MEASUREMENT OF THE AMPLITUDE-PROBABILITY DISTRIBUTION AND THE THREE MOMENTS FOR ATMOSPHERIC NOISE

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Abstract

The performance of a modified pulse height analyzer (P.H.A) is described. It is capable of measuring the occurrence frequency of the sample-amplitude-heights which are evaluated by sampling a continuous waveform within a given time interval. A measuring method of the amplitude probability distributions (A. P. D'. s.) is described referring to the atmospheric radio noise, where a 80 dB dynamic range of amplitude variations can be measured without saturation. The procedure of deriving A.P.D.'s, the average voltage, R.M.S. voltage and antilogarithm of the average logarithmic voltage are discussed from the data of the occurrence frequency of sample-amplitude-height. The measurements of atmospheric radio noise were made using a 3-dB bandwidth of 1 KHz at 50 KHz at Toyokawa in the summer, 1967. All the A.P.D.'s obtained have been compared with a series of theoretical distributions, the parameter r_1 which determines the amplitude characteristics of the impulses at an antenna have been deduced as the result of the comparison. The agreement found between the measured and theoretical distributions are, in most cases, very well. The occurrence frequency distributions of the parameter r_1 are shown for six 4h time blocks of a day and for a whole day. Diurnal variations of the three moments and the associated quantities are described. The plotting of the value of 20 $\log_{10}$ (R.M.S. voltage/ antilogarithm of the average logarithmic voltage) versus the value of 20 $\log_{10}$ (R.M.S. voltage / average voltage) is given for a 3-dB bandwidth of 1 KHz. It has been found that the present case significantly differs from an well known case obtained for various bandwidths.

1. Introduction

The amplitude probability distributions (A. P. D.' s) have been extensively measured by many workers (Clark 1960, 1962; Horner and Harwood 1956; Harwood 1958; Watt and Maxwell 1957; Yuhara et al. 1956; Nakai 1958) since the previous decade, to study the atmospheric radio-noise structure. The measurement of A. P. D.' s is believed to be indispensible to clarify the mechanism of noise interference on the communication systems. Though it is highly desirable to make a continuous measurement of A. P. D.' s of the atmospheric radio noise, it is not generally easy to carry it out. Continuous measurements, in most cases, which have been made in the world are limited to such a single paramenter as average voltage, R. M. S. voltage of atmospheric radio noise and so on because they can easily be measured in comparison to A. P. D.' s. Lately, the method of deriving A. P. D.' s has been developed from average voltage, R. M. S. voltage and antilogarithm of the average logarithmic voltage (Crichlow et al. 1960; Spaulding et al . 1962).

A modified P. H. A. has been used for the first time to measure A. P. D.' s of the atmospheric radio noise at Toyokawa, which is capable of measuring the occurrence frequency of sample-amplitude-height for the noise envelope. The profit of using this apparatus lies in that not only A.P.D. but three moments, i. e., average voltage, R. M. S. voltage and antilogarithm of the average logarithmic voltage can be calculated from the same occurrence frequency data of the sample-amplitude-height. It is especially useful to investigate the relation between the A.P.D. and these three moments.

The measurements with the P. H. A. were made for the atmospheric radio noise in a 3 dB-bandwidth of 1 KHz at 50 KHz at Toyokawa, in the summer 1967. In this paper, firstly the performance of the P. H. A., the procedure of deriving A. P. D. and three moments will be described and secondly, the results derived from the data of the measurements will be discussed.

2. Measurement of the atmospheric radio noise

Purposing a study of atmospheric radio noise structure, the noise measurements were made at Toyokawa during August 19 to 27, 1967. Fig. 1 shows a block diagram of the measuring system. The vertical component of the atmospheric noise field is received using a 8 metre vertical antenna in a bandwidth 1 KHz between 3 db points at 50 KHz.

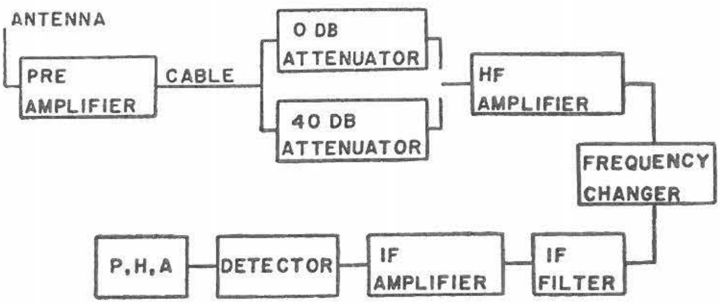


Fig. 1 Block diagram of measuring system

After the atmospheric radio noise received at the antenna has passed through the amplifier stage, mixer stage and so on, the IF radio noise at 30 KHz is peak-detected and the output of the receiver, i.e., the noise envelope is applied to the input of the P.H.A.. As to the description of the measuring system, we place an emphasis on the P.H.A., because it has been used for the first time to obtain the A.P.D. of atmospheric radio noise envelope.

