

**ECONOMIC RESEARCH CENTER  
DISCUSSION PAPER**

*E-Series*

No.E21-4

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Labor Market Fluctuations**

by

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**June 2021**

**ECONOMIC RESEARCH CENTER  
GRADUATE SCHOOL OF ECONOMICS  
NAGOYA UNIVERSITY**

# General Equilibrium Effects and Labor Market Fluctuations\*

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March 20, 2021

## Abstract

Business cycle models with search-matching frictions are studied to evaluate the importance of general equilibrium effects generated by movements in the stochastic discount factor and the income effect on labor supply. Without variable hours of work, the general equilibrium effect works only through the stochastic discount factor and is quantitatively very weak. With variable hours of work, the income effect operates to generate procyclical movements in the value of leisure and the marginal hourly wage rate. This effect is sizable and dampens labor market fluctuations. We also study discount factor shocks and find that capital formation strongly enhances labor market fluctuations.

JEL classification: E32, J20, J64.

Keywords: labor market search, stochastic discount factor, unemployment volatility.

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\*Part of this research is financially supported by KAKENHI (17K03619, 20H01476).

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# 1 Introduction

Explaining unemployment had been one of the greatest challenges in economics, until the search-matching model of the labor market, often referred to as the DMP model, was developed by Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000). The subsequent research such as Shimer (2005) explores labor market fluctuations over the business cycle using the DMP model.<sup>1</sup> Turning to the main-stream business cycle literature, Merz (1995) and Andolfatto (1996) integrate search-matching frictions of DMP-type into an otherwise standard dynamic stochastic general equilibrium (DSGE) framework and show that search-matching frictions can help improve the DSGE model.<sup>2</sup>

While the textbook DMP model such as Shimer (2005) and the DSGE model with search-matching frictions such as Merz (1995) and Andolfatto (1996) are seemingly identical, they employ different aggregation strategies. To avoid the potential difficulty in modeling wealth distribution across individuals and over time, the textbook DMP model assumes risk neutral individuals to rule out wealth accumulation altogether, while the DSGE model with search-matching frictions assumes a representative “big family” to suppress the issue of wealth distribution while keeping the intertemporal consumption choice.<sup>3</sup> As a result, the discount factor (i.e., the real interest rate) is endogenous and time-varying in the DSGE model, whereas it is exogenous and constant over time in the textbook DMP model.<sup>4</sup>

By directly comparing these aggregation strategies, this paper evaluates the importance of *general equilibrium effects* over the business cycle generated by intertemporal consumption choice. In the textbook DMP model, each agent discounts the future using a

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<sup>1</sup>Cole and Rogerson (1999) also bring the DMP model to the business cycle analysis, with special attention given to job creation and job destruction over the business cycle.

<sup>2</sup>Related contributions include den Haan et al. (2000), Trigari (2006, 2009), Krause and Lubik (2007, 2013), Gertler and Trigari (2009), and Christiano et al. (2016), to name a few. Ichiue et al. (2013) use a Bayesian DSGE framework to show that the labor market search model is superior to the sticky wage model often employed in the DSGE literature.

<sup>3</sup>In both models, consumption insurance is perfect because markets are complete. Krusell et al. (2010) and Nakajima (2012) introduce incomplete markets into the DSGE model with search-matching frictions. Krusell et al. (2010) ignore hours per worker to focus on the extensive margin of labor adjustment. Nakajima (2012) considers the intensive margin using the Greenwood-Hercowitz-Huffman (GHH) utility function to rule out the income effect. While the incomplete-markets approach is beyond the scope of this paper, it provides an important direction for further study.

<sup>4</sup>For convenience, in what follows the DMP model is referred to as the one with linear utility in consumption, and the DSGE model is referred to as the one with the big family assumption.

constant discount factor and (because of linear utility) the aggregate level of consumption is determined as a residual in the resource constraint. On the other hand, in the DSGE model, each individual agent discounts the future using the stochastic discount factor, which interacts with the aggregate level of consumption. This interaction is one of the general equilibrium effects we consider.

With variable hours of work, there is an additional general equilibrium effect—the *income effect* on labor supply. Because of concave utility, an increase in the level of consumption decreases the relative price of consumption within a big family, increasing the relative marginal disutility from work (Trigari, 2006, 2009). This effect is absent in models without the big family assumption.

How important is the income effect? This question is particularly relevant in countries in which the *intensive margin* of labor adjustment is sizable. While the conventional search-matching model focuses on the extensive margin, the importance of the intensive margin has increased in many major countries (Ohanian and Raffo, 2012). For instance, the intensive margin accounts for about 79 percent of variations in total labor input in Japan (Kudoh et al., 2019). Dossche et al. (2019) document the importance of the intensive margin in Europe.<sup>5</sup> In this paper, we target the Japanese economy for our quantitative analysis.

Throughout, we evaluate the importance of the general equilibrium effects in understanding the aggregate labor market dynamics. To address this question, we compare *stripped-down versions* of the DMP model, which is without the income effect, and the DSGE model, which captures the income effect among other effects. This analytical strategy is important because the modern DSGE models are often “medium scaled,” including nominal price stickiness and monetary policy. To identify the income effect over the business cycle, we consider minimum model elements that are common across the DMP model and the DSGE model, rather than introducing an unexplored new component.

We consider four models: the DMP model without variable hours of work, which is close to the textbook DMP model (Shimer, 2005; Hagedorn and Manovskii, 2008; Amaral Tasci, 2016); the DMP model with variable hours of work (Cooper et al., 2007; Trapeznikova, 2017; Kudoh et al., 2019); the DSGE model without variable hours of work (den Haan et al., 2000; Krause and Lubik, 2007; Costain and Reiter, 2008; Gertler and Trigari, 2009; Christiano et al., 2016; Atolia et al., 2018); and the DSGE model with variable hours of

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<sup>5</sup>A partial list of the recent literature on the intensive margin of labor adjustment is Wesselbaum (2016), Trapeznikova (2017), Kudoh et al. (2019), Dossche et al. (2019), Cacciatore et al. (2020), and Kolasa et al. (2021).

work (Trigari, 2006, 2009; Fang and Rogerson, 2009; Wesselbaum, 2016; Dossche et al., 2019; Cacciatore et al., 2020; Kolasa et al., 2021).<sup>6</sup> We compare these four models under two types of shocks: total factor productivity (TFP) shocks and discount factor shocks. The latter is motivated by Mukoyama (2009) and Hall (2017).

