ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

E-Series

No.E21-8

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by

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September 2021

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

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Abstract

Some recent studies assert that, in the US and some European countries, one-half of the growth in income inequality can be attributed to education. Therefore, focusing on performance-related pay for non-routine skilled workers and an opportunity for higher education, we present a mechanism to generate the polarization of wages. If the opportunity for higher education is limited, then wage distribution is polarized by the prevalence of performance pay. However, wage polarization may vanish in the case where higher education provides high-school graduates with a second chance at success. Our result highlights the important role of higher education in shaping the distribution of wages.

JEL Classification: I24, I26, J24, J42, L13

Keywords: Wage inequality, Higher education, Performance pay, Skill-biased technical change

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1 Introduction

Polarization is a phenomenon in the labor market in the United States and some European countries since the 1990s (Autor, Katz, and Kearney 2006; Goldin and Katz 2007; Lemieux 2008; Goos, Manning, and Salomons 2009). The term 'polarization' has two meanings. Autor, Katz, and Kearney (2006) explained that wage growth has been polarized since 1988, with higher wage growth rate occurring in the bottom quartile than in the middle two quartiles. However, the highest growth rate and sustained spreading out of the wage distributions occur in the top quartile. Employment growth in the 1990s was polarized with the highest employment growth rate occurring in high-skill jobs, the lowest growth rate occurring in middle-skill jobs, and the modest growth rate occurring in low-skill jobs.

In addition to the polarization, the top-end wage inequality in the United States that began at the end of the 1970s has also attracted considerable attention (Katz and Murphy 1992; Bound and Johnson 1992; Johnson 1997). Johnson (1997) points out two reasons for the rapid increase in the relative demand for skilled workers in the US economy during the 1980s. One is the skill-biased technological change (SBTC), and the other is increased openness. In the context of SBTC, some researchers attribute the top-end wage inequality to computerization or automation (Autor, Katz, and Krueger 1998; Autor, Levy, Murnane 2003; Autor and Dorn 2013; Cortes, Jaimovich, and Siu 2017; Cavenaile 2021). In the context of increased openness, some researchers attribute the top-end wage inequality to offshoring or capital mobility (Beladi, Marjit, and Broll 2011; Goel 2017; Cavenaile 2021).

Despite the emphasis on the SBTC and increased openness, the following two channels that could polarize the labor market have not been fully examined, that is, (1) performance-related pay such as bonuses and stock options (Piketty and Saez 2003; Lemieux, MacLeod, and Parent 2009; Alvaredo et al. 2013). Autor (2014) points out that, between 1980 and 2012, real hourly earnings of full-time college-educated US males rose anywhere from 20% to 56%, with the greatest gains going to those with a postbaccalaureate degree (p.849). The wage inequality within college-educated workers could be attributed to performance pay for non-routine skilled workers with professional degrees.

(2) Individuals' choice of education. Hoffmann, Lee, and Lemieux (2020) calculated the contribution rates of education and other factors to the growth in the variance of total income. They show that education accounted for 56% of the growth in income inequality over the late 1970s to the late 2010s in the United States. In the United Kingdom and Germany, the contribution rate of education is approximately 50%, although the sample periods are shorter than in the United States. They also show that occupation and place significantly cause the evolution of income inequality for women, but modest for men.

The purpose of the paper is to explain the mechanism of wage polarization by focusing on performance pay for non-routine skilled workers and postgraduate education. Our model is based on the following three assumptions.

First, individuals are heterogenous in a two-dimensional cost of higher education, that is, the cost of acquiring a college degree, and the cost of acquiring a postgraduate professional degree. This assumption enables us to examine the wage inequality within college-educated workers explicitly. Further, the cost approach has two merits from an empirical point of view. Because the education cost assumed corresponds proportionally to the rate of return on higher education, our model can be assessed by using available data on the rate of return on higher education. The other merit is related to the measurement problem of wages. Most literature assume that individuals are heterogeneous in a skill that would be effective only if they were employed in a specific task or occupation. Under this assumption, the wage rate in the specific occupation is measured in efficiency units, which implies that an additional assumption on the skill distribution is necessary to calculate hourly wage rates. The top 1% income distribution would follow the Pareto distribution, but the distribution of the other 99% is difficult to specify (Jones and Kim 2018). Our cost approach is free from the measurement problem, assuming that working time is the same for all workers.

Second, we assume that the three educational statuses correspond directly with three occupations, that is, high-school graduates are employed in the services sector as unskilled workers, and college graduates are employed in the final goods sector as routine skilled workers. Their wages are determined by the marginal product of labor in the corresponding competitive labor market. We assume that non-routine skilled workers who get a postgraduate professional degree finance foreign capital to produce intermediate goods. In a monopolistic competitive market, the earnings of non-routine skilled workers are tied to their firm's profits.

We admit that occupations in the real world are not segmented by education status. Ph.D. graduates may work in the service sector because they like to spend more time with their families or talk with customers. Our assumption is a natural extension of the literature that assumes that occupations are segmented by skill (Autor and Dorn 2013; Cortes, Jaimovich, and Siu 2017; Cavenaile 2021).

Finally, we assume that the performance pay for non-routine skilled workers increases with the degree of product differentiation of intermediate goods. Specifically, we examine the relationship between performance pay and the top-end and low-end wage inequality by parameterizing the elasticity of substitution between intermediated goods. It would be better if the pay system for managers was endogenized by incorporating some forms of incentive contract (Lemieux, MacLeod, and Parent 2009), or that the degree of product differentiation was endogenized by incorporating the firm's investment strategy for research and development (Jones and Kim 2018; Aghion et al. 2019; Akcigit, Pearce, and Prato 2020). These important micro-foundations of our model are left for future research.

Based on the above assumptions, we obtained two results. First, we derive a simple formula that suggests that the top-end wage inequality is negatively related to the low-end wage inequality. Suppose that the profit rate in the intermediate goods sector increases. Then, the rich become richer, which induces the poor to be richer. This is because the rich will increase demand for services that are a substitute for household production. This will, in turn, increase the wage rate of workers employed in the services sector (Manning 2004; Mazzolari and Ragusa 2013). Therefore, when the share of performance pay increases, the top-end wage inequality increases and the low-end wage inequality decreases. This is a possible story in our model, which could explain the labor polarization observed in the US, the UK, and Germany.