3. The performance of P. H. A.

The perfomance inherent in a P.H.A. is a capability of measuring the probability density of the amplitude of a signal applied to the P.H.A. input. Let us give some explanation about it.

The principle of its operation is as follows. The amplitude height of noise envelope applied to the input of the P. H. A. is evaluated with a preset sampling period. The sample-amplitude-height is then brought to a A.D. converter, and converted into a train of pulse, which have a pulse-repetition period 4 MHz, so that its duration is proportional to the sample-amplitude-height. The number of pulses included in such a pulse train is measured by a particular pulse counter.

The P. H. A. is provided with 100 memory elements, each of which is capable of memorizing 5 figures 00000 to 99999, and correspondingly measuring range of amplitude height of incoming signal is divided into 100 channels of equal amount at the input of the P. H. A.. The remaining function of the P.H.A. is to select a channel into which the sample-amplitude-height falls, and add 1 to the content of the memory element corresponding to the selected channel. All the contents are usually printed out on a paper after ending a measurement.

4. Probability and cumulative probability

Shown are two kinds of graphs in Figs. 2 (a) and (b), which represent a pair of frequency data on the sample-amplitude-height. The data here are obtained from the measurement of the atmospheric radio noise using the system shown in Fig. 1. The graphs each reproduces all the contents of 100 memory elements after a given time interval measurement. The abscissa represents the number of the amplitude channel, and the ordinates represent the occurrence frequency of a sample-amplitude-height which falls onto the amplitude channel of magnitude of 20 millivolts during a given time interval 50 seconds. The abscissa, incidentally, reprsents also the threshold which are expressed by a quantity equal to

(magnitude of an amplitude channel) × {(given channel number) − (number of zero level channel)} where interpretation of the meaning of "zero level channel" will be given in a following section. The threshold for Fig. 2 (b) is 100 times larger than that Fig. 2 (a).

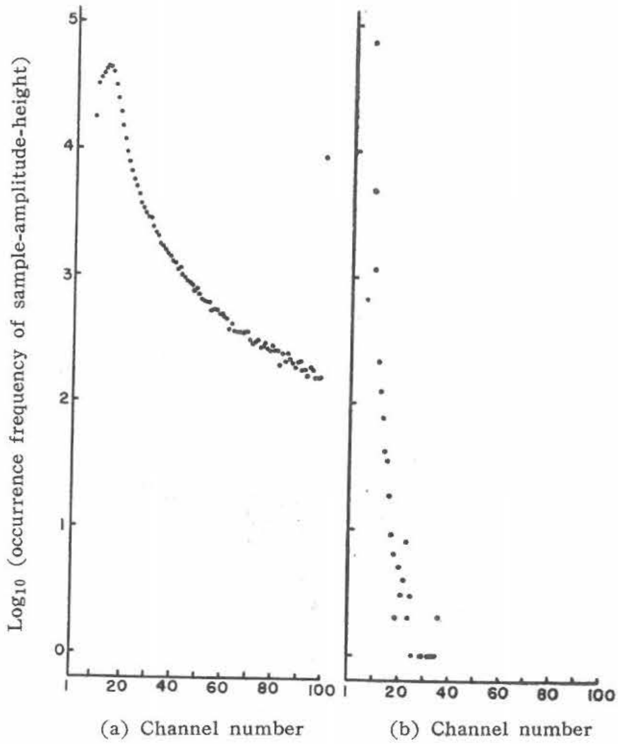


Fig. 2 Occurrence frequency data of sample-amlitude-height for atmospheric radio noise envelope
(a) 0 dB-attenuator measurement (b) 40 dB-attenuater measurement

The measurements for each of the two graphs in Fig. 2 have been made successively with some time interval between them keeping the attenuator settings at 0 dB and 40 dB respectively.

Almost all of the points on the occurrence frequency graph represent the occurrence frequency with which a given sample-amplitude-height falls into a channel on the abscissa. But, the point corresponding to the 100-th channel on the graph has the other meaning, because what is memorized with a memory element corresponding to the 100-th channel is the occurrence frequency with which the sample-amplitude-height exceeds the lower limit of this channel interval. The measurement relating to this point is required for the derivation of A. P. D. of the radio noise envelope with a dynamic range larger than 40 dB of amplitude variations.