When labor market fluctuations are driven by TFP shocks, without variable hours of work per employee, the two models are identical in the steady state, and their business cycle properties are nearly identical. The intuition is as follows. An increase in TFP increases the marginal product of capital in both models. In the DSGE model, part of the increase in the demand for capital is offset by an increase in the rental price of capital because the loan market becomes tight. However, we find that this effect is quantitatively weak. Thus, we conclude that the general equilibrium effect through the stochastic discount factor is not very important in accounting for labor market fluctuations.

However, with variable hours of work, the two models generate significantly different outcomes, both in and out of steady-state. For instance, the unemployment volatility in response to TFP shocks in the DSGE model is about 2/3 of that of the DMP model. Although the general equilibrium effect through the stochastic discount factor is weak, the income effect on labor supply, which works through a decline in the relative price of consumption, is quantitatively strong. Interestingly, this result is not explained by the surplus argument (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017). Indeed, under the same calibration strategy and targets, the steady-state value of the opportunity cost of employment is exactly the *same* for the two models.

Our results are complementary to those of Chodorow-Reich and Karabarbounis (2016), who measure the opportunity cost of employment (denoted by  $z$ ) over the business cycle for the U.S. and find that it is procyclical and volatile, and argue that the cyclicity of  $z$  dampens unemployment fluctuations because “[r]elative to the constant  $z$  case, a procyclical  $z$  increases the surplus from accepting a job at a given wage during a recession, which puts downward pressure on equilibrium wages and ameliorates the increase in unemployment” (p.1567). However, we show that the procyclicality of  $z$  as well as its volatility are both *stronger* in the DMP model than those in the DSGE model, while the DMP model generates a *higher* (not lower) unemployment volatility than the DSGE model does. Further, Kudoh et al. (2019) find that the DMP model with a *constant*  $z$  and

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<sup>6</sup>Kudoh et al. (2019) show that the DMP model with variable hours of work naturally implements Hagedorn Manovskii’s (2008) small surplus calibration through a *low* Frisch elasticity. This is in sharp contrast with the standard real business cycle literature, in which a high Frisch elasticity is necessary for labor market fluctuations of a realistic magnitude.

the one with a procyclical  $z$  (due to variable hours) generate the *same* unemployment volatility. On the surface, these results are at odd with Chodorow-Reich and Karabarbounis (2016).

Our key finding is that the procyclicality of  $z$  alone does not dampen unemployment volatility. The income effect is essential for this channel. The procyclicality of consumption implies that the relative weight on disutility of work is also procyclical. As a result, the marginal hourly wage rate increases in response to a positive productivity shock, which dampens the increase in hours of work during expansions. This effect plays a crucial role, and is missing in the DMP model, which is without the intertemporal consumption choice. While Chodorow-Reich and Karabarbounis (2016) consider a rich DSGE model including the income effect that we study, they treat the procyclicality of  $z$  as the key driving force. Our contribution is to highlight the role of the income effect in generating their result by comparing the models with and without general equilibrium effects.

Mukoyama (2009) and Hall (2017) ask whether variations in the discount factor can account for labor market fluctuations.<sup>7</sup> This questions is important on its own because financial turbulence is often associated with a massive job loss (Reinhart and Rogoff, 2009; Hall, 2017).<sup>8</sup> Motivated by these contributions, we study discount factor shocks as an alternative driver of business cycle. With discount factor shocks, the general equilibrium effects significantly dampen labor market fluctuations, with or without variable hours of work. Without variable hours of work, the volatility of unemployment in the DSGE model is about 1/6 of that of the DMP model.

The remainder of the paper is organized as follows. Section 2 presents the four models we consider, followed by characterization of the equilibrium conditions in Section 3. In Section 4, we calibrate the model parameters, and the quantitative results are presented in Section 5. Section 6 concludes.

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<sup>7</sup>Using a variant of the textbook DMP model, Mukoyama (2009) shows that discount factor shocks cannot account for labor market fluctuations in the U.S., and Hall (2017) shows that with wage stickiness, fluctuations in the discount factor can be a major source of U.S. labor market fluctuations. We find that the absence of capital explains the difficulty in accounting for discount-factor-driven labor market fluctuations.

<sup>8</sup>Note that the *long-run* relationship between the discount factor and unemployment has been extensively investigated in the literature. A change in the growth rate of technological progress influences job creation through its impact on the *effective* discount factor, known as the “capitalization effect” (Pissarides, 2000). It is well-known that the capitalization effect is quantitatively weak (Pissarides and Vallanti, 2007; Miyamoto and Takahashi, 2011).

## 2 Models

We consider two closely related models for understanding the aggregate labor market, the DMP model and the DSGE model with search-matching frictions. To facilitate comparisons, we consider a common model environment as much as possible. As in the standard DSGE model, we consider total factor productivity (TFP) as the driver of business cycles. This requires the standard neoclassical production technology with capital. To introduce capital, we assume that the representative final-good firm accumulates capital in the DMP model and the representative family accumulates capital in the DSGE model.<sup>9</sup> While large-firms models with intra-firm bargaining are useful for understanding the *composition* of labor demand over the business cycle (Kudoh et al., 2019), intra-firm bargaining is not an essential component of our analysis as we focus on the general equilibrium effects from intertemporal consumption choice. Thus, we assume that the production unit (i.e., the bargaining party) is each worker-vacancy pair.<sup>10</sup>

### 2.1 DMP Model

The standard DMP model and its variants emphasize the disaggregated aspect of the market economy and assume no representative household. To rule out the issue of wealth distribution over time, DMP models assume risk neutral individuals. Thus, in our first model, which we refer to as the DMP model, each individual has the following objective function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} D_t [I_t - e(h_t)], \quad (1)$$

where  $D_t$  is the cumulative discount factor,  $I_t$  is income, and  $e(h)$  represents the level of disutility from working for  $h$  hours. The (common) cumulative discount factor evolves according to  $D_{t+1} = \beta_t D_t$  with  $D_0 = 1$ , where  $\beta_t = 1/(1 + \rho_t)$ . Evidently,  $D_t = \beta^t$  when  $\beta_t = \beta$  for all  $t$ .

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<sup>9</sup>In the DMP model, risk-neutral individuals do not have incentives to hold capital. In the DSGE model, on the other hand, the representative family needs capital (i.e., means of savings) to smooth consumption over time.

