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Unemployment and Endogenous Choice on Tax Instruments in a Tax Competition Model: Unit Tax versus Ad Valorem Tax

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Abstract

This paper examines the endogenous choice of the government's tax instrument between the unit and ad valorem taxes under tax competition with unemployment. Governments seek to maximize their objective functions that are the weighted sum of employment and revenue levels. Considering the tax competition model, a high fixed wage rate generates not only unemployment but also employment externalities. This effect can be either positive or negative because of the presence of capital freely mobile among regions. Without unemployment, the revenue-maximizing governments choose unit tax as their tax instruments to avoid revenue loss from intense tax competition under ad valorem taxes. However, with unemployment, positive employment externalities generate additional benefits to use ad valorem taxes for stimulating employment. Therefore, the present study shows that one region chooses an ad valorem tax and the other selects a unit tax, or both governments use ad valorem taxes depending on employment externalities.

Keywords: Tax competition; Unit tax; Ad valorem tax; Unemployment
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1. Introduction

Numerous studies have examined interregional tax competition following the pioneering studies of Zodrow and Mieszkowski (1986) and Wilson (1986).[1] One of the recently focused issues in the literature is to investigate which of the unit tax and the ad valorem tax is chosen as the government's tax instruments (e.g., Lockwood, 2004; Akai et al., 2011; Aiura and Ogawa, 2013; Hoffmann and Runkel, 2016; Ogawa, 2016). Existing studies treated this issue by assuming a perfect labor market, and therefore, there is no unemployment. However, in reality, developed and developing countries have competed in their tax rates to encourage investment and create employment (OECD, 2017). The present study tackles the issue of the government's choice of tax instruments under unemployment by focusing on such an aspect of tax competition.

In an outstanding work, assuming constant returns to scale (CRS), competitive market, and perfect labor markets, Lockwood (2004) examined the nature of tax competition equilibrium with two different tax instruments by comparing the equilibrium outcomes in terms of welfare. Under full employment, the author showed that unit taxes are superior to ad valorem taxes because if all governments choose ad valorem taxes, a harmful tax competition can lead to underprovision of public goods. Akai et al. (2011) showed that a unit tax regime is endogenously chosen by all governments using two-stage games with two symmetric countries. By contrast, Aiura and Ogawa (2013) demonstrated that some countries might choose the ad valorem tax regime if countries' asymmetries are sufficiently large. Departing from CRS production technology, Hoffmann and Runkel (2016) showed that the tax competition under an ad valorem taxation might be less harmful than that under unit taxation with decreasing returns to scale. Furthermore, Ogawa (2016) revealed that asymmetric countries do not compete in the same tax instrument.

Respective studies examined the effects of tax competition with unemployment (e.g., Ogawa et al., 2006; Sato, 2009; Eichner and Upmann, 2012; Exbrayat et al., 2012). Although almost all of them assumed tax competition in the unit tax regime, unemployment can be illustrated in several ways: fixed wage, search friction, and labor union models.[2] A key factor affecting the tax competition equilibrium is the employment externality, which is caused by capital mobility and the technological relationship between capital and labor in production. If capital and labor are a supplement to each other, then attracting capital improves employment for own region, leading to the export of unemployment from own region to the other. In other words, the presence of unemployment derives employment externality.[3] This additional source of inefficiency generates

[1] See Zodrow (2010) for general review of the broad literature on tax competition.

[2] See Sato (2009) for search friction model and Eichner and Upmann (2012) and Exbrayat et al. (2012) for labor union model.

[3] Numerous studies found empirical evidence to support the effect of taxes on employment (e.g., Feld and Kirchgassner, 2003; Harden and Hoyt, 2003; Bettendorf et al., 2009; Felix, 2009; Feldmann, 2011; Zirgulis and Šarapovas, 2017). Almost all of the literature found that employment is negatively associated with tax rates, implying positive employment externalities. On the contrary, Feldmann (2011) showed empirical evidence that higher corporate taxes might lower the unemployment rate, leading to a negative employment externality. We will discuss the possibility of negative employment externalities in Section 4.

new channels of the effects of different tax instruments. Therefore, the equilibrium outcomes differ from the existing literature, which assumed full employment.

