THREE-DIMENTIONAL PHOTOELASTIC EXAMINATION OF A HELICAL SPRING

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I. Introduction

The stresses in a wire of a helical spring with a section as shown in Fig. 1 have been obtained mathematically in a previous paper.[1] The object of this paper is to examine the theoretical results by the frozen stress method.

Since for the preparation of a model having accurate dimensions of section and helix angle considerable technical difficulties are felt, a ring sector with a section as shown in Fig. 1 is cut as a model of the spring for the first time. The deformation at the freezing temperature is rather large and this large deformation is unfavourable in the usual tests. But in this experiment, by the large deformation at the freezing stage a model of spiral is readily made from the ring sector which is in simpler shape, since the ring sector submitted to an axial load deforms into a spiral at the freezing stage. Taking a suitable axial load, we have a ring sector deformed into a spiral with assigned helix angle.

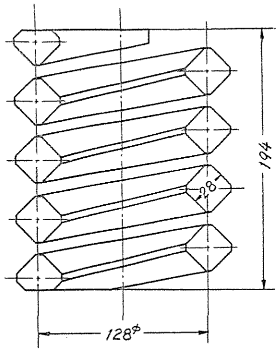

FIG. 1. Helical spring with special section.

II. Experimental Procedure

The model used in this experiment is a ring sector of epoxy-resin with dimensions as shown in Fig. 2. The model is placed horizontally and is pulled vertically at the center of the ring by means of two horizontal arms fixed at both ends of the ring.

With increase of the deformation of the model, the ring sector deforms into a spiral. Consequently, if the length of the ring remains unchanged, the length of the arms R and the central angle contained between the arms both change slightly and the arms rotate about their axes. Precautions are taken against every resistance which will accompany with the large deformation of the model.

The arm with a circumferential groove at its end is supported by a knife edge which can rotate freely about the axis of the ring. The length of

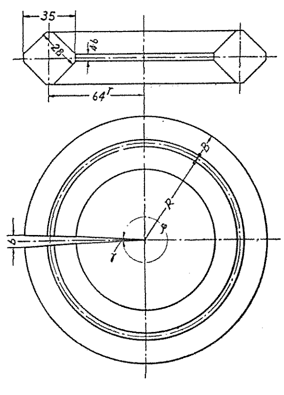

FIG. 2. Model of epoxy-resin.

the arm is adjustable to coincide with the calculated value of radius based on the spiral into which the ring sector will be deformed at the freezing stage. Hence, the model is always submitted to a purely axial load.

The temperature of the freezing bath which contains the model and the loading apparatus is elevated above the secondary transition of epoxy-resin and after the temperature distribution in the model is sufficiently equalized, the bath is cooled so slowly as to leave no residual stresses.

As was described, the deformation at the freezing stage is usually large but the deformation remains within the elastic range of the material and the frozen stresses in the model are those of the last stage, namely stresses for the spiral rod, and are independent of the form of model at progressing stages.

When a ring sector of radius R containing the angle φ at the center, Fig. 2, deforms into a spiral of radius R' with the helix angle α, ignoring the elongation of the wire, we have the relation that

$$\varphi' R' = \varphi R \cos \alpha \tag{1}$$

where φ' is the projection on the horizontal plane of the angle contained at the center by both ends of the spiral. If b denotes the initial distance between the points marked at the ends of the model and h, b' the vertical and the horizontal components of the distance of the marked points after deformation, it follows that

$$\alpha = \sin^{-1} \frac{h}{\varphi R} \tag{2}$$

From Eq. (1), it becomes

$$R'\left\{\pi - \sin^{-1}\frac{b'}{2(R'+B)}\right\} = R\left\{\pi - \sin^{-1}\frac{b}{2(R+B)}\right\}\cos\alpha \tag{3}$$

where B is the distance from the centroid of the cross section of the model to the marked point. Observing b, b' and h by a measuring microscope, we can easily evaluate the value of α, R' from Eqs. (2) and (3).

In this experiment, normal slices and oblique incidence are used. The thickness of slice is 2 ± 0.01 mm. In the case of oblique incidence the fringe order N is given by the following equation since the stresses are not constant along the optical path through the slice.

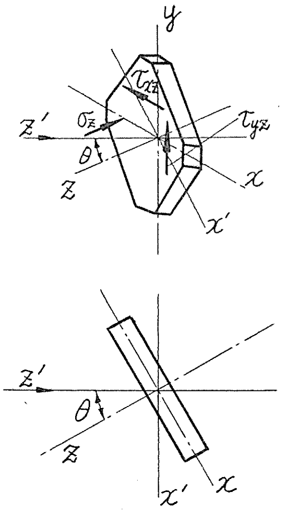

FIG. 3

$$N = \int_0^t C \sin\theta \{(\sigma_z \sin\theta + 2\tau_{xz}\cos\theta)^2 + 4\tau_{yz}^2\} \frac{dt}{\cos\theta} \tag{4}$$

where θ is the angle of incidence, Fig. 3, and C denotes the photoelastic sensitivity of epoxy-resin and t the thickness of the slice.

