\frac{\tau_3}{2}}\tau_2 e^{-i\theta\frac{\tau_3}{2}} = \tau_1\sin\theta + \tau_2\cos\theta \tag{4.5}$$

$$e^{i\theta\frac{\tau_3}{2}}\tau_3 e^{-i\theta\frac{\tau_3}{2}} = \tau_3 \tag{4.6}$$

The next step is decomposing α_μ into unbroken generator part and broken generator part like

$$\alpha_\mu = \alpha_{\parallel\mu} + \alpha_{\perp\mu}\,, \tag{4.13}$$

where their transformation law under $SU(2)_L \times U(1)_Y$ is given as

$$\alpha'_{\parallel\mu} = \mathfrak{h}(\pi, \theta_\mathfrak{g})\alpha_{\parallel\mu}\mathfrak{h}^\dagger(\pi, \theta_\mathfrak{g}) + \frac{1}{i}\mathfrak{h}(\pi, \theta_\mathfrak{g})\partial_\mu\mathfrak{h}^\dagger(\pi, \theta_\mathfrak{g})\,, \tag{4.14}$$

$$\alpha'_{\perp\mu} = \mathfrak{h}(\pi, \theta_\mathfrak{g})\alpha_{\perp\mu}\mathfrak{h}^\dagger(\pi, \theta_\mathfrak{g})\,. \tag{4.15}$$

Note that when we choose the broken generators X^a as Eq.(4.2), X^1 and X^2 are orthogonal to S, but X^3 is not.

$$\mathrm{Tr}\left[X^3 S\right] \neq 0 \tag{4.16}$$

Because the unbroken generator S and broken generators X^a are not orthogonal, the projection of α_μ to the unbroken generator S does not give $\alpha_{\parallel\mu}$, and the projection of α_μ to the broken generator X does not give $\alpha_{\perp\mu}$:

$$\alpha_{\parallel\mu} \neq \mathrm{Tr}\left[\alpha_\mu S\right] S\,, \tag{4.17}$$

$$\alpha_{\perp\mu} \neq \mathrm{Tr}\left[\alpha_\mu X^a\right] X^a\,. \tag{4.18}$$

The HEFT Lagrangian introduced in Sec. 3 is constructed with an assumption that its global symmetry breaking pattern is $SU(2)_L \times SU(2)_R \to SU(2)_V$. In that case, the decomposition of Cartan one-form into unbroken and broken generator parts is not so difficult as we show inappendix A, because the coset space $SU(1)_L \times SU(2)_R/SU(2)_V$ is symmetric: commutation relations of the generators have the parity symmetry τ_p defined by Eq.(A.7). This parity symmetry gave us guidelines on how we can decompose the Cartan one-form into unbroken and broken generator parts.

In the GHEFT case, we will treat new physics effects as model-independent as possible, so we do not assume any global symmetries: a global symmetry breaking pattern of the GHEFT is identical to that of the SM gauge symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. Because a coset space $SU(2)_L \times U(1)_Y / U(1)_{\rm em}$ is not symmetric space, commutation relations of the unbroken generator S and the broken generators X do not have parity symmetry τ_p any longer.

The decomposition of Cartan one-form into unbroken and broken generator part is not so easy, so we will consider extracting the unit of transformation. when we consider the basis of the unbroken and broken generator, we identify the upper-left two by two matrix to be $SU(2)_L$ subspace, and the lower-right two by two matrix to be that of $U(1)_Y$. In this notation, ξ can be expressed as

$$\xi = \begin{pmatrix} \hat{\xi}_W & \\ & \hat{\xi}_Y \end{pmatrix}, \tag{4.19}$$

with

$$\hat{\xi}_W = \exp\left(\frac{i}{2v} \sum_{\alpha=1,2} w^\alpha \tau^\alpha\right), \qquad \hat{\xi}_Y = \exp\left(\frac{i}{2v_Z} z\tau^3\right). \tag{4.20}$$

If we express transformation matrices $\mathfrak{g} \in SU(2)_L \times U(1)_Y$ and $\mathfrak{h} \in U(1)_{\rm em}$ as

$$\mathfrak{g} = \begin{pmatrix} \hat{\mathfrak{g}}_W & \\ & \hat{\mathfrak{g}}_Y \end{pmatrix}, \qquad \mathfrak{h} = \begin{pmatrix} \hat{\mathfrak{h}} & \\ & \hat{\mathfrak{h}} \end{pmatrix}, \tag{4.21}$$

with

$$\hat{\mathfrak{g}}_W = \exp\left(i \sum_a^3 \theta_W^a \frac{\tau^a}{2}\right), \tag{4.22}$$

$$\hat{\mathfrak{g}}_Y = \exp\left(i\theta_Y \frac{\tau^3}{2}\right), \tag{4.23}$$

$$\hat{\mathfrak{h}} = \exp\left(i\theta_h(\omega^\alpha, z, \theta_W, \theta_Y)\frac{\tau^3}{2}\right), \tag{4.24}$$

then the transformation law of $\hat{\xi}_W$ and $\hat{\xi}_Y$ under $SU(2)_L \times U(1)_Y$ can be expressed given as

$$\hat{\xi}_W \to \hat{\mathfrak{g}}_W \hat{\xi}_W \hat{\mathfrak{h}}^\dagger \,, \tag{4.25}$$

$$\hat{\xi}_Y \to \hat{\mathfrak{g}}_Y \hat{\xi}_Y \hat{\mathfrak{h}}^\dagger \,. \tag{4.26}$$

Then we find that the following quantities have simple transformation law.

$$\hat{\alpha}_{\parallel\mu} := -\frac{1}{i}\hat{\xi}_Y^\dagger \partial_\mu \hat{\xi}_Y \,, \tag{4.27}$$

$$\hat{\alpha}_{\perp\mu} := \sum_{a=1}^{3} \hat{\alpha}_{\perp\mu}^a \frac{\tau^a}{2} \,, \tag{4.28}$$

with each component of $\alpha_{\perp\mu}^a$ $(a = 1 \sim 3)$ given as

$$\hat{\alpha}_{\perp\mu}^\alpha = \mathrm{Tr}\left[\frac{1}{i}\hat{\xi}_W^\dagger \partial_\mu \hat{\xi}_W \tau^\alpha\right] \,, \qquad \Big(\alpha = 1, 2\Big) \,, \tag{4.29}$$

$$\hat{\alpha}_{\perp\mu}^3 = \mathrm{Tr}\left[\frac{1}{i}\hat{\xi}_W^\dagger \partial_\mu \hat{\xi}_W \tau^3\right] - \mathrm{Tr}\left[\frac{1}{i}\hat{\xi}_Y^\dagger \partial_\mu \hat{\xi}_Y \tau^3\right] \,. \tag{4.30}$$

We can easily check that, under $SU(2)_L \times U(1)_Y$ symmetry, $\alpha_\parallel$ transforms like gauge fields and $\alpha_\perp$ transforms homogeneously:

$$\hat{\alpha}'_{\parallel\mu} = \hat{\mathfrak{h}} \hat{\alpha}_{\parallel\mu} \hat{\mathfrak{h}}^\dagger + \frac{1}{i}\hat{\mathfrak{h}} \partial_\mu \hat{\mathfrak{h}}^\dagger \,, \tag{4.31}$$

$$\hat{\alpha}'_{\perp\mu} = \hat{\mathfrak{h}} \hat{\alpha}_{\perp\mu} \hat{\mathfrak{h}}^\dagger \,. \tag{4.32}$$

Note that $\{\alpha_{\perp\mu}^\alpha\}$ and $\alpha_{\perp\mu}^3$ comprise irreducible representation:

$$\mathfrak{h} \, \left(\{\hat{\alpha}_{\perp\mu}^\alpha\}, \ \hat{\alpha}_{\perp\mu}^3\right) \, \mathfrak{h}^\dagger = \left(\{\hat{\alpha}_{\perp\mu}^\beta\}, \ \hat{\alpha}_{\perp\mu}^3\right) \begin{pmatrix} \rho_{\beta\alpha}(\mathfrak{h}) & 0 \\ 0 & 1 \end{pmatrix} \,, \tag{4.33}$$

with

$$\rho(\mathfrak{h}) = \exp\Big(i\theta_{\mathfrak{h}}(\pi, \mathfrak{g})\tau_2\Big) \,. \tag{4.34}$$

Now we can write down the operator respecting non-linearly realized $SU(2)_L \times U(1)_Y$ using $\alpha_{\perp\mu}$:

$$\mathcal{L}_\pi = \frac{1}{2}\sum_{a,b}^{3} G^{(0)}_{ab}\hat{\alpha}^a_{\perp\mu}\hat{\alpha}^{b\mu}_{\perp}\,, \tag{4.35}$$

where $G^{(0)}_{ab}$ is a three by three matrix with its explicit form given as

$$G^{(0)}_{ab} = \frac{1}{4}\begin{pmatrix} v^2 & & \\ & v^2 & \\ & & v_Z^2 \end{pmatrix}\,. \tag{4.36}$$

Further calculation lead Eq.(4.35) to simpler form

$$\mathcal{L}_\pi = \frac{v^2}{4}\mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U\right] - \frac{v_Z^2 - v^2}{8}\mathrm{Tr}\left[U^\dagger(\partial_\mu U)\tau_3\right]\mathrm{Tr}\left[U^\dagger(\partial_\mu U)\tau_3\right]\,, \tag{4.37}$$

with

$$U := \xi_W \xi_Y^\dagger\,. \tag{4.38}$$

4.2.2 Matter fields

Next, we will consider adding new scalar matter fields to the HEFT Lagrangian. Here we consider adding n_C charged scalar fields and n_N neutral scalar fields to the HEFT matter contents.

Firstly, we will consider charged scalar fields.Note that the charged particles are expressed by complex fields, and if we add a complex scalar field with its electromagnetic charge $+q$, we must add its antiparticle with the same mass but opposite charge $-q$, which is expressed by Hermitian conjugation of the original field. The n_C pairs of charged scalar particles and anti-particles are written in terms of the $2n_C$ number of real scalar degrees of freedom as

$$\frac{\phi^1 \pm i\phi^2}{\sqrt{2}},\ \frac{\phi^3 \pm i\phi^4}{\sqrt{2}},\ \cdots,\frac{\phi^{2n_C-1} \pm i\phi^{2n_C}}{\sqrt{2}}, \tag{4.39}$$

where $\phi^1 \cdots \phi^{2n_C}$ are real scalar fields. If we assign electromagnetic charge $\mp q_k$ to the charged scalar fields $(\phi^k \pm \phi^{k+1})/\sqrt{2}$, then, its transformation law under $U(1)_{\rm em}$ is given by

$$\frac{1}{\sqrt{2}}\begin{pmatrix} \phi^k - i\phi^{k+1} \\ \\ \phi^k + i\phi^{k+1} \end{pmatrix} \to \begin{pmatrix} e^{+iq_k\theta} & \\ & \\ & e^{-iq_k\theta} \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^k - i\phi^{k+1} \\ \\ \phi^k + i\phi^{k+1} \end{pmatrix} , \tag{4.40}$$

with k runs from 1 to n_C.

Combining with the remaining n_N real scalar fields which are labeled as

$$\phi^{2n_C+1},\ \phi^{2n_C+2},\ \cdots ,\ \phi^{2n_C+n_N} , \tag{4.41}$$

we can express all the charged and neutral scalar matter fields in terms of real scalar degrees of freedom as

$$\phi^I = \{\overbrace{\phi^1,\ \cdots ,\ \phi^{2n_C}}^{\text{Charged}},\ \ \overbrace{\phi^{2n_C+1},\ \cdots ,\ \phi^{2n_C+n_N}}^{\text{Neutral}}\} . \tag{4.42}$$

The transformation law of scalar matter fields ϕ^I under non-linearly realized $SU(2)_L\times U(1)_Y$ is quite simple. Because charged scalars' transformation law under $U(1)_{\rm em}$ is given by Eq.(4.40) and neutral scalar fields do not transform under $U(1)_{\rm em}$, the transformation law of ϕ^I is given by

$$\phi^I \to [\rho_\phi(\mathfrak{h}(\pi, g_{G_1}))]^I{}_J \phi^J , \tag{4.43}$$

where

$$\rho_\phi(h(\pi, g_{G_1})) = \exp[i\theta_h(\pi, g_{G_1})Q_\phi] , \tag{4.44}$$

with Q_ϕ being $(2n_C + n_N) \times (2n_C + n_N)$ matrix given by

$$Q_\phi = \begin{pmatrix} q_1\sigma_2 & & & & & \\ & \ddots & & & & \\ & & -q_{n_C}\sigma_2 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}, \tag{4.45}$$

with $\sigma_2 = \tau_2$. Because the transformation matrix (4.44) depends on NG fields, it is a local transformation. Therefore, in order to write down invariant operators under $SU(2)_L \times U(1)_Y$, we must define a covariant derivative. As we see at the beginning of this section, unbroken generator part of Cartan one-form, $\alpha_\parallel$, transforms like gauge field, so we can construct covariant derivative as

$$(\mathcal{D}_\mu \phi)^I = \partial_\mu \phi^I + i\mathcal{V}_\mu^3 [Q_\phi]^I{}_J \phi^J\,, \tag{4.46}$$

with $\mathcal{V}_\mu^3$ given as

$$\mathcal{V}_\mu^3 = -\mathrm{Tr}\left[\frac{1}{i}\hat{\xi}_Y^\dagger (\partial_\mu \hat{\xi}_Y)\tau_3\right] + c\,\hat{\alpha}_{\perp\mu}^3\,. \tag{4.47}$$

where c is an arbitrary constant.

Now we can write down the GHEFT Lagrangian respecting the non-linearly realized $SU(2)_L \times U(1)_Y$ symmetry. The building blocks are given as

$$\hat{\alpha}_{\perp\mu}, \qquad (\mathcal{D}_\mu \phi)^I\,. \tag{4.48}$$

The most general $\mathcal{O}(p^2)$ Lagrangian invariant under non-linearly realized $SU(2)_L \times U(1)_Y$ is given by

$$\mathcal{L} = \frac{1}{2}G_{ab}\hat{\alpha}_{\perp\mu}^a \hat{\alpha}^{b\mu} + G_{aI}\hat{\alpha}_{\perp\mu}^a (\mathcal{D}^\mu \phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu \phi)^I (\mathcal{D}^\mu \phi)^J - V\,, \tag{4.49}$$

where G_{ab}, G_{aI}, G_{IJ} are functions of NG bosons and scalar matter fields, which transform homogeneously under $SU(2)_L \times U(1)_Y$.

Chapter 5

Geometry of the Scalar Sector

The origin of various physical phenomena can be understood from the behavior of the elementary particles, and what controls the rules of these elementary particles is quite simple in theoretical particle physics: interaction operators in Lagrangian. Interaction operators are quite important in the sense that they determine how particles interact with each other: if there is an operator that connects a particle A and a particle B with a large coupling constant, these particles are expected to interact strongly with each other, and if there are no operators connecting particle A to B, they never interact with each other in the tree level. They may interact from the higher order of the perturbation, but their interactions are expected to be weak. Furthermore, basic observables such as decay rate and scattering cross section are written in terms of the coupling constants, which is the coefficients of the interaction operators.

There is one subtle point, however. The forms of the interaction terms are affected by a choice of the field basis of the particles: if we take the different field basis, the appearance of the interaction operators may be drastically changed. The change of the coordinate system is, however, a quite artificial process and it should not affect the physical observables such as decay rates or scattering cross sections. Considering these situations, one simple question may arise: how can it be possible to get identical results under the different choice of field basis? In this chapter, we will focus on this topic.

5.1 Different interactions for different coordinate systems

One of the difficulties in EFT analysis is that the physical properties such as forms of interaction vertices look different depending on a choice of the field basis. Let's consider a simple example and check how the identical results are obtained under the different choice of field basis. In this section, we again consider the $O(N)$ linear sigma model as a concrete example, which is already described in Sec. 3.2. Lagrangian of $O(N)$ linear sigma model is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} - V(\phi)\,, \tag{5.1}$$

$$V(\phi) = \frac{1}{2}\mu^2(\vec{\phi})^2 + \frac{\lambda}{4}(\vec{\phi})^4\,. \tag{5.2}$$

where $\vec{\phi}$ is a N component scalar field in the fundamental representation of the global $O(N)$ symmetry. As we previously mentioned in Sec. 3.2, if the scalar potential V has a negative mass term ($\mu^2 < 0$), then the potential V becomes a wine bottle type potential, just like the SM, and the scalar field takes a nonzero vacuum expectation value

$$\langle\phi\rangle = \sqrt{\frac{-\mu^2}{\lambda}} =: v\,. \tag{5.3}$$

As a result, $O(N)$ symmetry is spontaneously broken to its subgroup $O(N-1)$. The scalar field $\vec{\phi}$ should now be replaced by the broken phase fields such as NG bosons denoted by $\vec{\pi}$ and a massive scalar matter field denoted by σ. The way how to express the symmetric phase field $\vec{\phi}$ in terms of the broken phase fields $\vec{\pi}$ and σ is not unique. In Sec. 3.2, we show two kinds of parametrization as examples. One is the Cartesian coordinate system, and the other is the polar coordinate system.

In the Cartesian coordinate system, $\vec{\phi}$ is expressed by $\vec{\pi}$ and σ as

$$\vec{\phi} = (\pi^1, \pi^2, \cdots, \pi^{N-1}, v+\sigma)^T\,. \tag{5.4}$$

and Lagrangian after the spontaneous symmetry breaking is expressed as

$$\mathcal{L}_L = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\vec{\pi},\sigma)\,, \tag{5.5}$$

with

$$V(\vec{\pi},\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \lambda v\sigma^3 + \lambda v(\vec{\pi}\,)^2\sigma + \frac{\lambda}{4}(\vec{\pi}\,)^4\,. \tag{5.6}$$

The important features we want to emphasize at this point is that, in the Cartesian coordinate system, the scalar potential V depends on both NG boson π and scalar matter field σ: $V = V(\vec{\pi},\sigma)$. The scalar potential should depend on the scalar matter field σ because σ has a nonzero mass, but V also depends on NG boson π through self-interaction terms such as $\pi\pi\pi\pi$ interaction. Also, NG boson π and scalar matter field σ interact with each other through the three-point $\pi\pi\sigma$ interaction term in the fourth term in Eq.(5.6). As we will see later, these features are absent in the polar coordinate system.

On the other hand, in the polar coordinate system, the interaction terms look quite different from those in the Cartesian coordinate system. Remember that, in the polar coordinate system, the symmetric phase field $\vec{\phi}$ is expressed in terms of broken phase fileds π and σ as

$$\vec{\phi} = (v+\sigma)\,\xi(\pi)\cdot\vec{F}\,, \tag{5.7}$$

where ξ is the $N\times N$ unitary matrix living in coset space G/H and $\vec{F}$ is a unit vector directing to the vacuum,

$$\xi(\pi) = \exp\left(\frac{i}{v}\pi^a X^a\right), \tag{5.8}$$

$$\vec{F} = (0,\cdots,0,1)^T\,. \tag{5.9}$$

with X^a denoting the broken generator. In this case, Lagrangian in the broken phase can be expressed as

$$\mathcal{L}_{NL} = \frac{1}{2}\left(1+\frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{1}{3v^2}\big\{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - (\vec{\pi}\cdot\vec{\pi})(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})\big\} + \mathcal{O}\big((\pi)^6\big)\right] \\ + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)\,, \tag{5.10}$$

with

$$V(\sigma) = \frac{\lambda}{4}(\sigma^2 + 2v\sigma)^2\,. \tag{5.11}$$

An important feature we want to emphasize at this point is that the scalar potential V depends only on the scalar matter field σ, $V = V(\sigma)$. Contrary to the case of the Cartesian coordinate system, NG boson π does not have any self-interactions in the scalar potential. Alternatively, π has self-interaction terms in the kinetic term. As you see in Eq.(5.10), there are the infinity numbers of self-interaction terms in the kinetic term.

