ON THE BEHAVIOR OF ELECTRON TEMPERATURE IN HIGH FREQUENCY FIELD*

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1. Introduction

A profound interest in the fundamental process of high frequency gas discharge has been taken along with the development of the application of high frequency technique.[1]

The electron temperature is the basis of the discharge phenomenon and here will be treated as the case with H.F. field, in such a simple way applying the equation of energy balance at each instant, as made in D.C. field.[2]

The calculation shows that the electron temperature pulsates with the alternating field, and also includes the transient term.

2. Approximate Calculation (I)

We first designate some notations for electron: λ = mean free path, m = mass, u = energy in volt, $\bar{c}$ = mean thermal velocity, k_e = mobility, $t_0 = \lambda/\bar{c}$ = mean time between impacts; and besides for gas molecule: M = mass, Ω = energy in volt; $E\sin\omega t$ is applied H.F. field strength.

According to the Compton's assumption for mobility, we have

$$k_e = 0.75\left(\frac{e}{m}\right)\left(\frac{\lambda}{\bar{c}}\right). \tag{1}$$

The average advance of an electron between impacts in the field direction may be written as

$$s = \int_{t-\frac{t_0}{2}}^{t+\frac{t_0}{2}} k_e E\sin\omega t\cdot dt. \tag{2}$$

We can thus make a simple calculation on the condition that the thermal velocity of the electron maintains almost constant value between impacts, when frequency of collision is much larger than that of field, that is

$$\omega t_0/2 \ll I \qquad (\text{or } \sin\omega t_0/2 \approx \omega t_0/2). \tag{3}$$

Then we have

$$s = 0.75\, e/m(\lambda/\bar{c})^2 E\sin\omega t.$$

Cravath and Compton[3] showed that the average energy lost by an electron of mass m, in an elastic impact with a spherical molecule of mass M, is a frac-

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tion of its energy given by

$$\kappa=\frac{8}{3}\left(\frac{mM}{(m+M)^2}\right)\left[1-\frac{\Omega}{u}\right],$$

as long as the energy of the electron is enough to give no excitation.

Taking the impact loss into considaration, the energy change edu of an electron advancing the distance dx in the field $E\sin\omega t$ is given by

$$edu=eE\sin\omega t\cdot dx-\kappa eu\frac{dx}{s}.$$

Substituting the values of κ and s in the above equation, we get

$$\frac{du}{dx}=E\sin\omega t-\frac{\frac{8}{3}\cdot\frac{m}{M}\left(1-\frac{\Omega}{u}\right)u}{0.75\frac{e}{m}\left(\frac{\lambda}{\bar{c}}\right)^2 E\sin\omega t}$$

Since $$eu=\frac{1}{2}mC^2=\frac{1}{2}\frac{3\pi}{8}m\bar{c}^2$$

or $$\bar{c}=\sqrt{\frac{16}{3\pi}\cdot\frac{e}{m}\cdot u}$$

and $$\frac{dx}{dt}=k_e E\sin\omega t=0.75\frac{e}{m}\cdot\frac{\lambda}{\bar{c}}E\sin\omega t,$$

replacing the variable x by t, we get at once

$$\sqrt{u}\frac{du}{dt}=a\lambda\left(E^2\sin^2\omega t-\frac{bu}{\lambda^2}(u-\Omega)\right), \tag{4}$$

where $$a=0.75\sqrt{\frac{3\pi}{16}\frac{e}{m}},$$

$$b=\frac{128\frac{m}{M}}{0.75\times 9\pi}.$$

The second term of the right hand in the equation (4) represents the term related to loss.

On behalf of caluculation, the following transaction may be permitted, if the values of the constants g and h shall be chosen for the given range of u,

$$u(u-\Omega)\cong g(u^{3/2}-h). \tag{5}$$

This is more accurate, when the values of g and h are settled for each range of u which are divided into several sections with respect to electron energy.

This is rather of convenience and practice, because the mean free path λ and the loss fraction of energy κ, which are generally considered as gaseous constants, varies with the electron energy u. (Usually g is nearly of unity for $u=0.2\sim1.6$ eV. and 4 for $u=4\sim18$ eV.) Then, positing $u=\Omega$ at $t=0$, we can get a complete solution of the above equation,

$$u(u-\Omega)=\frac{E^2}{2b/\lambda}[1-\cos\varphi\cdot\cos(2\omega t-\varphi)-\sin^2\varphi\, e^{-At}], \tag{6}$$

where $$A = 3\,abg/2\,\lambda, \qquad \tan\varphi = 2\,\omega/A.$$

Usually we may take $u \gg \Omega$. The transient term may be often neglected after one cycle.

For D.C. field, it becomes at once

$$u_{dc}\,(u_{dc} - \Omega) = E^2\left(\frac{\lambda^2}{b}\right)$$

which agrees of course with the result of Compton's calculation.[4]

We can see from this result, as show in Figs. 1 and 2:

(i) The electron temperature or the electron energy pulsates with twice the frequency of the field.

