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報告番号	※　　　　第　　　　号

主　論　文　の　要　旨

論 文 題 目

On Soliton Solutions of the Anti-Self-Dual Yang-Mills Equations from the Perspective of Integrable Systems
(可積分系の視点に基づく反自己双対ヤン・ミルズ方程式のソリトン解について)

氏　　名　HUANG Shan-chi

論　文　内　容　の　要　旨

Yang-Mills theory, which is constructed based on the unitary gauge theory, forms the foundation of the Standard Model for elementary particles. On the other hand, classical solitons in Yang-Mills theory, such as instanton, monopole, vortex, and domain wall, play important roles in the study of different fields of physics and mathematics. All of the above solitons are known as the exact solutions of anti-self-dual Yang-Mills (ASDYM) equations (or dimensionally reduced equations of ASDYM). In this thesis, we construct a class of ASDYM 1-solitons and multi-solitons that are different from the already known solitons.

In section 2, we review some necessary knowledge of quasideterminants which is required for the discussion in section 4, section 6, and section 7. Quasideterminats can be considered roughly as a noncommutative generalization of determinants. More than the meaning of mathematical generalization, quasideterminants are naturally fit for the description of noncommutative integrable systems. We introduce some elementary operation rules of quasideterminants, noncommutative Jacobi identity, homological relation, and a derivative formula of quasideterminants as useful mathematical tools in this section.

In section 3, we review some necessary knowledge of ASDYM theory mainly from the perspective of integrable systems. We consider the ASDYM theory in 4-dimensional complex space and introduce several equivalent descriptions of ASDYM equation. Especially, the anti-self-dual gauge fields can be expressed in terms of so-called J-matrix which is the solution of Yang equation. Combining the J-matrix formulation with Lax representation of ASDYM, it forms almost the theoretical foundation of our solutions in this thesis.

In section 4, we introduce a Darboux transformation which is considered firstly by Nimmo, Gilson, and Ohta. After n iterations of Darboux transformation, the resulting solutions (J-matrix) can be expressed beautifully in terms of Wronskian type (n+1)-th order quasideterminants. More precisely, each element of Wronskian type quasideterminant is a ratio of ordinary Wronskian determinants. We call them quasi-Wronskian for short in this thesis. This section is written mainly based on 副論文3 (Journal of Physics A: Mathematical and Theoretical, 53 (2020) 404002.), and partially by some recent results (Proposition 4.7 and Theorem 4.8).

In section 5, we construct ASDYM 1-solitons by applying 1 iteration of Darboux transformation. Firstly, we discuss general cases in 4-dimensional complex space and then impose some conditions to obtain 1-solitons on real spaces with Euclidean signature (+,+,+,+), Minkowski signature (+,−,−,−), and split signature(+,+,−,−)（Ultrahyperbolic space). In particular, the principal peak of Lagrangian density of 1-soliton is localized on a 3-dimensional hyperplane in 4-dimensional space. Therefore, we use the term soliton walls to distinguish them from domain walls because domain walls are described by scalar fields. Furthermore, we give an ansatz to obtain real-valued Lagrangian density even if the gauge group is non-unitary. For split signature case, we realized the gauge group to G=SU(2) successfully and hence the solion walls could be the candidates of physically interesting results on Ultrahyperbolic space. This section is written based on 副論文2 (Journal of High Energy Physics 2010 (2020) 101) and some revisions.

In section 6, we construct a special class of ASDYM n-solitons by applying n iterations of Darboux transformation. The resulting n-solitons are in the form of quasi-Wronskian. Furthermore, we use the techniques of quasideterminants to show that in the asymptotic region, n-soliton possesses n isolated Lagrangian densities (with phase shifts). Therefore, we can interpret it as n intersecting soliton walls. We calculate the phase shift factors explicitly and find that the Lagrangian densities can be real-valued for three kinds of signature. Especially for split signature, we show that the gauge group can be realized to G=SU(2). This section is written based on 副論文1 ([arXiv：2106.01353]) and some revisions.

In section 7, we construct an example of G=SU(3) 1-soliton on Ultrahyperbolic space and it can be interpreted as soliton wall as well.　After applying n iterations of Darboux transformation, we show that the resulting n-solitons can be interpreted as n intersecting soliton walls as well. As for gauge group, it can be realized to G=SU(3) for each soliton wall in the asymptotic region. This section is written based on some unpublished results recently.