Comparing the Lagrangian in the Cartesian coordinate system Eq.(5.5) and that in the polar coordinate system Eq.(5.10), we can find many differences in their interaction terms. As we mentioned previously, physical observables should not be affected by the choice of the coordinate system, but it seems to be quite nontrivial to get the same results in the Cartesian and polar coordinate Lagrangians when we calculate the physical observables such as the decay rate or the scattering cross section in each coordinate system. For examples, if we calculate the NG boson's four-point $\pi\pi \to \pi\pi$ scattering cross section, we expect that the results will show completely different behaviors in each coordinate system. If we calculate $\pi\pi \to \pi\pi$ scattering amplitude in the Cartesian coordinate system, because $\pi\pi\pi\pi$ self-interaction is included in the potential term, momentum dependence may only come from the scalar field's propagator and we do not expect the energy growing behavior of the scattering amplitude. On the other hand, if we calculate $\pi\pi \to \pi\pi$ scattering amplitude in the polar coordinate system, we expect that the amplitude shows energy growing behavior, because $\pi\pi\pi\pi$ self-interaction is included in the kinetic term and the resulting amplitude picks up the momentum dependence through space-time derivative. Therefore, scattering amplitude in the Cartesian coordinate system may not show the energy growing behavior, but the same amplitude calculated in the polar coordinate system will grow up as the center of mass energy increases. As we will see below, the expressions of $\pi\pi \to \pi\pi$ scattering amplitude are exactly the same: the expected energy growing behavior in polar coordinate are canceled out when we sum up all the contributions from relevant Feynman diagrams. In the remaining of this section, we will calculate NG boson's four-point scattering cross section in both the Cartesian and polar coordinate system and find that we actually obtain the same results.

Firstly, we will calculate $\pi\pi \to \pi\pi$ scattering amplitude in the Cartesian coordinate system. $O(N)$ linear sigma model Lagrangian in the Cartesian coordinate system is given as Eq.(5.5). The contribution from the contact interaction is calculated as

$$i\mathcal{A}^c = -2i\lambda(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\,, \tag{5.12}$$

and contributions from scalar exchange diagrams is calculated as

$$i\mathcal{A}^\sigma = i\mathcal{A}^s + i\mathcal{A}^t + i\mathcal{A}^u\,, \tag{5.13}$$

with

$$i\mathcal{A}^s = -4\lambda^2 v^2 \frac{i}{s - m_\sigma^2}\delta^{ab}\delta^{cd}\,, \tag{5.14}$$

$$i\mathcal{A}^t = -4\lambda^2 v^2 \frac{i}{t - m_\sigma^2}\delta^{ac}\delta^{bd}\,, \tag{5.15}$$

$$i\mathcal{A}^u = -4\lambda^2 v^2 \frac{i}{u - m_\sigma^2}\delta^{ad}\delta^{bc}\,. \tag{5.16}$$

Combining the result of the contact interaction (5.12) and that of scalar exchange (5.13), we get the final result given as

$$i\mathcal{A}^{\text{total}} = \; -2i\lambda\left(1 + \frac{m_\sigma^2}{s - m_\sigma^2}\right)\delta^{ab}\delta^{cd}$$

$$
\begin{aligned}
&- 2i\lambda \left(1 + \frac{m_\sigma^2}{t - m_\sigma^2}\right) \delta^{ac}\delta^{bd} \\
&- 2i\lambda \left(1 + \frac{m_\sigma^2}{u - m_\sigma^2}\right) \delta^{ad}\delta^{bc} \,.
\end{aligned}
\tag{5.17}
$$

As we expected, the momentum dependence of the scattering amplitude only comes from the scalar matter field exchange, and the resulting Mandelstam variables s, t, and u only appear in the denominator. Therefore, there is no energy growing behavior in the scattering amplitude calculated in the Cartesian coordinate system.

Next, we will calculate $\pi\pi \to \pi\pi$ scattering amplitude in polar coordinate system. $O(N)$ linear sigma model Lagrangian in the polar coordinate system is given as Eq.(5.10). The contribution from the contact interaction is calculated as

$$
i\mathcal{A}^c = \frac{i}{v^2}(s\,\delta^{ab}\delta^{cd} + t\,\delta^{ac}\delta^{bd} + u\,\delta^{ad}\delta^{bc}) \,. \tag{5.18}
$$

As we expected, contact interaction diagram pick up momentum dependence through self interactions in the NG boson's kinetic term, and show the energy growing behavior.

On the other hand, contributions from scalar exchange diagrams is calculated as

$$
i\mathcal{A}^\sigma = i\mathcal{A}^s + i\mathcal{A}^t + i\mathcal{A}^u \,, \tag{5.19}
$$

with

$$
i\mathcal{A}^s = -\frac{s^2}{v^2}\frac{i}{s - m_\sigma^2}\delta^{ab}\delta^{cd} \,, \tag{5.20}
$$

$$
i\mathcal{A}^t = -\frac{t^2}{v^2}\frac{i}{t - m_\sigma^2}\delta^{ac}\delta^{bd} \,, \tag{5.21}
$$

$$
i\mathcal{A}^u = -\frac{u^2}{v^2}\frac{i}{u - m_\sigma^2}\delta^{ad}\delta^{bc} \,. \tag{5.22}
$$

As we see, each scalar exchange diagram is proportional to the square of the Mandelstam variables. This is because three-point $\sigma\pi\pi$ interaction picks up space-time derivative two times and there are two $\sigma\pi\pi$ vertices in one scalar exchange diagram. Therefore, we find that not only the contact interaction diagram but also the scalar exchange diagram show energy growing behavior. Combining the result of the contact interaction (5.18) and that of scalar exchange (5.19), however, we find that the energy growing behaviors are completely canceled out. For examples, the energy growing behavior proportional to s is canceled as

$$s - \frac{s^2}{s - m_\sigma^2} = s - \frac{s^2 - m_\sigma^4 + m_\sigma^4}{s - m_\sigma^2} = s - \left(s + m_\sigma^2 - \frac{m_\sigma^4}{s - m_\sigma^2} \right) . \tag{5.23}$$

We finally find that the resulting expression,

$$\begin{aligned} i\mathcal{A}^{\text{total}} = & -2i\lambda \left(1 + \frac{m_\sigma^2}{s - m_\sigma^2}\right) \delta^{ab}\delta^{cd} \\ & -2i\lambda \left(1 + \frac{m_\sigma^2}{t - m_\sigma^2}\right) \delta^{ac}\delta^{bd} \\ & -2i\lambda \left(1 + \frac{m_\sigma^2}{u - m_\sigma^2}\right) \delta^{ad}\delta^{bc}, \end{aligned} \tag{5.24}$$

is exactly the same form as Eq.(5.17), which is obtained in the Cartesian coordinate system.

We now confirm that even if we adopt a different field basis, we get identical results when we calculate the physical observables such as scattering cross sections. At the same time, we reveal problems about the choice of coordinate system: if we choose a bad coordinate system, we will misunderstand the behavior of the observables just like we did in the previous examples of the polar coordinate system. How can we avoid this ambiguity coming from a choice of the coordinate system? How can we extract the coordinate independent properties of the observables? One of the solutions is focusing on the geometry of the field space, which will be explained in the next section.

5.2 Geometry of the scalar sector

In the previous section, we show that the bad choice of the coordinate system may course misunderstanding of the observables' behavior. Observables should not be affected by a choice of the coordinate system. What should we do if we want to extract coordinate-independent information?

Before considering the coordinate system in the field space, let us focus on the more familiar example, the space-time coordinate system. The line element of the space-time is expressed as

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \,. \tag{5.25}$$

In the case of space-time coordinate, the information of the coordinate system is restored in the functional form of the metric tensor, $g_{\mu\nu}$. Here, we want to emphasize that we cannot extract the information about space-time manifold from the functional form of the metric tensor: even if space is flat, $g_{\mu\nu}$ change its functional form depending on a choice of the coordinate system. To make sure space is flat or curved, we should focus on the other quantities which reflect the information of geometry in a more direct way and are independent of a choice of the coordinate system. What is the best quantity reflecting the information of the space-time geometry? One way to extract the information about the space-time manifold is calculating geometrical quantities such as the Riemann curvature tensor: even if we choose a different coordinate system, the Riemann curvature tensor is always zero in the flat space.

The same thing may happen in the case of the field space. As we mentioned in the previous section, observables should not depend on a choice of the field coordinate. Therefore, it is natural to anticipate that the physical property can be expressed in terms of the geometrical quantities such as the Riemann curvature tensor in the field space. To check this idea, let's consider the scalar field Lagrangian in the following form,

$$\mathcal{L} = \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi)\,, \tag{5.26}$$

with ϕ^i scalar field. Note that index i runs all kinds of scalar fields in the theory: if there are n_π NG bosons and n_σ scalar matter fields in the theory, i runs from 1 to $n_\pi + n_\sigma$ and ϕ^i can be expressed as

$$\phi^i = (\pi^1, \cdots, \pi^{n_\pi}, \sigma^1, \cdots, \sigma^{n_\sigma}) . \tag{5.27}$$

Here we want to emphasize that any type of scalar field Lagrangian can be expressed in the form of Eq.(5.26). This is just like what we see in the space-time coordinate: even if we have any type of space-time geometry, the world line in the geometry can always be expressed in the form of Eq.(5.25). By the analogy of space-time coordinate (5.25), we naively anticipate that observables can be expressed in terms of the geometrical quantity calculated from "metric tensor" g_{ij}. This assumption turns out to be true as we will see in chapter 6. Detailed explanation on this topic will be given in chapter 6 and in this section, we just demonstrate that the Riemann curvature tensor is a good quantity for extracting coordinate-independent information and gives the same results even if we adopt a different coordinate system in field space.

In order to demonstrate that the Riemann curvature tensor is useful for extracting coordinate-independent information, we will calculate the Riemann curvature tensor in $O(N)$ linear sigma model and show that the results calculated in both the Cartesian and polar coordinate system are the same. The Cartesian coordinate system is given by Eq.(5.4) and the metric tensor is given by N times N identity matrix:

$$g_{ij} = \mathbf{1}_N . \tag{5.28}$$

the Riemann curvature tensor is defined in terms of metric tensor g_{ij} as

$$R_{ijkl} := \frac{1}{2}\left(\frac{\partial^2 g_{il}}{\partial\phi^j\partial\phi^k} + \frac{\partial^2 g_{jk}}{\partial\phi^i\partial\phi^l} - \frac{\partial^2 g_{ik}}{\partial\phi^j\partial\phi^l} - \frac{\partial^2 g_{jl}}{\partial\phi^i\partial\phi^k}\right) + g_{mn}\left(\Gamma^m_{il}\Gamma^n_{jk} - \Gamma^m_{ik}\Gamma^n_{jl}\right) , \tag{5.29}$$

where Γ^i_{jk} denotes Affine connection in field space with its explicit form given by

$$\Gamma^i_{jk} := \frac{1}{2}g^{il}\left(\frac{\partial g_{lj}}{\partial\phi^k} + \frac{\partial g_{lk}}{\partial\phi^j} - \frac{\partial g_{jk}}{\partial\phi^l}\right) . \tag{5.30}$$

In the Cartesian coordinate system, because metric tensor is independent of the scalar fields, the Riemann curvature tensor is trivially zero.

On the other hand, calculation in polar coordinate system is a little bit complicated. Polar coordinate system is given by Eq.(3.40) and its Lagrangian can be rewritten as

$$\begin{aligned}
\mathcal{L}_{NL} &= \frac{1}{2}\left(1+\frac{\sigma}{v}\right)^2\left[\delta_{ab}+\frac{1}{3v^2}\left\{\pi^a\pi^b-(\vec{\pi}\cdot\vec{\pi})\delta^{ab}\right\}+\mathcal{O}\big((\pi)^4\big)\right]\partial_\mu\pi^a\partial^\mu\pi^b \\
&\qquad +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma) \\
&= \frac{1}{2}(\partial_\mu\pi^a,\,\partial_\mu\sigma)\begin{pmatrix}\tilde{g}_{ab}(\pi,\sigma) & \\ & 1\end{pmatrix}\begin{pmatrix}\partial^\mu\pi^b \\ \partial^\mu\sigma\end{pmatrix} - V(\sigma)\,,
\end{aligned} \tag{5.31}$$

with

$$\tilde{g}_{ab}(\pi,\sigma) = \left(1+\frac{\sigma}{v}\right)^2\left[\delta_{ab}+\frac{1}{3v^2}\left\{\pi^a\pi^b-(\vec{\pi}\cdot\vec{\pi})\delta^{ab}\right\}+\mathcal{O}\big((\pi)^4\big)\right]. \tag{5.32}$$

Comparing the last line of Eq.(5.31) with Eq.(5.26), we can read out ϕ^i, g_{ij} and V as

$$\phi^i = (\pi^a,\sigma)\,,\qquad g_{ij}(\phi) = \begin{pmatrix}\tilde{g}_{ab}(\pi,\sigma) & \\ & 1\end{pmatrix},\qquad V(\phi) = V(\sigma)\,. \tag{5.33}$$

Now let's calculate the Riemann curvature tensor with its index taken to all NG bosons, R_{abcd}. The first term of the Riemann curvature tensor (5.29) can be easily calculated as

$$\frac{1}{2}\left(\frac{\partial^2 g_{ad}}{\partial\phi^b\partial\phi^c}+\frac{\partial^2 g_{bc}}{\partial\phi^a\partial\phi^d}-\frac{\partial^2 g_{ac}}{\partial\phi^b\partial\phi^d}-\frac{\partial^2 g_{bd}}{\partial\phi^a\partial\phi^c}\right) = -\frac{1}{v^2}(\delta_{ad}\delta_{bc}-\delta_{ac}\delta_{bd})\,. \tag{5.34}$$

Calculating each component of Affine connection from the metric tensor given in Eq.(5.33), we can easily find that the nonzero component of Affine connection is

$$\bar{\Gamma}^\sigma_{ab} = -\frac{1}{v}\delta_{ab}\,, \tag{5.35}$$

and the other components are all equal to zero. Therefore, the second term of the Riemann curvature tensor (5.29) is calculated as

$$\begin{aligned} g_{mn}\left(\Gamma^m_{il}\Gamma^n_{jk} - \Gamma^m_{ik}\Gamma^n_{jl}\right) &= g_{\sigma\sigma}\left(\Gamma^\sigma_{ad}\Gamma^\sigma_{bc} - \Gamma^\sigma_{ac}\Gamma^\sigma_{bd}\right) \\ &= \frac{1}{v^2}\left(\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\right). \end{aligned} \tag{5.36}$$

Combining Eq.(5.34) and Eq.(5.36), we find that NG boson component of the Riemann curvature tensor is equal to zero:

$$\bar{R}_{abcd} = 0\,. \tag{5.37}$$

Similarly, we find

$$\bar{R}_{\sigma\sigma\sigma\sigma} = 0\,, \tag{5.38}$$

$$\bar{R}_{\sigma a\sigma b} = 0\,. \tag{5.39}$$

The other components of the Riemann curvature tensor are trivially zero. Now we confirm that the Riemann curvature tensor in $O(N)$ sigma model equal to zero in both the Cartesian and polar coordinate system.

5.3 Symmetry of the scalar sector

Before closing this chapter, we will mention the symmetry of the scalar sector. If we rewrite the scalar sector into the form of Eq.(5.26), it becomes difficult to see its underlying symmetry. In Eq.(5.26), we express all the scalar fields by ϕ^i regardless of which multiplet each component belongs to, so the underlying symmetry becomes implicit. How can we extract the information about underling symmetry from Lagrangian in the geometrical form, Eq.(5.26)? As we see later, the underlying symmetry can be expressed by the Killing vector in the field space.

The Killing vector is the infinitesimal field variation under the global symmetry transformation. At first, we consider $SU(2)_L$ Killing vectors in the SM. If rotation angle θ_w is much small, $\theta_w \ll 1$, then the transformation law of ϕ^i can be written as

$$\phi^i \to \phi'^i = \phi^i + \theta_w^a\, w_a^i(\phi)\,. \tag{5.40}$$

The Killing vector for $SU(2)_L$ symmetry is the field variation w_a^i in (5.40).

Let's search the conditions the Killing vectors should satisfy. Variating the $SU(2)_L \times U(1)_Y$ symmetric Lagrangian, we get

$$\begin{aligned}\delta_w \mathcal{L}[\phi] &= \mathcal{L}[\phi + \theta_w^a w_a(\phi)] - \mathcal{L}[\phi] \\ &= \frac{1}{2}\theta_w^a \Big(w_a^k\, g_{ij,k} + (w_a^k)_{,i}\, g_{kj} + (w_a^k)_{,j}\, g_{ik} \Big) \partial_\mu \phi^i \partial^\mu \phi^j - \theta_w^a V_{,k} w_a^k\,,\end{aligned} \tag{5.41}$$

where we use a comma-derivative notation,

$$g_{ij,k} := \frac{\partial}{\partial \phi^k} g_{ij}\,, \tag{5.42}$$

$$(w_a^k)_{,i} := \frac{\partial}{\partial \phi^i} w_a^k\,, \tag{5.43}$$

$$V_{,k} := \frac{\partial}{\partial \phi^k} V\,. \tag{5.44}$$

In order to satisfy

$$\delta_w \mathcal{L}[\phi] = 0 \tag{5.45}$$

in arbitrary θ_w, the Killing vector should satisfy the following equations:

$$0 = w_a^k\, g_{ij,k} + (w_a^k)_{,i}\, g_{kj} + (w_a^k)_{,j}\, g_{ik}\,, \tag{5.46}$$

$$0 = w_a^k\, V_{,k}\,. \tag{5.47}$$

The first condition (5.46) is coming from the invariance of the kinetic term, and is the usual conditions for the Killing vectors. This is coming from the fact that the kinetic term of Lagrangian can be regarded as the line element of the target space manifold. In Lagrangian (5.26), we have the other term which cannot be related to the line element of the target space manifold: the scalar potential $V(\phi)$. Therefore, in addition to Eq.(5.46), we have a new condition (5.47) for the Killing vectors, which is coming from the invariance of the scalar potential under $SU(2)_L$ global symmetry.

In the same way, we get conditions for the $U(1)_Y$ Killing vectors y^i, which is the infinitesimal field variation under the $U(1)_Y$ global symmetry:

$$\phi^i \to \phi'^i = \phi^i + \theta_y \, y^i(\phi) \,. \tag{5.48}$$

In order to satisfy

$$\delta_y \mathcal{L}[\phi] = \mathcal{L}[\phi + \theta_y y(\phi)] - \mathcal{L}[\phi] = 0 \tag{5.49}$$

in arbitrary θ_y, the Killing vector should satisfy the following equations.

$$0 = y^k g_{ij,k} + (y^k)_{,i} g_{kj} + (y^k)_{,j} g_{ik} \,, \tag{5.50}$$

$$0 = y^k V_{,k} \,. \tag{5.51}$$

Chapter 6

Perturbative Unitarity Conditions

As we mentioned previously, couplings among the observed Higgs boson and electroweak gauge fields, shortly the hVV couplings, are planned to be measured at $\mathcal{O}(1)\%$ accuracy in the future collider experiments. If hVV couplings turn out to deviate from the SM prediction, the observed Higgs boson fails to unitarize the amplitude of the longitudinal gauge boson scattering $V_L V_L \to V_L V_L$ and the unitarity violation will occur: remember that the cancellation of the energy growing behavior in the SM is realized due to the appropriately tuned value of the hVV couplings. In order to avoid the unitarity violation, we must expect that the new heavy degrees of freedom appear at some energy scale and cancel the energy growing behavior to unitarize $V_L V_L \to V_L V_L$ scattering amplitude completely, together with the observed Higgs boson.