Second, if the increased performance pay changes individuals' education choices, both top-end and low-end wage inequalities may decrease. In our model, the ultimate aim of working high-school graduates who later acquire a college degree is to also acquire a postgraduate professional degree. Therefore, the polarization of wages may vanish because the increased supply of non-routine skilled workers depresses their earnings. This scenario could explain the relatively flat wage distribution observed in France and Japan. Our result suggests that differences in the accessibility to higher education reflect the differences in the shape of the wage distribution.

The reminder of this paper is organized as follows: In Section 2, we briefly analyze the related literature. In Section 3, we introduce the basic model. In Section 4, we conduct a comparative statics exercise concerning the elasticity of substitution between intermediate goods. In Section 5, we provide numerical examples. The final section concludes the paper.

2 Related literature

This research is closely related to Beladi, Marjit, and Broll (2011), Autor and Dorn (2013), Cortes, Jaimovich, and Siu (2017), and Cavenaile (2021).

Beladi, Marjit, and Broll (2011) assume a static economy in which three goods, that is, high-, middle-, and unskilled goods, are produced by four inputs, high-, middle-, and unskilled labor and physical capital in competitive markets. Production of domestic high-skilled and middle-skilled labor is subject to the endowment of educational capital. In this setting, the paper analyzes the response of sectoral employment to a transition from a closed to an open economy. The paper shows that either the high-skilled or middle-skilled sector vanishes when foreign capital flows into the country and that the middle-skilled sector may vanish when foreign high-skilled labor flows into the country.

Our model is the same as Beladi, Marjit, and Broll (2011) in that education resources are allocated to the production of high-skilled and middle-skilled labor. However, the theoretical method and the purpose are quite different. Our research focuses on the wage polarization in a closed economy, while Beladi, Marjit, and Broll (2011) focuses on the employment polarization in a transitional economy.

Autor and Dorn (2013) is one of the most influential papers, which proves that computerization (a fall of computer prices) generates labor market polarization. In a closed competitive economy, goods are produced by abstract labor, routine labor, and computer capital. Computer capital is assumed to be a relative complement to abstract labor and a relative substitute for routine labor. When technical progress decreases the price of computer capital, the relative demand for routine labor to abstract labor decreases, which results in an expansion of the top-end wage inequality, that is, the relative wage rate of routine labor to abstract labor falls. Services are produced by manual labor. The number of high-skill workers who supply abstract labor in the goods sector is assumed to be constant. Low-skill workers supply either routine labor in the goods sector or manual labor in the services sector, depending on their intrinsic skill that is effective only if they are employed in the goods sector. Under those assumptions, employment is polarized in the sense that low-skill workers move from the goods sector to the services sector when computer prices fall.

Our model economy and the mechanism of generating wage polarization have characteristics similar to Autor and Dorn (2013). However, our model is different from Autor and Dorn (2013) in the following two aspects. First, we focus on performance pay for non-routine skilled workers as a factor of labor market polarization, while Autor and Dorn (2013) focus on computerization in the context of skill-biased technological change. Second, the supply of non-routine skilled labor is endogenized in our model by incorporating education choice. The assumption of a constant supply of high-skill labor in Autor and Dorn (2013) may overestimate the top-end wage inequality.

Cortes, Jaimovich, and Siu (2017) assume an aggregate production function with five inputs (non-routine cognitive workers, non-routine manual workers, routine workers, automation capital, and capital other than automation) when examining the effect of introducing automation capital on occupational choice. The number of non-routine cognitive workers is assumed to be constant. The other workers differ in work ability and disutility of labor, which divides them into routine workers, non-routine manual workers, and unemployed workers. Cortes, Jaimovich, and Siu (2017) show that introduction of automation capital, a substitute for routine workers generates employment polarization. In other words, when automation capital is introduced, the number of routine workers decreases, while the number of non-routine manual workers and non-employment increases.

Our model economy is the same as Cortes, Jaimovich, and Siu (2017) in that non-routine cognitive workers earn the share of profits. However, we differ from Cortes, Jaimovich, and Siu (2017) in that the objective of our research is wage polarization while theirs is employment polarization. In addition, our model endogenizes the supply of non-routine cognitive workers, which allows routine workers to become non-routine cognitive workers.

Cavenaile (2021) assumes a small open economy in which domestic workers choose their occupation from four alternatives: two jobs (manager or worker) in two sectors (goods or services). In equilibrium, occupations are segmented by intrinsic human capital. The highest pay occupation is a manager in the goods sector, followed by a manager in the services sector. The third is a worker in the goods sector, and the lowest pay occupation is a worker in the services sector because it is assumed that human capital of workers is not effective in the services sector. In this model economy, Cavenaile (2021) examines the effects of offshoring and computerization on the distributions of employment and wages. When the wage rate of foreign skilled workers decreases, both employment and wages of domestic skilled workers decrease. A reduction in the price of computer capital delivers the same result. With some empirical evidence, the paper concludes that both offshoring and computerization are the main factors of labor market polarization.

Our model economy is the same as that of Cavenaile (2021) in that the managers in the goods and the services sectors earn a share of profits. Another parallel is we both consider a segmented labor market. In our model, three occupations are segmented by a two-dimensional cost of higher education. In Cavenaile (2014), four occupations are segmented by human capital.

A crucial difference between Cavenaile (2021) and our research is the mechanism of polarization. The mechanism proposed in Cavenaile (2021) is rather conservative in the sense that both the skill-biased technological change and increased openness can be traced back to Johnson (1997). Our research is motivated by Hoffmann, Lee, and Lemieux (2020), who indicate that there is a sizable contribution of education to income inequality in some developed countries.

3 The model

The model economy consists of three production sectors (services, final consumption goods, and intermediate goods) (See Table 1). Individuals are heterogeneous with respect to a two-dimensional cost of higher education. Labor markets are education-segmented in the sense that high-school graduates work as unskilled labor in the services sector, college graduates work as routine-skilled labor in the final goods sector, and individuals with postgraduate professional degrees work as non-routine skilled labor in the intermediate goods sector. We assume that markets of services and final goods are perfectly competitive and that the market of intermediate goods is monopolistic. Because our focus is on wage inequality, we assume that firms in the intermediate goods sector can access the international capital markets, which implies that capital income inequality is beyond the scope of the paper.

