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Chicago Price Theory Meets Imperfect Competition: A Conduct Parameter Approach*

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Abstract

In this paper, we provide a formulation of how imperfect competition in the product market is incorporated into the industry model described in Chapter 11 of *Chicago Price Theory* (Jaffe, Minton, Mulligan, and Murphy 2019, *CPT*). A generalized version of the perturbed system of the industry is shown and used to analyze the effect of a wage increase in the long-run as well as in the short-run in relation to the intensity of competition.

Keywords: *Chicago Price Theory*; Industry Model; Imperfect Competition; Conduct Parameter Approach.

JEL classification: A22, A23, D43, L13.

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"Friedman became the leader of the Chicago School .... For the Chicagoans, Alfred Marshall was still the bible. .... *The Theory of Games* was itself a mathematical game. And imperfect competition was a snare and a delusion." (Warsh 2006, Ch. 10, p. 134)

"The reliance on price theory and inability to appreciate oligopoly account for much of the skepticism that Chicago School antitrust writing has had towards theories of harm that interact with market structure." (Hovenkamp and Scott Morton 2020, p. 1856)

"Can we, can we get along?" (Rodney King, 1992)

1 Introduction

Market competition is one of the key concepts at the core of price theory. Following this tradition, *Chicago Price Theory* (Jaffe, Minton, Mulligan, and Murphy 2019, *CPT*)—a recently published textbook for graduate microeconomics based on the legendary course taught at the University of Chicago—emphasizes the importance of studying market equilibrium *at the aggregate level.* This methodological stance stands in sharp contrast to game theory, which "typically focuses on interactions among small numbers of agents" (*CPT*, p. 3). Nowadays, it is widely held that *imperfect competition*, one of the most prominent characteristics in modern economy,[1] is fairly an advanced topic, and therefore should be taught only after game theory is introduced; see, e.g.., Tirole (1988) for an early manifesto that might have contributed to the formation of such a popular belief.

Perhaps, this common view is too narrow. To convince the reader of why, this paper proposes a *conduct parameter approach* à la Weyl and Fabinger (2013) to provide a synthesis of imperfect competition in the product market and price theory in its traditional style. In this way, it becomes possible to teach imperfect competition from the outset when the interaction of supply and demand is studied at the aggregate level, as in *CPT*'s Chapter 11, "The Industry Model." Additionally, this methodology enables one to escape from the (mis)belief that imperfect competition is one of the market failures and hence should be treated only as a special case. Rather, the conduct parameter approach makes it clear that it *is* perfect competition that should be treated as a special case of imperfect competition.

The rest of the paper is organized as follows. The next section illustrates how the industry model of single product under the assumption of perfect competition is generalized to include imperfect competition. In particular, the generalized version of the perturbed system of the

[1] For example, *common ownership*—a situation in which a small number of institutional investors own large shares of big firms—may weaken competition between firms in an industry, resulting in non-negligible markup (see, e.g., Schmalz (2018) for an introductory survey and references therein).

industry is presented. Then, in Section 3, we use this result to analyze the effect of a wage increase in the short-run as well as in the long-run. Specifically, we obtain the following testable prediction: when the industry faces an increase in the (perfectly competitive) price of labor or capital, a *weaker intensity of competition* in the product market, *ceteris paribus*, facilitates *more substitution* toward the use of the other input in the long-run, and in the short-run, *stronger reaction* of the other input's price. Lastly, Section 4 concludes. The accompanying Appendix provides some discussions on the relationship with Weyl and Fabinger (2013) and the case of the multi-product industry.

2 The Industry Model

We start with the description of the Industry Model under perfect competition in Chapter 11 of *CPT*. Then, we argue how imperfect competition is incorporated into this Industry Model, and show how the perturbed system of the Industry Model is generalized.

2.1 Preliminaries

Let $D(P)$ be the industry's demand, where $P > 0$ is the industry-level price. Throughout this paper, we consider the optimal production of one "representative firm" à la Marshall (1890/1920), which is a conceptual entity consisting of symmetric firms.[2] Then, this industry/firm's marginal cost is denoted by $MC(Y)$, where $Y > 0$ is the aggregate output in the industry. We assume that both $D(P)$ and $MC(Y)$ satisfy the standard restrictions. There are no fixed costs for this production.

Given the wage rate $w > 0$ and the rental rate $r > 0$, and under the assumption of *Constant Returns to Scale* (CRS), the cost function, $C(w, r, Y)$ satisfies: $C(w, r, Y) = Y \cdot C(w, r, 1)$, and thus the marginal cost of production is *constant*: $MC(Y) = C(w, r, 1)$ for any $Y > 0$. In addition, let $L > 0$ and $K > 0$ be labor and capital inputs, respectively, for the production process that is summarized by the production function, $Y = F(L, K)$. We also assume that the regular properties hold for this production function.

Under this setting, *CPT* presents the Industry Model under perfect competition, which is

[2] Firm heterogeneity would be readily incorporated, although the main thrust would not change significantly, whereas the notation would become heavier (see, e.g., Adachi and Fabinger 2020).

described by the following system of “four ingredients” (p. 131):

$$\begin{cases} 1. \dfrac{P - MC}{p} = 0 \\ 2. Y = D(P) \\ 3. L = \dfrac{\partial C(w, r, Y)}{\partial w} \text{ and } K = \dfrac{\partial C(w, r, Y)}{\partial r} \\ 4. Y = F(L, K), \end{cases}$$

in which the only modification from *CPT*'s original description appears in the first equation: here, it explicitly states that the markup rate is zero under perfect competition, whereas *CPT* simply writes this condition by $P = MC$. The second equation requires that the demand and the supply in the product market be equal in equilibrium, and the third implies that the firm is a price taker in the input market. Finally, the last equation describes the connection between output Y and inputs L and K.