In the previous section, we add the new scalar degrees of freedom to the HEFT. Thanks to these new scalar fields, $V_L V_L \to V_L V_L$ scattering amplitude can remain perturbative unitary even if the hVV couplings turns out to deviate from the SM prediction: the new contribution coming from the new scalar exchange diagrams cancels the energy growing behavior together with the Higgs exchange diagrams. In order to restore the perturbative unitarity completely, however, we must tune the coupling among these scalar fields and gauge fields, shortly, $\phi^I VV$ couplings. At this point, one naive question may arise: What conditions should we impose on the $\phi^I VV$ couplings in order to restore the perturbative unitarity? The conditions that should be satisfied by $\phi^I VV$ couplings in order to respect the perturbative unitarity are called *unitarity sum rules* [37–39]. In this chapter, we will calculate the $V_L V_L \to V_L V_L$ scattering amplitude in the GHEFT and derive unitarity sum rules.

6.1 Geometry of the Generalized HEFT

Feynman diagrams of $V_L V_L \to V_L V_L$ scattering amplitude are composed of two parts as we showed in Fig. 2.1, pure gauge diagrams and scalar exchange diagrams. Calculating the first part, pure gauge diagrams, we must read out relevant Feynman rules from the gauge sector given in Eq.(3.3). Because the gauge self-interactions are a little bit complicated, it is not so easy to calculate gauge contributions from VVV and $VVVV$ vertices. In order to calculate pure gauge diagrams in a more simple manner, we will rely on the useful theorem called equivalence theorem [32–36]. Remember that the longitudinal mode of massive gauge boson comes from absorption of an additional degree of freedom, provided by NG boson. Reflecting this fact, the longitudinal component becomes increasingly similar to NG bosons as the center of mass energy increases. In the high energy limit, the scattering amplitude with V_L appearing in the initial of the final state becomes equal to the scattering amplitude with V_L replaced with would-be NG boson. Therefore, checking the energy growing behavior of $V_L V_L \to V_L V_L$, we simply calculate $vv \to vv$, where v denotes would be NG boson absorbed by gauge field V.

As we briefly explained in Sec.5, there are some difficulties in calculating scattering amplitude using effective Lagrangian: depending on a choice of the field coordinate system, interaction operators look different, and this makes it difficult to extract the relevant information involving observables' behavior. To avoid this ambiguity, we introduced a useful technique in Sec.5. Rewriting the scalar Lagrangian to the following form,

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi) \,, \tag{6.1}$$

and focusing the geometry of the field space implicated by "metric tensor" g_{ij}, we can extract the coordinate-independent information. We expect that the geometry defined by g_{ij} may give us some implication on the observables' behavior, which also should be coordinate-independent. As we will see later in this section, the geometrical quantity calculated from g_{ij} definitely gives us physical information, the perturbative unitarity conditions of the various scattering amplitudes.

In order to focus on the geometry of the GHEFT in the field space, we first need to rewrite the GHEFT Lagrangian (4.49) into the form of Eq.(5.26). Expressing g_{ij} in terms of G_{ab}, G_{aI}, and G_{IJ} in Eq.(4.49) is a little bit complicated, so we skip the detailed calculation and only show the results in appendix B.

In the remaining of this section, we will calculate the energy growing amplitude from the Lagrangian (6.1). We firstly decompose ϕ^i into its vacuum expectation value $\bar{\phi}^i$ and an excitation fields φ^i: $\phi^i = \bar{\phi}^i + \varphi^i$. Now we need to rewrite the scalar Lagrangian (6.1) in terms of excitation fields φ^i. The metric tensor in Eq.(6.1) can be expanded around the vacuum $\bar{\phi}$ as

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{g}_{ij,k}\,\varphi^k + \frac{1}{2}\bar{g}_{ij,kl}\,\varphi^k\varphi^l + \cdots\,, \tag{6.2}$$

where each coefficient of Taylor expansion is given as

$$\bar{g}_{ij} := g_{ij}(\bar{\phi})\,, \tag{6.3}$$

$$\bar{g}_{ij,k} := \frac{\partial}{\partial\phi^k} g_{ij}(\phi)\bigg|_{\phi=\bar{\phi}}\,, \tag{6.4}$$

$$\bar{g}_{ij,kl} := \frac{\partial^2}{\partial\phi^k\partial\phi^l} g_{ij}(\phi)\bigg|_{\phi=\bar{\phi}}\,, \tag{6.5}$$

$$\vdots$$

Similarly, we can expand the potential $V(\phi)$ around the vacuum $\bar{\phi}$ as

$$V(\phi) = \bar{V} + \bar{V}_{,i}\varphi^i + \frac{1}{2}\bar{V}_{,ij}\varphi^i\varphi^j + \frac{1}{3!}\bar{V}_{,ijk}\varphi^i\varphi^j\varphi^k + \frac{1}{4!}\bar{V}_{,ijkl}\varphi^i\varphi^j\varphi^k\varphi^l\cdots\,, \tag{6.6}$$

where each coefficient of Taylor expansion is given as

$$\bar{V} := V(\bar{\phi})\,, \tag{6.7}$$

$$\bar{V}_{,i} := \frac{\partial}{\partial \phi^i} V(\phi)\bigg|_{\phi=\bar{\phi}} , \tag{6.8}$$

$$\bar{V}_{,ij} := \frac{\partial^2}{\partial \phi^i \partial \phi^j} V(\phi)\bigg|_{\phi=\bar{\phi}} , \tag{6.9}$$

$$\vdots$$

Because the scalar potential $V(\phi)$ should be minimized at the vacuum $\bar{\phi}$, we impose that

$$\bar{V}_{,i} = 0 \,. \tag{6.10}$$

Also, we assume that the kinetic and mass terms of the scalar fields are diagonalized, namely,

$$\bar{g}_{ij} = \delta_{ij} \,, \tag{6.11}$$

and

$$\bar{V}_{,ij} = \delta_{ij} m_i^2 \,. \tag{6.12}$$

6.2 Three-point amplitude

We start with the evaluation of the three-point scalar scattering amplitude

$$i\mathcal{M}(123) \tag{6.13}$$

at the tree level. The interaction vertices relevant for this amplitude are give as

$$\mathcal{L}_3 = \frac{1}{2}\bar{g}_{ij,k}\, \varphi^k (\partial_\mu \varphi^i)(\partial^\mu \varphi^j) - \frac{1}{3!}\bar{V}_{,ijk}\, \varphi^i \varphi^j \varphi^k \,. \tag{6.14}$$

The on-shell amplitude can be evaluated as

$$i\mathcal{M}(123) = \frac{i}{2}(\bar{g}_{i_1 i_2, i_3} + \bar{g}_{i_2 i_1, i_3})(-p_1 \cdot p_2) + \frac{i}{2}(\bar{g}_{i_2 i_3, i_1} + \bar{g}_{i_3 i_2, i_1})(-p_2 \cdot p_3)$$

$$
\begin{aligned}
&+\frac{i}{2}(\bar{g}_{i_3 i_1, i_2} + \bar{g}_{i_1 i_3, i_2})(-p_3 \cdot p_1) - i\bar{V}_{,i_1 i_2 i_3} \\
&= \frac{i}{2}\bar{g}_{i_1 i_2, i_3}\left(m_{i_1}^2 + m_{i_2}^2 - s_{12}\right) + \frac{i}{2}\bar{g}_{i_2 i_3, i_1}\left(m_{i_2}^2 + m_{i_3}^2 - s_{23}\right) \\
&+\frac{i}{2}\bar{g}_{i_3 i_1, i_1}\left(m_{i_3}^2 + m_{i_1}^2 - s_{31}\right) - i\bar{V}_{,i_1 i_2 i_3}\,.
\end{aligned}
\tag{6.15}
$$

From the conservation of the total momentum

$$
p_1 + p_2 + p_3 = 0\,,
$$

it is easy to see

$$
s_{12} = (p_1 + p_2)^2 = p_3^2 = m_{i_3}^2\,,
$$

and similarly

$$
s_{23} = m_{i_1}^2\,, \qquad s_{31} = m_{i_2}^2\,.
$$

The on-shell three-point amplitude, Eq.(6.15), can therefore be rewritten as

$$
\begin{aligned}
i\mathcal{M}(123) &= \frac{i}{2}\bar{g}_{i_1 i_2, i_3}\left(m_{i_1}^2 + m_{i_2}^2 - m_{i_3}^2\right) + \frac{i}{2}\bar{g}_{i_2 i_3, i_1}\left(m_{i_2}^2 + m_{i_3}^2 - m_{i_1}^2\right) \\
&+\frac{i}{2}\bar{g}_{i_3 i_1, i_1}\left(m_{i_3}^2 + m_{i_1}^2 - m_{i_2}^2\right) - i\,\bar{V}_{,i_1 i_2 i_3} \\
&= \frac{i}{2}m_{i_1}^2\left(\bar{g}_{i_1 i_2, i_3} + \bar{g}_{i_1 i_3, i_2} - \bar{g}_{i_2 i_3, i_1}\right) + \frac{i}{2}m_{i_2}^2\left(\bar{g}_{i_2 i_3, i_1} + \bar{g}_{i_2 i_1, i_3} - \bar{g}_{i_3 i_1, i_2}\right) \\
&+\frac{i}{2}m_{i_3}^2\left(\bar{g}_{i_3 i_1, i_2} + \bar{g}_{i_3 i_2, i_1} - \bar{g}_{i_1 i_2, i_3}\right) - i\,\bar{V}_{,i_1 i_2 i_3}\,.
\end{aligned}
\tag{6.16}
$$

The first three terms of Eq.(6.16) have the same tensor structure with that of Affine connection Γ^l_{jk},

$$
g_{il}\Gamma^l_{jk} := \frac{1}{2}\left[g_{ij,k} + g_{ki,j} - g_{jk,i}\right]\,. \tag{6.17}
$$

Rewriting the first three terms of Eq.(6.16) in terms of Affine connection and using Eq.(6.11) and Eq.(6.12), we finally get the following expression,

$$
i\mathcal{M}(123) = i\,\bar{V}_{,i_1 l}\bar{\Gamma}^l_{i_2 i_3} + i\,\bar{V}_{,i_2 l}\bar{\Gamma}^l_{i_3 i_1} + i\,\bar{V}_{,i_3 l}\bar{\Gamma}^l_{i_1 i_2} - i\,\bar{V}_{,i_1 i_2 i_3}\,. \tag{6.18}
$$

The final form of the three-point amplitude (6.18) looks quite similar to the third covariant derivative of the scalar potential, $\bar{V}_{ijk}$. Let's calculate the third covariant derivative evaluated at the vacuum, $\bar{V}_{ijk}$, explicitly and compare the result with the RHS of Eq.(6.18). The first, second, and third order of covariant derivative are given by

$$
\begin{aligned}
V_{;i} &= V_{,i}\,, && (6.19)\\
V_{;ij} &= V_{,ij} - \Gamma^l_{ij} V_{,l}\,, && (6.20)\\
V_{;ijk} &= (V_{;ij})_{,k} - \Gamma^l_{ki} V_{;lj} - \Gamma^l_{jk} V_{;li}\,,\\
&= V_{,ijk} - (\Gamma^l_{ij})_{,k} V_{,l} - \Gamma^l_{ij} V_{,lk}\\
&\quad -\Gamma^l_{ik} V_{,lj} - \Gamma^l_{jk} V_{,li} + \Gamma^l_{ik}\Gamma^m_{lj} V_{,m} + \Gamma^l_{jk}\Gamma^m_{li} V_{,m}\,. && (6.21)
\end{aligned}
$$

Applying Eq.(6.10) to the third order of covariant derivative (6.21), we find that the third order of covariant derivative $V_{;ijk}$ evaluated at the vacuum can simply given as

$$\bar{V}_{;ijk} = \bar{V}_{,ijk} - \bar{\Gamma}^l_{ij} \bar{V}_{,lk} - \bar{\Gamma}^l_{ik} \bar{V}_{,lj} - \bar{\Gamma}^l_{jk} \bar{V}_{,li}\,, \tag{6.22}$$

which is exactly the same form as the RHS of Eq.(6.18). It is easy to see that $\bar{V}_{;ijk}$ is symmetric under the $i \leftrightarrow j$, $i \leftrightarrow k$ and $j \leftrightarrow k$ exchanges, so we get

$$V_{;ijk} = V_{;(ijk)}\,, \tag{6.23}$$

with $V_{;(ijk)}$ being the totally symmetrized derivative of the potential. We now find the on-shell three-point amplitude formula given in a simple form,

$$i\mathcal{M}(123) = -i\bar{V}_{;(i_1 i_2 i_3)}\,. \tag{6.24}$$

6.3 Four-point amplitude

Next, we will calculate the four-point scalar scattering amplitude. Here we skip the details of the calculation and only show the results. The final result of the four-point scalar scattering amplitude $i\mathcal{M}(1234)$ is given as

$$\begin{aligned} i\mathcal{M}(1234) = \; & iM(1234) \\ & + i\mathcal{M}(125)[D(s_{12})]_{i_5 i_6} i\mathcal{M}(346) \\ & + i\mathcal{M}(135)[D(s_{13})]_{i_5 i_6} i\mathcal{M}(246) \\ & + i\mathcal{M}(145)[D(s_{14})]_{i_5 i_6} i\mathcal{M}(236) \,, \end{aligned} \tag{6.25}$$

with

$$\begin{aligned} iM(1234) = & -i\bar{V}_{;(i_1 i_2 i_3 i_4)} - \frac{i}{3}\left(\bar{R}_{i_1 i_3 i_4 i_2} + \bar{R}_{i_1 i_4 i_3 i_2}\right) s_{12} \\ & - \frac{i}{3}\left(\bar{R}_{i_1 i_2 i_4 i_3} + \bar{R}_{i_1 i_4 i_2 i_3}\right) s_{13} - \frac{i}{3}\left(\bar{R}_{i_1 i_2 i_3 i_4} + \bar{R}_{i_1 i_3 i_2 i_4}\right) s_{14} \,, \end{aligned} \tag{6.26}$$

and

$$[D(s)]_{ij} := \frac{i}{s - m_i^2} \bar{g}_{ij} \,. \tag{6.27}$$

The first line of Eq.(6.25) comes from the contact interaction. The second, third, and fourth line of Eq.(6.25) comes from scalar exchange diagrams with $i\mathcal{M}(125)$, $i\mathcal{M}(346)$, $\cdots$, $i\mathcal{M}(236)$ in Eq.(6.25) denoting the three point amplitude derived in the previous section. Note that the Riemann curvature tensor evaluated at the vacuum, $\bar{R}_{ijkl}$ is defined as

$$\bar{R}_{ijkl} = \frac{1}{2}\left(\bar{g}_{il,jk} + \bar{g}_{jk,il} - \bar{g}_{ik,jl} - \bar{g}_{jl,ik}\right) + \bar{g}_{mn}\left(\bar{\Gamma}^m_{il}\bar{\Gamma}^n_{jk} - \bar{\Gamma}^m_{ik}\bar{\Gamma}^n_{jl}\right) \,, \tag{6.28}$$

and the symmetrized fourth covariant derivative of the scalar potential evaluated at the vacuum, $\bar{V}_{;(ijkl)}$ is given as

$$\begin{aligned} \bar{V}_{;(ijkl)} = \; & \bar{V}_{,ijkl} - \bar{V}_{,ijm}\bar{\Gamma}^m_{kl} - \bar{V}_{,klm}\bar{\Gamma}^m_{ij} - \bar{V}_{,ikm}\bar{\Gamma}^m_{jl} - \bar{V}_{,jlm}\bar{\Gamma}^m_{ik} - \bar{V}_{,ilm}\bar{\Gamma}^m_{jk} - \bar{V}_{,jkm}\bar{\Gamma}^m_{il} \\ & + \bar{V}_{,mn}\left[\bar{\Gamma}^m_{ij}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^m_{ik}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^m_{il}\bar{\Gamma}^n_{jk}\right] \\ & + A_{ijkl} + A_{jikl} + A_{kijl} + A_{lijk} \,, \end{aligned} \tag{6.29}$$

with

$$
\begin{aligned}
A_{ijkl} :=& \frac{1}{6}\bar{V}_{,im}\bar{g}^{mn}\left[\bar{g}_{jk,nl} + \bar{g}_{kl,nj} + \bar{g}_{jl,nk} - 2(\bar{g}_{nj,kl} + \bar{g}_{nk,jl} + \bar{g}_{nl,jk})\right] \\
&+ \bar{V}_{,im}\left[\bar{\Gamma}^m_{jn}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^m_{kn}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^m_{ln}\bar{\Gamma}^n_{jk}\right] \\
&+ \frac{1}{3}\bar{V}_{,im}\bar{g}^{mp}\left[\bar{\Gamma}^q_{pj}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^q_{pk}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^q_{pl}\bar{\Gamma}^n_{jk}\right]\bar{g}_{qn} \,.
\end{aligned}
\tag{6.30}
$$

6.4 Unitarity sum rules

Applying the on-shell condition

$$
s_{12} + s_{13} + s_{14} = m^2_{i_1} + m^2_{i_2} + m^2_{i_3} + m^2_{i_4} \,, \tag{6.31}
$$

we can eliminate one of the s_{ij}. Eliminating s_{14}, we get

$$
\begin{aligned}
iM(1234) =& -\frac{i}{3}(\bar{R}_{i_1i_3i_4i_2} + \bar{R}_{i_1i_4i_3i_2} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4})s_{12} \\
&- \frac{i}{3}(\bar{R}_{i_1i_2i_4i_3} + \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4})s_{13} + \mathcal{O}(E^0).
\end{aligned}
\tag{6.32}
$$

Because s_{12} and s_{13}, amplitude grows up as center of mass energy increases. In order to avoid perturbative unitarity violation, we must impose the coefficients of the energy growing terms to be equal to zero. Therefore, we get the following unitarity sum rule conditions [37–39]:

$$
\bar{R}_{i_1i_3i_4i_2} + \bar{R}_{i_1i_4i_3i_2} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4} = 0 \,, \tag{6.33}
$$

$$
\bar{R}_{i_1i_2i_4i_3} + \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4} = 0 \,. \tag{6.34}
$$

Because the Riemann curvature tensor satisfy

$$
R_{ijkl} \equiv -R_{ijlk} \,, \tag{6.35}
$$

the unitarity sum rules (6.33) and (6.34) can be rewritten as

$$
2\bar{R}_{i_1i_3i_4i_2} - \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} = 0 \,. \tag{6.36}
$$

Applying Bianchi identity

$$R_{ijkl} + R_{iklj} + R_{iljk} \equiv 0\,, \tag{6.37}$$

Eq.(6.36) can be simplified as

$$3\bar{R}_{i_1 i_3 i_4 i_2} = 0\,. \tag{6.38}$$

Here we get the important conclusion: in order for four point scattering amplitude to be perturbative unitary, the Riemann curvature tensor of scalar manifold should satisfy the following conditions:

$$\bar{R}_{ijkl} = 0\,, \tag{6.39}$$

namely, the Riemann curvature tensor should be flat at the vicinity of the vacuum.