<sup>10</sup>Kudoh and Sasaki (2011) and Kudoh et al. (2019) consider multi-worker firms to study hours of work in the context of intra-firm bargaining. Kudoh and Sasaki (2011) show that because of the well-known overhiring effect, the equilibrium level of hours of work is below the optimal level and this inefficiency cannot be corrected by the Hosios condition. For a quantitative investigation on this issue, see Dossche et al. (2019).

Throughout, we specify the disutility function as

$$e(h) = e_0 \frac{h^{1+\mu}}{1+\mu}, \quad (2)$$

where  $e_0 \geq 0$  and  $1/\mu$  is the Frisch elasticity parameter. We shall set  $e_0 = 0$  to study a version of the model without variable hours of work per employee.

We introduce a representative final consumption good firm with the production technology

$$y_t = A_t X_t^\alpha k_t^{1-\alpha}, \quad (3)$$

where  $y_t$  is output (i.e., consumption good),  $A_t$  is total factor productivity (TFP), and  $X_t$  is the amount of intermediate inputs supplied by intermediate goods firms.

We assume that the final output firm accumulates capital, as in Cahuc et al. (2008) and Kudoh et al. (2019). This assumption is necessary because the risk-neutral individuals do not make saving. Another important role of the firm is to aggregate intermediate inputs. Each intermediate input is produced by an intermediate firm, which employs a single worker and bargains over the terms of trade. This market structure conveniently rules out the issue of intra-firm bargaining.<sup>11</sup>

The final output firm behaves competitively. The price of the final consumption good is normalized to unity. Taking the (sequence of) input price  $p_t$  as given, the firm solves the following problem,

$$\max_{\{X_t, i_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} D_t \left[ A_t X_t^\alpha k_t^{1-\alpha} - p_t X_t - i_t \right],$$

subject to  $k_{t+1} = (1 - \delta) k_t + i_t$ , where  $\delta$  is the capital depreciation rate and  $i_t$  is the level of capital formation in period  $t$ . Let  $F(k)$  be the value of the firm. Thus, the problem can be stated recursively as

$$F(k) = \max_{X, k'} \left[ A X^\alpha k^{1-\alpha} - p X - k' + (1 - \delta) k + \mathbb{E} \beta F(k') \right],$$

from which the first-order conditions and the envelope condition are  $\alpha A X^{\alpha-1} k^{1-\alpha} = p$ ,  $1 = \mathbb{E} \beta F'(k')$ , and  $F'(k) = (1 - \alpha) A X^\alpha k^{-\alpha} + 1 - \delta$ . Thus, we obtain  $\alpha A_t X_t^{\alpha-1} k_t^{1-\alpha} = p_t$  and  $\mathbb{E}_t \beta_t [(1 - \alpha) A_{t+1} X_{t+1}^\alpha k_{t+1}^{-\alpha} + 1 - \delta] = 1$ . In any equilibrium,  $X_t = h_t l_t$  holds because  $l_t$  is the number of firms (which equals the number of employees) and  $h_t$  is the production

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<sup>11</sup>See Cahuc et al. (2008), Kudoh and Sasaki (2011), Krause and Lubik (2013), and Kudoh et al. (2019) for models with intra-firm bargaining.



intensity in each production unit. In a model without hours of work per employee, we replace  $X_t = h_t l_t$  with  $X_t = l_t$ .

The number of matches in period  $t$  is determined by  $m_0 U_t^\xi V_t^{1-\xi}$ , where  $m_0 > 0$  and  $0 < \xi < 1$  are parameters,  $U_t$  is the total number of job seekers, and  $V_t$  is the number of aggregate job vacancies. The vacancy filling rate is given by

$$q_t = m_0 U_t^\xi V_t^{1-\xi} / V_t = m_0 \theta_t^{-\xi} \equiv q(\theta_t), \quad (4)$$

where  $V_t / U_t \equiv \theta_t$ . Similarly, the job finding rate is given by  $m_0 U_t^\xi V_t^{1-\xi} / U_t = m_0 \theta_t^{1-\xi} \equiv \theta_t q(\theta_t)$ . The labor force is normalized to unity, so the unemployment rate  $u_t$  satisfies  $u_t = U_t$ .

The values of employment and unemployment, denoted respectively by  $J_t^E$  and  $J_t^U$ , are standard:

$$J_t^E = W(h_t) - e(h_t) + \lambda \mathbb{E}_t \beta_t J_{t+1}^U + (1 - \lambda) \mathbb{E}_t \beta_t J_{t+1}^E, \quad (5)$$

$$J_t^U = b + \theta_t q(\theta_t) \mathbb{E}_t \beta_t J_{t+1}^E + [1 - \theta_t q(\theta_t)] \mathbb{E}_t \beta_t J_{t+1}^U, \quad (6)$$

where  $W(h_t)$  is the amount of earnings,  $\lambda$  is the separation rate, and  $b$  is the unemployment benefit. In a model without variable hours per employee, we replace  $W(h_t) - e(h_t)$  with the hourly wage rate  $w_t$ .

The values of a filled job and a vacancy are given respectively as  $J_t^F = p_t h_t - W(h_t) + \lambda \mathbb{E}_t \beta_t J_{t+1}^V + (1 - \lambda) \mathbb{E}_t \beta_t J_{t+1}^F$  and  $J_t^V = -c + q(\theta_t) \mathbb{E}_t \beta_t J_{t+1}^F + [1 - q(\theta_t)] \mathbb{E}_t \beta_t J_{t+1}^V$ , where  $c$  is the cost of posting a vacancy. Note that the firm's revenue is  $p_t h_t$ . Thus, each intermediate firm sells its employee's labor input to the final output firm at the competitive price  $p_t$ . In a model without variable hours per employee, we replace  $p_t h_t - W(h_t)$  with  $p_t - w_t$ .

The equilibrium number of vacancies is determined by the free entry condition  $J_t^V = 0$ , from which we obtain  $c = q(\theta_t) \mathbb{E}_t \beta_t J_{t+1}^F$ . Thus, we obtain the job-creation condition:

$$\frac{c}{q(\theta_t)} = \mathbb{E}_t \beta_t \left[ p_{t+1} h_{t+1} - W(h_{t+1}) + \frac{(1 - \lambda)c}{q(\theta_{t+1})} \right], \quad (7)$$

where  $W(h_{t+1})$  is replaced with  $w_{t+1}$  in a model without variable hours of work.

As in the textbook DMP model, we assume Nash bargaining to determine both earnings and hours of work. We follow Cooper et al. (2007) and Kudoh et al. (2019) to assume that a firm and a worker bargain over a state-contingent contract that specifies an earnings schedule, which maps hours of work into an amount of compensation.