2.2 Incorporating Imperfect Competition into the Industry Model

Now, we introduce the *conduct parameter*, $\theta \in [0, 1]$, measuring the intensity of imperfect competition in the industry: the industry is characterized by monopoly if $\theta = 1$ (there is only one producer or all firms form a perfect cartel) whereas $\theta = 0$ results in marginal cost pricing. Thus, the first ingredient for the markup rate above is now generalized as:

$$\frac{P - MC}{P} = \frac{\theta}{(-\epsilon^D)}, \tag{1}$$

where $\epsilon^D \equiv \frac{P}{Y}\frac{dD(P)}{dP} < 0$ is the *price elasticity of demand*. Let Δ denote the percentage change (e.g., $\Delta P \equiv \frac{dP}{P} = d \ln P$). Then, the perturbed system of the industry under imperfect competition, which generalize that of *CPT*, is given as follows.

Proposition 1. *The perturbed system of the industry with the intensity of competition being characterized by $\theta \in [0, 1]$ is described by the following system of four ingredients:*

$$\begin{cases} \Delta P = \dfrac{s_L \Delta w + s_K \Delta r}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} \\ -\Delta Y = (-\epsilon^D)\Delta P \\ \Delta L - \Delta K = \sigma \cdot (\Delta r - \Delta w) \\ \Delta Y = s_L \Delta L + s_K \Delta K, \end{cases}$$

where $s_L \equiv \frac{wL}{PY}$ and $s_K \equiv \frac{rK}{PY}$ are the labor share and the capital share of aggregate income *(excluding corporate profit), respectively,* $\alpha^D(P) \equiv -\frac{PD''(P)}{D'(P)}$ *is the demand curvature, and* $\sigma > 0$ *is the elasticity of substitution, defined by* $\Delta\frac{L}{K} = \sigma \cdot \Delta\frac{r}{w}$*.*

Proof. Recall that under constant returns to scale (CRS), $MC = C(w, r, 1)$. Thus, Equation (1) is further formulated as:

$$P - C(w,r,1) = \theta \cdot \frac{P}{(-\epsilon^D)}$$

$$\Rightarrow dP - \underbrace{\frac{\partial C(w,r,1)}{\partial w}}_{=\frac{L}{Y}} dw - \underbrace{\frac{\partial C(w,r,1)}{\partial r}}_{=\frac{K}{Y}} dr = \theta \cdot \left[\frac{1}{(-\epsilon^D)} dP + P \cdot \frac{1}{(\epsilon^D)^2} d\epsilon^D\right], \tag{2}$$

which implies that

$$\underbrace{\frac{dP}{P}}_{\equiv \Delta P} - \underbrace{\frac{wL}{PY}}_{\equiv s_L} \underbrace{\frac{dw}{w}}_{\equiv \Delta w} - \underbrace{\frac{rK}{PY}}_{\equiv s_K} \underbrace{\frac{dr}{r}}_{\equiv \Delta r} = \theta \cdot \left[\frac{1}{(-\epsilon^D)} \Delta P - \underbrace{\frac{d\epsilon^D}{\epsilon^D}}_{\equiv \Delta\epsilon^D} \frac{1}{(-\epsilon^D)}\right],$$

and hence

$$\Delta P - s_L \Delta w - s_K \Delta r = \frac{\theta}{(-\epsilon^D)} [\Delta P - \Delta\epsilon^D]$$

$$\Rightarrow \left[1 - \frac{\theta}{(-\epsilon^D)}\right] \Delta P = s_L \Delta w + s_K \Delta r - \frac{\theta}{(-\epsilon^D)} \Delta\epsilon^D. \tag{3}$$

Now, it is observed that

$$d\epsilon^D = \left(\frac{\partial\left(\frac{dD(P)}{dP} \cdot \frac{P}{Y}\right)}{\partial P}\right) dP + \left(\frac{\partial\left(\frac{dD(P)}{dP} \cdot \frac{P}{Y}\right)}{\partial Y}\right) dY,$$

where

$$\frac{\partial\left(\frac{dD}{dP} \cdot \frac{P}{Y}\right)}{\partial P} = \frac{1}{Y} \cdot \left(\frac{dD}{dP} + P\frac{d^2D}{dP^2}\right)$$

and

$$\frac{\partial\left(\frac{dD}{dP} \cdot \frac{P}{Y}\right)}{\partial Y} = -\frac{P}{Y^2}\frac{dD}{dP} = -\frac{\epsilon^D}{Y}.$$