Speaking of the ordinary points on the graph, the probability with which the sample-amplitude height falls onto a given channel is equal to (occurrence frequency corresponding to the channel) / A

where $A =$ (sampling period in second) / (measuring time interval)

$= 50 / (100 \times 10^{-6})$ (in present case)

While for the point corresponding to the 100th channel, the occurrence frequency divided by the factor A becomes equal to a cumulative probability with which the sample-amplitude-height exceeds the threshold corresponding to the 100th channel.

It is simple to calculate the cumulative probability, with which a saple-amplitude-height exceeds a given threshold, from such a occurrence frequency data of the sample-amplitude-height as is shown Fig. 2 (a) or (b), i.e.,

$$P(E_{sa} \geq E_t) = \left\{ \sum_{i=r}^{99} N(E_i) + N(E_{100}) \right\} \cdot t/T \qquad (1)$$

where

E_{sa} : sample-amplitude height

E_t : threshold

T : measuring time interval

t : sampling period

E_i : voltage corresponding to the central point in the i th channel

e : magnitude of an amplitude channel

$N(E_i)$: occurrence frequency with which the sample-amplitude-height falls into the i th channel

r : r is determined by the relation

$$E_t = E_r - e/2$$

5. Measurement of A.P.D.'s

The amplitude variations of the atmospheric radio noise envelope are expected from our experience to have a dynamic range nearly 80 dB wide or larger than this provided we measure them in a 3-dB-bandwidth of 1 KHz at 50 KHz. Therefore, for the purpose of a measurement of noise envelope spreading for such a large dynamic range, made with a 100 element P.H.A., a consideration must be paid so that the A.P.D.'s become obtainable without any saturation over a wide dynamic range of 80 dB.

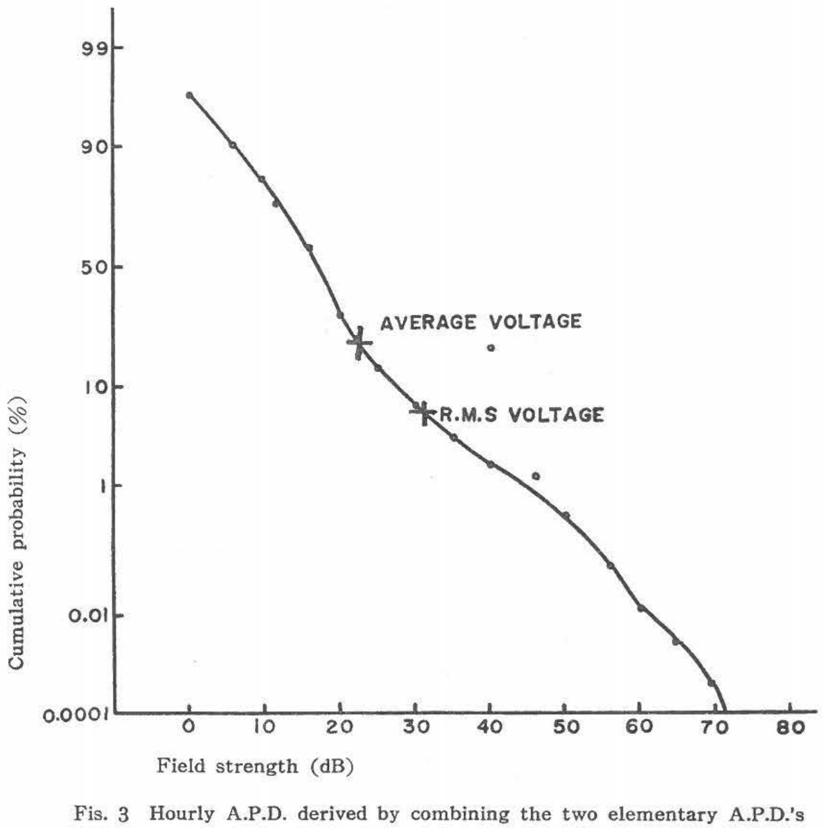
To get the A.P.D.'s of atmospheric radio noise envelope with the new technique, the measurement of them was made at 50 KHz on hourly base of a day, where the hourly measurement consisted of two 50 seconds measurements made respectively on the two settings of attenuator 0 dB and 40 dB at the HF stage in the measuring system. This was done, because the two dynamic ranges, low level 40 dB and high level 40 dB, can be measured during the lst 50 second interval and during the 2 nd 50 second interval, without any saturation with the 100 element P.H.A..

The detailed programme for measuring the A.P.D. on each hour is :

(1) noise measurement with 0 dB attenuator for 50 seconds
(2) printing out all the contents of 100 memory elements and changing attenuator setting from 0 dB to 40 dB for 30 seconds,
(3) noise envelope measurements with 40 dB attenuator for 50 seconds
(4) printing out all the contents of 100 memory elements and changing attenuator setting from 40 dB to 0 dB for 30 seconds. All the steps given above were made automatically, it took 160 seconds to carry out an hourly measurement. Figs. 2 (a) and (b) are the occurrence frequency data of the sample-amplitude-height obtained following the programme for hourly A.P.D. measurement.