A caveat is that with variable hours of work, we need to distinguish between earnings (denoted by  $W_t$ ) and the wage rate (denoted by  $w_t$ ). As is made clear by Kudoh and

Sasaki (2011) and Kudoh et al. (2019), the (average) hourly wage rate,  $w_t = W_t/h_t$ , has *no allocative role* because under any bilateral bargaining protocol, what matters is the division of total surplus, which is earnings  $W_t$ , not the (average) wage rate that can only be calculated ex-post by dividing  $W_t$  by  $h_t$ .<sup>12</sup> This is in sharp contrast with the perfectly competitive labor market, in which all agents take the wage rate as a given numerical value when make choices. To allow for a general earnings structure, we specify  $W_t = W(h_t)$  rather than  $W_t = w_t h_t$ .

Consider the following bargaining problem:

$$\max_{W_t, h_t} \left( J_t^F - J_t^V \right)^{1-\eta} \left( J_t^E - J_t^U \right)^\eta,$$

where  $\eta$  is the exogenous bargaining power for the worker. The first order conditions imply  $(1 - \eta)(J_t^E - J_t^U) = \eta(J_t^F - J_t^V)$  and  $p_t = W'(h_t)$ , from which we obtain  $W(h_t) = \eta p_t h_t + (1 - \eta)[e(h_t) + b] + \eta c \theta_t$ . Hours per employee are determined in bargaining so that  $p_t = \eta p_t + (1 - \eta)e'(h_t)$ , from which we obtain  $\alpha A_t (h_t l_t)^{\alpha-1} k_t^{1-\alpha} = e_0 h_t^\mu = e'(h_t)$ .

It is important to note that in our model, who chooses hours of work is irrelevant for equilibrium conditions because the earnings schedule  $W(h)$  guarantees the efficient bargaining outcome for each level of  $h$ .<sup>13</sup> With intra-firm bargaining and concave production, however, it matters who makes the choice of hours (Kudoh and Sasaki, 2011; Kudoh et al., 2019; Dossche et al., 2019). The vertical industry structure employed in this paper and in many recent DSGE models is a useful device that bridges the aggregate production technology and each production unit.

## 2.2 DSGE Model

DSGE models with search-matching frictions typically allow for a richer preference structure such as concavity and non-separability. However, to rule out the issue of wealth distribution, all agents are assumed to be members of a single large family (Merz 1995; Andolfatto, 1996). All incomes of all individuals are pooled in each period and intertemporal consumption choice is made by the big family so that all individuals consume the same units of the consumption good (i.e., perfect risk sharing among individuals).

<sup>12</sup>Kudoh and Sasaki (2011) and Kudoh et al. (2019) therefore emphasize the *marginal* hourly wage rate,  $\partial W_t / \partial h_t$ .

<sup>13</sup>This is in sharp contrast with the right-to-manage literature, in which the firm chooses hours of work given the predetermined hourly wage rate as a numerical value by implicitly assuming a linear relationship between hours and earnings (Trigari, 2006; Sunakawa, 2012). Inefficiency arises because the true relationship between hours and earnings is nonlinear.

Thus, in our second model, the objective function for the large family is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} D_t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - e(h_t) l_t \right], \quad (8)$$

where  $C_t$  is the level of consumption and  $1/\sigma$  is the intertemporal elasticity of substitution (and  $\sigma$  is the coefficient of relative risk aversion). This family structure works as a risk-sharing device among the individuals. With concave utility in consumption, the family chooses to make saving. Thus, unlike the previous model, capital is held by the family and lent to the final output firm.

Since all incomes are pooled within the family, the budget constraint for the family is

$$C_t + k_{t+1} - k_t = W_t l_t + (1 - l_t) b + (r_t - \delta) k_t + T_t + \Pi_t, \quad (9)$$

where  $k_t$  is the stock of capital (so that  $k_{t+1} - k_t$  is capital formation in period  $t$ ),  $r_t$  is the rental price of capital (so that  $r_t - \delta$  is that rate of return on capital),  $W_t$  denotes earnings per employed,  $l_t$  is the proportion of employed individuals,  $b$  is the unemployment benefit from the government,  $T_t$  is transfer,  $\Pi_t$  is dividends (profits). In equilibrium, transfer  $T_t$  is determined so that the government's budget is balanced. Thus,  $T_t + (1 - l_t) b = 0$ .

Given that employment, earnings, the rate of returns on capital, and dividends are determined as part of search equilibrium, the big family chooses the paths of consumption and capital to maximize (8) subject to (9). Let  $\Lambda_t$  be the Lagrange multiplier associated with (9). Then, the first-order conditions are

$$\Lambda_t = C_t^{-\sigma}, \quad (10)$$

$$\Lambda_t = \mathbb{E}_t \beta_t \Lambda_{t+1} [1 - \delta + r_{t+1}]. \quad (11)$$

The transversality condition is  $\lim_{t \rightarrow \infty} \Lambda_t D_t k_t = 0$ .

To evaluate all decentralized variables in terms of the marginal utility of the final consumption good, we shall introduce the stochastic discount factor, defined by

$$S_t = \beta_t \frac{\Lambda_{t+1}}{\Lambda_t}, \quad (12)$$

from which (11) reduces to

$$1 = \mathbb{E}_t S_t [1 - \delta + r_{t+1}]. \quad (13)$$

In any steady state, (13) implies  $\beta = 1/(1 + \tilde{r})$ , where  $\tilde{r}$  is the market real interest rate net of capital depreciation. In contrast to the DMP model, the real interest rate in the DSGE model is endogenous.

The final consumption good is produced by the (perfectly competitive) final output firm with production technology (3). The price of the final consumption good is normalized to one. Since the big family owns the entire stock of capital, the final output firm uses capital at the rental price  $r_t$ . Thus, the final output firm's problem essentially static, and is given by

$$\max_{X_t, k_t} \left[ A_t X_t^\alpha k_t^{1-\alpha} - p_t X_t - r_t k_t \right],$$

from which we obtain the first-order conditions  $\alpha A_t X_t^{\alpha-1} k_t^{1-\alpha} = p_t$  and  $(1-\alpha) A_t X_t^\alpha k_t^{-\alpha} = r_t$ . In equilibrium,  $X_t = h_t l_t$  in the model with variable hours of work and  $X_t = l_t$  in the model without variable hours per employee.