Therefore, it is verified that

$$\underbrace{\frac{d\epsilon^D}{\epsilon^D}}_{\equiv \Delta\epsilon^D} = \frac{\frac{P}{Y} \cdot \left(\frac{dD}{dP} + P\frac{d^2D}{dP^2}\right) \underbrace{\frac{dP}{P}}_{=\Delta P}}{\epsilon^D} - \frac{\epsilon^D}{\epsilon^D} \underbrace{\frac{dY}{Y}}_{\equiv \Delta Y}$$

$$\begin{aligned}
\Rightarrow \Delta\epsilon^D &= \frac{\epsilon^D + P\frac{P}{Y}\frac{d^2D}{dP^2}}{\epsilon^D}\Delta P - \Delta Y \\
&= \left(1 + \frac{P\frac{d^2D}{dP^2}}{\frac{dD}{dP}}\right)\Delta P - \Delta Y \\
&= (1 - \epsilon^D - \alpha^D)\Delta P \qquad (4)
\end{aligned}$$

because $-\Delta Y = (-\epsilon^D)\Delta P$. Combining Equations (3) and (4), one can verify that

$$\Delta P = \frac{1}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} \cdot (s_L \Delta w + s_K \Delta r), \qquad (5)$$

whereas no changes are made for the third ($\Delta L - \Delta K = \sigma \cdot (\Delta r - \Delta w)$) and fourth ($\Delta Y = s_L \Delta L + s_K \Delta K$) ingredients because $(\Delta P, \Delta\epsilon^D)$ does not appear in these equations. □

Note here that Equation (5) is degenerated into $\Delta P = s_L \Delta w + s_K \Delta r \equiv \Delta MC$ for the case of perfect competition (i.e., $\theta = 0$). Note also that the denominator of Equation (5) is less than one if and only if the demand is *sufficiently convex* that $\alpha^D > (-\epsilon^D)$ around the equilibrium, In contrast, the denominator is greater than one if only if $\alpha^D < (-\epsilon^D)$ in equilibrium. Hence, the following result is obtained.

Corollary 1. *The marginal cost pass-through rate is absorbing (i.e., $\frac{\Delta P}{\Delta MC} < 1$) if and only if the demand is not too convex such that $\alpha^D < (-\epsilon^D)$. In contrast, it is complete (i.e., $\frac{\Delta P}{\Delta MC} = 1$) if and only if $\alpha^D = (-\epsilon^D)$, and is amplifying (i.e., $\frac{\Delta P}{\Delta MC} > 1$) if and only if $\alpha^D > (-\epsilon^D)$.*

The role of θ is not to determine the sign, but is related to the significance of absorption or amplification: when absorption takes place, $\left|\frac{\Delta P}{\Delta MC}\right|$ is smaller for a larger value of θ. This is probably a well-known result in intermediate microeconomics; an analogy that comes from the basic fact that under monopoly with linear demand and constant marginal cost, the cost pass-through is one half. However, in the case of amplification, the opposite is true: $\frac{\Delta P}{\Delta MC}$ is *larger* for a larger value of θ. Note here that these results are expressed in terms of change in *rate, not value.*[3] In the Appendix, we discuss how our formula is related to Weyl and Fabinger's (2013) which is expressed in terms of value.

Finally, note that under imperfect competition (i.e., when $\theta > 0$), the firm's profits are positive:

$$\Pi = PY - wL - rK > 0,$$

[3]See Ritz (2020) for an analysis of how non-constant marginal cost changes the results of pass-through under the assumption of constant marginal cost.

which implies that

$$s_L + s_K = 1 - s_\Pi,$$

where $s_\Pi \equiv \frac{\Pi}{PY}$ is the *profit share* of aggregate product value, PY. Obviously, $s_\Pi = 0$ under perfect competition (i.e., when $\theta = 0$).

3 Analysis of the Perturbed System

Following *CPT*, this section provides both long-run and short-run analyses using the perturbed system in Proposition 1.

Long-Run We suppose that in the long-run, $\Delta r = 0$ holds. Then, given Δw, one can solve the system of four equations for four unknowns, ΔP, ΔY, ΔL, and ΔK. From the first two equations of single product industry's perturbed system, it is observed that

$$-\Delta Y = \frac{s_L \cdot (-\epsilon^D)}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} \Delta w,$$

which captures the *scale effect*: this measures to what extent an increase in the competitive wage, w, reduces output, Y.

Then, it is observed that $(\Delta L, \Delta K)$ satisfies:

$$\begin{pmatrix} 1 & -1 \\ s_L & s_K \end{pmatrix} \begin{pmatrix} \Delta L \\ \Delta K \end{pmatrix} = \begin{pmatrix} -\sigma \cdot \Delta w \\ -\frac{s_L \cdot (-\epsilon^D)}{1+\theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} \Delta w \end{pmatrix}$$

and thus,

$$\begin{pmatrix} \Delta L \\ \Delta K \end{pmatrix} = \frac{1}{s_L + s_K} \begin{pmatrix} s_K & 1 \\ -s_L & 1 \end{pmatrix} \begin{pmatrix} -\sigma \cdot \Delta w \\ -\frac{s_L \cdot (-\epsilon^D)}{1+\theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} \Delta w \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{\Delta L^{LR}}{\Delta w} \\ \frac{\Delta K}{\Delta w} \end{pmatrix} = \frac{1}{1 - s_\Pi} \begin{pmatrix} -\left(s_K \cdot \sigma + \frac{s_L \cdot (-\epsilon^D)}{1+\theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]}\right) \\ \left(\sigma - \frac{(-\epsilon^D)}{1+\theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]}\right) \cdot s_L \end{pmatrix},$$

where LR stands for the long-run. Here, $-s_K \cdot \sigma$ captures *the substitution effect*, which measures to what extent an increase in the competitive wage, w, increases capital input, K.