(ii) Its phase lags by $\varphi/2$ behind the field.

(iii) With increasing frequency, the phase lag of the electron temperature behind the field increases, and the amplitude of its ripple diminishes, until the electron temperature becomes such a constant value as is obtained in the case of D.C. field, whose strength is equal to the effective value of H.F. fieid.

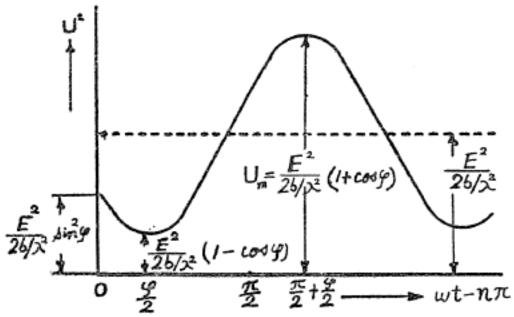

FIG. 1

3. Discussion

In the equation (6) taking $\kappa = 4 \times 10^{-2}$, and the mean free path at I mm Hg $\cong 3 \times 1^{-2}$ and $g = 4$, we get

$$A = \frac{3}{2}\,\frac{ab}{\lambda}\,g = 8.2 \times 10^7\,\frac{\kappa g}{\lambda}$$

$$\cong pg \times 10^8 \quad (p = \text{gaseous pressure in mm Hg.})$$

$$\tan\varphi = \frac{2\,\omega}{A} \cong \left(\frac{f}{p}\right)\frac{1}{g} \times 10^{-7}$$

(f=cycle per sec.)

and also,

$$\text{tna}\,\varphi = \frac{2\,\omega}{A} = \frac{8\sqrt{u}}{3\,g\kappa}\,\frac{\omega t_0}{2} = 33\,\frac{\omega t_0}{2}.$$

According to the assumption (3) in the outset of the calculation, the frequency may be permitted up to $\omega t_0/2 = 0.3\sim0.5$ ($\omega t_0/2 = 0.35$, $\sin\omega t_0/2 = 0.34$; $\omega t_0/2 = 0.52$, $\sin\omega t_0/2 = 0.57$), that is $\tan\varphi = 10\sim15$. Thus, the

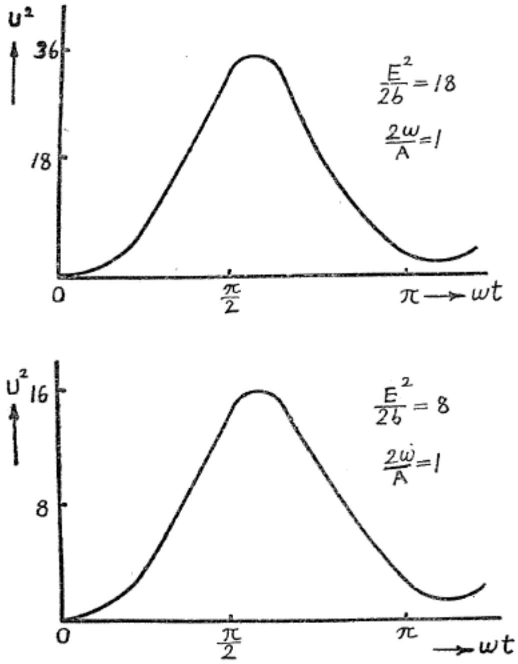

FIG. 2. Precise calculation carried out by dividing the electron energy into four range, $U \leqq 1$, $1 \leqq U \leqq 2$, $2 \leqq U \leqq 4$, $4 \leqq U$.

equation (6) shall be adoptable up to the frequency of $f/p \cong 2\times10^{8} \sim 10^{9}$. Even at the limit of the condition: $\omega t_0/2=\pi/2$ or t_0=half a period, the equation can be used with some compensation.

In the application of this equation (6) we may take the constants $g=1$ and $h=0.047$ in general purpose ($u \ll 2$) and $g=4$, $h=5$ for high energy. Usually, the former is applicable to the stationary discharge.

At the medium energy, we must choose proper values of g and h, and connect these graphically with the former equation which is derived with the values of $g=1$, $h=0.047$.

4. Displacement of Electron and Its Energy Relation

The maximum displacement of oscillating electron (twice in the amplitude) is

$$d=\frac{2}{\omega}\int_{\omega t=\varphi/2}^{(\pi/2)+(\varphi/2)} k_e E\sin\omega t\cdot d(\omega t)\cong 1.13\frac{e\lambda E}{\omega\sqrt{m}\sqrt{\frac{3k}{2}T_{em}}}\sqrt{1-\frac{\pi}{12g^2}\frac{\omega^2 m\lambda^2}{e\kappa}}, \quad (7)$$

where T_{em}=maximum electron temperature.