This paper aims to examine the equilibrium properties under different tax regimes by endogenizing the government's choice of tax instruments. Hence, we develop a tax competition model with unemployment and two tax instruments, along with the theoretical framework based on Akai et al. (2011). Our model differs from the existing literature with respect to the labor market structure; the fixed wage is assumed in accordance with Ogawa et al. (2006).[4] We investigate how the equilibrium outcome is affected by the employment externality using a two-stage game where each government chooses either a unit tax or an ad valorem tax in the first stage and determines the tax rate in the second stage. Furthermore, we examine the robustness of our results through numerical analyses assuming regions' asymmetries.

The key factor in the government's choice of tax instrument is employment externality. If positive employment externalities are sufficiently small and the government's preference for employment is sufficiently weak, then each government chooses a unit tax as the tax instrument. For a small degree of positive employment externality and preference for employment, the economy with unemployment is closed to that under a perfect labor market or without preference for employment. Hence, the result is consistent with that of Lockwood (2004) and Akai et al. (2011). However, suppose positive employment externalities are sufficiently large, and the government's preference for employment is sufficiently strong. In such a case, ad valorem taxes are chosen as their tax instruments, similar to that of Aiura and Ogawa (2013) and Hoffmann and Runkel (2016). Furthermore, surprisingly, if positive employment externalities and a taste for employment are intermediate levels between the former two cases, one government chooses a unit tax, and the other opts for an ad valorem tax.

Using ad valorem tax induces governments to reduce their tax rate because the factor price of capital (i.e., the marginal product of capital) decreases with capital inflow. Thus, the government facing capital price down must lower the tax rate to keep the tax revenue (Lockwood, 2004).[5] Without unemployment and the government's preference for employment, this side effect causes severe tax-cutting competition. Therefore, both governments choose unit taxes as their tax instruments to avoid their loss in such a difficult situation (Akai et al., 2011). However, if a positive externality exists and the regional government has an interest in creating employment, this unfavorable side effect could be converted into a favorable one that generates employment benefits. Therefore, all combinations of tax instruments can be strategically chosen based on the degree of employment externality and the government's preference for employment. The

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[4] Several recent studies examined the nature of tax competition equilibrium with unemployment assuming fixed wage (e.g., Gillet and Pauser, 2018; Kikuchi and Tamai, 2019; Tamai and Myles, 2022).

[5] Focusing on a tariff war, Lockwood and Wong (2000) showed that the country switching from specific tariff to ad valorem tax has an incentive to lower its tariff.

robustness of these results is verified through extensive analyses.

The remainder of this paper is organized as follows. Section 2 presents the basic framework of our theoretical analyses. Section 3 characterizes the tax competition equilibrium under the unit tax and ad valorem tax regimes. Section 4 presents extensions of our basic model by considering negative tax, negative employment externalities, and asymmetricity of regions. Finally, Section 5 concludes the study.

2. The model

Consider two-region economy where each region has the population measured as N_i $(i = 1,2)$. In region i, a continuum of identical firms competitively produces a homogenous good using capital, labor, and land inputs (K_i, L_i, and Z_i, respectively). Let be Y_i as the output in region i. The production function in region i is formulated as follows:

$$Y_i = F^i(K_i, L_i, Z_i).$$

Assuming that F^i is a homogenous of degree 1, the production function can be rewritten as

$$Y_i = F^i\left(\frac{K_i}{Z_i}, \frac{L_i}{Z_i}, 1\right) Z_i = f^i(k_i, l_i) Z_i,$$

where $k_i \equiv K_i/Z_i$ and $l_i \equiv L_i/Z_i$. Capital is freely mobile between two regions, whereas labor and land are stuck to the original regions. Without loss of generality, the land input in each region is normalized to unity (i.e., $Z_i = 1$).