Hence, it is verified that $\Delta w > 0$ imply $\Delta K > 0$ if and only if

$$\sigma > \frac{(-\epsilon^D)}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]}.$$

Note that the threshold under perfect competition ($\theta = 0$) is greater than that under imperfect competition ($\theta > 0$):

$$(-\epsilon^D) > \frac{(-\epsilon^D)}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]}$$

if the demand is not too convex: $\alpha^D < (-\epsilon^D)$. Thus, if σ is less than $(-\epsilon^D)$ but greater than $\frac{(-\epsilon^D)}{1+\theta\cdot[1-\alpha^D/(-\epsilon^D)]}$, then the amount of capital used *decreases* ($\Delta K < 0$) under perfect competition, whereas it increases ($\Delta K > 0$) under imperfect competition. Therefore, *in the long-run, imperfect competition is more likely to expand the use of capital (*$\Delta K > 0$*), ceteris paribus, in response to an increase in the wage (*$\Delta w > 0$*).*

Short-Run In the short-run, capital is held fixed, $\Delta K = 0$, whereas the rental rate can change: $\Delta r \neq 0$. Now, unknown variables are ΔP, ΔY, ΔL, and Δr. Then, $(\Delta L, \Delta r)$ satisfies:

$$\begin{pmatrix} 1 & -\sigma \\ s_L & \frac{(-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} s_K \end{pmatrix} \begin{pmatrix} \Delta L \\ \Delta r \end{pmatrix} = \begin{pmatrix} -\sigma \cdot \Delta w \\ -\frac{s_L \cdot (-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} \Delta w \end{pmatrix}$$

and thus,

$$\begin{pmatrix} \Delta L \\ \Delta r \end{pmatrix} = \frac{1}{\sigma \cdot s_L + \frac{(-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} s_K} \begin{pmatrix} \frac{(-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} s_K & \sigma \\ -s_L & 1 \end{pmatrix} \begin{pmatrix} -\sigma \cdot \Delta w \\ -\frac{s_L \cdot (-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} \Delta w \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{\Delta L^{SR}}{\Delta w} \\ \frac{\Delta r}{\Delta w} \end{pmatrix} = \frac{1}{\sigma \cdot s_L + \frac{(-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]} s_K} \begin{pmatrix} -\left(\frac{1-s_\Pi}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]}\right) \cdot (-\epsilon^D)\sigma \\ \left(\sigma - \frac{(-\epsilon^D)}{1+\theta\cdot\left[1-\frac{\alpha^D}{(-\epsilon^D)}\right]}\right) \cdot s_L \end{pmatrix},$$

where SR stands for the short-run.

Therefore, $\Delta w > 0$ imply $\Delta r > 0$ if and only if

$$\sigma > \frac{(-\epsilon^D)}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]}.$$

This is the *same* condition for $\frac{\Delta K}{\Delta w} > 0$ in the long-run, although in general $\frac{\Delta K}{\Delta w}$ and $\frac{\Delta r}{\Delta w}$ take different values. As in the long-run case, *imperfect competition is more likely in the short run to raise the rental price of capital (*$\Delta r > 0$*), ceteris paribus, in response to an increase in the wage (*$\Delta w > 0$*).*

Summary Based on these results, we obtain the following testable prediction: when the industry faces an increase in the price of one of the two inputs, a *weaker intensity of competition* (a greater value of θ) in the product market, *ceteris paribus*, facilitates *more substitution* toward the use of the other input in the long-run, and in the short-run, *stronger reaction* of the other input's price. In the Appendix, we show that our arguments can be extended to the case of the multi-product industry.

4 Concluding Remarks

In this paper, we have argued that the conduct parameter approach to modeling imperfect competition is useful to generalize the industry model presented in Chapter 11 of *Chicago Price Theory*. It is shown that imperfect competition in the product market matters to the prediction of how the pattern of substitution between labor and capital is affected by a change in the (perfectly competitive) wage. Throughout this paper, we have assumed that imperfect competition exists *only in the product market*, and Δw or Δr is treated as an exogenous change as if the *labor* market as well as the *rental/capital* market are perfectly competitive. Incorporating imperfect competition into these markets ("imperfect competition in *general equilibrium*") is left for future research, and this note intends to be a small step toward this direction (but see, e.g., Azar and Vives (2021) for such an attempt).

Appendix: Discussions

In this appendix, we first show the equivalence of our expression and Weyl and Fabinger's (2013). Then, we argue that how our single-product firm model can be generalized to the case of the multi-product industry. We also use this multi-product firm model to analyze the long-run and the short-run.

A1. Relationship with Weyl and Fabinger (2013)

Note that Equation (2) in the text is also expressed by:

$$dP - dMC = \theta \cdot \left[\frac{1}{(-\epsilon^D)} dP + P \cdot \frac{1}{(\epsilon^D)^2} d\epsilon^D \right],$$

which implies that

$$\Delta P - \frac{MC}{P} \Delta MC = \frac{\theta}{(-\epsilon^D)} [\Delta P - \Delta \epsilon^D]$$

and thus

$$\frac{dP}{dMC} = \frac{1}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)} \right]}.$$

This is a special case of Weyl and Fabinger's (2013, p. 548) equation (2):

$$\frac{dP}{dMC} = \frac{1}{1 + \frac{\theta}{\epsilon^{\theta}} + \frac{(-\epsilon^D) - \theta}{\epsilon^S} + \frac{\theta}{\epsilon^{ms}}},$$

where ϵ^S is the elasticity of marginal cost (or market supply under perfect competition), which is $\epsilon^S = \infty$ under CRS, and $\epsilon^{\theta} \equiv \frac{\theta}{Q \cdot \frac{d\theta}{dQ}}$ takes into account the possibility of θ varying with Q, which is $\epsilon^{\theta} = \infty$ as θ is assumed to be constant. Thus, according to Weyl and Fabinger's (2013, p. 548), the marginal cost pass-through is expressed as:

$$\frac{dP}{dMC} = \frac{1}{1 + \theta \cdot \frac{1}{\epsilon^{ms}}},$$

where $\epsilon^{ms} \equiv \frac{ms}{\frac{dms}{dY} Y}$ is the "elasticity of the inverse marginal surplus function" (Weyl and Fabinger 2013, p. 539) and $ms \equiv -D/D'$ is the "negative of the marginal consumer surplus" (Weyl and Fabinger 2013, p. 538).

