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主論文の要旨

論文題目　**Robust topology optimisation for wideband acoustic devices based on the boundary element method**
（境界要素法に基づく音響機器の広帯域化のためのロバストトポロジー最適化）

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論文内容の要旨

For designing high-performance functional structures, topology optimisation (TO) is well-known as one of the most flexible methods. The prosperous expansion of TO in various technologies, however, does not mean substantial progress in engineering-design. This may be due to the lack of concerns about uncertainties stemming from the real world. Usually, during a standard topology optimisation (which is called "deterministic topology optimisation" (DTO) in this thesis), the structure is optimised to an ideal configuration with one or multiple fixed design parameters assumed. However, even a slight perturbation in the design parameter can, in turn, lead to a drastic deterioration in performance since the optimum works in the inevitable presence of uncertainty encountered in practice. It is, therefore, of importance to take the uncertainties into account at the design stage.

Robust topology optimisation (RTO) is developed based on DTO for addressing uncertainties. In RTO, the design parameter is assumed as a stochastic variable so that its response also becomes stochastically distributed. By optimising the expectation and minimising the variance of a stochastic objective function, RTO can efficiently produce the desired configuration insensitive to environmental variations. So far, most of the existing RTO researches are concerned with compliance minimisation for mechanical structural design. There is hardly RTO study on wave devices.

On designing acoustic devices, to date, many authors have investigated TO for conventional and innovative acoustic applications. Meanwhile, most of the existing studies aim to control a “monochrome” sound wave oscillating with a single frequency. Thus, we cannot guarantee the high performance of a design by the existing topology optimisations if the frequency of the incident sound wave is different from the one assumed at the design stage. On the other hand, for practical applications, it is desirable to develop an acoustic device that works for a wideband sound wave.

Considering the advantages in both efficacy and fidelity, we are interested in developing an RTO for designing acoustic devices insensitive to frequency perturbations. TO methods for wideband wave devices often consider an “interior” problem, where the state variables are governed by a boundary value problem defined in a bounded domain. Since wave problems are often defined in the infinite domain, it may be worthwhile to develop a robust topology optimisation for “exterior” acoustics, which is, however, a challenging problem. This is because we cannot explicitly write conditions corresponding to the “radiation condition” for the frequency derivatives of the solution of the exterior acoustic problem. An effective method is to deal with the infinite domain by truncating it with a finite domain using the perfectly matched layer boundary conditions. However, the frequency derivative with this approach is expressed by a domain integral over the truncated large domain. The evaluation of such an integral may be a computationally expensive task and thus is impractical in the exterior problem. In this study, we describe the behaviour of the frequency derivative of the solution at infinity in the boundary integral form, which gives a formulation that does not include any domain integral. This enables us to deal with the frequency derivatives of a state variable defined in the unbounded domain.

For realising RTO handling a wide bandwidth of interest, another difficulty is raised by the approximation of frequency response. In most of the robust optimisations, the variance of the response is often approximated by 1st− or 2nd−order Taylor’s expansion, which can be insufficient for an accurate approximation of frequency response over a wide range. The inaccurate uncertainty modelling may result in either a narrow working bandwidth of an optimal wave design. To resolve this, we develop a robust topology optimisation using a higher-order approximation aiming at further enhancing the robustness of the optimised acoustic device with respect to incident frequency variation.

To evaluate the Nth order frequency derivative of the acoustic response, we need to solve N + 1 boundary integral equations. The systems of the algebraic equation obtained by discretising them share the coefficient matrix. Yet, assembling the matrix is a heavy task because it ends up with a dense matrix due to the BEM nature. Composing N +1 right-hand sides' vectors is also numerically demanding because it involves matrix-vector products with dense matrices. All these issues can result in low efficiency of computation, and thus may not realise desirable optimisation for a large scale problem. To overcome the aforementioned difficulties, we use the H-matrix method to LU-decompose the dense coefficient matrix and the fast multipole method (FMM) to accelerate the matrix-vector multiplications. Moreover, automatic differentiation is employed to reduce the efforts and possible mistakes of manual programming. The full acceleration helps our method find an optimal design within a feasible time.

In summary, we propose robust topology optimisations (RTOs) for wideband acoustic devices in two-dimensional unbounded regions. Each chapter is summarised as follows:

In Chapter 1, we compare various kinds of topology optimisations concerning uncertainties including RTO. After a survey of related studies. We emphasise the necessity of developing RTO for wave devices based on BEM as well as the advantage of the high-order approximation used for modelling frequency uncertainty.

In Chapter 2, we state a deterministic topology optimisation problem as the preliminary of RTO. Under the assumption that the excitation frequency is subject to the normal distribution, we approximate the frequency response over a bandwidth of interest with the help of high-order Taylor series expansion. The objective function for RTO is defined as a linear combination between the expectancy and standard deviation of a deterministic objective. The general expression of the corresponding topological derivative is given thereafter. We also introduce the level-set method employed for operating topology optimisations.

In Chapter 3, we propose an RTO for acoustically rigid material characterised by the homogeneous Neumann boundary condition. To evaluate frequency derivatives of state variables in the Helmholtz problem, we come up with a boundary integral representation of them. By using a boundary element method equipped with automatic differentiation, the high-order derivatives can be conveniently evaluated. The Monte-Carlo simulations are implemented to examine the minimal truncation order required for a given approximation accuracy over different bandwidth ranges. We derive the topological derivative for a 2D exterior Neumann problem. After numerically validating the topological derivative, we carry out topology optimisations for two different wideband acoustic applications and give some concluding remarks on the robustness attained.

In Chapter 4, we extend our RTO for soft materials described by the impedance boundary condition. The topological derivative of an impedance structure in an unbounded domain is rigorously derived and numerically validated. Two numerical examples of wideband acoustic designs are illustrated to show the feasibility of our extended method.

In Chapter 5, to deal with materials having more realistic acoustic properties, we further develop an RTO for viscoelastic structures coupled by acoustic-elastodynamic waves in two dimensions. We propose another boundary integral representation for frequency derivatives of state variables governed by Navier's equation. The topological derivative for a coupled structure in an unbounded domain is presented and numerically validated. On the other hand, since the discretised coupling problem has more degree of freedom than that of a pure Helmholtz problem, the more expensive BEM needs acceleration for evaluating high-order derivatives. To this end, we incorporate the H-matrix method and FMM our BEM. The proposed method is numerically exemplified by two robust acoustic designs. We also compare the efficiency of the original and the accelerated BEM.

In Chapter 6, we draw some conclusions and comment on the future directions as well as the remaining problems.