An hourly A.P.D. is shown in Fig. 3. This has been obtained from the occurrence frequency data of sample-amplitude-height shown in Fig. 2 (a) and (b). The abscissa represents the threshold, the ordinate represents the cumulative probability. The procedure for deriving an hourly A.P.D. from the occurrence frequency data is as follows : Firstly, two A.P.D.'s are calculated from their respective occurrence frequency data of sample-amplitude-height at a given hour using the formula (1), and finally, an hourly A.P.D. is obtained for 80 dB dynamic range of amplitude variations by combining the two elementary A.P.D.'s, which have each their respective 40 dB dynamic range.

It can not be avoided that an error is introduced on the shape of hourly A.P.D., so



Fis. 3 Hourly A.P.D. derived by combining the two elementary A.P.D.'s

long as we follow the above procedure. Such error is supposed to be due to the drift of actual zero point from a reference zero, which is taken at the lower limit of the zero level channel interval. We set the zero point in unit of voltage at the lower limit of a channel with small channel number at the beginning of measurement. This gives the definition of a zero level channel.

The drift might, even though small, cause a finite error on the value of a cumulative probability, and also on the shape of a part of A.P.D. curve corresponding to amplitude channel intervals in the neighbourhood of zero level. Thus the error can appear on its lower threshold part of an A.P.D. which is measured with an attenuator setting at 0 dB or 40 dB.

Fortunately, the erroneous parts of the A. P. D. s can reasonably be corrected as follows : (1) the erroneous part of an A. P. D. derived from the 0 dB attenuator measurement can be corrected by comparing with a Rayleigh distribution curve, and (2) the erroneous part of an A.P.D. derived from 40 dB attenuator measurement can be corrected by drawing a smooth curve which connect the two elementary A. P. D.'s obtained from an hourly measurement.

6. Parameters calculated from A.P.D.

The profit brought by the use of P. H. A. for the study of the noise is that (1) not only A. P. D. but also average voltage, R. M. S. voltage and antilogarithm of average logarthmic voltage can be calculated from the same occurrence frequency data of sample-amplitude-height for radio noise envelope, and (2) the procedure of calculation of the three moments is simple and plain in comparison with what is based on reading out the values of cumulative probabilities on a measured A.P.D. curve on a graph paper.

The following expressions may be used in calculating the three moments from the occurrence frequency data of sample-amplitude-height in low level and in high level of 40 dB, when the atmospheric radio noise with a dynamic range larger than 40 dB is measured following such a programme of hourly measurement of A.P.D. as shown in the preceding section.

$$\text{average voltage} = \left\{ \sum_{i=1}^{99} N_0(E_i) \cdot E_i + N_0(E_{99}) \cdot E_{100} + \sum_{i=2}^{99} N_{40}(E_i) \cdot 100E_i + N_{40}(E_{100}) \cdot 100E_{100} \right\} \cdot t/T \quad (2)$$

$$\text{R.M.S. voltage} = \sqrt{\left\{ \sum_{i=1}^{99} N_0(E_i) \cdot E_i^2 + N_0(E_{99}) \cdot E_{100}^2 + \sum_{i=2}^{99} N_{40}(E_i) \cdot 100^2 E_i^2 + N_{40}(E_{100}) \cdot 100^2 \cdot E_{100}^2 \right\} \cdot t/T} \quad (3)$$

$$\text{average logarithmic voltage} = \log_{10} Y$$

$$= \left\{ \sum_{i=1}^{99} N_0(E_i) \cdot \log_{10} E_i + N_0(E_{99}) \cdot \log_{10} E_{100} + \sum_{i=2}^{99} N_{40}(E_i) \cdot \right.$$

$$\log_{10}(100E_i) + N_{40}(E_{100}) \cdot \log_{10}(100E_i)\Big\} \cdot t/T \qquad (4)$$

where Y is the antilogarithm of average logarithmic voltage. $N_0(E_i)$, and $N_{40}(E_i)$ both represent the occurrence frequency with which sample-amplitude-height falls onto the i th element, where subscripts 0-, 40-stands for 0 dB-, 40 dB-attenuator settings of the measurements respectively. The remaining symbols follow the notation described in section 4. We also assume that the channel number for the zero level channel is equal to 1, and $N_0(E_{99}) = N_0(E_{100})$. Some modifications must be done for three expressions (2), (3) and (4), if the other number than 1 is alloted to the zero level channel.