To show the equivalence of these two expressions (i.e., $1 - \frac{\alpha^D}{(-\epsilon^D)} = \frac{1}{\epsilon^{ms}}$) in equilibrium, note first that:

$$\epsilon^{ms} = \frac{ms}{\frac{dms}{dp} \frac{1}{D'} D}$$

$$\Rightarrow \frac{1}{\epsilon^{ms}} = \frac{ms' \cdot D}{ms \cdot D'} = -ms'.$$

Now, from the definition of ms, it is verified that

$$\begin{aligned} -ms' &= \frac{[D']^2 - D \cdot D''}{[D']^2} \\ &= 1 - \frac{D \cdot D''}{[D']^2}. \end{aligned}$$

On the other hand, from the definitions of α^D and ϵ^D, it is also verified that

$$\begin{aligned} \frac{\alpha^D}{(-\epsilon^D)} &= \left(-\frac{P \cdot D''}{D'}\right)\left(-\frac{D}{P \cdot D'}\right) \\ &= \frac{D \cdot D''}{[D']^2}. \end{aligned}$$

Therefore, it turns out that the two expressions are equivalent:

$$\frac{1}{1 + \theta \cdot \left[1 - \frac{\alpha^D}{(-\epsilon^D)}\right]} = \frac{dP}{dMC} = \frac{1}{1 + \theta \cdot \frac{1}{\epsilon^{ms}}}.$$

However, our expression is more useful in that it consists of estimable objects such as ϵ^D and α^D as well as θ.

A2. Multi-Product Representative Firm

It is also possible to consider the case of the representative firm producing multiple products. We assume that it produces symmetrically differentiated products, and thus the number of product is, without loss of generality, two. In this case, the representative firm's profit function is given by

$$\Pi = P_1 \cdot D_1(P_1, P_1) + P_2 \cdot D_2(P_1, P_2) - C[D_1(P_1, P_2), D_2(P_1, P_2)].$$

We also assume that *there is no economies or diseconomies of scope* (i.e., $\frac{\partial^2 C}{\partial Y_1 \partial Y_2} = 0$), and the CRS holds for each product. Then, the cost of production is given by

$$C(w, r, \mathbf{Y}) = Y_1 \cdot C_1(w, r, 1) + Y_2 \cdot C_2(w, r, 1),$$

where $C_j(w, r, Y_j)$ is the cost of producing Y_j units of product j. Hence, the profit function is now simply written as:

$$\Pi = \sum_{j=1,2} [P_j - C_j(w, r, 1)] D_j(P_1, P_2).$$

The firm's pricing is characterized by the conduct parameter approach in the following manner:

$$\begin{pmatrix} \theta D_1 \\ \theta D_2 \end{pmatrix} = -\begin{pmatrix} \frac{\partial D_1}{\partial P_1} & \frac{\partial D_2}{\partial P_1} \\ \frac{\partial D_1}{\partial P_2} & \frac{\partial D_2}{\partial P_2} \end{pmatrix} \begin{pmatrix} P_1 - MC_1 \\ P_2 - MC_2 \end{pmatrix},$$

which is rewritten as

$$\begin{pmatrix} \theta D_1 \\ \theta D_2 \end{pmatrix} = -\begin{pmatrix} D_1 \epsilon_{11}^D & \frac{P_2 D_2}{P_1} \epsilon_{12}^D \\ \frac{P_1 D_1}{P_2} \epsilon_{21}^D & D_2 \epsilon_{22}^D \end{pmatrix} \begin{pmatrix} \dfrac{P_1 - MC_1}{P_1} \\ \dfrac{P_2 - MC_2}{P_2} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \dfrac{P_1 - MC_1}{P_1} \\ \dfrac{P_2 - MC_2}{P_2} \end{pmatrix} = \frac{\theta}{\epsilon_{11}^D \epsilon_{22}^D - \epsilon_{12}^D \epsilon_{21}^D} \begin{pmatrix} (-\epsilon_{22}^D) + \frac{P_2 D_2}{P_1 D_1} \epsilon_{12}^D \\ (-\epsilon_{11}^D) + \frac{P_1 D_1}{P_2 D_2} \epsilon_{21}^D \end{pmatrix},$$

where $\epsilon_{jk}^D \equiv \frac{P_j}{Y_k} \frac{\partial D_k}{\partial P_j}$ is the price elasticity of demand for product $k = 1, 2$ with the same product (for $j = k$) or across the products (for $j \neq k$).