III. Experimental Results

The photoelastic sensitivity is measured for a specimen cut from the same material of the model by the freezing method. The value of C is 48.92 mm/kg for $\lambda = 5{,}461$ Å. The fringe patterns obtained by the oblique incidence method are shown in Figs. 4-6. The dark lines in the figures are the fringes of integral orders since the fringe patterns have been obtained in a dark field. It can be seen from the figures that any fringe does not appear at $\theta = 0°$, and the number of fringes increases with increase of θ. Figures 1 and 3 described in Fig. 4 indicate the inside and outside portions of the ring, respectively, and 2-4 the direction of the axial load.

The fringe patterns thus obtained are not symmetrical with respect to the axis 1-3, as are shown in the figures. This shows the effect of the normal stress σ_z occurring in the spiral rod. The distribution of the fringe orders at the angle of incidence $\theta = 46°25'$ is shown by curves in Fig. 7 where dashed lines are those in a bright field.

The distribution of fringe orders along y_n axis, Fig. 8, is shown by curves in Fig. 9.

Using the previous theoretical results, the stresses are evaluated for dimensions of the model, and the theoretical values of N are calculated from Eq. (4) for the value of θ described. The values of N thus obtained are given in the following table.

The distribution of N taken from the table is also shown by dashed lines in Fig. 9.

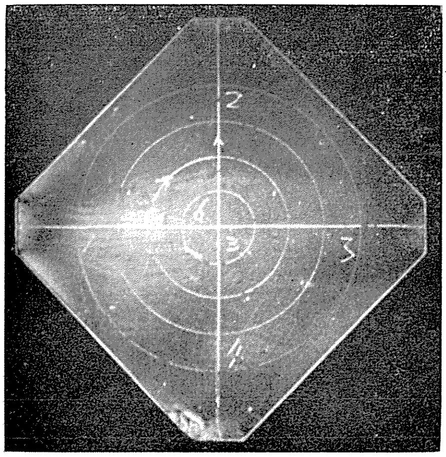

FIG. 4. Fringe pattern. $P = 2$ kg, $\theta = 0°$.

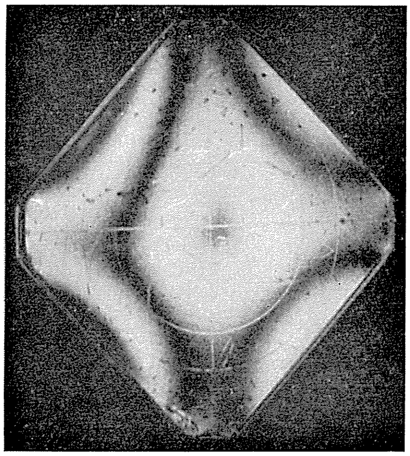

FIG. 5. Fringe pattern. $P = 2$ kg, $\theta = 26°30'$

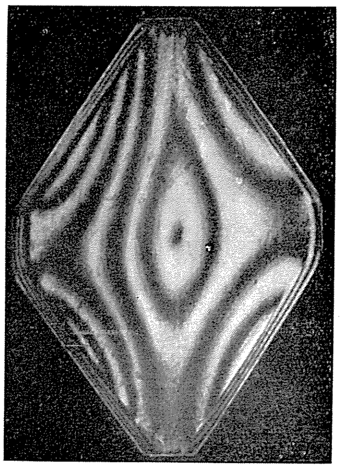

FIG. 6. Fringe pattern. $P = 2$ kg, $\theta = 46°25'$

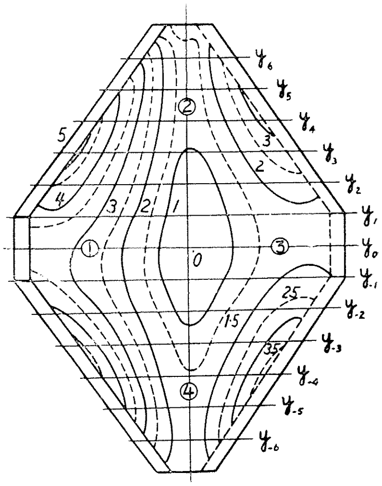

FIG. 7. Fringe order distribution.