Table 1. Production sectors

Output	Inputs	
Services	Unskilled (L)	
Final goods	Routine skilled (M)	Composite goods
Intermediate goods	Non-routine skilled (H)	Capital

3.1 Individuals

We assume that the total number of individuals is constant, and is denoted by N. Individuals are classified into three groups: high-school graduates (denoted by L), college graduates (M), and college graduates with a postgraduate professional degree (H).

Individuals choose their education and the allocation of consumption. The allocation problem of individual $i = L$, M, and H is formulated as

$$\max_{c_i,\, d_i} u_i = (1-\alpha)\ln c_i + \alpha \ln d_i \quad \text{subject to} \quad y_i = c_i + p_d d_i \tag{1}$$

where c_i and d_i represent goods consumption and services consumption, respectively. y_i represents the disposable income of an individual i, which is specified below. Assuming that goods are numeraire, p_d represents the relative price of services. $0 < \alpha < 1$ is a preference parameter attached to services consumption.

Solving the problem, the demand functions are given by

$$c_i = (1-\alpha)y_i \tag{2}$$

$$d_i = \alpha \frac{y_i}{p_d} \tag{3}$$

Individuals are heterogenous with respect to a pair of education costs $(e_1, e_2) \in \mathbf{R}_+^2$. e_1 represents the cost incurred in acquiring a college degree, and e_2 represents the cost incurred in acquiring a postgraduate professional degree. The disposable income of high-school graduates (say, unskilled labor), college graduates (routine-skilled labor), and college graduates with a professional degree (non-routine skilled labor) are respectively given by

$$\begin{cases} y_L = w_L \\ y_M = w_M - e_1(w_M - w_L) \\ y_H = w_H - e_1(w_M - w_L) - e_2(w_H - w_M) \end{cases} \tag{4}$$

where w_L, w_M, and w_H represent the wage rates for unskilled labor, routine skilled labor, and non-routine skilled labor, respectively. We assume, in equilibrium,

$$w_H > w_M > w_L$$

Our setup is not so specific. Eq.(4) yields

$$\frac{y_M}{y_L} - 1 = (1-e_1)\left(\frac{w_M}{w_L} - 1\right)$$

$$\frac{y_H}{y_M} - 1 = (1-e_2)\left(\frac{w_H}{w_M} - 1\right)$$

Note that $(w_M/w_L - 1)$ represents the rate of return on college education, and $(w_H/w_M - 1)$ represents the rate of return on postgraduate education. The greater e_1 is, the smaller the net rate of return on college education. The greater e_2 is, the smaller the net rate of return on postgraduate education. Therefore, the pair of (e_1, e_2) represents the heterogeneity among individuals with respect to the rates of return on college and postgraduate education.

Formally, the optimal choice of education is specified as follows, an individual (e_1, e_2) becomes:

(i) Unskilled labor L, if

$$\begin{cases} y_L > y_M \\ y_L > y_H \end{cases} \Rightarrow \begin{cases} e_1 > 1 \\ e_2 > \phi(e_1; \mathbf{w}) \end{cases}$$

where

$$\phi(e_1; \mathbf{w}) = \frac{w_H - w_L}{w_H - w_M} - \frac{w_M - w_L}{w_H - w_M} e_1 \tag{5}$$

(ii) Routine skilled labor M, if

$$\begin{cases} y_M > y_L \\ y_M > y_H \end{cases} \Rightarrow \begin{cases} e_1 < 1 \\ e_2 > 1 \end{cases}$$

or (iii) non-routine skilled labor H if

$$\begin{cases} y_H > y_L \\ y_H > y_M \end{cases} \Rightarrow \begin{cases} e_2 < \phi(e_1; \mathbf{w}) \\ e_2 < 1 \end{cases}$$

[Figure 1 here]

Figure 1 illustrates the regions of L, M, and H on (e_1, e_2). The region above the horizontal line $e_2 = 1$ corresponds to a traditional two-class economy: Individuals $e_1 < 1$ go to college, and individuals $e_1 > 1$ do not. The region below line $e_2 = 1$ is the core of our model. Low-cost individuals ($e_1 < 1$) go to college to obtain a postgraduate degree. High-cost individuals on the right side of the border $e_2 = \phi(e_1; \mathbf{w})$ become high-school graduates. In the middle triangle region ($e_1 > 1$ and $e_2 < \phi(e_1; \mathbf{w})$), individuals go to college even though the net rate of return on college education is negative. The reason is that they have a chance to become non-routine skilled workers by getting a postgraduate degree. Notice that the triangle is widened when w_H and w_L increase, or when w_M decreases. In this sense, wage polarization encourages a second chance at success provided by higher education.

3.2 Firms

3.2.1 Services sector

For simplicity, we assume a linear technology of the services sector,

$$Y_d = f(L) = L \tag{6}$$

where L is unskilled labor, and Y_d is the output of the services.

Perfect competition makes the wage rate of unskilled labor equal to the price of services,

$$w_L = p_d \tag{7}$$

3.2.2 Final goods sector

Final consumption goods are produced by routine-skilled labor and composite goods which include intermediate goods. Following Dixit and Stiglitz (1977), the technology is specified by

$$Y = M^{1-\beta} Z^{\beta} \tag{8}$$

$$Z = \left[\int_0^H x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \tag{9}$$

where Y is the output of the final goods, and M and Z represent the inputs of routine-skilled labor and the composite goods, respectively. $x(j)$ represents the input of an intermediate good $j \in [0, H]$, where H is the variety of intermediate goods. $0 < \beta < 1$ is a constant production share of the composite goods, and $\sigma > 1$ is a constant elasticity of substitution between the different intermediate goods.

The optimization problem is broken up into two parts. First, firms minimize expenditure for intermediated goods, taking the level of composite goods as given,

$$e(Z) = \min_{x(j)} \int_0^H p(j)x(j)dj \quad \text{subject to} \quad Z = \left[\int_0^H x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

where $p(j)$ is the price of the intermediate goods j, and $e(Z)$ is the expenditure function.