Chapter 7

Oblique Parameters

7.1 One-loop corrections

In order to compare the predictions with precisely-measured observables, we need to calculate loop corrections. The authors of [41] calculated the scalar one-loop corrections to the Lagrangian (5.26) using the background field method [44–47, 52], and derive a one-loop effective Lagrangian in a coordinate-independent form. They showed a divergent part of the scalar one-loop corrections whose explicit form is given as

$$\Delta\mathcal{L}^{\varphi-\mathrm{loop}}_{\mathrm{div}} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}\mathrm{tr}(Y_{\mu\nu}Y^{\mu\nu}) + \frac{1}{2}\mathrm{tr}(X^2)\right] , \tag{7.1}$$

where ϵ is dimensional regularization parameter and is written in terms of spacetime dimension D as

$$\epsilon := 4 - D\,. \tag{7.2}$$

$Y_{\mu\nu}$ and X are given as

$$[Y_{\mu\nu}]^i{}_j = R^i{}_{jkl}(D_\mu\phi)^k(D_\nu\phi)^l + W^a_{\mu\nu}(w^i_a)_{;j} + B_{\mu\nu}(y^i)_{;j}\,, \tag{7.3}$$

$$[X]^i{}_k = R^i{}_{jkl}(D_\mu\phi)^j(D^\mu\phi)^l + g^{ij}V_{;jk}\,, \tag{7.4}$$

where covariant derivatives for $SU(2)_L$ Killing vectors w_a^i and $U(1)_Y$ Killing vectors y^i are given by

$$(w_a^i)_{;j} := w_{a,j}^i + \Gamma^i_{jk} w_a^k \,, \tag{7.5}$$

$$(y^i)_{;j} := y^i_{,j} + \Gamma^i_{jk} y^k \,. \tag{7.6}$$

As we show in Eq.(D.29), $Y_{\mu\nu}$ includes $SU(2)_L$ and $U(1)_Y$ gauge field strength. In Eq.(7.1), one-loop correction to the following quantity

$$W_{\mu\nu}W^{\mu\nu}\,, \qquad B_{\mu\nu}B^{\mu\nu}\,, \qquad W_{\mu\nu}B^{\mu\nu}\,, \tag{7.7}$$

are evaluated. The last quantity in Eq.(7.7) corresponds to the Peskin Takeuchi S parameter, so from Eq.(7.1), we can evaluate the effect of the new scalar fields to parameter S [51]. The remaining oblique parameters T and U are not yet evaluated in [41], and also, only the scalar-loop contributions are calculated and not the gauge fields' contributions. In order to complete the evaluation, we must calculate the whole set of oblique parameters S, T, U, and also evaluate the gauge bosons' contributions to these parameters.

In this section, we will evaluate the Peskin Takeuchi S, T, U parameters in the GHEFT framework. To calculate oblique parameters respecting gauge symmetry, we rely on background field method [52]. The Peskin Takeuchi's S, T, and U parameters are defined as

$$S := 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0))\,, \tag{7.8}$$

$$U := 16\pi(\Pi'_{11}(0) - \Pi'_{33}(0))\,, \tag{7.9}$$

$$\alpha T := 4\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)\,, \tag{7.10}$$

where $\Pi_{AB}(0)$ is a vacuum polarization evaluated at $p^2 = 0$,

$$\int d^4q J_A(q^2) J_B(0) = i g_A g_B g_{\mu\nu} \Pi_{AB} + p_\mu p_\nu \,, \tag{7.11}$$

and $\Pi'_{AB}(0)$ is a derivative function with respect to p^2 evaluated at $p^2 = 0$,

$$\Pi'_{AB}(0) := \frac{d}{dp^2}\Pi_{AB}(p^2)\bigg|_{p^2=0} . \tag{7.12}$$

In the remaining of this section, we will calculate vacuum polarization diagrams relevant to the S, T and U oblique parameters,

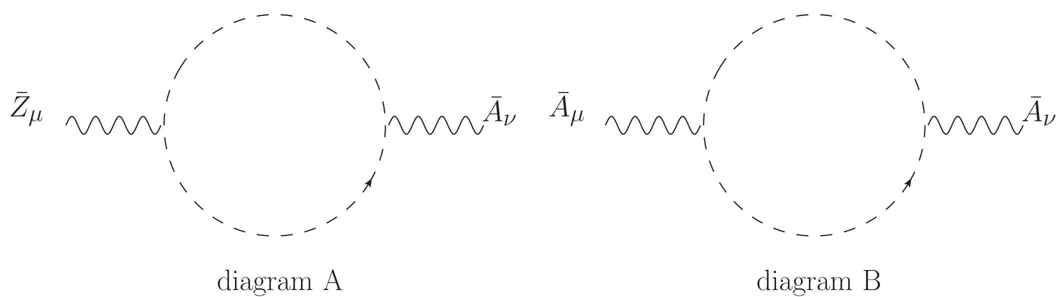
$$\Pi_{3Q} := \frac{1}{gg_Y}\Pi_{ZA} + \frac{1}{g^2}\Pi_{AA}\,, \qquad \Pi_{11}\,, \qquad \Pi_{33}\,. \tag{7.13}$$

7.2 Scalar loop

In this subsection, we will evaluate a scalar loop contribution to the vacuum polarization Π_{3Q}, Π_{11} and Π_{33} in turn.

Π_{3Q} vacuum polarization

Firstly, we will evaluate the one-loop scalar contribution to Π_{3Q}, denoted by $\Pi^{\xi\xi}_{3Q}$. Relevant diagrams are listed in Fig.7.1, with gauge and scalar fields expressed by wavy and dashed lines, respectively.



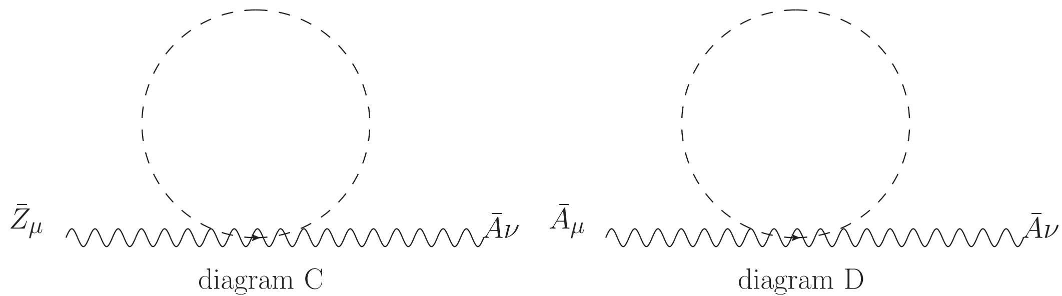


Figure 7.1: Feynman diagrams contributing to $\Pi^{\xi\xi}_{3Q}$

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.1, we get the following result,

$$
\begin{aligned}
g^2\Pi^{\xi\xi}_{3Q}(p^2) = &-\frac{2}{(4\pi)^2}(G_Z)^i{}_j(G_A)^j{}_iB_{22}(p^2;M_i^2,M_j^2)\\
&-\frac{2}{(4\pi)^2}(G_A)^i{}_j(G_A)^j{}_iB_{22}(p^2;M_i^2,M_j^2)\\
&-\frac{1}{(4\pi)^2}(G_Z)^k{}_i(G_A)^l{}_j\delta_{kl}\delta^{ij}A(M_i^2)\\
&-\frac{1}{(4\pi)^2}(G_A)^k{}_i(G_A)^l{}_j\delta_{kl}\delta^{ij}A(M_i^2)\,,
\end{aligned}
\tag{7.14}
$$

Note that G_Z and G_A in Eq.(7.14) are defined as

$$
(G_Z)^i{}_j = g_W c_W(\bar{w}_3^i)_{;j} - g_Y s_W(\bar{y}^i)_{;j}\,, \tag{7.15}
$$

$$
(G_A)^i{}_j = g_W s_W(\bar{w}_3^i)_{;j} + g_Y c_W(\bar{y}^i)_{;j}\,, \tag{7.16}
$$

where $(\bar{w}_3^i)_{;j}$ and $(\bar{y}^i)_{;j}$ are the covariant derivatives of $SU(2)_W$ and $U(1)_Y$ Killing vectors evaluated at the vacuum, respectively, and s_W and c_W defined as

$$
c_W := \frac{g_W}{\sqrt{g_W^2+g_Y^2}}\,, \qquad s_W := \frac{g_Y}{\sqrt{g_W^2+g_Y^2}}\,. \tag{7.17}
$$

B_{22} and A are loop functions [53] and their explicit form is given as

$$\frac{i}{(4\pi)^2} A(m^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2}\,, \tag{7.18}$$

$$\frac{i}{(4\pi)^2} B_{22}(p^2; m_1^2, m_2^2) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 - m_1^2)\{(k+p)^2 - m_2^2\}}\bigg|_{g_{\mu\nu}}\,, \tag{7.19}$$

Note that the notation for the loop functions is identical to that of [54]. We finally get

$$\begin{aligned}\Pi^{\xi\xi}_{3Q}(p^2) = & \frac{-2}{(4\pi)^2}\Big\{(\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} + (\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\Big\} B_{22}(p^2; M_i^2, M_j^2) \\ & + \frac{1}{(4\pi)^2}\Big\{(\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} + (\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\Big\} A(M_M^2)\,.\end{aligned} \tag{7.20}$$

For the estimation of the UV divergences, we regularize the loop functions A, B_0, and B_{22} by employing the dimensional regularization. The loop functions are expanded as

$$A(m^2) = -\Lambda^2 + m^2 \ln\frac{\Lambda^2}{\mu^2} - (4\pi)^2 A_r(m)\,, \tag{7.21}$$

$$B_{22}(p^2, m_1^2, m_2^2) = -\frac{1}{2}\Lambda^2 + \frac{1}{4}\left(m_1^2 + m_2^2 - \frac{p^2}{3}\right)\ln\frac{\Lambda^2}{\mu^2} + \frac{1}{4}(4\pi)^2 B_{0r}(m_1, m_2, p^2)\,, \tag{7.22}$$

where the terms proportional to Λ^2 and $\ln\Lambda^2$ correspond to the terms proportional to $1/(2-D)$ and $1/(4-D)$, respectively. D and μ denote the spacetime dimension and the renormalization scale, respectively. A_r and B_r are Λ-independent (μ-dependent) functions and their explicit forms are given as

$$A_r(m) = -\frac{m^2}{(4\pi)^2}\left[\ln\frac{\mu^2}{m^2} + 1\right]\,, \tag{7.23}$$

$$B_r(m_1, m_2; p^2) = \frac{1}{(4\pi)^2}\int_0^1 dx\, \ln\left(\frac{\mu^2}{m_1^2 x + m_2^2(1-x) - p^2 x(1-x)}\right)\,. \tag{7.24}$$

Substituting Eq.(7.21) and Eq.(7.22) into Eq.(7.20), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\Pi'^{\xi\xi}_{3Q}(0)\Big|_{\rm div} = \frac{1}{6(4\pi)^2}\Big\{(\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} + (\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\Big\}\ln\frac{\Lambda^2}{\mu^2}\,. \tag{7.25}$$

$\Pi_{\rm ab}$ vacuum polarization

Next, we will evaluate the one-loop scalar contribution to Π_{ab}, $(a, b = 1 \sim 3)$, denoted by $\Pi^{\xi\xi}_{ab}$. Relevant diagrams are listed in Fig.7.2.

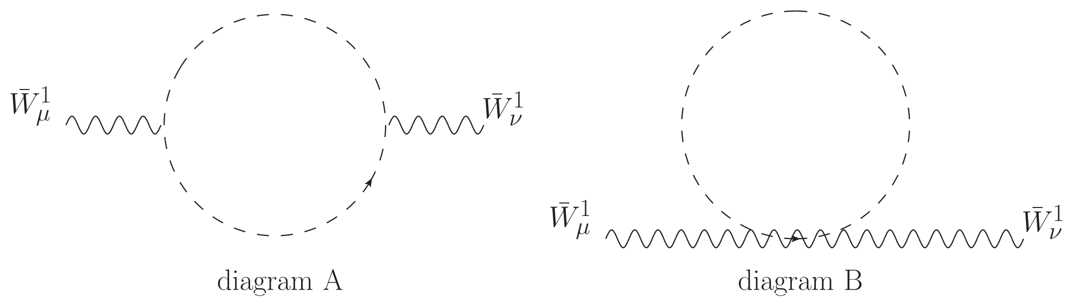


Figure 7.2: Feynman diagrams contributing to $\Pi^{\xi\xi}_{11}$

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.2, we get the following result,

$$\begin{aligned}\Pi^{\xi\xi}_{ab}(p^2) = {} & \frac{-2}{(4\pi)^2}(\bar{w}_a^i)_{;j}(\bar{w}_b^j)_{;i}B_{22}(p^2; M_i^2, M_j^2) \\ & - \frac{1}{(4\pi)^2}\Big((\bar{w}_a^k)_{;i}(\bar{w}_b^l)_{;j}\delta_{kl} - (\bar{w}_a^k)(\bar{w}_b^l)\bar{R}_{kilj}\Big)\delta^{ij}A(M_m^2)\,, \end{aligned} \tag{7.26}$$

where the first line comes from loop diagram in left side of Fig.7.2, and the second line comes from loop diagram in right side of Fig.7.2.

Substituting Eq.(7.21) and Eq.(7.22) into Eq.(7.26), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\Pi^{\xi\xi}_{ab}(0)\Big|_{\Lambda^2} = -\frac{1}{(4\pi)^2}(\bar{w}^k_a)(\bar{w}^l_b)\bar{R}_{kilj}\delta^{ij}\Lambda^2\,, \tag{7.27}$$

$$\Pi^{\xi\xi}_{ab}(0)\Big|_{\ln\Lambda^2} = \frac{1}{(4\pi)^2}(\bar{w}^k_a)(\bar{w}^l_b)\bar{R}_{kilj}M^2_{ij}\ln\frac{\Lambda^2}{\mu^2}\,. \tag{7.28}$$

7.3 Scalar gauge loop

In this subsection, we will evaluate a scalar and gauge loop contribution to the vacuum polarization Π_{3Q}, Π_{11} and Π_{33} in turn.

Π_{3Q} vacuum polarization

Firstly, we will evaluate the one-loop scalar and gauge contribution to Π_{3Q}, denoted by $\Pi^{\mathcal{V}\xi}_{3Q}$. Relevant diagrams are listed in Fig.7.3, with gauge and scalar fields expressed by wavy and dashed lines, respectively.

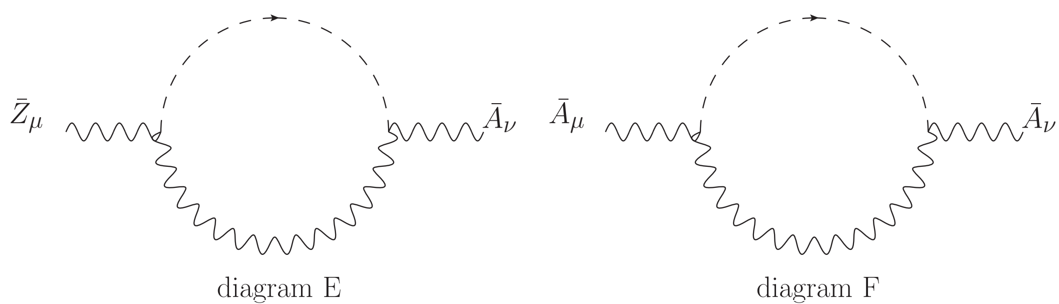


Figure 7.3: Feynman diagrams contributing to $\Pi^{\mathcal{V}\xi}_{3Q}$

Diagrams in Fig.7.3 contain B_0 and do not contain any other loop functions, so $\Pi^{\mathcal{V}\xi}_{3Q}$ remains finite at one-loop level.

Π_{ab} vacuum polarization

Next, we will evaluate the one-loop scalar and gauge contribution to Π_{ab}, denoted by $\Pi_{ab}^{\mathcal{V}\xi}$. Relevant diagrams are listed in Fig.7.4.

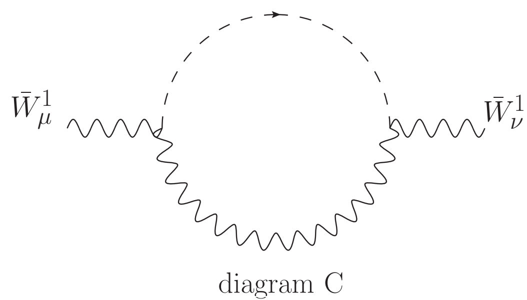


Figure 7.4: Feynman diagrams contributing to $\Pi_{ab}^{\mathcal{V}\xi}$

Calculating the vacuum polarization diagram shown in Fig. 7.4, we get the following result,

$$\begin{aligned}\Pi_{ab}^{\mathcal{V}\xi}(p^2) = -\frac{4}{(4\pi)^2}(\bar{w}_a^m)(\bar{w}_b^n)\Bigg[g^2\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2}&(\bar{w}_\alpha^m)_{;i}(\bar{w}_\beta^n)_{;j}\delta^{ij}\delta^{\alpha\beta}B_0(p^2;M_i^2,M_W^2)\\ &+(G_Z)^m{}_k(G_Z)^n{}_l\delta^{kl}B_0(p^2;M_Z^2,M_i^2)\\ &+(G_A)^m{}_k(G_A)^n{}_l\delta^{kl}B_0(p^2;0,M_i^2)\Bigg]\,. \end{aligned} \tag{7.29}$$

B_0 is loop function and its explicit form is given as

$$\frac{i}{(4\pi)^2}B_0(p^2;m_1^2,m_2^2) = \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_1^2)\{(k+p)^2-m_2^2\}}\,. \tag{7.30}$$

For the estimation of the UV divergences, we regularize the loop functions B_0 by employing the dimensional regularization. The loop function B_0 is expanded as

$$B_0(p^2,m_1^2,m_2^2) = \ln\frac{\Lambda^2}{\mu^2} + (4\pi)^2B_r(m_1,m_2,p^2)\,, \tag{7.31}$$

where B_{0r} is Λ-independent (μ-dependent) functions and their explicit forms are given as

$$B_{0r}(m_1, m_2; p^2) = \frac{2}{(4\pi)^2}\int_0^1 dx\,[m_1^2 x + m_2^2(1-x) - p^2 x(1-x)]\times \\ \times\left[\ln\left(\frac{\mu^2}{m_1^2 x + m_2^2(1-x) - p^2 x(1-x)}\right) + 1\right] . \tag{7.32}$$

Substituting Eq.(7.31) into Eq.(7.29), we can evaluate the UV divergence of the vacuum polarization, which is given by

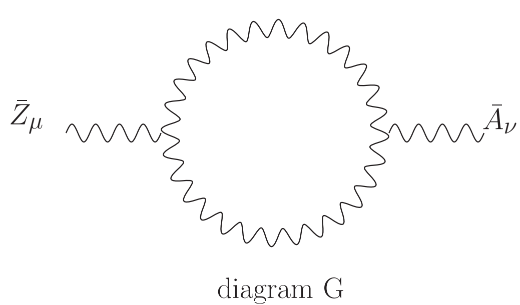
$$\Pi_{ab}^{\mathcal{V}\xi}(0)\Big|_{\text{div}} = -\frac{4}{(4\pi)^2}(\bar{w}_a^m)(\bar{w}_b^n)\Big[g^2\sum_{c=1}^{3}(\bar{w}_c^m)_{;i}(\bar{w}_c^n)_{;j} + g_Y^2(\bar{y}^m)_{;i}(\bar{y}^n)_{;j}\Big]\delta^{ij}\ln\frac{\Lambda^2}{\mu^2} . \tag{7.33}$$

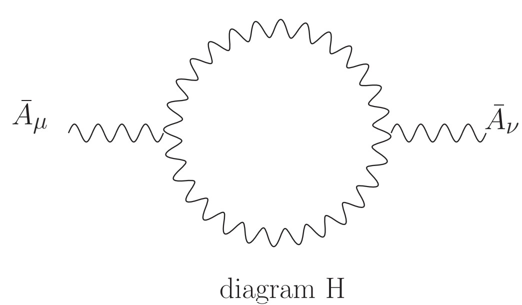
7.4 Gauge loop

In this subsection, we will evaluate a gauge loop contribution to the vacuum polarization Π_{3Q}, Π_{11} and Π_{33} in turn.