7. Results of measurement

7.1 General

In total, 180 A. P. D.'s of the atmospheric radio noise envelope at 50 KHz were measured with the 100 element P.H.A., about 80 percent of which has been found to spread over the probability range from 95 percent down to percent less than 0.001. The three moments, i.e., avereage voltage Vave, R.M.S. voltage Vrms and antilogarithm of average logarithmic voltage $V_{\log}$ have also been evaluated with accuracy, from the occurrence frequency data of sample-amplitude-height, according to the expressions (2), (3) and (4).

The A.P.D. and the three moments have been considered to be indispensible to the study of noise structure, especially to that of its amplitude characteristics. Next descriptions will be given about the following items.

(1) comparison between experimental, and theoretical, A.P.D.'s
(2) frequency distribution of the parameter r_1 which determines the amplitude characteristics of impulses on the antenna
(3) diurnal variations of the three moments
(4) relation between average voltage and R.M.S. voltage and that between antilogarithm of average logarithmic voltage and R.M.S. voltage (Clark 1962).

7.2 Comparison between experimental, and theoretical, A.P.D's

All the measured A.P.D.'s have been compared with a series of theoretical A.P.D.'s. These theoretical A.P.D.'s, as reported by Nakai (1966), have been calculated on the assumption that the peak amplitude voltage distribution of atmospheric impulses appearing on an antenna can reasonably be approximated by two different power functions over the whole range of antenna voltage, that is, the lower-voltage-range distribution p_1 can be expressed by a power function with an exponent r_1, and the higher-voltage-range distribution p_2 can be expressed by a power function with an exponent r_2. The parameters values r_1 and r_2 and a intersection of the two distributions p_1 and p_2 have been determined so as they agree with the experimental distributions of the atmospheric radio noise, that is a series of values 0.8 to 1.8 with 0.5 intervals for r_1, a constant value (the most probable value) for r_2 and several values at 12 dB intervals, beginning from the point of crossing rate of 1 for the intersection.

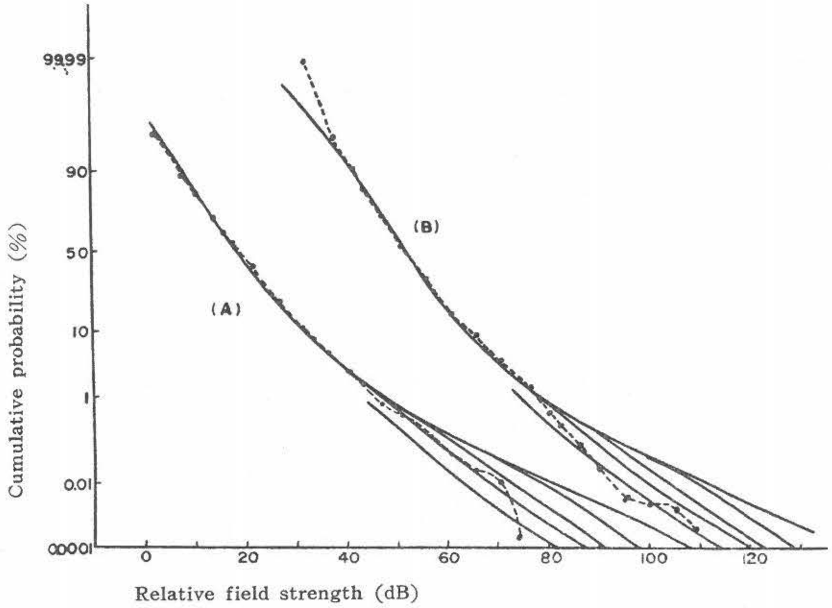


Fig. 4 Comparison between measured and theoretical A.P.D.'s
(a) a series of theoretical curves calculated for parameters $r_1 = 1.1$, $r_2 = 2.0$ of the two power functions and for various intersections at 12 dB intervals of two power functions
(b) a series of theoretical curves for $r_1=1.15$, $r_2=2.0$ and various intersections

Two comparisons of A.P.D.'s are shown on a log-normal graph in Fig.4, where broken lines represent A.P.D.'s measured with P.H.A., while full lines represent theoretical curves. Such theoretical curves which match the measured A.P.D. curves, have been selected from a series of theoretical A.P.D. curves so that differences between the two A.P.D.'s to be compared may be brought to the smallest.

Now, it is difficult to illustrate the result of comparison for all cases, on such a graph as shown in Fig. 4, so that it is desirable to introduce a quantity representing the degree of agreement between the two distributions to be compared. Let us, therefore, introduce an average departure which is defined by averaging the amount of departure, in dB unit of measured A.P.D. from theoretinal A.P.D. at a series of threshold with separation of 5 dB. All result of comparison is shown by the histogram in Fig. 5, where the abscissa represents the average departure expressed as a multiple of 0.5 dB as a unit. The ordinate represents the occurrence percentage of an average departure given on the abscissa. It is noticed that large number of all of the measured A.P.D.'s will coincide with the theoretical A.P.D.'s over a wide range of the probability, and the departure from theory is trivial on average.