The value functions for workers are given as

$$J_t^E = W(h_t) - \Lambda_t^{-1} e(h_t) + \lambda \mathbb{E}_t S_t J_{t+1}^U + (1-\lambda) \mathbb{E}_t S_t J_{t+1}^E, \quad (14)$$

$$J_t^U = b + \theta_t q(\theta_t) \mathbb{E}_t S_t J_{t+1}^E + [1 - \theta_t q(\theta_t)] \mathbb{E}_t S_t J_{t+1}^U, \quad (15)$$

which are nearly identical to (5) and (6), respectively. The big family's concave utility introduces two important differences. One is that workers discount the future at rate  $S_t$ , which is the stochastic discount factor. To see how it introduces the general equilibrium effect, suppose for now that utility from consumption is  $\ln C$  so that  $S_t = \beta_t C_t / C_{t+1}$ . This implies that an increase in the consumption growth rate increases the discount factor because future consumption becomes less costly. This mechanism is absent in the model with linear utility.

The other difference the big family assumption introduces is the evaluation of disutility from longer hours,  $\Lambda_t^{-1} e(h_t)$ . The adjustment through  $\Lambda_t$  is necessary because the value of employment  $J_t^E$  is measured in terms of the utility units of the final consumption good (Trigari, 2006, 2009; Sunakawa, 2015; Wesselbaum, 2016; Dossche et al., 2019). What this implies is that as the marginal utility from consumption goes up (when the level of consumption declines), the relative value of disutility decreases, inducing longer hours of work. This mechanism is absent in the textbook DMP model because  $\Lambda_t^{-1} = 1$  under the linear utility.<sup>14</sup> The term  $\Lambda_t^{-1}$  captures the strength of the wealth effect on the supply of hours.

The value of a filled job and the value of a vacancy are  $J_t^F = p_t h_t - W(h_t) + \lambda \mathbb{E}_t S_t J_{t+1}^V + (1-\lambda) \mathbb{E}_t S_t J_{t+1}^F$  and  $J_t^V = -c + q(\theta_t) \mathbb{E}_t S_t J_{t+1}^F + [1 - q(\theta_t)] \mathbb{E}_t S_t J_{t+1}^V$ , respectively. Free en-

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<sup>14</sup>Cooley and Quadrini (1999) develop a monetary general equilibrium model with variable hours in which the household's utility is linear in consumption. We view that the Cooley-Quadrini model belongs to the DMP paradigm because the discount factor is constant over time.

try of jobs implies  $J_t^V = 0$ , from which we obtain the job-creation condition:

$$\frac{c}{q(\theta_t)} = \mathbb{E}_t S_t \left[ p_{t+1} h_{t+1} - W(h_{t+1}) + \frac{(1-\lambda)c}{q(\theta_{t+1})} \right]. \quad (16)$$

The bargaining outcome is given by  $W(h_t) = \eta p_t h_t + (1-\eta)[\Lambda_t^{-1} e(h_t) + b] + \eta c \theta_t$ . The optimal level of hours satisfies  $\alpha A_t (h_t l_t)^{\alpha-1} k_t^{1-\alpha} = \Lambda_t^{-1} e_0 h_t^\mu = \Lambda_t^{-1} e'(h_t) = p_t$ .

## 3 Equilibria

### 3.1 Models without Variable Hours of Work

Consider first the models without variable hours of work. The equilibrium conditions for the DSGE model are summarized by

$$1 = \mathbb{E}_t S_t [1 - \delta + (1-\alpha) A_{t+1} l_{t+1}^\alpha k_{t+1}^{-\alpha}], \quad (17)$$

$$\frac{c}{q(\theta_t)} = \mathbb{E}_t S_t \left[ \alpha A_{t+1} l_{t+1}^{\alpha-1} k_{t+1}^{1-\alpha} - w_{t+1} + \frac{(1-\lambda)c}{q(\theta_{t+1})} \right], \quad (18)$$

$$w_t = \eta \alpha A_t l_t^{\alpha-1} k_t^{1-\alpha} + (1-\eta)b + \eta c \theta_t, \quad (19)$$

$$u_{t+1} = u_t - \theta_t q(\theta_t) u_t + \lambda(1-u_t), \quad (20)$$

$$y_t - c \theta_t u_t = C_t + k_{t+1} - (1-\delta)k_t, \quad (21)$$

and

$$S_t = \beta_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}. \quad (22)$$

For the DMP model, the stochastic discount factor  $S_t$  in (17) and (18) is replaced with the time-varying discount factor  $\beta_t$ .

In the DSGE model, the level of consumption  $C_t$  is *jointly* determined by the resource constraint (21) and stochastic discount factor (22). In contrast, in the DMP model, the equilibrium is determined by (17)-(20), and  $C_t$  is determined by the recourse constraint (21) as a residual.

Because  $S = \beta$  in any steady state, the steady-state conditions for the two models are exactly the same:

$$(1-\alpha)AK^{-\alpha} = \rho + \delta, \quad (23)$$

$$\frac{(\rho + \lambda)c}{q(\theta)} + \eta c \theta = (1-\eta) \left[ \alpha AK^{1-\alpha} - b \right], \quad (24)$$

with  $u = \lambda/[\lambda + \theta q(\theta)]$ ,  $l = 1 - u$ ,  $K = k/l$ , and  $y - c\theta u = C + \delta k$ . It is easy to establish the uniqueness of the steady-state equilibrium from these expressions. Thus, the two models are identical in any steady state.

### 3.2 Models with Variable Hours of Work

Consider the models with variable hours of work. The equilibrium conditions for the DSGE model are summarized by

$$1 = \mathbb{E}_t S_t [1 - \delta + (1 - \alpha) A_{t+1} (h_{t+1} l_{t+1})^\alpha k_{t+1}^{-\alpha}], \quad (25)$$

$$\alpha A_t (h_t l_t)^{\alpha-1} k_t^{1-\alpha} = \Lambda_t^{-1} e_0 h_t^\mu, \quad (26)$$

$$\frac{c}{q(\theta_t)} = \mathbb{E}_t S_t \left[ \alpha A_{t+1} (h_{t+1} l_{t+1})^{\alpha-1} k_{t+1}^{1-\alpha} h_{t+1} - W_{t+1} + \frac{(1 - \lambda) c}{q(\theta_{t+1})} \right], \quad (27)$$

$$W_t = \eta \alpha A_t (h_t l_t)^{\alpha-1} k_t^{1-\alpha} h_t + (1 - \eta) \left[ \Lambda_t^{-1} e_0 \frac{h_t^{1+\mu}}{1 + \mu} + b \right] + \eta c \theta_t, \quad (28)$$

with (20), (21), and (22). For the DMP model, the stochastic discount factor  $S_t$  is replaced with the time-varying discount factor  $\beta_t$ , and  $\Lambda_t$  is replaced with 1.