Solving the problem, the demand for the intermediate good j is given by

$$x(j) = Z \left[\frac{P}{p(j)} \right]^{\sigma} \tag{10}$$

where P represents a price index

$$P = \left[\int_0^H p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \tag{11}$$

Eqs.(10) and (11) yield $e(Z) = PZ$, which implies that P represents the price of the composite goods.

Second, firms minimize total cost of production, taking the output as given,

$$\min_{M,\, Z} w_M M + PZ \quad \text{subject to} \quad Y = M^{1-\beta} Z^{\beta}$$

where w_M is the wage rate of routine-skilled labor.

The optimality condition is given by

$$\frac{w_M}{P} = \frac{1-\beta}{\beta}\frac{Z}{M} \tag{12}$$

which implies that the factor price ratio is equal to the marginal rate of transformation.

Solving the problem, the factor demand is given by

$$\begin{aligned} M &= \left(\frac{\beta}{1-\beta}\frac{w_M}{P}\right)^{-\beta} Y \\ Z &= \left(\frac{\beta}{1-\beta}\frac{w_M}{P}\right)^{1-\beta} Y \end{aligned}$$

Finally, the zero-profit condition yields the factor price frontier,

$$1 = \left(\frac{w_M}{1-\beta}\right)^{1-\beta}\left(\frac{P}{\beta}\right)^{\beta} \tag{13}$$

3.2.3 Intermediate goods sector

Intermediate goods are produced by managers (non-routine skilled labor) and capital. Focusing on wage income, we assume that capital is owned by foreigners or domestic owners outside of the model.[1] We assume a linear technology in sector $j \in [0, H]$ as

$$x(j) = k(j) \tag{14}$$

where $x(j)$ and $k(j)$ represent output and capital, respectively.

Denoting a constant world interest rate by $r > 0$, the production cost is $rk(j) = rx(j)$. The manager in sector j earns $w_H(j)$ as performance pay. Specifically, the optimization problem is formulated by

$$w_H(j) = \max_{p(j),\, x(j)} \ \pi(j) = [p(j) - r]x(j)$$

subject to Eq.(10).

Solving the problem, the monopoly price is given by the markup pricing

$$\frac{p(j) - r}{p(j)} = \frac{1}{\sigma} \tag{15}$$

for all $j \in [0, H]$.

From Eq.(15), the profit share, that is, the ratio of earnings of the manager to total revenue is σ^{-1}, which implies that the performance pay is negatively related to the elasticity of factor substitution. Therefore, we interpret a reduction of σ as meaning that performance pay for managers increases.

Omitting the index of the intermediate good j, price index P, output of intermediate goods x, and the wage rate of non-routine skilled labor w_H are given by

$$P = H^{\frac{1}{1-\sigma}} \times \frac{\sigma}{\sigma-1} r \tag{16}$$

$$x = H^{\frac{\sigma}{1-\sigma}} \times Z \tag{17}$$

$$w_H = H^{\frac{\sigma}{1-\sigma}} \times \frac{rZ}{\sigma-1} \tag{18}$$

Because $\sigma > 1$, the price index and wage rate of the non-routine skilled labor decrease with the number of non-routine skilled labor, H.

3.3 Market equilibrium

Let us denote the numbers of non-routine skilled labor, routine skilled labor, and unskilled labor by n_H, n_M, and n_L, respectively. The population constraint is given by

$$N = n_H + n_M + n_L$$

[1] We can extend the basic model to incorporate capital endowments among individuals. Because capital income is positively associated with labor income (Hoffmann, Lee, and Lemieux 2020), the extension would strengthen our result.

where N represents a constant total number of individuals.

Because labor markets are education-segmented, the market clearing conditions are given by

$$\begin{aligned} L &= n_L \\ M &= n_M \\ H &= n_H \end{aligned}$$

The services market clearing condition is given by

$$Y_d = \sum_{i \in L,M,H} d_i \tag{19}$$

Since the model is closed, the goods market clearing condition,

$$Y = \sum_{i \in L,M,H} c_i + (w_M - w_L) \sum_{i \in M,H} e_1 + (w_H - w_M) \sum_{i \in H} e_2 + r \int_0^H k(j)dj$$

can be derived by Walas' law. Notice that the distribution of education cost $(e_1, e_2) \in \mathbf{R}_+^2$ is given exogenously.

3.4 Equilibrium

In our model, individuals' choice of education depends on the distribution of wages, while the equilibrium wages depend on the outcome of individuals' education choice. Therefore, we define the equilibrium formally in the following way.

Denote the distribution of wages by $\mathbf{w} = (w_H, w_M, w_L) \in \mathbf{R}_+^3$ and the outcome of education choice by $\mathbf{n} = (n_H, n_M, n_L) \in \mathbf{R}_+^3$.

1. A mapping of the wage space $\mathbf{R}_+^3$ onto the population space $\mathbf{R}_+^3$, denoted by $\mathbf{n} = \boldsymbol{\varphi}(\mathbf{w})$, is given by

$$\begin{aligned} n_M &= \# \{(e_1, e_2) | \, e_1 < 1 \text{ and } e_2 > 1\} \\ n_H &= \# \{(e_1, e_2) | \, e_2 < \phi(e_1; \mathbf{w}) \text{ and } e_2 < 1\} \\ n_L &= N - n_H - n_M \end{aligned}$$

where the symbol # represents the number of points (e_1, e_2) in the corresponding region.

2. A mapping of the population space onto the wage space, denoted by $\mathbf{w} = \boldsymbol{\psi}(\mathbf{n})$, is given by

$$\begin{aligned} w_H &= \psi_H(\mathbf{n}) \\ w_M &= \psi_M(\mathbf{n}) \\ w_L &= \psi_L(\mathbf{n}) \end{aligned}$$

3. Equilibrium distribution of wages, denoted by $\mathbf{w}^*$ is defined as a fixed point in the composite mapping: $\mathbf{w}^* = \boldsymbol{\psi} \circ \boldsymbol{\varphi}(\mathbf{w}^*)$.