Π_{3Q} vacuum polarization

Firstly, we will evaluate the one-loop gauge contribution to Π_{3Q}, denoted by $\Pi_{3Q}^{\mathcal{V}\mathcal{V}}$. Relevant diagrams are listed in Fig.7.5, with gauge fields expressed by wavy line.





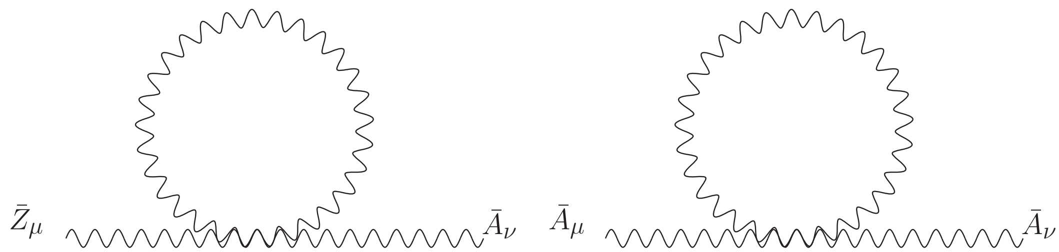


Figure 7.5: Feynman diagrams contributing to $\Pi_{3Q}^{\mathcal{V}\mathcal{V}}$

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.5, we get the following result,

$$\begin{aligned}
\Pi_{3Q}^{\xi\mathcal{V}} = & \frac{4}{(4\pi)^2}\left[2p^2 s_W c_W B_0(p^2; M_W^2, M_W^2) + 4 s_W c_W B_{22}(p^2, M_W^2, M_W^2)\right] \\
& + \frac{4}{(4\pi)^2}\left[2p^2 s_W^2 B_0(p^2; M_W^2, M_W^2) + 4 s_W^2 B_{22}(p^2, M_W^2, M_W^2)\right] \\
& - \frac{8}{(4\pi)^2} s_W c_W A(M_W^2) \\
& - \frac{8}{(4\pi)^2} s_W^2 A(M_W^2) \\
= & \frac{4}{(4\pi)^2}\left[2p^2 B_0(p^2; M_W^2, M_W^2) + 4 B_{22}(p^2, M_W^2, M_W^2) - 2A(M_W^2)\right] . \qquad (7.34)
\end{aligned}$$

Substituting Eq.(7.21), Eq.(7.22) and Eq.(7.31) into Eq.(7.26), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\Pi_{3Q}^{\prime\mathcal{V}\mathcal{V}}(0)\Big|_{\text{div}} = \frac{1}{(4\pi)^2}\frac{20}{3}\ln\frac{\Lambda^2}{\mu^2} \, . \qquad (7.35)$$

$\Pi_{\mathbf{ab}}$ vacuum polarization

Next, we will evaluate the one-loop gauge contribution to Π_{ab}, denoted by $\Pi_{ab}^{\mathcal{V}\mathcal{V}}$. Relevant diagrams are listed in Fig.7.6.

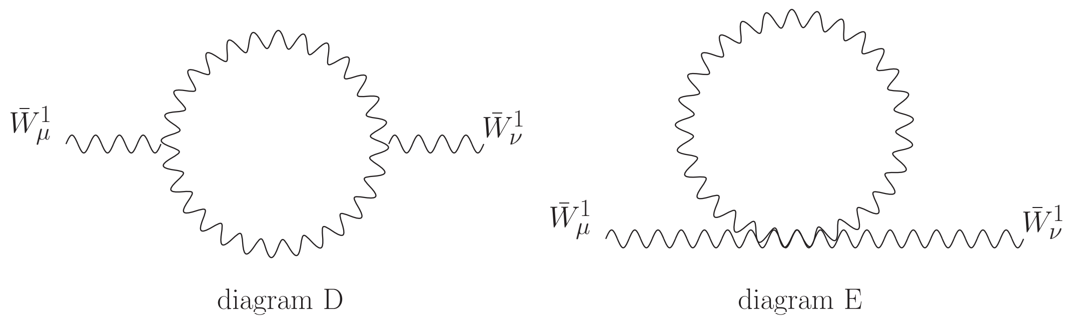


Figure 7.6: Feynman diagrams contributing to $\Pi^{\mathcal{V}\mathcal{V}}_{11}$

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.6, we get the following result,

$$\begin{aligned}\Pi^{\mathcal{V}\mathcal{V}}_{11}(p^2) = \frac{4}{(4\pi)^2}\Big[&2p^2 c_W^2 B_0(p^2; M_Z^2, M_W^2) + 4c_W^2 B_{22}(p^2, M_Z^2, M_W^2)\\ &+ 2p^2 s_W^2 B_0(p^2; 0, M_W^2) + 4s_W^2 B_{22}(p^2, 0, M_W^2)\\ &- A(M_W^2) - c_W^2 A(M_Z^2) - s_W^2 A(0)\Big]\,, \end{aligned} \tag{7.36}$$

$$\Pi^{\mathcal{V}\mathcal{V}}_{33}(p^2) = \frac{4}{(4\pi)^2}\Big[2p^2 B_0(p^2; M_W^2, M_W^2) + 4B_{22}(p^2; M_W^2, M_W^2) - 2A(M_W^2)\Big]\,. \tag{7.37}$$

Substituting Eq.(7.21), Eq.(7.22) and Eq.(7.31) into Eq.(7.36) and Eq.(7.37), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\begin{aligned}\Pi^{\mathcal{V}\mathcal{V}}_{11}(0)\Big|_{\rm div} &= \frac{4}{(4\pi)^2}\Big[4c_W^2\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{4}(M_W^2 + M_Z^2)\ln\frac{\Lambda^2}{\mu^2}\Big) + 4s_W^2\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{4}M_W^2\ln\frac{\Lambda^2}{\mu^2}\Big)\\ &\quad -\Big(-\Lambda^2 + M_W^2\ln\frac{\Lambda^2}{\mu^2}\Big) - c_W^2\Big(-\Lambda^2 + M_Z^2\ln\frac{\Lambda^2}{\mu^2}\Big) - s_W^2\Big(-\Lambda^2 + 0\times\ln\frac{\Lambda^2}{\mu^2}\Big)\Big]\\ &= 0\,, \end{aligned} \tag{7.38}$$

and

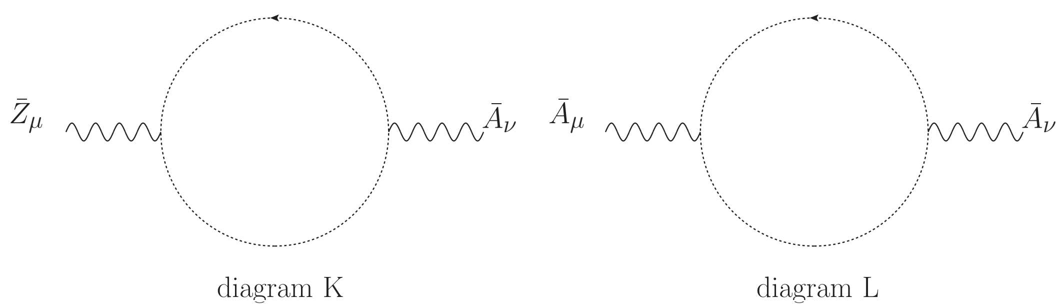
$$\Pi_{33}^{\mathcal{V}\mathcal{V}}(0)\Big|_{\rm div} = \frac{4}{(4\pi)^2}\Big[4\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{2}M_W^2 \ln\frac{\Lambda^2}{\mu^2}\Big) - 2\Big(- \Lambda^2 + M_W^2 \ln\frac{\Lambda^2}{\mu^2}\Big)\Big] = 0\,. \tag{7.39}$$

7.5 Ghost loop

In this subsection, we will evaluate a scalar and ghost loop contribution to the vacuum polarization Π_{3Q}, Π_{11} and Π_{33} in turn.

$\mathbf{\Pi_{3Q}}$ vacuum polarization

Firstly, we will evaluate the one-loop ghost contribution to Π_{3Q}, denoted by Π_{3Q}^{cc}. Relevant diagrams are listed in Fig.7.7, with gauge and ghost fields expressed by wavy and dotted lines, respectively.



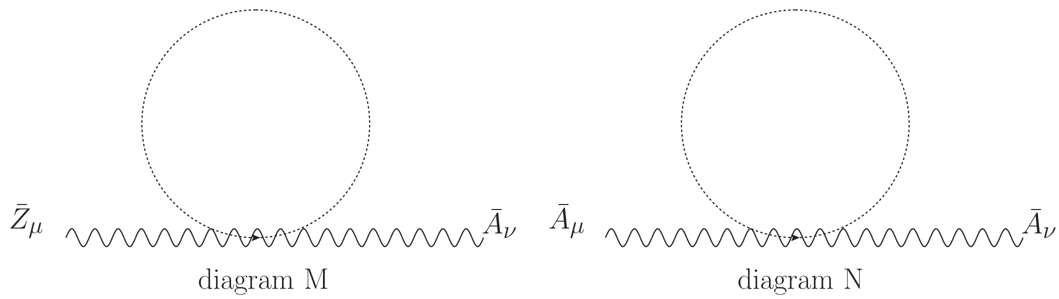


Figure 7.7: Feynman diagrams contributing to Π^{cc}_{3Q}

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.7, we get the following result,

$$\begin{aligned} g^2\Pi^{cc}_{3Q}(p^2) = & \frac{1}{(4\pi)^2}\Big[-8s_Wc_WB_{22}(p^2;M_W^2,M_W^2)\Big] \\ & + \frac{1}{(4\pi)^2}\Big[-8s_W^2B_{22}(p^2;M_W^2,M_W^2)\Big] \\ & + \frac{4}{(4\pi)^2}s_Wc_WA(M_W^2) \\ & + \frac{4}{(4\pi)^2}s_W^2A(M_W^2)\,. \end{aligned} \tag{7.40}$$

Eq.(7.41) can be further simplified as

$$\Pi^{cc}_{3Q}(p^2) = -\frac{4}{(4\pi)^2}\Big[2B_{22}(p^2;M_W^2,M_W^2) - A(M_W^2)\Big]\,. \tag{7.41}$$

Substituting Eq.(7.21) and Eq.(7.22) into Eq.(7.41), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\Pi'^{cc}_{3Q}(0)|_{\rm div} = \frac{1}{(4\pi)^2}\frac{2}{3}\ln\frac{\Lambda^2}{\mu^2}\,. \tag{7.42}$$

Π_{ab} vacuum polarization

Next, we will evaluate the one-loop ghost contribution to Π_{ab}, denoted by Π_{ab}^{cc}. Relevant diagrams are listed in Fig.7.8.

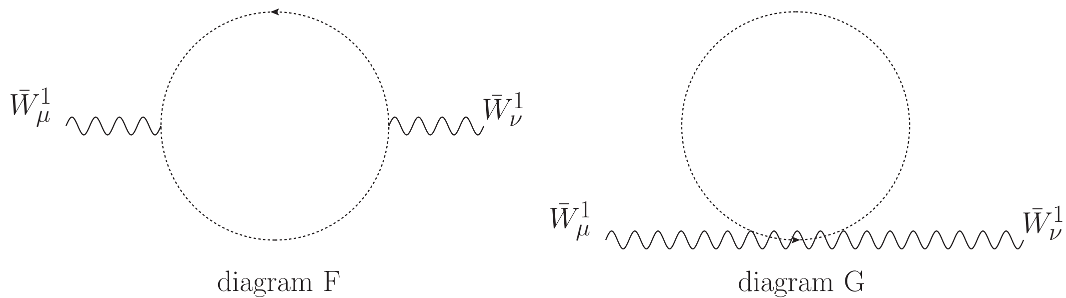


Figure 7.8: Feynman diagrams contributing to Π_{11}^{cc}

Summing up the contributions coming from vacuum polarization diagrams shown in Fig. 7.8, we get the following result,

$$\Pi_{11}^{cc}(p^2) = \frac{2}{(4\pi)^2}\Big[-4c_W^2 B_{22}(p^2; M_Z^2, M_W^2) - 4s_W^2 B_{22}(p^2; 0, M_W^2) + A(M_W^2) + c_W^2 A(M_Z^2) + s_W^2 A(0)\Big]. \tag{7.43}$$

$$\Pi_{33}^{cc}(p^2) = \frac{1}{(4\pi)^2}\Big[-8B_{22}(p^2; M_W^2, M_W^2) + 4A(M_W^2)\Big]. \tag{7.44}$$

Substituting Eq.(7.21) and Eq.(7.22) into Eq.(7.43) and Eq.(7.44), we can evaluate the UV divergence of the vacuum polarization, which is given by

$$\begin{aligned}\Pi_{11}^{cc}(0)\Big|_{\mathrm{div}} &= \frac{2}{(4\pi)^2}\Big[-4c_W^2\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{4}(M_W^2 + M_Z^2)\ln\frac{\Lambda^2}{\mu^2}\Big) - 4s_W^2\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{4}M_W^2\ln\frac{\Lambda^2}{\mu^2}\Big) \\ &\quad + \Big(-\Lambda^2 + M_W^2\ln\frac{\Lambda^2}{\mu^2}\Big) + c_W^2\Big(-\Lambda^2 + M_Z^2\ln\frac{\Lambda^2}{\mu^2}\Big) + s_W^2\Big(-\Lambda^2 + 0\times\ln\frac{\Lambda^2}{\mu^2}\Big)\Big] \\ &= 0\,,\end{aligned} \tag{7.45}$$

and

$$\Pi^{cc}_{33}(0)\Big|_{\rm div} = \frac{4}{(4\pi)^2}\left[-2\Big(-\frac{1}{2}\Lambda^2 + \frac{1}{2}M_W^2 \ln\frac{\Lambda^2}{\mu^2}\Big) + \Big(-\Lambda^2 + M_W^2\ln\frac{\Lambda^2}{\mu^2}\Big)\right] = 0\,. \tag{7.46}$$

7.6 S, U parameters

Definitions of S and U parameters are given as follows,

$$S := 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0))\,, \tag{7.47}$$

$$U := 16\pi(\Pi'_{11}(0) - \Pi'_{33}(0))\,, \tag{7.48}$$

with Π_{AB} is the momentum integral of the product of conserved current J_A and J_B, given by Eq.(7.11). The divergent parts of S and U are given as

$$S_{\rm div} = -\frac{1}{12\pi}(\bar{w}^i_3)_{;j}(\bar{y}^j)_{;i}\ln\frac{\Lambda^2}{\mu^2}, \tag{7.49}$$

$$U_{\rm div} = \frac{1}{12\pi}\left((\bar{w}^i_1)_{;j}(\bar{w}^j_1)_{;i} - (\bar{w}^i_3)_{;j}(\bar{w}^j_3)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}. \tag{7.50}$$

The above results are quite reasonable and we can easily understand the origin of their structures from the current algebra. Because the Killing vector is the infinitesimal variation under the conserved symmetry, it is natural that it relates to the conserved current of the associated symmetry. Actually, The conserved current of $SU(2)_L$ and $U(1)_Y$ symmetry are roughly related to the associated Killing vectors as

$$J^\mu_a \sim \phi^i(w^i_a)_{;j}\partial^\mu\phi^j\,, \qquad (a = 1\sim 3)\,, \tag{7.51}$$

$$J^\mu_Q \sim \phi^i(w^i_3 + y^i)_{;j}\partial^\mu\phi^j\,. \tag{7.52}$$

Combining the above relation to the definition of S parameter (7.47), we get

$$S_{\rm div} \propto (\bar{w}^i_3)_{;j}(\bar{w}^j_3)_{;i} - (\bar{w}^i_3)_{;j}(w^j_3 + y^j)_{;i} = -(\bar{w}^i_3)_{;j}(y^j)_{;i}\,. \tag{7.53}$$

Similarly we find

$$U_{\rm div} \propto (\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i}\,. \tag{7.54}$$

Chapter 8

Unitarity v.s. Oblique Corrections

In this chapter, we will check the relation between the perturbative unitarity and the oblique correction parameters S, T and U.

8.1 Perturbative unitarity v.s. S, U parameters

As we see in Sec. 7.6, divergent part of oblique parameters S and U can be expressed in terms of the Killing vectors of $SU(2)_L$ and $U(1)_Y$ as

$$S_{\text{div}} = -\frac{1}{12\pi}(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} \ln \frac{\Lambda^2}{\mu^2} , \tag{8.1}$$

$$U_{\text{div}} = \frac{1}{12\pi}\left((\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i}\right) \ln \frac{\Lambda^2}{\mu^2} . \tag{8.2}$$

In this section, we consider relating the coefficient of the UV divergence of oblique parameters, appearing in Eq.(8.1) and Eq.(8.2), to the perturbative unitarity of the four-point scalar scattering amplitude. Firstly, we will focus on the equations the Killing vectors should satisfy.

As I mentioned previously, the invariance of the Lagrangian under $SU(2)_L \times U(1)_Y$ is reflected to the invariance of the metric tensor under infinitesimal $SU(2)_L \times U(1)_Y$

transformation, which is expressed as the Killing equations,

$$0 = (w_a^k) g_{ij,k} + (w_a^k)_{,i} g_{kj} + (w_a^k)_{,j} g_{ik} \,, \tag{8.3}$$

$$0 = (y^k) g_{ij,k} + (y^k)_{,i} g_{kj} + (y^k)_{,j} g_{ik} \,. \tag{8.4}$$

Now let's consider expanding the Killing equations (8.3) and Eq.(8.4) around the vacuum, $\phi^i = \bar{\phi}^i$, If we take Riemann normal coordinate system, the metric tensor g_{ij} can be expanded around the vacuum as

$$g_{ij}(\phi) = \delta_{ij} - \frac{1}{3} \bar{R}_{ikjl} \varphi^k \varphi^l + \cdots \,, \tag{8.5}$$

with

$$\delta_{ij} = \bar{g}_{ij} = g_{ij}(\phi) \bigg|_{\phi=\bar{\phi}} \,, \qquad \bar{R}_{ijkl} = R_{ijkl} \bigg|_{\phi=\bar{\phi}} \,. \tag{8.6}$$

Also, we can expand $SU(2)_L$ and $U(1)_Y$ Killing vectors w_a^i and y^i around the vacuum as

$$w_a^i = \bar{w}_a^i + (\bar{w}_a^i)_{,j} \varphi^j + \frac{1}{2!} (\bar{w}_a^i)_{,jk} \varphi^j \varphi^k + \cdots \,, \tag{8.7}$$

$$y^i = \bar{y}^i + (\bar{y}^i)_{,j} \varphi^j + \frac{1}{2!} (\bar{y}^i)_{,jk} \varphi^j \varphi^k + \cdots \,. \tag{8.8}$$

As we see in Eq.(8.5), Taylor expansion of the metric tensor includes the Riemann curvature tensor in its expansion coefficient, so it seems that by substituting the Taylor expansions of the metric tensor and the Killing vectors into the Killing equation (8.3) and (8.4), we can relate the Killing vectors to the Riemann curvature tensor.