Each jurisdictional government taxes on capital as a unit tax T_i or an ad valorem tax t_i after the choice of the tax instruments. Profit maximization and perfect mobility of capital conditions lead to

$$\begin{cases} r = f_K^i - T_i, \\ r = (1 - t_i) f_K^i, \end{cases} \tag{1}$$

where r is the post-tax return on capital that is common between two regions and f_K^i is the partial derivative with respect to K_i (i.e., $f_K^i \equiv \partial f^i / \partial K_i$). Following Ogawa et al. (2006), we assume the imperfect labor market and the imperfection are symbolized as the fixed wage.[6] Let us denote the fixed wage rate in region i by $\overline{w}_i$. The labor demand satisfies the following:

$$\overline{w}_i = f_L^i. \tag{2}$$

The capital market equilibrium condition is

$$K = K_1 + K_2. \tag{3}$$

In the labor market with a sufficiently high level of $\overline{w}_i$, we have $L_i < N_i$.

[6] Several assumptions about labor market imperfection have been made in previous studies. For instance, the efficiency wage model presented by Yellen (1984) is the simplest way to justify our model.

Each jurisdictional government's budget equation becomes

$$\begin{cases} G_i = T_i K_i, \\ G_i = t_i f_K^i K_i, \end{cases} \tag{4}$$

where G_i stands for region i government's tax revenue (or government spending which does not affect residents' utility and firms' productivity).

The previous literature used the specified production function for the equilibrium analysis (e.g., Wildasin, 1991; Brueckner, 2004; Akai et al., 2011). In particular, we use the following production function:

$$Y_i = \left[1 + \frac{\alpha_i K_i + \beta_i L_i}{Z_i} - \frac{A_i K_i^2 + B_i L_i^2}{2Z_i^2} + \frac{\gamma_i K_i L_i}{Z_i^2}\right] \phi_i Z_i, \tag{5}$$

which leads to

$$f_K^i = \left[\frac{\alpha_i}{Z_i} - \frac{A_i K_i}{Z_i^2} + \frac{\gamma_i L_i}{Z_i^2}\right] \phi_i = [\alpha_i - A_i K_i + \gamma_i L_i] \phi_i,$$

$$f_L^i = \left[\frac{\beta_i}{Z_i} - \frac{B_i L_i}{Z_i^2} + \frac{\gamma_i K_i}{Z_i^2}\right] \phi_i = [\beta_i - B_i L_i + \gamma_i K_i] \phi_i,$$

$$f_Z^i = \left[1 + \frac{A_i K_i^2 + B_i L_i^2}{2Z_i^2} - \frac{\gamma_i K_i L_i}{Z_i^2}\right] \phi_i = \left[1 + \frac{A_i K_i^2 + B_i L_i^2}{2} - \gamma_i K_i L_i\right] \phi_i.$$

Using Eqs. (2) and (5) yields the following:

$$L_i = \left[\beta_i + \gamma_i K_i - \frac{\overline{w}_i}{\phi_i}\right] \frac{1}{B_i} = \frac{\beta_i + \gamma_i K_i - \omega_i}{B_i} = c_i + \mu_i K_i, \tag{6}$$

where

$$\omega_i \equiv \frac{\overline{w}_i}{\phi_i}, c_i \equiv \frac{\beta_i - \omega_i}{B_i}, \text{ and } \mu_i \equiv \frac{\gamma_i}{B_i}.$$