The steady state of the DMP model satisfies

$$(1 - \alpha) AK^{-\alpha} = \rho + \delta, \quad (29)$$

$$\alpha AK^{1-\alpha} = e_0 h^\mu = e'(h), \quad (30)$$

$$\frac{(\rho + \lambda) c}{q(\theta)} = (1 - \eta) [e'(h) h - e(h) - b] - \eta c \theta, \quad (31)$$

with  $u = \lambda/[\lambda + \theta q(\theta)]$ ,  $l = 1 - u$ , and  $K = k/hl$ . The uniqueness of the steady state is easily verified. First, (29) determines the capital-labor ratio  $K$ . Given  $K$ , (30) determines hours of work per employee,  $h$ . Finally, given  $h$ , (31) determines tightness  $\theta$ .

The steady state of the DSGE model is a pair  $(h, \theta)$  satisfying

$$\alpha AK^{1-\alpha} = [C(h, \theta)]^\sigma e_0 h^\mu, \quad (32)$$

$$\frac{(\rho + \lambda) c}{q(\theta)} + \eta c \theta = (1 - \eta) \left[ \frac{\mu}{1 + \mu} \alpha AK^{1-\alpha} h - b \right], \quad (33)$$

where, from resource constraint,

$$C = y - c\theta u - \delta k = \left( AK^{1-\alpha} - \delta K \right) \frac{\theta q(\theta)}{\lambda + \theta q(\theta)} h - \frac{c\theta \lambda}{\lambda + \theta q(\theta)} \equiv C(h, \theta).$$

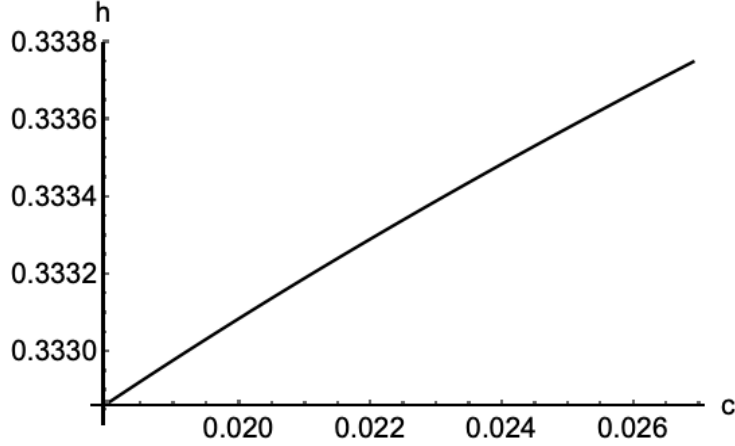


Figure 1: Vacancy Cost and Equilibrium Hours of Work

Interestingly, through the term  $(\Lambda^{-1})$ , we now have a rich interaction between  $h$  and  $\theta$ . As shown in Figure 1, an increase in  $c$  increases  $h$ .<sup>15</sup> The same result has been shown in a related environment by Fang and Rogerson (2009). As is evident from (30), in the DMP model, an increase in  $c$  has no effect on  $h$ , even though (31) implies a positive relationship between  $h$  and  $\theta$ .

## 4 Calibration

We calibrate our models to match Japanese labor market facts. We use Japan's economy as our target because the intensive margin adjustment over the business cycle is sizable in Japan (Kudoh et al., 2019). Our calibration procedure closely follows Kudoh et al. (2019) and therefore we keep the description here to be brief.<sup>16</sup>

We choose the model period to be a quarter and set the steady-state subjective discount rate at  $\rho = 0.01$ , or  $\beta = 1/(1 + \rho) = 0.99$ . In the production function, we set  $\alpha = 2/3$ . The depreciation rate is set at  $\delta = 0.0094 \times 3$ . The matching elasticity with respect to the number of job seekers is set at  $\xi = 0.6$ . Following the convention, we set  $\eta = \xi = 0.6$ .

<sup>15</sup>The parameter values are those calibrated in Section 4. We plot the equilibrium levels of hours for  $c$  from 20% below to 20% above its calibrated value.

<sup>16</sup>Kudoh et al. (2019) also provide some empirical facts on the Japanese labor market. Miyamoto (2011) and Lin and Miyamoto (2012) study worker flow data to estimate various transition probabilities necessary for calibration.

We target the labor market tightness of 0.78 and the quarterly job finding rate of  $0.142 \times 3$ . These targets with  $\zeta = 0.6$  pin down the scale parameter  $m_0 = 0.471$ . We also set the exogenous separation rate to be  $\lambda = 0.0048 \times 3$ . As in Kudoh et al. (2019), we set  $\mu = 1.8$  or the Frisch elasticity  $1/\mu$  to be 0.56. We target the unemployment benefit  $b$  to satisfy  $b = 0.6W$ , which is consistent with Japan’s replacement ratio reported in Nickell (1997). We target the steady-state working hours per employee to be  $1/3$ . With  $\theta = 0.78$ ,  $h = 1/3$ , and  $b = 0.6W$ , we determine the remaining parameters  $b$ ,  $c$ , and  $e_0$  by solving the steady-state equilibrium conditions.

We set the relative risk aversion parameter to be  $\sigma = 2$ . Robustness analysis for this parameter value will be presented.

Table 1: Fixed Parameters

Parameter	Description	Value
$\rho$	Steady-state discount rate	0.01
$\beta$	Steady-state discount factor $1/(1 + \rho)$	0.99
$\sigma$	Relative risk aversion	2.0
$\alpha$	Parameter in production function	2/3
$\zeta$	Matching elasticity	0.6
$\lambda$	Exogenous separation rate	0.014
$\delta$	Depreciation rate	0.028
$\mu$	Inverse of Frisch elasticity	1.8
$\eta$	Worker’s bargaining power	0.6
$A$	Steady-state productivity level	1.0

Table 1 summarizes the fixed parameters and Table 2 presents the calibrated parameter values. Without variable hours of work, the two models share the same parameter values because the steady-state equations for the two models are identical. With variable hours, for the same set of calibration targets, the two models require distinct sets of calibrated parameters. In the DMP model,  $e_0 = 14.23$  while in the DSGE model  $e_0 = 27.64$ . The calibrated value for the vacancy cost differs significantly for the models with and without variable hours. Without hours,  $c$  is calibrated to be 0.628 and this amounts to  $c/Y$  is 22.0% of output. On the other hand, with hours of work,  $c = 0.022$  and  $c/Y$  is 2.4% of output. The latter figure is more consistent with the evidence.