Now, utilizing the symmetry, we define the *own* and *cross elasticities* by $\epsilon_{own}^D \equiv \frac{P_j}{Y_j} \frac{\partial D_j}{\partial P_j} =$

$\epsilon_{11}^D = \epsilon_{22}^D$ and $\epsilon_{cross}^D \equiv \frac{P_k}{Y_j}\frac{\partial D_j}{\partial P_k} = \epsilon_{12}^D = \epsilon_{21}^D$, respectively. Then, the Lerner formula given above is simplified to

$$\frac{P_j - MC_j}{P_j} = \frac{\theta}{(-\epsilon_{own}^D) - \epsilon_{cross}^D}$$

for $j = 1, 2$, which generalizes Equation (1) in the text.

Accordingly, the industry model for the case of imperfect competition with multiple products is given by

$$\begin{cases} 1.\ \dfrac{P_j - MC_j}{P_j} = \dfrac{\theta}{(-\epsilon_{own}^D) - \epsilon_{cross}^D} \\ 2.\ Y_j = D_j(P_1, P_2) \\ 3.\ L_j = \dfrac{\partial C(w, r, Y_j)}{\partial w} \text{ and } K_j = \dfrac{\partial C(w, r, Y_j)}{\partial r} \\ 4.\ Y_j = F_j(L_j, K_j) \end{cases}$$

for $j = 1, 2$.

The perturbed system of the multi-product industry under imperfect competition, and its analysis is given below. It is verified that qualitatively similar results are obtained for the long-run and the short-run.

A2.1 Derivation of the Perturbed System

First, we define the *own curvature evaluated at the own first-order derivative* (oo) by

$$\alpha_{oo}^D \equiv -\frac{P \cdot \frac{\partial^2 D_j}{\partial P_j^2}}{\frac{\partial D_j}{\partial P_j}}$$

and the *twice-crossing curvature evaluated at the cross first-order derivative* (tc) by

$$\alpha_{tc}^D \equiv -\frac{P \cdot \frac{\partial^2 D_j}{\partial P_k^2}}{\frac{\partial D_j}{\partial P_k}}.$$

Similarly, the *single-crossing curvature evaluated at the own first-order derivative* (so),

$$\alpha_{so}^D \equiv -\frac{P \cdot \frac{\partial^2 D_j}{\partial P_j \partial P_k}}{\frac{\partial D_j}{\partial P_j}},$$

and the *single-crossing curvature evaluated at the cross first-order derivative* (sc),

$$\alpha_{sc}^D \equiv -\frac{P \cdot \frac{\partial^2 D_j}{\partial P_j \partial P_k}}{\frac{\partial D_j}{\partial P_k}},$$

are also defined. Accordingly, we define the following two indices as follows:

$$\begin{cases} A_{own}^D = \alpha_{oo}^D + \alpha_{so}^D \\ A_{cross}^D = \alpha_{tc}^D + \alpha_{sc}^D, \end{cases}$$

in which $A_{own}^D = \alpha^D$ and $A_{cross}^D = 0$ in the case of the single-product industry (i.e., $\epsilon_{cross}^D = 0$). Then, the following proposition is obtained.

Proposition. *The perturbed system of the two-product industry with the intensity of competition being characterized by* $\theta \in [0, 1]$ is described by the following system of four ingredients:

$$\begin{cases} \Delta P_j = \dfrac{s_{L_j}\Delta w + s_{K_j}\Delta r}{1 + \theta \cdot \left[1 - \frac{\epsilon_{own}^D A_{own}^D + \epsilon_{cross}^D A_{cross}^D}{-(\epsilon_{own}^D + \epsilon_{cross}^D)^2}\right]} & \text{(A1)} \\ -\Delta Y_j = [(-\epsilon_{own}^D) - \epsilon_{cross}^D]\Delta P_j & \text{(A2)} \\ \Delta L_j - \Delta K_j = \sigma \cdot (\Delta r - \Delta w) & \text{(A3)} \\ \Delta Y_j = s_{L_j}\Delta L_j + s_{K_j}\Delta K_j & \text{(A4)} \end{cases}$$

for $j = 1, 2$.

Proof. First, it is observed that

$$Y_j = D_j(P_1, P_2)$$

$$\Rightarrow \underbrace{\frac{dY_j}{Y_j}}_{\equiv \Delta Y_j} = \frac{P_j}{Y_j} \cdot \frac{\partial D_j}{\partial P_j} \cdot \underbrace{\frac{dP_j}{P_j}}_{\equiv \Delta P_j} + \frac{P_k}{Y_j} \cdot \frac{\partial D_j}{\partial P_k} \cdot \underbrace{\frac{dP_k}{P_j}}_{\equiv \Delta P_k}$$

for $j, k = 1, 2$ and $k \neq j$. By using the symmetry, this equation is written as

$$-\Delta Y_j = [(-\epsilon_{own}^D) - \epsilon_{cross}^D]\Delta P_j$$

for $j = 1, 2$, which is the second equation of the system described above. The third and fourth equations are easily obtained.