Substituting the above Taylor expansions into Killing equations (8.3) and (8.4), and comparing both sides of the equation in each order of ϕ^i, we can obtain series of relations which holds only at the vacuum,

$$0 = \bar{g}_{ik} (\bar{w}_a^k)_{,j} + \bar{g}_{jk} (\bar{w}_a^k)_{,i} \,, \tag{8.9}$$

$$0 = \bar{g}_{ik} (\bar{y}^k)_{,j} + \bar{g}_{jk} (\bar{y}^k)_{,i} \,, \tag{8.10}$$

$$(\bar{w}_a^i)_{,jk} = \frac{1}{3} \left(\bar{R}^i{}_{jkl} + \bar{R}^i{}_{kjl} \right) \bar{w}_a^l \,, \tag{8.11}$$

$$(\bar{y}^i)_{,jk} = \frac{1}{3}\left(\bar{R}^i{}_{jkl} + \bar{R}^i{}_{kjl}\right)\bar{y}^l\,, \tag{8.12}$$

$$\vdots$$

In Eq.(8.11) and Eq.(8.12), we can see that we successfully relate the Killing vector to the Riemann curvature tensor, but as we can see, the Killing vector begins to relate to the Riemann curvature tensor from the second-order of the derivative. This means that, as long as we focus on the Killing equation only, we cannot obtain the desired equations relating the first covariant derivative of the Killing vector to the Riemann curvature tensor. We need another equation.

Actually, commutation relation of $SU(2)_L \times U(1)_Y$ Killing vectors play this role.

$$[w_a\,, w_b] = \varepsilon_{abc} w_c\,, \tag{8.13}$$

$$[w_a\,, y] = 0\,, \tag{8.14}$$

with

$$w_a := w_a^i \frac{\partial}{\partial\phi^i}\,, \qquad y := y^i \frac{\partial}{\partial\phi^i}\,. \tag{8.15}$$

Taylor expanding the both side of the commutation relation, we get

$$\bar{w}_a^j(\bar{w}_b^i)_{,j} - \bar{w}_b^j(\bar{w}_a^i)_{,j} = \epsilon_{abc}\bar{w}_c^i\,, \tag{8.16}$$

$$\bar{w}_a^j(\bar{y}^i)_{,j} - \bar{y}^j(\bar{w}_a^i)_{,j} = 0\,, \tag{8.17}$$

$$\bar{w}_a^j(\bar{w}_b^i)_{,jk_1} + (\bar{w}_a^j)_{,k_1}(\bar{w}_b^i)_{,j} - \bar{w}_b^j(\bar{w}_a^i)_{,jk_1} - (\bar{w}_b^j)_{,k_1}(\bar{w}_a^i)_{,j} = \epsilon_{abc}(\bar{w}_c^i)_{,k_1}\,, \tag{8.18}$$

$$\bar{w}_a^j(\bar{y}^i)_{,jk_1} + (\bar{w}_a^j)_{,k_1}(\bar{y}^i)_{,j} - \bar{y}^j(\bar{w}_a^i)_{,jk_1} - (\bar{y}^j)_{,k_1}(\bar{w}_a^i)_{,j} = 0\,, \tag{8.19}$$

$$\vdots$$

Substituting the solution of the Killing equation (8.11) into the commutation relation (8.18), we get

$$(T_a)_j{}^i = \frac{1}{2}\varepsilon_{abc}\left([T_b, T_c]\right)_j{}^i + \frac{1}{2}\varepsilon_{abc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,, \tag{8.20}$$

with

$$(T_a)_j{}^i := (\bar{w}_a^i)_{,j}\,. \tag{8.21}$$

Now we can see that in Eq.(8.20), the first derivative of the Killing vector w_a is successfully related to the Riemann curvature tensor. Similarly, substituting Eq.(8.12) into the commutation relation (8.19), we get

$$0 = ([T_a, T_Y])_j{}^i + (\bar{w}_a^k)\,(\bar{y}^l)\,\bar{R}^i{}_{jkl}\,, \tag{8.22}$$

with

$$(T_Y)_j{}^i := (\bar{y}^i)_{,j}\,. \tag{8.23}$$

Again, we can see that in Eq.(8.22), the first derivative of the Killing vector y is successfully related to the Riemann curvature tensor.

Now we are ready to relate the coefficient of the S parameter's divergence in Eq.(8.1), which is expressed by the first derivative of the Killing vector, to the perturbative unitarity, which is expressed by the Riemann curvature tensor.

$$\begin{aligned}
(\bar{w}_3^i)_{,j}(y^j)_{,i} &= \mathrm{tr}(T_3 T_Y) \\
&= \frac{1}{2}\varepsilon_{3bc}\,\mathrm{tr}([T_b, T_c]T_Y) + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\
&= \frac{1}{2}\varepsilon_{3bc}\,\mathrm{tr}([T_c, T_Y]T_b) + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\
&= -\frac{1}{2}\varepsilon_{3bc}(\bar{w}_c^k)\,(\bar{y}^l)\,\bar{R}^i{}_{jkl}(T_b)_i{}^j + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\
&= \frac{1}{2}\varepsilon_{3bc}(\bar{w}_c^k)\,(\bar{w}_3^l)\,\bar{R}^i{}_{jkl}(T_b)_i{}^j + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}(T_Y)_i{}^j\,.
\end{aligned} \tag{8.24}$$

To get the second line from the first line, we used Eq.(8.20). To get the third line, we used the invariance of trace under cyclic permutations. To get the forth line, we used Eq.(8.20). To get from the last line, we used

$$0 = \bar{w}_3^i + \bar{y}^i\,. \tag{8.25}$$

which is ensured by the $U(1)_{\mathrm{em}}$ symmetry conserved in the broken phase.

Eq.(8.24) can be rewritten in a covariant form

$$(\bar{w}_3^i)_{;j}\,(\bar{y}^j)_{;i} = \frac{1}{2}\left(\varepsilon_{3bc}\,(\bar{w}_c^k)\,(\bar{w}_3^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_b^j)_{;i} + \varepsilon_{3bc}\,(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{y}^j)_{;i}\right)\,. \tag{8.26}$$

$$(\bar{w}_1^i)_{;j}\,(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}\,(\bar{w}_3^j)_{;i} = \frac{1}{2}\left(\varepsilon_{1bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_1^j)_{;i} - \varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_3^j)_{;i}\right)\,. \tag{8.27}$$

Substituting Eq.(8.26) into Eq.(8.1), we can rewrite the coefficient of the UV divergence of the parameter S as

$$S_{\rm div} = -\frac{1}{12\pi}\left(\varepsilon_{3bc}(\bar{w}_c^k)\,(\bar{w}_3^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_b^j)_{;i} + \varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}\,. \tag{8.28}$$

As we previously mentioned in Sec. 6, the Riemann curvature tensor corresponds to the unitarity sum rules in the $\phi^i\phi^j \to \phi^k\phi^l$ scattering amplitude. Looking carefully at the expression (8.1), we can see that the divergent part of the S parameter is nontrivially related to the perturbative unitarity.

Similarly, the divergent part of the parameter U can be expressed in terms of the Riemann curvature tensor by substituting Eq.(8.27) into Eq.(8.2),

$$U_{\rm div} = \frac{1}{12\pi}\left(\varepsilon_{1bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_1^j)_{;i} - \varepsilon_{3bc}(\bar{w}_b^k)\,(\bar{w}_c^l)\,\bar{R}^i{}_{jkl}\,(\bar{w}_3^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}\,. \tag{8.29}$$

Before closing this section, we will mention the implication of Eq.(8.28) and Eq.(8.29). As we found in Sec.6.4, the Riemann curvature tensor gives conditions for respecting the perturbative unitarity. The unitarity sum rules are given as the flatness condition at the vicinity of the vacuum,

$$\bar{R}_{ijkl} = 0\,. \tag{8.30}$$

$S_{\rm div}$ and $U_{\rm div}$ are both proportional to the Riemann curvature tensor, so we get the following relation,

$$\bar{R}_{ijkl} = 0 \quad \Rightarrow \quad S_{\rm div} = U_{\rm div} = 0\,. \tag{8.31}$$

This means that once the unitarity sum rules are satisfied, the divergent part of the parameter S and U is automatically canceled out, so the perturbative unitarity ensures the one-loop finiteness of the oblique parameters S and U.

How about the inverse relation of Eq.(8.31)? Remember that the $\bar{w}^i_a$ denotes the ith component of the $SU(2)_L$ Killing vector, and the Killing vector corresponds to the scalar fields' variation under the infinitesimal symmetry transformation. The index i includes both NG boson indices and scalar matter fields' indices, but $\bar{w}^i_a$ takes nonzero values only when index i corresponds to NG bosons' index:

$$\begin{aligned}
&\bar{w}^i_a \neq 0, \qquad (i = \text{NG fields' index})\,,\\
&\bar{w}^i_a = 0, \qquad (i = \text{scalar matter fields' index})\,.
\end{aligned} \tag{8.32}$$

This is because non-linearly realized $SU(2)_L$ acts on NG bosons as some kinds of shift symmetry, and even if we substitute zero field value, $w^\alpha = z = 0$, to the infinitesimal variation, it still has nonzero value coming from a constant shift. On the other hand, scalar matter variation under infinitesimal $SU(2)_L$ transformation can be expressed by the generator acting on the scalar matter fields, so if we substitute $\phi^I = 0$, it vanishes. Therefore, the Riemann curvature component $\bar{R}^i{}_{jkl}$ with its indices k or l corresponding to the matter field indices does not contribute to the $S_{\rm div}$ and $U_{\rm div}$ Therefore, we can say that the perturbative unitarity conditions are stronger than finiteness conditions.

8.2 Perturbative unitarity v.s. T parameter

From Eq.(7.27), we get

$$\left.\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)\right|_{\Lambda^2} = \left.\left(\frac{1}{v^2}\Pi^{\xi\xi}_{11}(0) - \frac{1}{v_Z^2}\Pi^{\xi\xi}_{33}(0)\right)\right|_{\Lambda^2}$$

$$= -\frac{1}{(4\pi)^2}\left(\frac{1}{v^2}(\bar{w}_1^k)(\bar{w}_1^l) - \frac{1}{v_Z^2}(\bar{w}_3^k)(\bar{w}_3^l)\right)\bar{R}_{kilj}\delta^{ij}\Lambda^2\,, \tag{8.33}$$

and from Eq.(7.28) and Eq.(7.33), we get

$$\begin{aligned}
&\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)\Bigg|_{\ln\Lambda^2} \\
&= \left(\frac{1}{v^2}\Pi_{11}^{\xi\xi}(0) - \frac{1}{v_Z^2}\Pi_{33}^{\xi\xi}(0)\right)\Bigg|_{\ln\Lambda^2} + \left(\frac{1}{v^2}\Pi_{11}^{\mathcal{V}\xi}(0) - \frac{1}{v_Z^2}\Pi_{33}^{\mathcal{V}\xi}(0)\right)\Bigg|_{\ln\Lambda^2} \\
&= \frac{1}{(4\pi)^2}\left(\frac{1}{v^2}(\bar{w}_1^k)(\bar{w}_1^l) - \frac{1}{v_Z^2}(\bar{w}_3^k)(\bar{w}_3^l)\right)\times \\
&\qquad \times\left[\bar{R}_{kilj}M_{ij}^2 - 4\Big(g^2\sum_{a=1}^{3}(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j} + g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\Big)\delta^{ij}\right]\ln\frac{\Lambda^2}{\mu^2}\,,
\end{aligned} \tag{8.34}$$

with

$$M_{ij}^2 := g^2\sum_{a=1}^{3}(\bar{w}_a^i)(\bar{w}_a^j) + g_Y^2(\bar{y}^i)(\bar{y}^j) + V_{;ij}\,. \tag{8.35}$$

As we see from Eq.(8.34), the coefficient of the T parameter divergence is partially expressed by the Riemann curvature tensor, but there is part independent of the Riemann curvature tensor. This means that we need additional conditions to ensure the one-loop finiteness of the parameter T. In addition to the perturbative unitarity of four scalar scattering amplitude, we need the following conditions:

$$\left(\frac{1}{v^2}(\bar{w}_1^k)(\bar{w}_1^l) - \frac{1}{v_Z^2}(\bar{w}_3^k)(\bar{w}_3^l)\right)\left(g^2\sum_{a=1}^{3}(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j} + g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\right)\delta^{ij} = 0\,. \tag{8.36}$$

Chapter 9

Summary and Conclusions

In this thesis, we have done two things.

1. We extended the Higgs Effective Field Theory (HEFT) so that it includes the arbitrary number of neutral and charged scalar fields, and formulated "Generalized Higgs Effective Field Theory (GHEFT)."

2. We derived the perturbative unitarity condition and oblique parameters S, T, U in the context of the GHEFT in the covariant form. Focusing on the geometry of the GHEFT, we find correlations between unitarity sum rules and the oblique parameters.

We firstly generalized the Higgs Effective Field Theory (HEFT) so that it includes the arbitrary number of neutral and charged scalar fields. Because $SU(2)_L \times U(1)_Y$ symmetry is non-linearly realized in the broken phase, we relied on the CCWZ method to add new scalar fields to the HEFT in a consistent manner with underlying symmetry. We named the extension of the HEFT as "Generalized Higgs Effective Field Theory (GHEFT)." The explicit form of the GHEFT Lagrangian is given by Eq.(4.49). Thanks to the GHEFT, we can calculate the physical process with new scalar particles appearing in the initial of the final state, such as the production cross sections and decay rates of the new scalars. These processes cannot be calculated in the HEFT framework because new particles are integrated out from the theory.

Furthermore, we rewrote the GHEFT Lagrangian to avoid the ambiguity coming from a choice of the field coordinate system. The dictionary for converting the original GHEFT Lagrangian (4.49) into its geometrical form (6.1) is given in appendix B.

Secondly, We derived the perturbative unitary conditions and the expression of the Peskin Takeuchi S, T, U parameters in the framework of the GHEFT. Thanks to the geometrical form of the GHEFT (6.1), we derived these expressions in the covariant forms. Deriving the perturbative unitarity conditions, we found that the unitarity sum rules are expressed by the flatness conditions in the vicinity of the vacuum, $\bar{R}_{ijkl} = 0$. We also found that the coefficient of one-loop divergence of the oblique parameters S, T, and U can be written in terms of the Killing vectors and Riemann curvature tensors. Furthermore, using the Killing equations and the commutation relations of the Killing vectors, we find that the coefficients of the divergent parts of S and U are proportional to the Riemann curvature tensor evaluated at the vacuum. This ensures that once perturbative unitarity conditions are satisfied, a one-loop divergence of the oblique parameters S and U is automatically canceled out. The inverse relation does not hold, however: even if the S and U are one-loop finite, this does not ensure the perturbative unitarity. Therefore, we concluded that the perturbative unitarity conditions are sufficient conditions for the one-loop finiteness of S and U, but not the necessary conditions.

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Appendix A

Thanks to the CCWZ formalism introduced in Sec3.3, we can construct the HEFT Lagrangian. Here we will construct the HEFT $SU(2)_L \times SU(2)_R \to SU(2)_V$, where $SU(2)_V$ is the symmetry rotating same angle in $SU(2)_L$ and $SU(2)_R$. Generators of $SU(2)_L \times SU(2)_R$ can be expressed by four by four matrices as

$$T_L^a = \frac{1}{2}\begin{pmatrix} \tau^a & \\ & 0 \end{pmatrix}, \qquad T_R^a = \frac{1}{2}\begin{pmatrix} 0 & \\ & \tau^a \end{pmatrix}, \tag{A.1}$$

where τ^a denotes Pauli matrices given by Eq.(2.8). Upper-left two by two matrix denotes $SU(2)_L$ subspace, and lower-right two by two denotes that of $SU(2)_R$.

The unbroken generator S^a is rotate $SU(2)_L$ and $SU(2)_R$ by the same angle, so it can be expressed by sum of $SU(2)_L$ and $SU(2)_R$ generators. As for the broken generator X^a, it can be expressed simply by subtraction of $SU(2)_L$ and $SU(2)_R$ generators. Therefore unbroken and broken generators of $SU(2)_L \times SU(2)_R \to SU(2)_V$ can be written as

$$S^a = \frac{T_L^a + T_R^a}{2} = \frac{1}{2}\begin{pmatrix} \tau^a & \\ & \tau^a \end{pmatrix}, \qquad X^a = \frac{T_L^a - T_R^a}{2} = \frac{1}{2}\begin{pmatrix} \tau^a & \\ & -\tau^a \end{pmatrix}. \tag{A.2}$$

Note that the broken generators in general has an ambiguity of adding or subtracting the unbroken generators S^a. Therefore we can also adopt the following matrices X'^a as the broken generator, which is different from X^a in Eq.(A.8) by the sum of unbroken generators S^a:

$$X'^a = X^a + S^a = \begin{pmatrix} \tau^a & \\ & 0 \end{pmatrix} . \tag{A.3}$$

The two theories constructed by the generators $\{S^a, X^a\}$ and by $\{S^a, X'^a\}$ look different at the first look, but these theories are simply related by the field redefinitions. Though these two theories give different interaction vertices, we can show that they give exactly the same scattering cross sections. In this thesis, we adopt simpler parametrization, which is given by Eq.(A.8).

Unbroken and broken generators given by Eq.(A.8) satisfy following commutation relations

$$[S^a, S^b] = i\epsilon^{abc} S^c \,, \tag{A.4}$$
$$[S^a, X^b] = i\epsilon^{abc} X^c \,, \tag{A.5}$$
$$[X^a, X^b] = i\epsilon^{abc} S^c \,. \tag{A.6}$$

Here we define parity operation τ_p, under which unbroken and broken generator transform as

$$\begin{aligned} \tau_p : \quad & S^a \to S^a \,, \\ & X^a \to -X^a \,. \end{aligned} \tag{A.7}$$

This parity operation τ_p actually corresponds to the exchange of $SU(2)_L$ and $SU(2)_R$ generators. As we see in Eq.(A.8), unbroken generator S^a and broken generator X^a can be expressed in terms of $SU(2)_L$ and $SU(2)_R$ generators T_L^a, T_R^a by

$$S^a = \frac{T_L^a + T_R^a}{2} \,, \qquad X^a = \frac{T_L^a - T_R^a}{2} \,. \tag{A.8}$$

Exchanging T_L^a and T_R^a, unbroken generator S^a remains invariant, but broken generator X^a flip its sign, reproducing Eq.(A.7). The important thing is that commutation relations Eqs.(A.4)-(A.6) are invariant under the parity transformation τ_p. This property turns out to be quite useful when we construct the invariant operator under non-linearly realized $SU(2)_L \times U(1)_Y$.