The dependence of electron energy on the electron displacement x in the direction of the field can be calculated on the relation,

$$\frac{dx}{dt}=k_e E\sin\omega t,$$

and is given as,

$$u(u-\Omega)\cong\frac{E^2}{2b/\lambda^2}\left[1+\cos\varphi-2\cos\varphi\left(1-\frac{2x}{d}\right)^2\right]. \quad (8)$$

5. In Case of H.F. Field Superposed by D.C. Field ($E_0+E\sin\omega t$)

In this case, the culculation can be carried out in the same way as given in the previous section. The result is

$$u(u-\Omega)=\frac{E^2}{2b/\lambda^2}[1-\cos\varphi\cos(2\omega t-\varphi)-\sin^2\varphi\cdot e^{-At}]$$

$$+\frac{E_0}{b/\lambda^2}[E_0+2E\cos\varphi_0\sin(\omega t-\varphi_0)-(E_0-E\sin2\varphi_0)e^{-At}] \quad (9)$$

where $\tan\varphi_0=\omega/A$, $\tan\varphi=2\omega/A$.

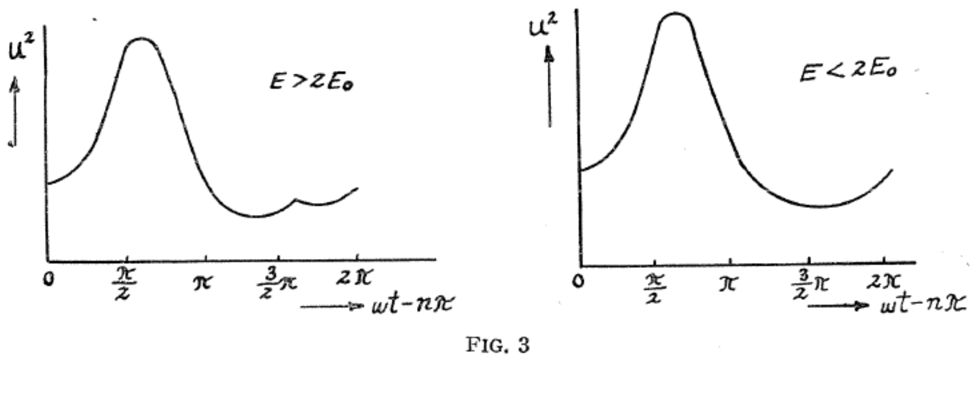

FIG. 3

As shown in Fig. 3, the features are somewhat different for $E>2E_0$ and $2E_0>E$.

6. Calculation of Electron Temperature (II)

When the mean free path of electron varies in proportion to the electron speed, that is $\lambda=\beta\sqrt{u}$, providing that β is constant, we get from the equation (4),

$$\frac{du}{dt}=a\beta\left[E^2\sin^2\omega t-\frac{b}{\beta^2}(u-\Omega)\right].$$

This is perfectlly soluble and taking $u=\Omega$ at $t=0$, we get

$$u-\Omega=\frac{E^2}{2b/\beta^2}\left[1-\cos\theta\cos(2\omega t-\theta)-\sin^2\theta\cdot e^{-(ab/\beta)t}\right], \tag{10}$$

where
$$\tan\theta=\frac{2\omega}{ab/}$$

7. The Number of Ionization per Second per Electron Generated by Impacts with Electron

The number of ions per second per electron created by impacts with electrons, Z, can be obtained from T_e. In case of H.F. field, owing to the pulsation of electron temperature, the mean value of the number of ionization Z may be calculated by integrating as follows,

$$\begin{aligned} Z_h &= 4f\times\frac{1}{\omega}\int_{\varphi/2}^{(\pi/2)+(\varphi/2)} Z\cdot d(\omega t) \\ &= 4f\frac{600\,apV_j}{\omega\pi}\sqrt{\frac{4e}{3m}}\left[\sqrt{U_m}\,e^{-3V_j/2U_m}\int_0^\infty\frac{e^{-(3V_j/2U_m)r}dr}{\sqrt{r(r+2)(r+1)^3}}\right. \\ &\quad \left.+\frac{4}{3V_j}e^{-3V_j/2U_m}\int_0^\infty\frac{e^{-(3V_j/2U_m)r}dr}{(r+1)\sqrt{r(r+2)(r+1)^3}}\right], \end{aligned} \tag{11}$$

where Z is the function of T_e as was given by Steenbeck,[5] V_j is the ionization potential and, U_m is the maximum electron energy.

In order to evaluate the effect of the ionization in H.F. field, we may compare with Z_D of D.C. field in which the electron can acquire the same energy U_m. From the result as shnwn in Table, one can see that the difference between Z_h and Z_D becomes larger as the pulsation becomes larger, espescially in case of small electron energy in comparison with the ionization energy.

$\frac{V}{U_m}=\frac{3}{2}\frac{V_j}{U_m}$	0.1	1	10	20
$\frac{Z_h}{Z_D}$	0.6	0.4	0.1	0.05

8. Conclusion

In this papes we have carried out the calculation of electron temperature in H.F. field with the method similar to that was derived by Dr. K. T. Compton.

Form these results, it is pointed out that the electron temperature pulsates and have exponential transient term and that in this case there are quite different ionization effect from the D.C. field.

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