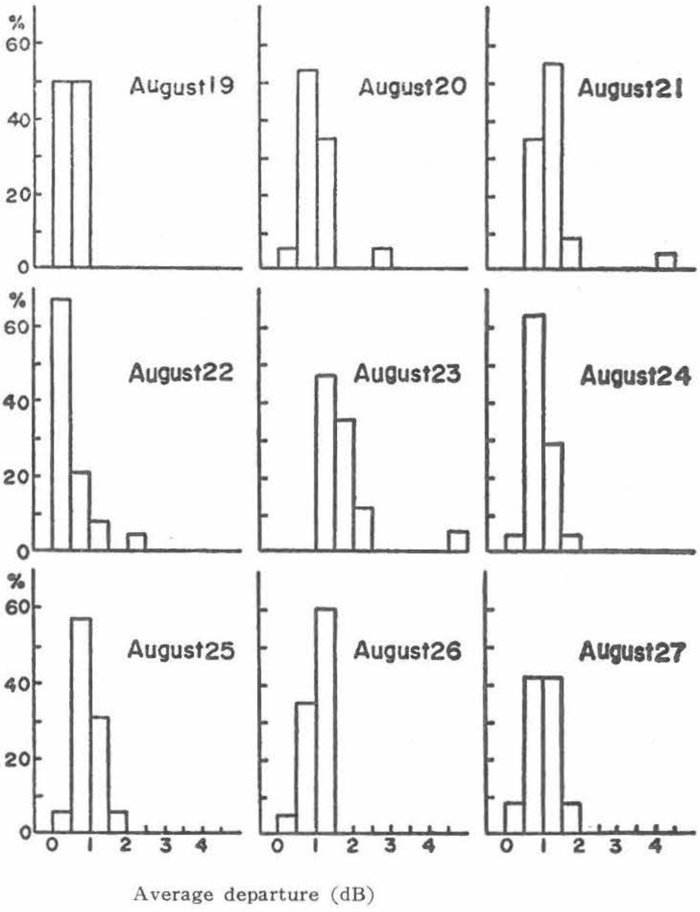


Fig. 5 Graphs showing the results of the comparisons between measured, and theoretical. A.P.D.'s in terms of the average departure (in the range 90 to 0.001 %)

7.3 Histograms of parameter r_1

The parameter r_1 has been estimated from a particular theoretical A.P.D., which is so selected as to make it to coincide with as much as possible with a given measured A.P.D.. The parameter r_1 is a quantity that determines the amplitude characteristics of atmospheric impulses appearing on an antenna, with an exception of the impulses of larger amplitude, which are determined by the parameter r_2. The parameter r_1 has been estimated by a few workers in their observations of crossing rate distributions (Harwood 1958; Nakai and Suzuki 1961).

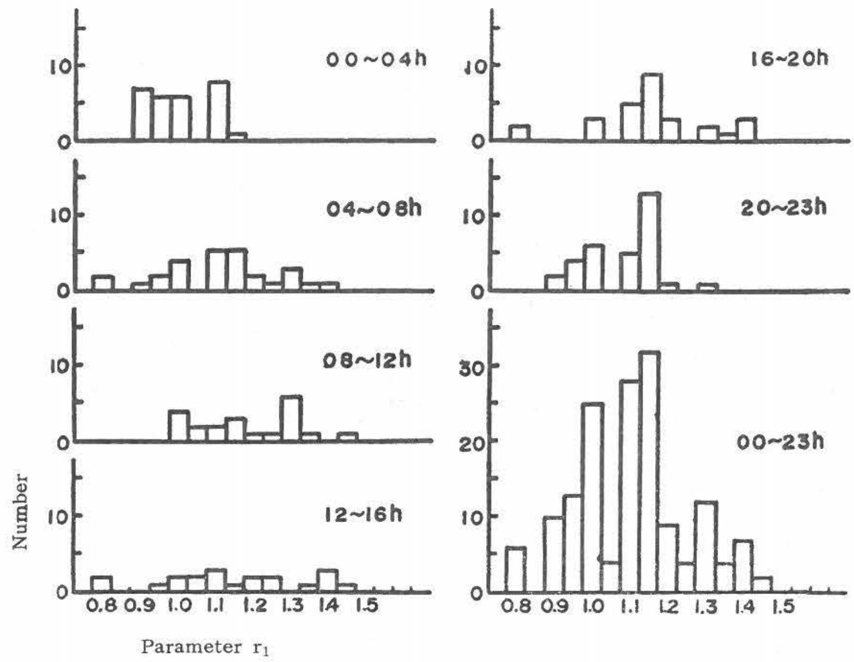


Fig. 6 Number-histogram of parameter r_1

A series of histograms of the parameter r_1 are shown in Fig. 6, where the abscissa shows the value of the parameter r_1, and the ordinate represents the occurrence frequency of the value on the abscissa. Of the seven histograms, first six show the successive four hours statistics of a day and the remaining one stands for a day.