Now, it is shown that

$$P_j - C_j(w, r, 1) = \theta \cdot \frac{P_j}{(-\epsilon_{own}^D) - \epsilon_{cross}^D}$$

$$\Rightarrow dP_j - \underbrace{\frac{\partial C_j(w,r,1)}{\partial w}}_{=\frac{L_j}{Y_j}} dw - \underbrace{\frac{\partial C_j(w,r,1)}{\partial r}}_{=\frac{K_j}{Y_j}} dr$$

$$= \theta \cdot \left[\frac{1}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} dP_j + P_j \cdot \frac{1}{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]^2} (d\epsilon^D_{own} + d\epsilon^D_{cross}) \right],$$

which implies that

$$\underbrace{\frac{dP_j}{P_j}}_{\equiv \Delta P_j} - \underbrace{\frac{wL_j}{P_j Y_j}}_{\equiv s_{L_j}} \underbrace{\frac{dw}{w}}_{\equiv \Delta w} - \underbrace{\frac{rK_j}{P_j Y_j}}_{\equiv s_{K_j}} \underbrace{\frac{dr}{r}}_{\equiv \Delta r} = \frac{\theta}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \left[\Delta P_j + \frac{d\epsilon^D_{own} + d\epsilon^D_{cross}}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \right],$$

and hence

$$\begin{aligned} \left[1 - \frac{\theta}{(-\epsilon^D_{own}) - \epsilon^D_{cross}}\right] \Delta P_j &= s_{L_j} \Delta w + s_{K_j} \Delta r \\ &+ \frac{\theta}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \cdot \left[\frac{\epsilon^D_{own}}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \Delta \epsilon^D_{own} + \frac{\epsilon^D_{cross}}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \Delta \epsilon^D_{cross} \right]. \end{aligned}$$

Now, it is observed that

$$\begin{cases} d\epsilon^D_{own} = \frac{1}{Y_j} \cdot \left(\frac{\partial D_j}{\partial P_j} + P_j \frac{\partial^2 D_j}{\partial P_j^2} \right) dP_j + \left(\frac{P_j}{Y_j} \cdot \frac{\partial^2 D_j}{\partial P_k \partial P_j} \right) dP_k - \epsilon^D_{own} \cdot \underbrace{\frac{dY_j}{Y_j}}_{\equiv \Delta Y_j} \\ d\epsilon^D_{cross} = \left(\frac{P_k}{Y_j} \cdot \frac{\partial^2 D_j}{\partial P_j \partial P_k} \right) dP_j + \frac{1}{Y_j} \cdot \left(\frac{\partial D_j}{\partial P_k} + P_k \frac{\partial^2 D_j}{\partial P_k^2} \right) dP_k - \epsilon^D_{cross} \cdot \underbrace{\frac{dY_j}{Y_j}}_{\equiv \Delta Y_j} \end{cases}$$

because $\epsilon^D_{own} = \frac{P_j}{Y_j} \frac{\partial D_j(P_j,P_k)}{\partial P_j}$ and $\epsilon^D_{cross} = \frac{P_k}{Y_j} \frac{\partial D_j(P_j,P_k)}{\partial P_k}$.

Then, it is verified that

$$\underbrace{\frac{d\epsilon^D_{own}}{\epsilon^D_{own}}}_{\equiv \Delta \epsilon^D_{own}} = \frac{\frac{P_j}{Y_j} \cdot \left(\frac{\partial D_j}{\partial P_j} + P_j \frac{\partial^2 D_j}{\partial P_j^2} \right) \underbrace{\frac{dP_j}{P_j}}_{=\Delta P_j} + \frac{P_j}{Y_j} \cdot \left(P_k \frac{\partial^2 D_j}{\partial P_j \partial P_k} \right) \underbrace{\frac{dP_k}{P_k}}_{=\Delta P_k}}{\epsilon^D_{own}} - \Delta Y_j$$

$$\Rightarrow \Delta \epsilon^D_{own} = \left(1 - \alpha^D_{oo}\right) \Delta P_j - \alpha^D_{so} \Delta P_k - \Delta Y_j$$

and similarly,

$$\Delta \epsilon^D_{cross} = -\alpha^D_{sc} \Delta P_j + (1 - \alpha^D_{tc}) \Delta P_k - \Delta Y_j.$$

If the symmetry is further imposed, then

$$\begin{cases} \Delta\epsilon^D_{own} = (1 - \epsilon^D_{own} - \epsilon^D_{cross} - A^D_{own})\Delta P_j \\ \Delta\epsilon^D_{cross} = (1 - \epsilon^D_{own} - \epsilon^D_{cross} - A^D_{cross})\Delta P_j \end{cases}$$

and hence

$$\begin{aligned} \left[1 - \frac{\theta}{(-\epsilon^D_{own}) - \epsilon^D_{cross}}\right]\Delta P_j &= s_{L_j}\Delta w + s_{K_j}\Delta r \\ &+ \frac{\theta}{(-\epsilon^D_{own}) - \epsilon^D_{cross}} \cdot \left[\frac{\epsilon^D_{own}(1 - \epsilon^D_{own} - \epsilon^D_{cross} - A^D_{own})}{(-\epsilon^D_{own}) - \epsilon^D_{cross}}\right. \\ &\qquad \left. + \frac{\epsilon^D_{cross}(1 - \epsilon^D_{own} - \epsilon^D_{cross} - A^D_{cross})}{(-\epsilon^D_{own}) - \epsilon^D_{cross}}\right]\Delta P_j, \end{aligned}$$

which yields the first equation of the perturbed system. □

Obviously, Equation (A1) generalizes Equation (5) in the text as the former coincides with the latter if $\epsilon^D_{cross} = 0$ and hence $A^D_{cross} = 0$.