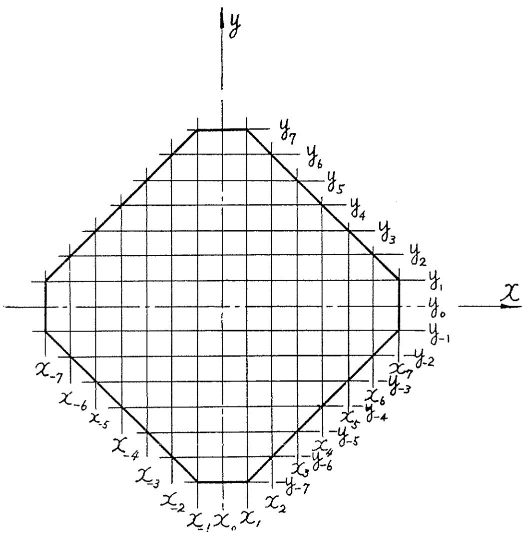

FIG. 8

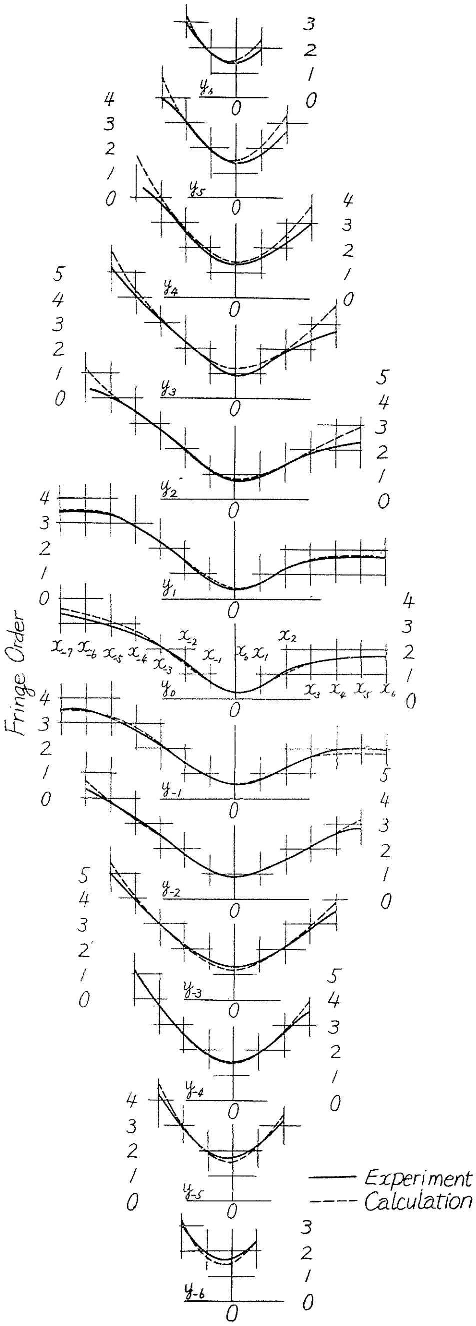

FIG. 9. Fringe order distribution.

Theoretical Values of N ($P=2$ kg, $\theta=46°25'$)

	x_{-7}	x_{-6}	x_{-5}	x_{-4}	x_{-3}	x_{-2}	x_{-1}	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
y_7															
y_6						3.23	1.73	1.50	2.38						
y_5					4.88	3.01	1.90	1.47	2.17	3.40					
y_4				5.58	3.83	2.70	1.79	1.46	1.69	2.62	3.70				
y_3			5.83	4.27	3.22	2.35	1.50	1.21	1.46	1.99	2.75	3.81			
y_2		5.13	4.13	3.49	2.78	1.90	1.22	0.82	1.04	1.48	1.96	2.57	3.05		
y_1	3.56	3.57	3.44	2.99	2.33	1.52	0.82	0.42	0.67	1.22	1.59	1.76	1.72	1.69	
y_0	3.61	3.37	3.09	2.72	2.07	1.36	0.72	0.21	0.51	1.11	1.54	1.70	1.75	1.76	
y_{-1}	3.40	3.49	3.27	2.88	2.24	1.46	0.88	0.60	0.82	1.30	1.71	1.82	1.76	1.80	
y_{-2}		4.85	3.89	3.09	2.61	1.83	1.14	0.85	1.14	1.61	2.13	2.72	3.25		
y_{-3}			5.47	4.00	3.15	2.21	1.48	1.19	1.54	2.10	3.08	3.97			
y_{-4}				5.29	3.63	2.58	1.76	1.48	1.92	2.77	4.06				
y_{-5}					4.68	2.89	1.82	1.50	2.24	3.59					
y_{-6}						3.13	1.69	1.50	2.47						
y_{-7}															

IV. Discussion and Conclusion

As is shown in Fig. 9, the theoretical and experimental results are fairly in good agreement. This shows that the theoretical solution previously obtained for a helical spring, taking the effect of helix angle into account, is accurate enough for engineering practice.

Since oblique incidence is used, the stresses are not constant along the optical path through the slice. The accuracy of the experiment may be improved if thinner slices be used. The difficulty of observing the fringe orders, however, increases as the thickness of slices decreases. The use of a microphotometric devices will overcome the difficulty and remarkably improve the accuracy of the experiment.

The values of the Poisson's ratio of epoxy-resin is about 0.5 at the freezing stage and is much larger than the value of steel. But the effect occurring from the discrepancy of the values of Poison's ratio is not perceptible, since the predominating stress for this case is the shearing stress due to torsion.

Reference

1) Yosio Ōhasi : On the Stress Distributions of Helical Spring with a Special Section. Temporary Report of Railway Technical Research Institute, No. 6–120, Sept. 1955.