Table 2: Calibrated Parameters

Parameter	Description	Models w/o $h$	DMP with $h$	DSGE with $h$
$m_0$	Matching efficiency	0.471	0.471	0.471
$b$	Unemployment benefits	1.168	0.393	0.393
$c$	Vacancy cost	0.628	0.022	0.022
$e_0$	Disutility	-	14.23	27.64

## 5 Results

### 5.1 TFP Shocks

TFP follows a first order autoregressive process:  $\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \varepsilon_t$ , where  $\varepsilon_t \sim N(0, \sigma_A^2)$ . We set  $\rho_A = 0.612$  and  $\sigma_A = 0.009$  to match the first-order autocorrelation and standard deviation of TFP in the data.

Table 3 reports the standard deviations of the unemployment rate, the vacancy rate, employment, hours of work per employee, the (average) hourly wage rate, and the aggregate consumption, scaled by the standard deviation of TFP, which is 0.011. The standard deviation of the stochastic discount factor  $S_t$  is 0.0003 for Model 2 and 0.0004 for Model 4, and it is 0.002 in the data.

Table 3: TFP Shocks

	Relative standard deviations					
	$\hat{U}$	$\hat{V}$	$\hat{l}$	$\hat{h}$	$\hat{w}$	$\hat{C}$
<b>Data</b>	<b>5.45</b>	<b>8.92</b>	<b>0.35</b>	<b>0.70</b>	<b>0.91</b>	<b>0.89</b>
Model 1: DMP w/o hours	0.46	1.37	0.02	-	0.97	8.88
Model 2: DSGE w/o hours	0.50	1.13	0.02	-	0.88	0.42
Model 3: DMP with hours	4.33	12.81	0.15	0.63	0.91	1.57
Model 4: DSGE with hours	2.77	7.29	0.09	0.39	0.84	0.41
DSGE with hours ( $\sigma = 10.0$ )	2.30	6.53	0.08	0.49	0.93	0.14

Table 3 gives us important insights. First, without the intensive margin, the DMP model and the DSGE model with search-matching frictions generate nearly identical quantitative results. Thus, we conclude that the general equilibrium effect through the stochastic discount factor does not matter in accounting for labor market fluctuations. Further, in both models, fluctuations in unemployment, vacancies and employment are

significantly smaller than those in the data (Shimer, 2005). This is primarily explained by the small opportunity cost of employment  $b$  in the models without hours of work per employee (Hagedorn and Manovskii, 2008; Chodorow-Reich and Karabarbounis, 2016).

Second, with the intensive margin, the DMP model generates realistic magnitudes of fluctuations in these labor market variables (Kudoh et al., 2019). Interestingly, while the DSGE model with the intensive margin outperforms the one without, the magnitudes of fluctuations are significantly below those of the DMP model with hours.

It is important to observe that the surplus argument (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017) cannot explain the differences in the magnitudes of labor market fluctuations between the two models because the two models imply the same level of surplus under the same calibration strategy and targets. To see this, we compute the opportunity cost of employment. In the DMP model, it is given by

$$z^{DMP} = b + e(h) = b + e_0 \frac{h^{1+\mu}}{1+\mu} = 0.39 + 14.23 \times \frac{(1/3)^{1+1.8}}{1+1.8} = 0.628.$$

In the DSGE model, the opportunity cost of employment is

$$z^{DSGE} = b + \Lambda^{-1}e(h) = b + C^\sigma e_0 \frac{h^{1+\mu}}{1+\mu} = 0.39 + 0.72^2 \times 27.64 \times \frac{(1/3)^{1+1.8}}{1+1.8} = 0.628.$$

The key to understanding the result is the propagation of shocks. Consider a positive TFP shock. An increase in TFP increases the marginal product of capital. In the DMP model, the demand for capital increases significantly because the rental price of capital is assumed to be constant. In the DSGE model, part of the increase in the demand for capital is offset by an increase in the rental price of capital because the loan market becomes tight. However, comparison of Model 1 and Model 2 suggests that this channel is quantitatively very weak.

The other general equilibrium effect, which works through a decline in the relative price of consumption  $\Lambda_t$ , induces individuals to work *less* in response to an increase in TFP by increasing the relative weight on disutility of work. The earnings functions for the two models  $W(h_t) = \eta p_t h_t + (1 - \eta)[e(h_t) + b] + \eta c \theta_t$  and  $W(h_t) = \eta p_t h_t + (1 - \eta)[\Lambda_t^{-1}e(h_t) + b] + \eta c \theta_t$  imply that the *marginal hourly wage rate* is  $W'(h_t) = \eta p_t + (1 - \eta)e'(h_t)$  for the DMP model and  $W'(h_t) = \eta p_t + (1 - \eta)\Lambda_t^{-1}e'(h_t) = \eta p_t + (1 - \eta)C_t^\sigma e_0 h_t^\mu$  for the DSGE model. The procyclicality of  $C_t$  implies that the marginal hourly wage rate to be *higher* and hence hours and output to be *lower* during expansions in the DSGE model than those in the DMP model. Through this effect, the DSGE model with the intensive

margin has the income effect of labor supply that dampens labor market fluctuations. Comparison of Model 3 and Model 4 suggests that this effect is sizable.

Chodorow-Reich and Karabarbounis (2016) measure the opportunity cost of employment, which is  $z$  in our model, over the business cycle and find that it is procyclical and volatile. They argue that the cyclical nature of  $z$  dampens unemployment fluctuations in the search and bargaining model of unemployment. However, Kudoh et al. (2019) find that the DMP model with a constant level of hours and the one with variable hours generate the *same* unemployment volatility. This implies that the procyclicality of  $z$  alone does not dampen unemployment volatility. Our results reveal that the income effect is essential for this channel.