In the next section, we derive the mapping $\mathbf{w} = \boldsymbol{\psi}(\mathbf{n})$ explicitly from equilibrium conditions.

4 Wage inequality

In this section, we examine the characteristics of the top-end and low-end wage inequality, focusing on the performance pay of non-routine skilled workers. Because the performance pay is associated with the degree of product differentiation of intermediate goods, we measure the effect of performance pay on the wage inequality by parameterizing the elasticity of factor substitution, σ.

4.1 Top-end wage inequality

The following proposition summarizes the top-end wage inequality.

Proposition 1 *The top-end wage inequality is given by*

$$\frac{w_H}{w_M} = \frac{\beta}{\sigma(1-\beta)} \times \frac{n_M}{n_H} \tag{20}$$

Proof. Combining Eqs.(12), (16), (18), $M = n_M$, and $H = n_H$, we obtain Eq.(20). ■

Eq.(20) implies that the relative demand for (non-) routine skilled labor is negatively related to the relative wage rate of (non-) routine skilled labor. Eq.(20) corresponds to Johnson's (1992) relative demand curve.

From Proposition 1, we obtain the following corollary.

Corollary 2 *Assume that the elasticity of factor substitution σ falls and that this technical change keeps both n_H and n_M constant. Then, the top-end inequality increases.*

From Eqs.(13), (16), and (20), the wage rates of routine labor and non-routine labor are explicitly given by

$$w_M = (1-\beta)\left[\frac{\beta(\sigma-1)}{\sigma r}(n_H)^{\frac{1}{\sigma-1}}\right]^{\frac{\beta}{1-\beta}} \tag{21}$$

$$w_H = \frac{\beta}{\sigma}\frac{n_M}{n_H}\left[\frac{\beta(\sigma-1)}{\sigma r}(n_H)^{\frac{1}{\sigma-1}}\right]^{\frac{\beta}{1-\beta}} \tag{22}$$

which represent $\psi_M(\mathbf{n})$ and $\psi_H(\mathbf{n})$ in Section 3.4, respectively.

The wage rate of routine skilled labor w_M increases with n_H and σ. When either n_H or σ increases, the price index P falls (See Eq.(16)), which increases w_M along the factor price frontier in Eq.(13). w_M is neutral to the number of routine skilled labor, n_M.

As for the wage rate of non-routine skilled labor w_H, we obtain following lemma.

Lemma 3 *The wage rate of non-routine skilled labor w_H is (i) hump-shaped with respect to n_H and σ,*

$$\frac{\partial w_H}{\partial n_H} \gtreqless 0, \frac{\partial w_H}{\partial \sigma} \gtreqless 0 \quad \textit{if} \quad \sigma \lesseqgtr \frac{1}{1-\beta} \tag{23}$$

and (ii) increasing in the number of routine-skilled labor, n_M.

Proof. The power n_H in Eq.(22) is $\beta/[(1-\beta)(\sigma-1)]-1$, which is positive (negative) if and only if $\sigma < (>) 1/(1-\beta)$.

For σ, w_H increases (decreases) with σ if and only if $(\sigma-1)^\beta/\sigma$ increases (decreases). Log-differentiation yields the condition in Eq.(23). ■

In a special case, the number of non-routine skilled labor n_H is equal to the number of routine skilled labor n_M, the equilibrium condition $w_H > w_M$ requires $\sigma < \beta/(1-\beta)$. In this case, Lemma 3 shows $\partial w_H/\partial n_H > 0$ and $\partial w_H/\partial \sigma > 0$, which implies that w_H and w_M move in the same direction when either n_H or σ changes.

If the non-routine skilled labor is scarce enough to satisfy $n_H < \beta n_M$, then $\sigma > 1/(1-\beta)$ is possible. In this case, we obtain that $\partial w_H/\partial \sigma < 0$, which implies that a reduction of σ increases w_H and decreases w_M. The higher the degree of product differentiation is, the larger the wage inequality among college graduates.

4.2 Low-end wage inequality

In this section, we derive the wage rate of unskilled labor, $w_L = \psi_L(\mathbf{n})$, in Section 2.4 explicitly.

Substituting Eqs.(3), (7), and (8) into Eq.(19), the wage rate of unskilled labor is given by

$$w_L = \frac{\alpha}{n_L}\sum_{i\in L,M,H} y_i \tag{24}$$

where the aggregate disposable income by type $i = L$, M, and H are given by

$$\sum_{i\in L} y_i = n_L w_L$$

$$\sum_{i\in M} y_i = n_M w_M - (w_M - w_L)\sum_{i\in M} e_1$$

$$\sum_{i\in H} y_i = n_H w_H - (w_M - w_L)\sum_{i\in H} e_1 - (w_H - w_M)\sum_{i\in H} e_2$$

respectively. Substituting them into Eq.(24), and rearranging terms, we obtain the following proposition.

Proposition 4 *Assume that*

$$(1-\alpha)n_L - \alpha \sum_{i\in M,H} e_1 > 0$$

Then, the low-end wage inequality is given by

$$\frac{w_L}{w_M} = \frac{\alpha\left[n_M - \sum_{i\in M,H} e_1 + \sum_{i\in H} e_2 + \left(n_H - \sum_{i\in H} e_2\right)\left(\frac{w_H}{w_M}\right)\right]}{(1-\alpha)n_L - \alpha\sum_{i\in M,H} e_1} \tag{25}$$

Using Eqs.(25), (21), and (22), we obtain $w_L = \psi_L(\mathbf{n})$. Eq.(25) implies that w_L/w_M is positively related to the top-end wage inequality w_H/w_M because the coefficient of w_H/w_M in Eq.(25) is positive (note that $e_2 < 1$ for $\forall i \in H$). The reasons for this are as follows: assume that the wage rate of the non-routine skilled labor w_H increases by one dollar. For an individual of type H, his/her disposable income increases with the net rate of return on graduate education, $1 - e_2$. Then, he/she increases expenditure for services by $\alpha(1 - e_2)$. Given the number of the non-routine skilled labor is constant, the aggregate expenditure for services is increased by $\alpha\sum_{i\in H}(1 - e_2) = \alpha\left(n_H - \sum_{i\in H} e_2\right)$. This demand-side effect increases the price of services, which increases the wage rate of unskilled labor. When the rich becomes richer, the poor also becomes richer.