We see a clear trend, (1) parameter r_1 varies over a larger range in day-time than night-time, (2) parameter r_1 is smaller in night-time than in the day time on average.

Table 1 shows medium, upper-decile and lower-decile of the parameter value r_1 for six of four hours time blocks and for one time block for a day.

Table 1

time block	h 00~04	h 04~08	h 08~12	h 12~16	h 16~20	h 20~00	h 00~24
upper-decile	1.10	1.30	1.35	1.40	1.40	1.15	1.30
medium	1.00	1.10	1.15	1.15	1.15	1.15	1.15
lower-decile	0.90	0.90	1.00	0.95	1.00	0.95	0.95
number of A.P.D.'s	28	27	21	20	28	32	156

7.4 Diurnal variations of the three moments

The three moments, i.e., average voltage V_{ave}, R.M.S. voltage V_{rms} and antilogarithm of the average logarithmic voltage V_{log} have been calculated from the occurrence frequency data of sample-amplitude-height for atmospheric radio noise.

Diurnal variations of these moments are shown in Figs. 7 (a), (b) and (c), where the ordinate represents $20 \log_{10} V_{ave}$, $20 \log_{10} V_{rms}$ and $20 \log_{10} V_{log}$ respectively, and the abscissa represents hours of a day. The full lines show the average of hourly values of each moment. We can see a similar trend among the three, especially between average voltage and R.M.S, voltage.

Figs. 8 (a) and (b) show the diurnal variation of V_d and L_d defined as follows (Spaulding et al. 1962).

$$V_d = 20 \log_{10} (V_{rms}/V_{ave})$$
$$L_d = 20 \log_{10} (V_{rms}/V_{log})$$

The full line shows the average of hourly values of V_d and L_d respectively, and it is seen from the figures that the variations of two averages of V_d and L_d are fairly small and kept within 4 or 5 dB throughout a day. This fact may be expected to have some applications, but it must be noted that the fluctuation of hourly values, as represented by a vertical line at each hour, are roughly as large as 10 dB for V_d and 17 dB for L_d.