The matrix ξ given by Eq.(3.54) can be written in this case as

$$\xi(\pi) = \begin{pmatrix} \hat{\xi}_L(\pi) & \\ & \hat{\xi}_R(\pi) \end{pmatrix}, \tag{A.9}$$

where $\hat{\xi}_L$ and $\hat{\xi}_R$ are given by

$$\hat{\xi}_L(\pi) := \exp\left(\frac{i}{2v}\pi^a\tau^a\right), \tag{A.10}$$

$$\hat{\xi}_R(\pi) := \exp\left(-\frac{i}{2v}\pi^a\tau^a\right). \tag{A.11}$$

In the remaining of this section, matrix M with hat symbol, $\hat{M}$, denote 2 by 2 matrix embedded in upper-left or lower-right of the four by four matrix. Note that, if we express the transformation matrix of $SU(2)_L \times SU(2)_R$ as

$$\mathfrak{g} = \begin{pmatrix} \hat{\mathfrak{g}}_L & \\ & \hat{\mathfrak{g}}_R \end{pmatrix} \tag{A.12}$$

where

$$\hat{\mathfrak{g}}_L = \exp\left(i\theta_L^a\frac{\tau^a}{2}\right), \tag{A.13}$$

$$\hat{\mathfrak{g}}_R = \exp\left(i\theta_R^a\frac{\tau^a}{2}\right), \tag{A.14}$$

then ξ_L and ξ_R in Eq.(A.9) transforms under $SU(2)_L \times SU(2)_R$ like

$$\hat{\xi}_L(\pi) \to \hat{\xi}_L(\pi') = \hat{\mathfrak{g}}_L \cdot \hat{\xi}_L(\pi) \cdot \hat{\mathfrak{h}}^\dagger(\pi, \theta_L, \theta_R), \tag{A.15}$$

$$\hat{\xi}_R(\pi) \to \hat{\xi}_R(\pi') = \hat{\mathfrak{g}}_R \cdot \hat{\xi}_R(\pi) \cdot \hat{\mathfrak{h}}^\dagger(\pi, \theta_L, \theta_R). \tag{A.16}$$

Note that $\hat{\mathfrak{h}}$ in Eq.(A.15) is not $\hat{\mathfrak{h}}(\pi, \theta_L)$ and $\hat{\mathfrak{h}}$ in Eq.(A.16) is not $\hat{\mathfrak{h}}(\pi, \theta_R)$, but they are identical matrix $\hat{\mathfrak{h}}(\pi, \theta_L, \theta_R)$. This fact is easily understood by coming back to four by four matrix notation, where the transformation law (A.15) and (A.16) are collectively expressed as

$$\begin{pmatrix} \hat{\xi}_L(\pi) & \\ & \hat{\xi}_R(\pi) \end{pmatrix} \to \begin{pmatrix} \hat{\xi}_L(\pi') & \\ & \hat{\xi}_R(\pi') \end{pmatrix} = \begin{pmatrix} \hat{\mathfrak{g}}_L & \\ & \hat{\mathfrak{g}}_R \end{pmatrix} \begin{pmatrix} \hat{\xi}_L(\pi) & \\ & \hat{\xi}_R(\pi) \end{pmatrix} \mathfrak{h}^\dagger \,. \tag{A.17}$$

Now it is clear that the upper-left matrix and the lower-right matrix in $\mathfrak{h}$ must be identical, otherwise $\mathfrak{h}$ is not a group element of unbroken symmetry $SU(2)_V$, which rotate $\hat{\xi}_L$ and $\hat{\xi}_R$ in the same angle.

Cartan one-form defined by Eq.(3.58) is written in terms of ξ given by Eq.(A.9) as

$$\alpha_\mu(\pi) = \frac{1}{i}\xi^\dagger(\pi)\partial_\mu\xi(\pi) \,. \tag{A.18}$$

The next step is decomposing α_μ into unbroken generator part and broken generator part like

$$\alpha_\mu = \alpha_{\parallel\mu} + \alpha_{\perp\mu} \,, \tag{A.19}$$

$$\alpha_{\parallel\mu} := \mathrm{Tr}\left[\alpha_\mu S^a\right] S^a \,, \tag{A.20}$$

$$\alpha_{\perp\mu} := \mathrm{Tr}\left[\alpha_\mu X^a\right] X^a \,. \tag{A.21}$$

At this point, it is important to focus on the commutation relation of generators.

$$\alpha_\mu = \begin{pmatrix} \frac{1}{i}\hat{\xi}_L^\dagger\partial_\mu\hat{\xi}_L & \\ & \frac{1}{i}\hat{\xi}_R^\dagger\partial_\mu\hat{\xi}_R \end{pmatrix} =: \begin{pmatrix} \hat{\alpha}_{L\mu} & \\ & \hat{\alpha}_{R\mu} \end{pmatrix} \tag{A.22}$$

α_μ can be trivially decomposed as

$$\alpha_\mu = \frac{1}{2}\begin{pmatrix} \hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} & \\ & \hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \hat{\alpha}_{L\mu} - \hat{\alpha}_{R\mu} & \\ & -\hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} \end{pmatrix} \,. \tag{A.23}$$

We consider operating the parity transformation τ_p introduced in Eq.(A.7) to the Cartan one form α_μ. As we previously commented, parity transformation τ_p corresponds to the exchange of $SU(2)_L$ and $SU(2)_R$ generators. For $\hat{\alpha}_{L\mu}$ and $\hat{\alpha}_{R\mu}$, τ_p operate as an exchange $\hat{\alpha}_{L\mu} \leftrightarrow \hat{\alpha}_{R\mu}$. Under this exchange, the first term of RHS of Eq.(A.23) remain invariant, but the second term of RHS of Eq.(A.23) flip its overall sign. Comparing it with Eq.(A.7), we can conclude

$$\alpha_{\parallel\mu} = \frac{1}{2}\begin{pmatrix} \hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} & \\ & \hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} \end{pmatrix}, \tag{A.24}$$

$$\alpha_{\perp\mu} = \frac{1}{2}\begin{pmatrix} \hat{\alpha}_{L\mu} - \hat{\alpha}_{R\mu} & \\ & -\hat{\alpha}_{L\mu} + \hat{\alpha}_{R\mu} \end{pmatrix}. \tag{A.25}$$

Now we can write down a kinetic term of NG bosons:

$$\mathcal{L}_\pi = \frac{v^2}{2}\mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right]. \tag{A.26}$$

Expressing the kinetic term (A.26) in terms of ξ_L and ξ_R, we get

$$\mathcal{L}_\pi = \frac{v^2}{2}\mathrm{Tr}\left[(\hat{\alpha}_{L\mu} - \hat{\alpha}_{R\mu})(\hat{\alpha}_L^\mu - \hat{\alpha}_R^\mu)\right] = \frac{v^2}{4}\mathrm{Tr}\left[\partial_\mu\left(\hat{\xi}_L\hat{\xi}_R^\dagger\right)^\dagger\partial^\mu\left(\hat{\xi}_L\hat{\xi}_R^\dagger\right)\right]. \tag{A.27}$$

Defining

$$U := \hat{\xi}_L\hat{\xi}_R^\dagger, \tag{A.28}$$

we finally get

$$\mathcal{L}_\pi = \frac{v^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger\partial^\mu U\right] \tag{A.29}$$

where the transformation law of U under $SU(2)_L \times SU(2)_R$ is given as

$$U \to U' = \mathfrak{g}_L U \mathfrak{g}_R. \tag{A.30}$$

Appendix B

In this section, we will treat the power counting formula for the electroweak chiral perturbation theory (EWChPT) [8, 13]. The EWChPT is the effective field theory written in terms of the SM fields other than the Higgs boson, and its leading order Lagrangian is obtained by setting $h \to 0$ in the leading order HEFT Lagrangian (3.2).

In the first raw of Table 1, we listed all types of the interaction vertices found in the leading order of the EWChPT. Note that φ denotes the would-be NG bosons $w^{\pm}$ and z, collectively. X_μ expresses weak gauge boson fields W_μ and B_μ, and $\psi_{L(R)}$ denotes left-handed (right-handed) SM fermions. From now on, we will consider L - loop diagram $\mathcal{D}_L$ containing a n_i number of φ^{2i}-vertices, ν_k Yukawa-type $\bar{\psi}_{L(R)}\psi_{R(L)}\varphi^k$-vertices, a m_l number of gauge-NG boson vertex $X_\mu\varphi^l$, a r_s numer of $X_\mu^2\varphi^s$-vertex, a x number of four point gauge self-interaction X_μ^4, a u number of three point gauge self-interaction X_μ^3, and a g number of $\bar{\psi}_{L(R)}\psi_{L(R)}X_\mu$-vertex.

Table 1: LO interactions, corresponding vertices, and the number of vertices in $\mathcal{D}_L$

interactions	φ^{2i}	$\bar{\psi}_{L(R)}\psi_{R(L)}\varphi^k$	$X_\mu\varphi^l$	$X_\mu^2\varphi^s$	X_μ^4	X_μ^3	$\bar{\psi}_{L(R)}\psi_{L(R)}X_\mu$
factors of vertices	p^2/v^{2i-2}	y/v^{k-1}	gp/v^{l-2}	g^2/v^{s-2}	g^2	gp	g
# of vertices in $\mathcal{D}_L$	n_i	ν_k	m_l	r_s	x	u	$z_L(z_R)$

The L - loop diagrams $\mathcal{D}_L$ with $\mathcal{B}$ NG boson propagators, $\mathcal{V}$ gauge boson propagators, and $\mathcal{F}_{L(R)}$ left-handed (right-handed) fermion propagators can be expressed as

$$\mathcal{D}_L \sim \frac{v^{2L}}{\Lambda^{2L}} \frac{y^{\Sigma_k \nu_k} g^{\Sigma_l m_l + \Sigma_s 2r_s + 2x + u + z}}{v^{\Sigma_i (2i-2) n_i + \Sigma_k (k-1)\nu_k + \Sigma_l (l-2) m_l + \Sigma_s (s-2) r_s}} \\ \times p^{4L + \Sigma_i 2n_i + \Sigma_l m_l + u - 2\mathcal{B} - 2\mathcal{V} - \mathcal{F}_L - \mathcal{F}_R - V} \bar{\psi}_L^{F_L^1} \psi_L^{F_L^2} \bar{\psi}_R^{F_R^1} \psi_R^{F_R^2} \varphi^B (X_{\mu\nu})^V , \quad \text{(B.1)}$$

where F_L^1, F_L^2, F_R^1, and F_R^2 denote the number of external fields of left-handed anti-fermion, left-handed fermion, right-handed anti-fermion, right-handed fermion, respectively. B denotes the number of external NG boson fields and V denotes the number of external gauge boson fields which should appear in the form of field strength.

The power of the momentum in (B.1) express the superficial degree of divergence for the diagram $\mathcal{D}_L$, which is denoted by "d" with

$$d = 4L + \sum_i 2n_i + \sum_l m_l + u - 2\mathcal{B} - 2\mathcal{V} - \mathcal{F}_L - \mathcal{F}_R - V \,. \quad \text{(B.2)}$$

Note that, in Eq.(B.2), $4L$ powers of momentum comes from the integration with respect to the loop momentum, $\sum_i 2n_i + \sum_l m_l + u$ powers of momentum comes from the derivative interactions listed in Table 1, $-2\mathcal{B} - 2\mathcal{V} - \mathcal{F}_L - \mathcal{F}_R$ powers of momentum comes from the NG boson, gauge boson, and fermion propagators, and $-V$ powers of momentum comes from the field strength of the external gauge boson fields $X_{\mu\nu}$, which can be obtained by replacing one of the external momentum with a derivative acting on the external gauge field.

We can eliminate the number of internal fields, $\mathcal{F}_{L(R)}$, $\mathcal{B}$, and $\mathcal{V}$ from Eq.(B.2) by using the following identities,

$$F_L + 2\mathcal{F}_L = \sum_k \nu_k + 2z_L \,, \quad \text{(B.3)}$$

$$F_R + 2\mathcal{F}_R = \sum_k \nu_k + 2z_R \,, \quad \text{(B.4)}$$

$$B + 2\mathcal{B} = \sum_i 2in_i + \sum_k k\nu_k + \sum_l lm_l + \sum_s sr_s \,, \quad \text{(B.5)}$$

$$V + 2\mathcal{V} = \sum_l m_l + \sum_s 2r_s + 4x + 3u + z \,. \quad \text{(B.6)}$$

Note that the LHS's of (B.3)-(B.6) express the sum of the external fields and the internal fields. The number of internal fields, $\mathcal{F}_{L(R)}$, $\mathcal{B}$, and $\mathcal{V}$, are multiplied by two because we need to contract two internal fields to get a propagator. The sum of the external fields and the internal fields should be equal to the total number of particle fields appearing in the diagram $\mathcal{D}_L$ through the interaction vertices, which are expressed by the RHS's of (B.3)-(B.6).

After eliminating the number of internal fields, $\mathcal{F}_{L(R)}$, $\mathcal{B}$, and $\mathcal{V}$ in (B.2), we get

$$\begin{aligned} d = 4L + B + \frac{F_L + F_R}{2} + \sum_i (2-2i) n_i - \sum_k (k+1)\nu_k - \sum_l l m_l - \sum_s (s+2) r_s \\ - 4x - 2u - 2z \,. \end{aligned} \tag{B.7}$$

Applying the topological identities for Feynman diagrams,

$$L = \mathcal{F}_L + \mathcal{F}_R + \mathcal{B} + \mathcal{V} - \sum_i n_i - \sum_k \nu_k - \sum_l m_l - \sum_s r_s - x - u - z + 1 \,, \tag{B.8}$$

we get the final expression for d,

$$d = 2L + 2 - \frac{F_L + F_R}{2} - V - \nu - m - 2r - 2x - u - z \,, \tag{B.9}$$

with

$$\nu := \sum_k \nu_k \,, \tag{B.10}$$

$$m := \sum_l m_l \,, \tag{B.11}$$

$$r := \sum_s r_s \,. \tag{B.12}$$

The important thing in (B.9) is that all the vertices have negative contributions to the superficial degree of divergence, d. Given the fixed value of the loop order "L," the diagram $\mathcal{D}_L$ has loop divergence when d takes the non-negative value, $d \geq 0$. Thanks to the negative contributions from the interaction vertices, however, d has the maximal value, and this means we can absorb all the divergence in the various loop diagrams by the finite number of counter terms: listing all the combinations of $(F_{L(R)}, V, \nu, m, r, x, u, z)$ for each non-negative value of d, and introducing corresponding operators to the L-th order HEFT Lagrangian, we can pursue perturbative calculations systematically.

The formula (B.7) gives the power counting rules for the HEFT, because once we give the loop order L, we can write down all the counter terms needed for absorbing the divergence by using the formula (B.7), which correspond to the L-th order operators in the loop expansion.

Appendix C

In this Appendix, we list the dictionary for converting the original GHEFT Lagrangian (4.49) into its geometrical form (6.1). We express the metric tensor g_{ij} in Eq.(6.1) in terms of G_{ab}, G_{aI}, and G_{IJ} in Eq.(4.49). Not that $\tilde{w}^a := w^a/v$ with $a = 1, 2$.

$(i, j) = (a, b)$

$$g_{11} = G_{11} - G_{13}\tilde{w}^2 + \frac{1}{3}\left(-G_{11}\tilde{w}^2\tilde{w}^2 + G_{12}\tilde{w}^1\tilde{w}^2\right) + \frac{1}{4}G_{33}\tilde{w}^2\tilde{w}^2 + \mathcal{O}((\pi)^3)\,, \tag{C.1}$$

$$g_{12} = G_{12} + \frac{1}{2}\left(G_{13}\tilde{w}^1 - G_{23}\tilde{w}^2\right) + \frac{1}{6}\left(G_{11}\tilde{w}^1\tilde{w}^2 + G_{22}\tilde{w}^1\tilde{w}^2 - G_{12}\tilde{w}^1\tilde{w}^1 - G_{12}\tilde{w}^2\tilde{w}^2\right) - \frac{1}{4}G_{33}\tilde{w}^1\tilde{w}^2 + \mathcal{O}((\pi)^3)\,, \tag{C.2}$$

$$g_{13} = G_{13} - \frac{1}{2}G_{33}\tilde{w}^2 + \frac{1}{6}\left(-G_{13}\tilde{w}^2\tilde{w}^2 + G_{23}\tilde{w}^1\tilde{w}^2\right) - G_{1I}[iQ_\phi]^I{}_J\phi^J\left(1 - \frac{1}{6}\tilde{w}^2\tilde{w}^2\right) + \frac{1}{2}G_{3I}[iQ_\phi]^I{}_J\phi^J\tilde{w}^2 - \frac{1}{6}G_{2I}[iQ_\phi]^I{}_J\phi^J\tilde{w}^1\tilde{w}^2 + \mathcal{O}((\pi)^3)\,, \tag{C.3}$$

$$g_{22} = G_{22} + G_{23}\tilde{w}^1 + \frac{1}{3}\left(-G_{22}\tilde{w}^1\tilde{w}^1 + G_{12}\tilde{w}^1\tilde{w}^2\right) + \frac{1}{4}G_{33}\tilde{w}^1\tilde{w}^1 + \mathcal{O}((\pi)^3)\,, \tag{C.4}$$

$$g_{23} = G_{23} + \frac{1}{2}G_{33}\tilde{w}^1 + \frac{1}{6}\left(G_{13}\tilde{w}^1\tilde{w}^2 - G_{23}\tilde{w}^1\tilde{w}^1\right) - G_{2I}[iQ_\phi]^I{}_J\phi^J\left(1 - \frac{1}{6}\tilde{w}^1\tilde{w}^1\right) - \frac{1}{2}G_{3I}[iQ_\phi]^I{}_J\phi^J\tilde{w}^1 - \frac{1}{6}G_{1I}[iQ_\phi]^I{}_J\phi^J\tilde{w}^1\tilde{w}^2 + \mathcal{O}((\pi)^3)\,, \tag{C.5}$$

$$g_{33} = G_{33} - 2G_{3I}[iQ_\phi]^I{}_J\phi^J + G_{IJ}[iQ_\phi]^I{}_K[iQ_\phi]^J{}_L\phi^K\phi^L\,, \tag{C.6}$$

$(i, j) = (a, I)$

$$g_{1I} = G_{1I} + \frac{1}{2} G_{3I} \tilde{w}^2 - \frac{1}{6} G_{1I} \tilde{w}^2 \tilde{w}^2 + \frac{1}{6} G_{2I} \tilde{w}^1 \tilde{w}^2 + \mathcal{O}((\pi)^3) \,, \tag{C.7}$$

$$g_{2I} = G_{2I} + \frac{1}{2} G_{3I} \tilde{w}^1 + \frac{1}{6} G_{1I} \tilde{w}^1 \tilde{w}^2 - \frac{1}{6} G_{2I} \tilde{w}^1 \tilde{w}^1 + \mathcal{O}((\pi)^3) \,, \tag{C.8}$$

$$g_{3I} = G_{3I} - G_{IJ} [i Q_\phi]^J{}_K \phi^K \,, \tag{C.9}$$

$$(i, j) = (I, J)$$

$$g_{IJ} = G_{IJ} \,. \tag{C.10}$$

Appendix D

In this appendix, we will derive one-loop correction to the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(D_\mu\phi)^i (D^\mu\phi)^j - V(\phi)\,, \tag{D.1}$$

with D_μ denoting gauge covariant derivative,

$$(D_\mu\phi)^i = \partial_\mu\phi^i + A_\mu^\alpha t_\alpha^i(\phi). \tag{D.2}$$

The authors of [41] derive the one loop correction to the Lagrangian (D.1), which is given by

$$\Delta\mathcal{L}_{\rm div}^{\varphi-\rm loop} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}\mathrm{tr}(Y_{\mu\nu}Y^{\mu\nu}) + \frac{1}{2}\mathrm{tr}(X^2)\right]\,, \tag{D.3}$$

where ϵ is dimensional regularization parameter and is written in terms of spacetime dimension D as

$$\epsilon := 4 - D\,. \tag{D.4}$$

$Y_{\mu\nu}$ and X are given as

$$[Y_{\mu\nu}]^i{}_j = R^i{}_{jkl}(D_\mu\phi)^k(D_\nu\phi)^l + F^a_{\mu\nu}(t^i_a)_{;j}\,, \tag{D.5}$$

$$[X]^i{}_k = R^i{}_{jkl}(D_\mu\phi)^j(D^\mu\phi)^l + g^{ij}V_{;jk}\,, \tag{D.6}$$

where covariant derivatives for Killing vectors t^i_a is given by

$$(t^i_a)_{;j} := t^i_{a,j} + \Gamma^i_{jk}t^k_a\,. \tag{D.7}$$

The aim of this appendix is to derive Eq.(D.3).

In order to calculate one-loop correction respecting gauge invariance, we use the background field method.