Combining Corollary 2 and Proposition 4, we obtain the following corollary.

Corollary 5 *Assume that the elasticity of factor substitution σ falls and that this technical change keeps the allocation of labor constant, then, we can observe that the top-end inequality increases, while the low-end inequality decreases.*

5 Numerical example

In this section, we present numerical examples. In Section 4.1, we examine the case where $\mathbf{n} = \boldsymbol{\varphi}(\mathbf{w})$ is a constant mapping, that is, individuals' education choice does not depend on the distribution of wages. As shown in Corollary 5, we can observe a negative relationship between the top-end and low-end wage inequality. In Section 4.2, we examine a more general case. When w_H/w_M and w_L/w_M increase, the triangle in Figure 1 is widened. This means that some individuals change their education status from high-school graduates to college graduates with a postgraduate degree. Therefore, the supply of non-routine skilled labor increases and the supply of unskilled labor decreases during the transition process. We show that both top-end and low-end wage inequalities decrease in transition. On the one hand, the reduction of unskilled labor shifts the supply curve to the left, which increases the wage rate of unskilled labor. On the other hand, an increased variety of intermediate goods depresses profits, which decreases the wage rate of non-routine skilled labor.

5.1 1-2-3 economy: An immobile case

[Figure 2 here]

Table 2. Immobile case

	n_i	e_1	e_2
H	1	1	$\frac{1}{2}$
M	2	$\frac{1}{2}$	(2)
L	3	$(\frac{3}{2})$	(1)

Figure 2 illustrates the distribution of population on the plane (e_1, e_2) (See also Table 2). There is one person whose education cost is $(e_1, e_2) = (1, 1/2)$. He/she optimally chooses to get a postgraduate degree. Two persons with education cost $(e_1, e_2) = (1/2, 2)$ become college graduates, and three persons with $(e_1, e_2) = (3/2, 1)$ are high-school graduates.

The aggregate costs of college education and postgraduate education are respectively given by

$$\begin{aligned}\sum_{i\in M,\,H} e_1 &= 1\times 1 + 2\times\frac{1}{2} = 2\\ \sum_{i\in H} e_2 &= 1\times\frac{1}{2} = \frac{1}{2}\end{aligned}$$

From Eqs.(20) and (25), the top-end and low-end wage inequalities are as follows:

$$\frac{w_H}{w_M} = \frac{2\beta}{\sigma(1-\beta)}$$
$$\frac{w_L}{w_M} = \frac{\alpha}{3-5\alpha}\left[\frac{1}{2}+\frac{\beta}{\sigma(1-\beta)}\right]$$

Table 3. Factor substitution and wage inequality

σ	$\frac{w_H}{w_M}$	$\frac{w_L}{w_M}$
1.1	7.27	0.414
1.2	6.67	0.383
1.3	6.15	0.358
1.4	5.71	0.336
1.5	5.33	0.317
2.0	4.00	0.250
2.5	3.20	0.210
3.0	2.67	0.183
3.5	2.29	0.164
4.0	2.00	0.150

Note. $\beta = 0.8$, $\alpha = 0.2$

Table 3 shows the relationship between the elasticity of factor substitution σ and both top-end and low-end wage inequalities. When $\sigma = 4$, the top-end wage inequality is $w_H/w_M = 2$ and the low-end wage inequality is $w_L/w_M = 0.15$. When σ drops to 2, the top-end wage inequality is doubled, and the low-end wage inequality decreases to $w_L/w_M = 0.25$. Notice that education choice is not affected by changes in the wage distribution in this example.

5.2 1-2-2-1 economy: A mobile case

[Figure 3 here]

Table 4. Mobile case

	n_i	e_1	e_2
H	1	1	$\frac{1}{2}$
M	2	$\frac{1}{2}$	(2)
L	2	$(\frac{3}{2})$	(1)
Q	1	$1+\varepsilon$	$\frac{1}{2}$

The previous section assumes that individuals do not respond to changes in wage distribution. This section allows for a case in which some individuals optimally change their education choice.

Figure 3 illustrates the distribution of the population (See also Table 4). The types $i = H$, M, and L are the same as in the previous section. We added a group Q, in which individuals do not want to be routine skilled workers because $e_1 = 1+\varepsilon > 1$, but may want to be non-routine skilled workers if ε is small and the slope of the border-line is flat.

Assume that θ persons in group Q become H and $(1-\theta)$ persons become L. Then, the aggregate education costs are given by

$$\sum_{i\in M,\, H} e_1 = 2+(1+\varepsilon)\theta$$
$$\sum_{i\in H} e_2 = \frac{1}{2}(1+\theta)$$

Substituting them into Eqs.(20) and (25), the top- and low-end wages inequality are respectively given by

$$\frac{w_H}{w_M} = \frac{\beta}{\sigma(1-\beta)} \times \frac{2}{1+\theta}$$
$$\frac{w_L}{w_M} = \frac{\alpha}{3-5\alpha-(1+\alpha\varepsilon)\,\theta}\left[\frac{1}{2}-\left(\frac{1}{2}+\varepsilon\right)\theta+\frac{1}{2}(1+\theta)\frac{w_H}{w_M}\right]$$

The disposable income of an individual in group Q, who becomes H, is given by

$$y'_H = w_H - (1+\varepsilon)(w_M - w_L) - \frac{1}{2}(w_H - w_M)$$

which yields

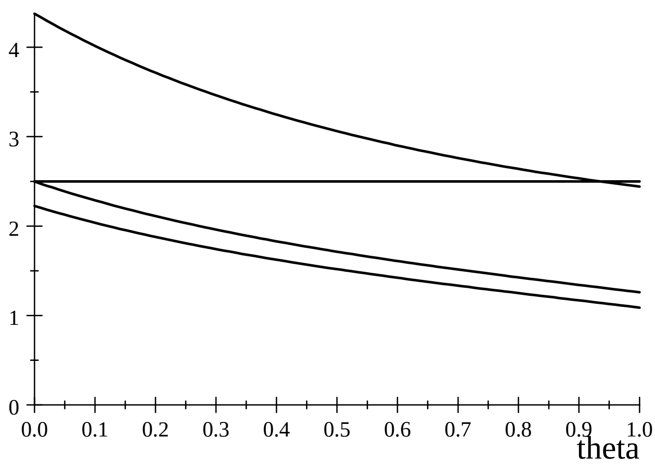
$$y'_H \gtreqless y_L \Leftrightarrow \frac{w_H}{w_M} + 2\varepsilon\frac{w_L}{w_M} \gtreqless 1 + 2\varepsilon \tag{26}$$

If an individual expects $y'_H > y_L$, then he/she becomes H. Otherwise, he/she becomes L. The equilibrium is given by $y'_H = y_L$, which determines θ, the share of H in group Q.