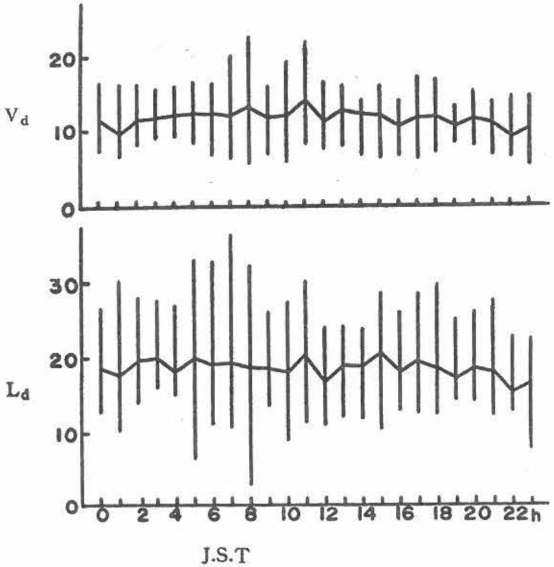


Fig. 8 Diurnal variations of V_d and L_d

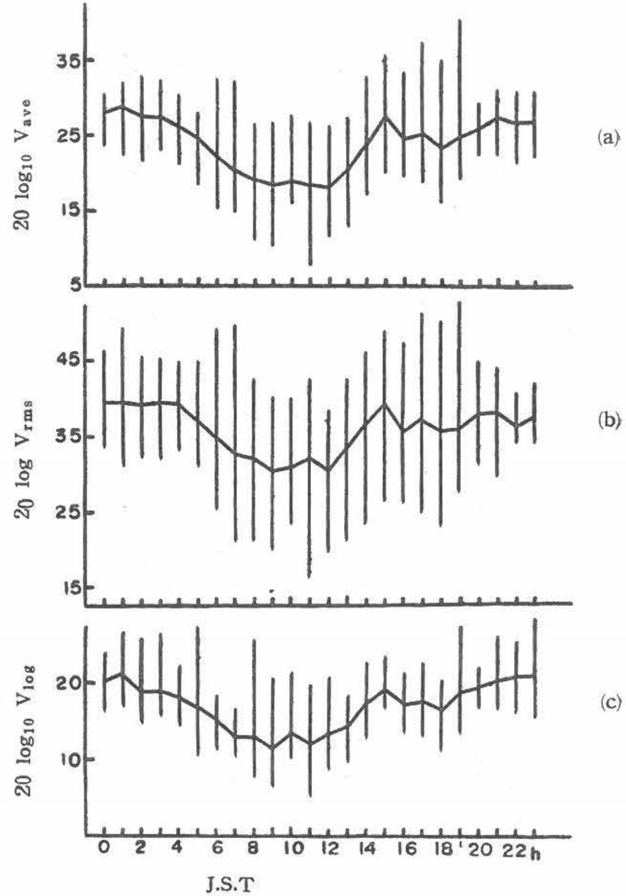


Fig. 7 Diurnal variations of V_{ave}, V_{rms} and V_{log}

7.5 The correlation data L_d versus V_d

It is well known that the ratio V_{rms}/V_{ave} is a useful index for a radio noise structure, because it is a measure of the departure of the noise structure from a thermal type distribution. The V_d, i.e., 20 $\log_{10} V_{rms}/V_{ave}$, in addition, can be considered to be a useful index for the shape of A.P.D. of the atmospheric radio noise envelope, because Crichlow and his colleagues (Crichlow and et al. 1960; Spaulding et al. 1962) succeeded in deriving a series of the most probable A.P.D. curves as a function of V_d. The derivation is on the basis of the fact that the linear relation

$$L_d = 1.69V_d + 0.72 \tag{5}$$

holds for various bandwidths, seasons and time of a day. The relation seems to have been established by studying the relation between L_d and V_d, which are calculated from the measured disributions.

Fig. 9 shows the relation of L_d vesus V_d derived from the measurement of

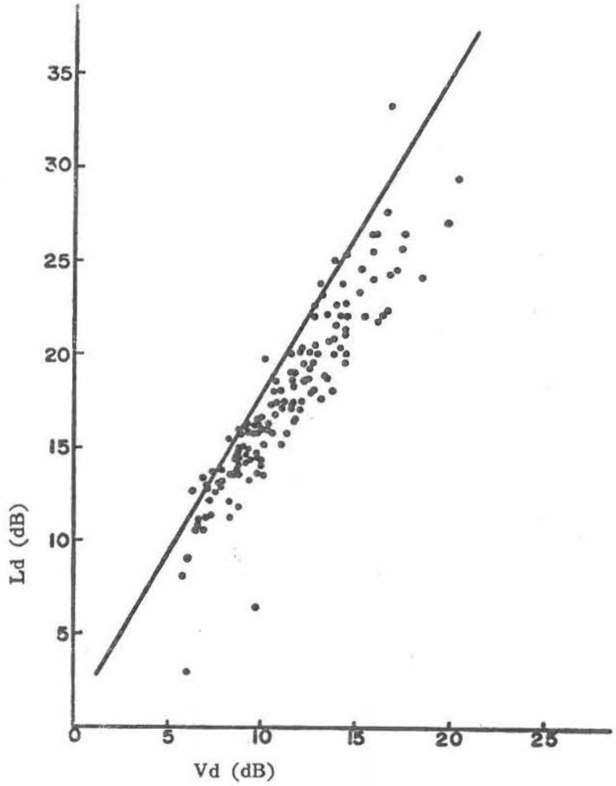


Fig. 9 Correlation of L_d and V_d

atmospheric radio noise at 50 KHz in a 3 dB-bandwidth. The individual point corresponds to a hourly A.P.D. and all the points have been calculated from the occurrence frequency data of the sample-amplitude-height for atmospheric noise envelope. A straight line on the graph in Fig.9 shows the relation (5) given by Spaulding et al (Spaulding et al. 1962)

Although we see significant fluctuations existing among the measured points in Fig. 9, a linear relation seems to hold between the two quantities except for several points. But it will be important to note that the measured points depart as a general trend, from the straight line given by Spaulding et al. Perhaps it will be necessary to collect the data L_d and V_d for another bandwidths, frequencies and seasons.

8. Conclusion

A technique of P.H.A. has been introduced to the measurement of atmospheric radio noise envelope. An amplitude probability distribution (A.P.D.) and the three moments, i.e., average voltage, R.M.S. voltage and antilogarithm of average logarithmic voltage have been calculated from the occurrence frequency data of the sample-amplitude-height. We believe that A.P.D. and the three moments have been obtained with a sufficient accuracy, judging from the method of measurement of atmospheric radio noise and from the procedures to calculate them from the occurrence frequency data of the sample-amplitude-height.

0 dB, as used in this paper, if transformed to the field strength, correspond to 2.4 microvolt / metre at the antenna.

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