Firstly, let us decompose ϕ^i into a background field $\tilde{\phi}^i$ and a fluctuation field ξ^i as

$$\phi^i = \tilde{\phi}^i + \xi^i - \frac{1}{2}\tilde{\Gamma}^i_{jk}\xi^j\xi^k + \cdots , \tag{D.8}$$

where $\tilde{\Gamma}^i_{jk}$ is Affine connection evaluated at $\phi^i = \tilde{\phi}^i$. For the definition of Affine connection, see Eq.(5.30). We consider variating the action

$$S = \int d^4x \left[\frac{1}{2} g_{ij}(\phi)(D_\mu\phi)^i(D^\mu\phi)^j - V(\phi) \right] , \tag{D.9}$$

with respect to the fluctuation field ξ. Expanding the metric tensor g_{ij}, the Killing vector t^i_a, and the scalar potential V around $\phi^i = \tilde{\phi}^i$, we get

$$g_{ij}(\phi) = \tilde{g}_{ij} - \frac{1}{3}\tilde{R}_{ikjl}\,\xi^k\xi^l + \cdots , \tag{D.10}$$

$$t^i_a(\phi) = \tilde{t}^i_a + \tilde{t}^i_{a;j}\xi^j - \frac{1}{3}\tilde{R}^i{}_{kjl}\,\tilde{t}^j_a\,\xi^k\xi^l + \cdots , \tag{D.11}$$

$$V(\phi) = \tilde{V} + \tilde{V}_{,i}\xi^i + \frac{1}{2}\tilde{V}_{;ij}\xi^i\xi^j + \cdots . \tag{D.12}$$

To get Eq.(D.11), we used the following relation coming from the Killing equation,

$$g_{il}(w^l_a)_{;jk} = R_{ikjl}w^l_a \, . \tag{D.13}$$

In order to make the callcuration easy, we consider the Riemann Normal Coordinate (R.N.C.) in the remaining of this appendix. Note that, in the R.N.C., the following relations are satisfied,

$$\tilde{\Gamma}^i_{(j_1 j_2)} = 0 \, , \tag{D.14}$$

$$\tilde{\Gamma}^i_{(j_1 j_2 ,\, j_3)} = 0 \, , \tag{D.15}$$

$$\tilde{\Gamma}^i_{(j_1 j_2 ,\, j_3 j_4)} = 0 \, , \tag{D.16}$$

$$\vdots$$

where parentheses indicate the symmetrization with respect to indices: $\mathcal{T}_{(j_1 j_2)} = \frac{1}{2!}(\mathcal{T}_{j_1 j_2} + \mathcal{T}_{j_2 j_1})$. Note that, in the R.N.C., Riemann curvature tensor $\bar{R}^i{}_{jkl}$ appearing in Eq.(D.11) can be expressed as

$$\tilde{R}^i{}_{jkl} = \tilde{\Gamma}^i_{jl,k} - \tilde{\Gamma}^i_{jk,l} \,, \tag{D.17}$$

because of Eq.(D.14).

Expanding the covariant derivative $(D_\mu \phi)^i$ defined by Eq.(D.2) around $\phi^i = \tilde{\phi}^i$ in the R.N.C. coordinate, we get

$$(D_\mu \phi)^i = (D_\mu \phi)^i \Big|_{\xi^0} + (D_\mu \phi)^i \Big|_{\xi^1} + \frac{1}{2!}(D_\mu \phi)^i \Big|_{\xi^2} + \cdots \,, \tag{D.18}$$

with

$$(D_\mu \phi)^i \Big|_{\xi^0} = \partial_\mu \tilde{\phi}^i + \tilde{A}^a_\mu \tilde{t}^i_a =: (\tilde{D}_\mu \tilde{\phi})^i \,, \tag{D.19}$$

$$(D_\mu \phi)^i \Big|_{\xi^1} = \partial_\mu \xi^i + \tilde{A}^a_\mu (\tilde{t}^i_a)_{;j} \xi^j =: (\tilde{D}_\mu \xi)^i \,, \tag{D.20}$$

$$(D_\mu \phi)^i \Big|_{\xi^2} = -\left(\frac{1}{2} (\tilde{\Gamma}^i_{jk})_{,l} (\partial_\mu \tilde{\phi}^l) + \frac{1}{3} \tilde{A}^a_\mu \tilde{t}^l_a \tilde{R}^i{}_{jlk} \right) \xi^j \xi^k \,. \tag{D.21}$$

Using Eq.(D.10), Eq.(D.12) and Eqs. D.19)-(D.21), we can calculate the second variation of the action (D.9) with respect to the fluctuation field ξ, which is denoted by $S^{\xi\xi}$. The explicit form of $S^{\xi\xi}$ is given as

$$\begin{aligned} S^{\xi\xi} = \frac{1}{2} \int d^4x \Big[& \tilde{g}_{ij} (\tilde{D}_\mu \xi)^i (\tilde{D}^\mu \xi)^j - \tilde{R}_{kilj} (\tilde{D}_\mu \tilde{\phi}^k)(\tilde{D}^\mu \tilde{\phi}^l) \xi^i \xi^j - \tilde{V}_{;ij} \xi^i \xi^j \\ & + \tilde{g}_{ik} \left(\frac{2}{3} \tilde{R}^k{}_{mjn} - (\tilde{\Gamma}^k_{mn})_{,j} \right) (\tilde{D}_\mu \tilde{\phi})^i (\partial^\mu \tilde{\phi}^j) \xi^m \xi^n \Big] \,. \end{aligned} \tag{D.22}$$

where covariant derivatives $(\tilde{D}_\mu \tilde{\phi})^i$ and $(\tilde{D}_\mu \xi)^i$ are defined by Eq.(D.19) and Eq.(D.20), respectively. We can easily show that the second line of Eq.(D.22) vanishes: focusing on the relevant part of the second line of Eq.(D.22), we get

$$\left(\frac{2}{3}\tilde{R}^k{}_{mjn} - (\tilde{\Gamma}^k_{mn})_{,j}\right)\xi^m\xi^n = -\frac{1}{3}\left((\tilde{\Gamma}^k_{mn})_{,j} + (\tilde{\Gamma}^k_{mj})_{,n} + (\tilde{\Gamma}^k_{mj})_{,n}\right)\xi^m\xi^n$$
$$= -\frac{1}{3}\left((\tilde{\Gamma}^k_{mn})_{,j} + (\tilde{\Gamma}^k_{jm})_{,n} + (\tilde{\Gamma}^k_{nj})_{,m}\right)\xi^m\xi^n\,. \qquad \text{(D.23)}$$

In the first line of Eq.(D.23), we used the expression of the Riemann curvature tensor in R.N.C., Eq.(D.17). To get the second line from the first line of Eq.(D.23), we exchanged the dummy indices $m \leftrightarrow n$ in the third term in the parenthesis. Applying Eq.(D.15), we can see that the last line of Eq.(D.23) vanishes.

The final expression of $S^{\xi\xi}$ is therefore given as

$$S^{\xi\xi} = \frac{1}{2}\int d^4x\xi^i\Big[-\tilde{g}_{ij}\tilde{D}_\mu\tilde{D}^\mu - \tilde{R}_{kilj}(\tilde{D}_\mu\tilde{\phi})^k(\tilde{D}^\mu\tilde{\phi})^l - \tilde{V}_{;ij}\Big]\xi^j\,. \qquad \text{(D.24)}$$

Adding Affine connections to Eq.(D.24) appropriately and rewriting Eq.(D.24) in the covariant form, we finally get

$$S^{\xi\xi} = \frac{1}{2}\int d^4x\xi^i\Big[-\tilde{g}_{ij}\tilde{\mathscr{D}}_\mu\tilde{\mathscr{D}}^\mu - \tilde{R}_{kilj}(\tilde{D}_\mu\tilde{\phi})^k(\tilde{D}^\mu\tilde{\phi})^l - \tilde{V}_{;ij}\Big]\xi^j\,. \qquad \text{(D.25)}$$

with

$$(\mathscr{D}_\mu\xi)^i := [\delta^i_j\partial_\mu + \Gamma^i_{jk}(\partial_\mu\phi^k)]\xi^j + A^\alpha_\nu[t^i_{\alpha,j} + \Gamma^i_{jk}t^k_\alpha]\xi^j\,. \qquad \text{(D.26)}$$

The second variation (D.25) enters the one-loop effective action,

$$\Gamma_{\text{one-loop}} = \frac{i}{2}\log\det\left(-g^{ik}\frac{\delta^2 S}{\delta\xi^k\delta\xi^j}\right)\,. \qquad \text{(D.27)}$$

Reading out the effective Lagrangian from Eq.(D.27), we finally get the one-loop correction to the action (D.1), whose explicit form is given by

$$\Delta\mathcal{L}^{\varphi-\text{loop}}_{\text{div}} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}\text{tr}(Y_{\mu\nu}Y^{\mu\nu}) + \frac{1}{2}\text{tr}(X^2)\right]\,, \qquad \text{(D.28)}$$

with

$$[Y_{\mu\nu}]^i{}_j = [\mathscr{D}_\mu, \mathscr{D}_\nu]^i{}_j \,, \tag{D.29}$$

$$[X]^i{}_k = R^i{}_{jkl}(D_\mu\phi)^j(D^\mu\phi)^l + g^{ij}V_{;jk} \,. \tag{D.30}$$

Calculating the commutation relation in RHS of Eq.(D.28) explicitly, which is done in appendix D, we get

$$[Y_{\mu\nu}]^i{}_j = R^i{}_{jkl}(D_\mu\phi)^k(D_\nu\phi)^l + F^a_{\mu\nu}(t^i_a)_{;j} \,. \tag{D.31}$$

Appendix E

In this appendix, we will express the commutation relation

$$Y_{\mu\nu} := [\mathscr{D}_\mu, \mathscr{D}_\nu]^i{}_j\,, \tag{E.1}$$

in terms of covariant quantities such as the Riemann curvature tensor, the Killing vectors, and the covariant derivative of the scalar fields. Note that $\mathscr{D}_\mu$ denotes a covariant derivative defined by Eq.(D.26).

Expressing the commutation relation in terms of tensors appearing in the RHS of Eq.(D.26), we get

$$\begin{aligned}[]
[\mathscr{D}_\mu, \mathscr{D}_\nu]^i{}_j\ \eta^j = &\Big[\Gamma^i_{jk,l} + \Gamma^i_{ml}\Gamma^m_{jk}\Big](\partial_\mu\phi^l)(\partial_\nu\phi^j)\eta^k \\
&+ A^\beta_\nu(t^i_{\beta;k})_{,j}(\partial_\mu\phi^j)\eta^k + \Gamma^i_{jl}(\partial_\mu\phi^j)A^\alpha_\nu t^l_{\alpha;k}\eta^k - A^\alpha_\nu t^i_{\alpha;l}\Gamma^l_{jk}(\partial_\mu\phi^j)\eta^k \\
&+ (\partial_\mu A^\alpha_\nu)t^i_{\alpha;k}\eta^k + A^\alpha_\mu t^i_{\alpha;l}A^\beta_\nu t^l_{\beta;k}\eta^k \\
&- (\mu \leftrightarrow \nu)\,.
\end{aligned} \tag{E.2}$$

We will rewrite the first, second, and third line of Eq.(E.2) in turn.

1. First line of Eq.(E.2)

It is easy to see that the square bracket in the first line of Eq.(E.2) leads to the Riemann curvature tensor:

$$\Big[\Gamma^i_{jk,l} + \Gamma^i_{ml}\Gamma^m_{jk}\Big](\partial_\mu\phi^l)(\partial_\nu\phi^j)\eta^k - (\mu \leftrightarrow \nu)$$

$$
\begin{aligned}
&= \Big[\Gamma^i_{jk,l} + \Gamma^i_{ml}\Gamma^m_{kj} - (l \leftrightarrow j)\Big](\partial_\mu\phi^l)(\partial_\nu\phi^j)\eta^k \\
&= R^i{}_{klj}(\partial_\mu\phi^l)(\partial_\nu\phi^j)\eta^k\,,
\end{aligned}
\tag{E.3}
$$

with

$$
R^i{}_{klj} := \Gamma^i_{jk,l} - \Gamma^i_{lk,j} + \Gamma^i_{ml}\Gamma^m_{kj} - \Gamma^i_{mj}\Gamma^m_{kl}\,. \tag{E.4}
$$

Note that the Riemann curvature tensor defined by Eq.(E.4) satisfy usual anti-symmetric relation,

$$
R^i{}_{klj} = -R^i{}_{kjl}\,. \tag{E.5}
$$

2. Second line of Eq.(E.2)

Next, we will focus on the second line of Eq.(E.2). It is easy to check that the second line of Eq.(E.2) can be rewritten as

$$
\begin{aligned}
&A^\beta_\nu(t^i_{\beta;k})_{,j}(\partial_\mu\phi^j\eta^k + \Gamma^i_{jl}(\partial_\mu\phi^j)A^\alpha_\nu t^l_{\alpha;k}\eta^k - A^\alpha_\nu t^i_{\alpha;l}\Gamma^l_{jk}(\partial_\mu\phi^j)\eta^k - (\mu \leftrightarrow \nu) \\
=&A^\alpha_\nu(\partial_\mu\phi^j)\eta^k\Big(t^i_{\alpha,kj} + \Gamma^i_{kl} + \Gamma^i_{jl}t^l_{\alpha,k} - \Gamma^l_{kj}t^i_{\alpha,l}\Big) \\
&+ A^\alpha_\nu(\Gamma^i_{kl})_{,j}t^l_\alpha(\partial_\mu\phi^j)\eta^k + \Gamma^i_{jl}(\partial_\mu\phi^j)A^\alpha_\nu\Gamma^l_{km}t^m_\alpha\eta^k - A^\alpha_\nu\Gamma^i_{lm}t^m_\alpha\Gamma^l_{jk}(\partial_\mu\phi^j)\eta^k \\
&- (\mu \leftrightarrow \nu)\,.
\end{aligned}
\tag{E.6}
$$

Applying the following formula

$$
0 = t^l_\alpha\Gamma^i_{kj,l} + t^l_{\alpha,k}\Gamma^i_{lj} + t^l_{\alpha,j}\Gamma^i_{kl} - t^i_{\beta,l}\Gamma^l_{kj} + t^i_{\beta,kj} \tag{E.7}
$$

to the parenthesis in the second line of Eq.(E.6), we get

$$
\begin{aligned}
Eq.(\mathrm{E.6}) &= A^\alpha_\nu t^l_\alpha(\partial_\mu\phi^j)\eta^k\Big(\Gamma^i_{kl,j} - \Gamma^i_{kj,l} + \Gamma^i_{jm}\Gamma^m_{kl} - \Gamma^i_{ml}\Gamma^m_{jk}\Big) - (\mu \leftrightarrow \nu) \\
&= -A^\alpha_\nu t^l_\alpha(\partial_\mu\phi^j)R^i_{klj}\eta^k - (\mu \leftrightarrow \nu) \\
&= 2A^\alpha_\mu t^l_\alpha(\partial_\nu\phi^j)R^i_{klj}\eta^k\,.
\end{aligned}
\tag{E.8}
$$

To get the second line from the first line of Eq.(E.8), we used the definition of the Riemann curvature tensor (E.4). To get the third line from the second line of Eq.(E.8), we used Eq.(E.5).

*3. Third line of Eq.(*E.*2)*

Finally, we will focus on the third line of Eq.(E.2),

$$(\partial_\mu A_\nu^\alpha)t^i_{\alpha;k}\eta^k + A_\mu^\alpha t^i_{\alpha;l}A_\nu^\beta t^l_{\beta;k}\eta^k - (\mu \leftrightarrow \nu)\,. \tag{E.9}$$

The second term of Eq.(E.9) can be rewritten as

$$\begin{aligned}
& A_\mu^\alpha t^i_{\alpha;l}A_\nu^\beta t^l_{\beta;k}\eta^k - (\mu \leftrightarrow \nu) \\
&= A_\mu^\alpha A_\nu^\beta \Big[t^i_{\alpha,l}t^l_{\beta,k} + \Gamma^l_{km}t^i_{\alpha,l}t^m_\beta + \Gamma^i_{lj}t^j_\alpha t^l_{\beta,k} + \Gamma^i_{lj}\Gamma^l_{km}t^j_\alpha t^m_\beta - (\alpha \leftrightarrow \beta)\Big] \\
&= A_\mu^\alpha A_\nu^\beta \Big[\Big(t^i_{\beta,lk} + \Gamma^i_{jl}t^j_{\beta,k} - \Gamma^m_{kl}t^i_{\beta,m}\Big)t^l_\alpha + \Gamma^i_{lj}\Gamma^l_{km}t^j_\alpha t^m_\beta - (\alpha \leftrightarrow \beta)\Big] - f_{\alpha\beta}{}^\gamma A_\mu^\alpha A_\nu^\beta t^i_{\gamma,k} \\
&= A_\mu^\alpha A_\nu^\beta \Big[\Big(\Gamma^i_{km,l} - \Gamma^i_{kl,m} + \Gamma^i_{jl}\Gamma^j_{km} - \Gamma^i_{jm}\Gamma^j_{kl}\Big)t^l_\alpha t^m_\beta - [t_\alpha, t_\beta]^m\Gamma^i_{km}\Big]\eta^k - f_{\alpha\beta}{}^\gamma A_\mu^\alpha A_\nu^\beta t^i_{\gamma,k} \\
&= A_\mu^\alpha A_\nu^\beta \Big(R^i{}_{klm}t^l_\alpha t^m_\beta - f_{\alpha\beta}{}^\gamma t^i_{\gamma;k}\Big)\eta^k\,.
\end{aligned} \tag{E.10}$$

To get the second term from the first term of Eq.(E.10), we replace $\mu \leftrightarrow \nu$ with $\alpha \leftrightarrow \beta$. To get the third line from the second line of Eq.(E.10), we used the following relation

$$t^i_{\alpha,l}t^l_{\beta,k} - t^i_{\beta,l}t^l_{\alpha,k} = t^l_\alpha t^i_{\beta,lk} - t^l_\beta t^i_{\alpha,lk} - f_{\alpha\beta}{}^\gamma\,, \tag{E.11}$$

which is obtained by differentiate the Killing vectors' commutation relation with respect to scalar index,

$$[t_\alpha, t_\beta]^i = t^l_\alpha t^i_{\beta,l} - t^l_\beta t^i_{\alpha,l} = f_{\alpha\beta}{}^\gamma t^i_\gamma\,. \tag{E.12}$$

To get the fourth line from the third line of Eq.(E.10), we used

$$t^i_{\beta,lk} + t^j_{\beta,k}\Gamma^i_{jl} - t^i_{\beta,m}\Gamma^m_{kl} = -t^m_\beta \Gamma^i_{kl,m} - t^m_{\beta,l}\Gamma^i_{km}\,, \tag{E.13}$$

which comes from Eq.(E.7). Finally, to get the forth line from the third line of Eq.(E.10), we used the commutation relation (E.12) and the definition of the Riemann curvature tensor (E.4).

Substituting the final result in Eq.(E.10) into the second term of Eq.(E.9), we get

$$(\partial_\mu A^\alpha_\nu) t^i_{\alpha;k}\eta^k + A^\alpha_\mu t^i_{\alpha;l} A^\beta_\nu t^l_{\beta;k}\eta^k - (\mu \leftrightarrow \nu) = R^i{}_{klm}(A^\alpha_\mu t^l_\alpha)(A^\beta_\nu t^m_\beta)\eta^k + F^\gamma_{\mu\nu} t^i_{\gamma;k}\eta^k\,, \tag{E.14}$$

with

$$F^\gamma_{\mu\nu} = \partial_\mu A^\gamma_\nu - \partial_\nu A^\gamma_\mu - f_{\alpha\beta}{}^\gamma A^\alpha_\mu A^\beta_\nu\,. \tag{E.15}$$

4. Total of Eq.(E.2)

Combining Eq.(E.3), Eq.(E.8), and Eq.(E.14), we finally get

$$[\mathscr{D}_\mu, \mathscr{D}_\nu]^i{}_j = R^i{}_{jkl}(D_\mu\phi)^k(D^\mu\phi)^l + F^\alpha_{\mu\nu} t^i_{\alpha;j}\,. \tag{E.16}$$

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