Figure 4 illustrates both sides of Eq.(26) in the plane with the θ axis. A downward sloped curve represents $w_H/w_M + 2\varepsilon w_L/w_M$, which shifts upward when σ decreases. The horizontal line is $1 + 2\varepsilon$. When $\sigma = 4$, the curve lies below the line, which implies that no one becomes H ($\theta = 0$). When $\sigma = 2$, the curve intersects with the line at $\theta = 0.934$. The equilibrium is stable because the excess demand for higher education is positive (negative) when $\theta < (>)0.934$.

Table 5 shows the relationship among the factor substitutions, the share of H in group Q, and both top-end and low-end wage inequalities. In cases of the corner solution ($\theta = 0$ or $\theta = 1$), the reduction in σ increases the top-end inequality and decreases the low-end inequality in the same manner as the previous section. However, in cases of the interior solution, a reduction of σ decreases both top-end and low-end wage inequalities. This is because some individuals change from unskilled labor to non-routine skilled labor, which shifts the supply curve of unskilled labor to the left, and that of non-routine skilled labor to the right. This example sheds light on the important role of a chance of higher education in shaping the distribution of wages.

Figure 4. Education choice



Note. Each curve represents $w_H/w_M + 2\varepsilon w_L/w_M$ as a function of θ when $\sigma = 4$ (bottom), $\sigma = 3.546$ (middle), or $\sigma = 2$ (top). The horizontal line is $1 + 2\varepsilon$. As σ decreases, the curve shifts upward, which implies that θ increases. $\beta = 0.8$, $\alpha = 0.2$, $\varepsilon = 0.75$.

Table 5. Factor substitution, education choice, and wage inequality

σ	θ	$\frac{w_H}{w_M}$	$\frac{w_L}{w_M}$
1.1	1	3.636	0.679
1.2	1	3.333	0.608
1.3	1	3.077	0.548
1.4	1	2.857	0.496
1.5	1	2.667	0.451
2.0	0.934	2.068	0.288
2.5	0.462	2.189	0.207
3.0	0.195	2.231	0.179
3.5	0.014	2.254	0.164
3.546	0	2.256	0.163
4.0	0	2.000	0.150

Note. $\beta = 0.8$, $\alpha = 0.2$, $\varepsilon = 0.75$

6 Conclusions

In a simple static model of higher education, we examine the relationship between performance pay for non-routine skilled workers and the distribution of wages. In an economy where the chance of higher education is limited, we can observe a negative relationship between the top-end and low-end wage inequalities. This result is consistent with the stylized facts presented in Lemieux (2008). However, top-end wage inequality could be positively related to low-end inequality if individuals can access a second chance at success provided by higher education. This scenario could explain the relatively flat wage distribution observed in France and Japan.

Our research can be extended in several directions. For example, we use the elasticity of substitution between intermediate goods as a proxy of the prevalence of performance pay. Research is necessary to build micro-foundations of both the performance pay (Lemieux, MacLeod, and Parent 2019) and the determinant of product differentiation (Jones and Kim 2018; Aghion et al. 2019; Akcigit, Pearce, and Prato 2020). Focusing on wage inequality, this research passes over the inequality of capital income. Incorporating capital ownership is a natural extension to capture the overall income inequality. We also assumed that occupations are segmented by education status, however, it would be plausible to alleviate the assumption by determining the relationship between education and occupation in a probabilistic setting. Our result highlights the important role of higher education in shaping the distribution of wages. Further, analyses on the public policy for higher education would complement this paper because the public policy for higher education will affect not only the top-end wage inequality but also the low-end wage inequality. These extensions are left for future research.

Acknowledgments

I would like to thank Toshiki Tamai, Akira Kamiguchi, and the seminar participants at Nagoya University for their useful comments. This research is financially supported by the Japan Society for the Promotion of Science (JSPS KAKENHI Grant No. JP21K01472). Any remaining errors were my own.