A2.2 The Long-Run Analysis

In the long-run, the rental price of capital is stabilized (i.e., $\Delta r = 0$). Hence, Equation (A3) of our perturbed system becomes:

$$\Delta L_j = \Delta K_j - \sigma\Delta w$$

for $j = 1, 2$. By substituting this result into Equation (A4) of our perturbed system, we can rewrite Equation (A4) as follows:

$$\Delta Y_j = s_{L_j}(\Delta K_j - \sigma\Delta w) + s_{K_j}\Delta K_j. \qquad \text{(A5)}$$

From Equation (A2), it is also true that:

$$-\Delta Y_j = [(-\epsilon^D_{own}) - \epsilon^D_{cross}]\Delta P_j. \qquad \text{(A6)}$$

Then, we combine Systems (A5) and (A6) to eliminate ΔY in the following manner:

$$\begin{aligned} \Delta P_j &= -\frac{s_{L_j}(\Delta K_j - \sigma\Delta w) + s_{K_j}\Delta K_j}{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]} \\ &= \frac{s_{L_j}\sigma\Delta w - \Delta K_j}{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]} \end{aligned}$$

as $s_{L_j} + s_{K_j} = 1$ for each $j = 1, 2$. Together with Equation (A1), this yields:

$$\frac{s_{L_j}\sigma\Delta w - \Delta K_j}{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]} = \frac{1}{1 + \theta \cdot \left[1 - \frac{\epsilon^D_{own}A^D_{own} + \epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own} + \epsilon^D_{cross})^2}\right]} s_{L_j}\Delta w$$

$$\Leftrightarrow \Delta K^{LR}_j = \left\{\sigma - \frac{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]}{1 + \theta \cdot \left[1 - \frac{\epsilon^D_{own}A^D_{own} + \epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own} + \epsilon^D_{cross})^2}\right]}\right\} s_{L_j}\Delta w$$

where LR stands for the long-run.

Hence, it is verified that $\Delta w > 0$ implies $\Delta K > 0$ if and only if

$$\sigma > \frac{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]}{1 + \theta \cdot \left[1 - \frac{\epsilon^D_{own}A^D_{own} + \epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own} + \epsilon^D_{cross})^2}\right]}.$$

Note that the threshold under perfect competition ($\theta = 0$) is greater than that under imperfect competition ($\theta > 0$):

$$(-\epsilon^D_{own}) - \epsilon^D_{cross} > \frac{[(-\epsilon^D_{own}) - \epsilon^D_{cross}]}{1 + \theta \cdot \left[1 - \frac{\epsilon^D_{own}A^D_{own} + \epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own} + \epsilon^D_{cross})^2}\right]}.$$

A2.3 The Short-Run Analysis

In the short-run, the amount of capital is fixed (i.e., $\Delta K_j = 0$ for each $j = 1, 2$). Then, Equations (A3) and (A4) are simplified to

$$\Delta Y_j = s_{L_j}\Delta L_j$$

and

$$\Delta L_j = \sigma \cdot (\Delta r - \Delta w),$$

respectively, which yields:

$$\Delta Y_j = s_{L_j}\sigma \cdot (\Delta r - \Delta w)$$

for $j = 1, 2$.

By substituting this expression into Equation (A2), we obtain:

$$-s_{L_j}\sigma \cdot (\Delta r - \Delta w) = [(-\epsilon^D_{own}) - \epsilon^D_{cross}]\Delta P_j,$$

which, together with Equation (A1), leads to:

$$-s_{L_j}\sigma\cdot(\Delta r-\Delta w)=\frac{[(-\epsilon^D_{own})-\epsilon^D_{cross}]}{1+\theta\cdot\left[1-\frac{\epsilon^D_{own}A^D_{own}+\epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own}+\epsilon^D_{cross})^2}\right]}s_{L_j}\Delta w+s_{K_j}\Delta r$$

$$\Leftrightarrow\frac{\Delta r}{\Delta w}=\frac{\left\{\sigma-\frac{[(-\epsilon^D_{own})-\epsilon^D_{cross}]}{1+\theta\cdot\left[1-\frac{\epsilon^D_{own}A^D_{own}+\epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own}+\epsilon^D_{cross})^2}\right]}\right\}s_{L_j}}{\sigma s_{L_j}+\frac{[(-\epsilon^D_{own})-\epsilon^D_{cross}]}{1+\theta\cdot\left[1-\frac{\epsilon^D_{own}A^D_{own}+\epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own}+\epsilon^D_{cross})^2}\right]}s_{K_j}}$$

for $j=1,2$.

Therefore, $\Delta w>0$ implies $\Delta r>0$ if and only if

$$\sigma>\frac{[(-\epsilon^D_{own})-\epsilon^D_{cross}]}{1+\theta\cdot\left[1-\frac{\epsilon^D_{own}A^D_{own}+\epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own}+\epsilon^D_{cross})^2}\right]}.$$

Note that the threshold under perfect competition ($\theta=0$) is greater than that under imperfect competition ($\theta>0$):

$$[(-\epsilon^D_{own})-\epsilon^D_{cross}]>\frac{[(-\epsilon^D_{own})-\epsilon^D_{cross}]}{1+\theta\cdot\left[1-\frac{\epsilon^D_{own}A^D_{own}+\epsilon^D_{cross}A^D_{cross}}{-(\epsilon^D_{own}+\epsilon^D_{cross})^2}\right]}.$$

This is the *same* condition for $\frac{\Delta K_j}{\Delta w}>0$ in the long-run, although in general $\frac{\Delta K_j}{\Delta w}$ and $\frac{\Delta r}{\Delta w}$ take different values. As in the long-run case, *imperfect competition is more likely in the short run to raise the rental price of capital ($\Delta r>0$), ceteris paribus, in response to an increase in the wage ($\Delta w>0$).*

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