Table 4: The Opportunity Cost of Employment

$z$	$std(\hat{z})$	$std(\hat{z})/std(\hat{A})$	$corr(\hat{z}, \hat{U})$	$corr(\hat{z}, \hat{y})$
$z_t^{DMP} = b + e(h_t)$	0.007169	0.662	-0.956	0.9985
$z_t^{DSGE} = b + \Lambda_t^{-1}e(h_t)$	0.005147	0.475	-0.847	0.9983

Table 4 presents some statistics that characterize the business cycle properties of the opportunity cost of employment  $z$  for the DMP model and the DSGE model. Consistent with Chodorow-Reich and Karabarbounis (2016),  $z$  is highly procyclical in both models. A caveat is that while the procyclicality of  $z$  as well as its volatility are both *stronger* in the DMP model than those in the DSGE model, the DMP model generates a *higher* unemployment volatility than the DSGE model does. This result supports our claim that the procyclicality of  $z$  alone cannot dampen unemployment volatility.

Finally, consider how our results depend on the risk aversion parameter  $\sigma$ . When  $\sigma = 0$ , the DSGE model becomes identical to the DMP model. With a risk averse household, consumption smoothing motive becomes strong and hence consumption fluctuations are smaller under a higher risk aversion. This implies that under a higher risk aversion, the stochastic discount factor fluctuates less, inducing less fluctuations in job creation. However, as Table 3 suggests, the change in the level of  $\sigma$  does not generate a quantitatively large impact on the results.

## 5.2 Discount Factor Shocks

To focus on variations in the discount factor (i.e., the real interest rate) as the driver of business cycles, we shut down TFP fluctuations altogether and set  $A_t = 1$  for all  $t$ . We

construct a quarterly series of the real interest rate to obtain a series of discount factor. We assume that the discount factor follows  $\log S_t - \log S = \rho_S(\log S_{t-1} - \log S) + \varepsilon_{S,t}$ , where  $\varepsilon_{S,t} \sim N(0, \sigma_S^2)$ . We set  $\rho_S = 0.913$  and  $\sigma_S = 0.002$  based on the basic loan rate obtained from the Bank of Japan.<sup>17</sup>

Table 5: Discount Factor Shocks

	Relative standard deviations					
	$\hat{U}$	$\hat{V}$	$\hat{l}$	$\hat{h}$	$\hat{w}$	$\hat{C}$
<b>Data</b>	<b>9.99</b>	<b>16.38</b>	<b>0.64</b>	<b>1.29</b>	<b>1.67</b>	<b>1.63</b>
Model 1: DMP w/o hours	12.13	23.71	0.41	-	12.87	193.76
No direct impact on capital	1.20	2.36	0.04	-	0.50	0.03
Model 2: DSGE w/o hours	2.06	3.31	0.07	-	1.97	3.65
Model 3: DMP with hours	103.09	201.51	3.48	7.34	12.28	11.46
Model 4: DSGE with hours	16.28	28.55	0.55	2.86	2.76	3.39
DSGE with hours ( $\sigma = 10.0$ )	11.44	21.60	0.39	3.22	2.40	0.76

Table 5 reports the standard deviations of the unemployment rate, the vacancy rate, employment, hours of work per employee, the (average) hourly wage rate, and the aggregate consumption, scaled by the standard deviation of the discount factor, which is 0.002.

Consider the models without variable hours of work. Contrary to the findings in Mukoyama (2009) and Hall (2017), our DMP model generates sufficiently large fluctuations in unemployment and vacancies. This is because our model includes capital formation. To see the importance of capital, consider the steady-state equilibrium conditions (23) and (24). According to (24), a reduction in the discount rate directly stimulates job creation through the capitalization effect. In our model, there is an additional effect at work: (23) implies that a reduction in the discount rate increases capital, raising the marginal product of labor and stimulates job creation. Once we drop the latter effect from our model, we confirm Mukoyama (2009) and Hall (2017) that discount factor shocks cannot account for labor market fluctuations. Thus, a quick remedy for the puzzle posed by Mukoyama (2009) and Hall (2017) is to introduce capital.

Even though introduction of capital enhances the model's responsiveness to varia-

<sup>17</sup>Providing the empirical analysis on the time series properties of the real interest rate in Japan is beyond the scope of this paper as we focus on the implications of the presence/absence of the general equilibrium effects.

tions in the discount factor, the general equilibrium effect through the endogenous real interest rate significantly reduces the magnitudes of labor market fluctuations. Because the loanable funds are constrained by the household's intertemporal consumption choice, any increase in the demand for capital is associated with an increase in the real interest rate, dampening the job-creation incentive. The same reasoning applies to the models with variable hours of work. In the DMP model, all variables fluctuate too much to explain the data. However, with the general equilibrium effects, the magnitudes of fluctuations are significantly reduced and we obtain somewhat reasonable results. As in the results for productivity-driven business cycles, the risk-aversion parameter plays a minor role in determining the magnitude of labor market fluctuations.

## 6 Conclusion

The general equilibrium effect through the stochastic discount factor (or the real interest rate) that potentially dampens TFP-driven labor market fluctuations is quantitatively very weak. With variable hours of work, however, there is an additional general equilibrium effect, namely the income effect on labor supply, and this effect is quantitatively strong. The cyclical nature of the opportunity cost of employment is procyclical and volatile, consistent with Chodorow-Reich and Karabarbounis (2016). However, it is not the cyclical nature of the opportunity cost of employment itself that dampens unemployment volatility. The key is the response of the marginal hourly wage rate driven by the income effect. Our analysis highlights the importance of the marginal hourly wage rate in the macro-labor literature. An important direction of future research is to build an appropriate notion of wage rigidity in models with the intensive margin to further explore the importance of the cyclical nature of the opportunity cost of employment for unemployment volatility.<sup>18</sup>

The general equilibrium effects matter very much in accounting for labor market dynamics driven by variations in the discount factor. We showed that, with or without variable hours of work, the general equilibrium effects significantly dampen labor market fluctuations. Even without variable hours, the volatility of unemployment in the DSGE model is about 1/6 of that of the DMP model. We also found that capital for-

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<sup>18</sup>With variable hours of work, there is a clear distinction between rigid wage rate and rigid earnings. Kudoh et al. (2019) introduce the notion of *contract rigidity* to allow the earnings schedule to imperfectly reflect the current level of TFP.

mation enhances the (otherwise weak) short-run capitalization effect. Our analysis suggests the importance of capital in magnifying labor market dynamics. This observation is somewhat new because capital itself is known to play very little in accounting for output fluctuations (Cooley and Prescott, 1995) and influential DMP models of labor market fluctuations such as Shimer (2005) do not consider capital. An important avenue for future research is to explore the role of capital over the business cycle.<sup>19</sup> For instance, the interaction between capital utilization and labor utilization over the business cycle is an open question.

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<sup>19</sup>Earlier contributions in this direction are Hornstein et al. (2007) and Miyamoto (2011).

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