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Figure 1. Education choice

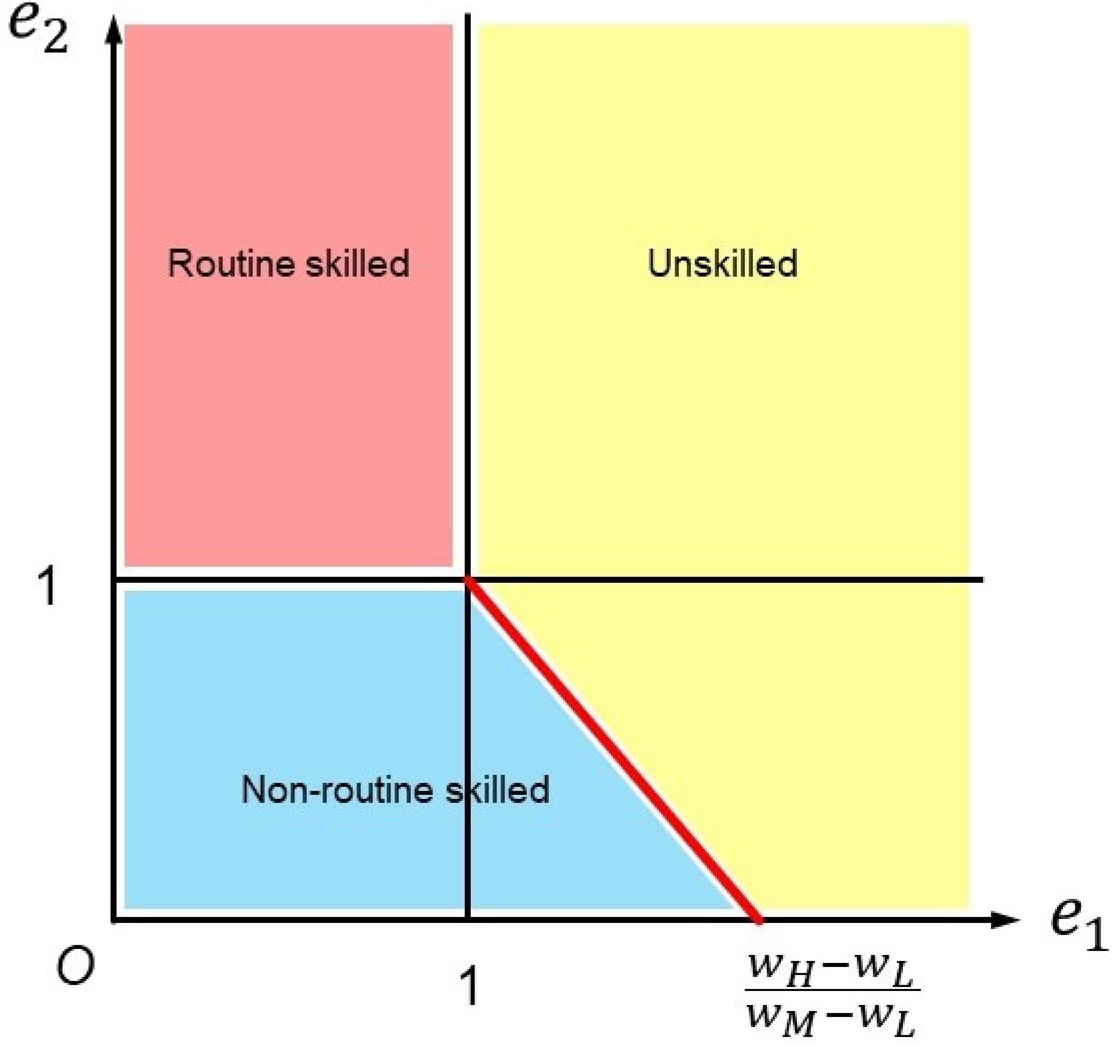

Figure 2. 1-2-3 economy

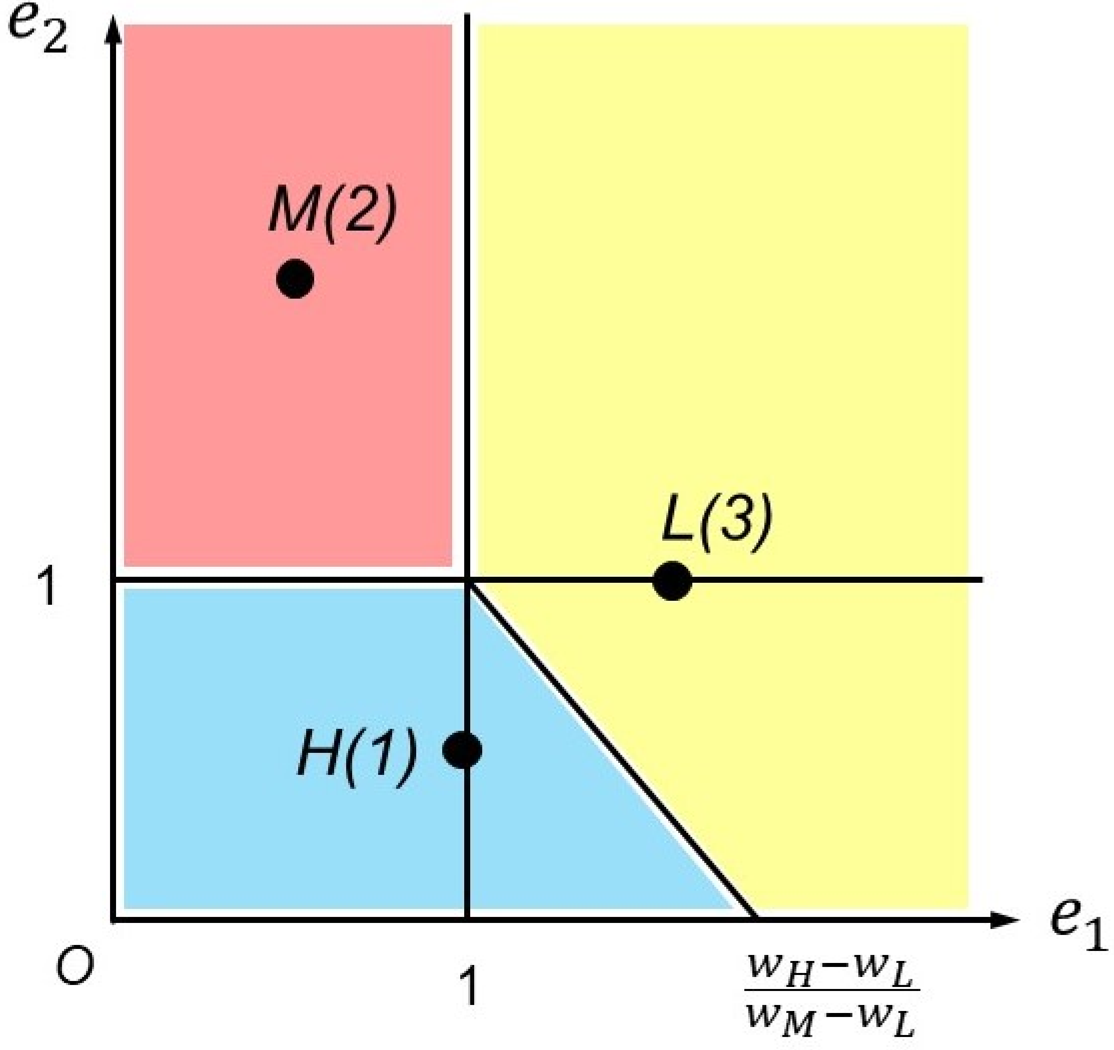

Figure 3. 1-2-2-1 economy