Eq. (6) implies that the employment level increases with capital. We assume that c_i is sufficiently small to ensure $L_i < 1$. Eqs. (5) and (6) provide

$$f_K^i = \left[\alpha_i - \left(A_i - \frac{\gamma_i^2}{B_i}\right) K_i + \left(\frac{\beta_i - \omega_i}{B_i}\right) \gamma_i\right] \phi_i = a_i - b_i K_i, \tag{7}$$

where

$$a_i \equiv \left[\alpha_i + \left(\frac{\beta_i - \omega_i}{B_i}\right) \gamma_i\right] \phi_i \text{ and } b_i \equiv \left(A_i - \frac{\gamma_i^2}{B_i}\right) \phi_i.$$

Eq. (7) has a functional form that is similar to that of Akai et al. (2011). Particularly, it is identical to $\gamma_i = 0$. With $\gamma_i \neq 0$, the intercept a_i and the coefficient b_i depend on γ_i. The employment externality will quantitatively affect the payoff of policy choice and, therefore, the equilibrium strategy. We focus on the case where positive employment externality $\gamma_i > 0$ until Section 4.

3. Competition in tax instruments and tax levels

In this section, we study a tax competition game with the choice of a tax instrument. The game is noncooperative and has two stages. In the first stage of the game (stage 1), two jurisdictional governments simultaneously choose their tax instruments out of a unit or an ad valorem tax. In the second stage (stage 2), the governments determine their tax levels to maximize the values of their objective functions. Throughout this section, we assume that the two regions are symmetric. Then, we will make the following assumption to ensure $Y_i > 0$:

Assumption 1 $a > bK$.

The same assumption is made in the previous literature. Assumption 1 ensures that the marginal product of capital cannot be negative if the entire stock of capital is used in a region.

3.1. Determination of tax levels

We now consider the determination of tax levels at stage 2 when (i) both regional governments choose a unit tax (*UU*), (ii) both governments choose an ad valorem tax (*AA*), or (iii) one jurisdictional government chooses a unit tax, whereas the other chooses an ad valorem tax (*UA* or AU). U and A denote the choice of unit tax and that of the ad valorem tax, respectively. The governments consider the following effects of their tax policy (see Appendix A for the derivation of the following equations):

$$UU\text{: } \frac{\partial K_i}{\partial T_i} = -\frac{1}{2b} < 0,$$

$$AA\text{: } \frac{\partial K_i}{\partial t_i} = -\frac{a - bK_i}{(2 - t_i - t_j)b} < 0,$$

$$AU \text{ or } UA\text{: } \frac{\partial K_i}{\partial T_i} = -\frac{1}{(2 - t_j)b} < 0 \text{ and } \frac{\partial K_i}{\partial t_i} = -\frac{a - bK_i}{(2 - t_i)b} < 0,$$

where $a_1 = a_2 = a$ and $b_1 = b_2 = b$ $(i = 1,2$ and $i \neq j)$.

Previous studies supposed that governments maximize their tax revenue like Leviathan. In addition to the Leviathan objective, we consider that the governments care about their regional employment levels. The region i government's objective function is formulated as follows:[7]

[7] The absentee owenership of capital and land justifies this functional form. The Benthamite welfare function should be formulated as $W_i = r\sigma K + \bar{w}_i L_i + \pi_i + v(G_i) = F^i(K_i, L_i, Z_i) - (r + T_i)K_i + r\sigma K + v(G_i)$. If $v(G_i) = (1 + \eta)G_i$, we have $W_i = f^i(K_i, L_i) + (\sigma K - K_i)r + \eta G_i$. Using Eq. (8) is a more simple way to illustrate the effects of tax competition on equilibrium outcome (the same as capital, employment, and government revenue).

$$V_i = \theta_i L_i + (1 - \theta_i) G_i, \tag{8}$$

where the weight for the regional employment level is $0 \leq \theta_i \leq 1$. If $\theta_i = 0$, the government's objective function is identical to that of Akai et al. (2011). Hence, the governments behave like a pure Leviathan. By the assumption of symmetric regions, we focus on $\theta_i = \theta > 0$ $(i = 1,2)$.

Regarding the equilibrium tax rates at stage 2, we obtain the following Lemma 1 (see Appendix B for the proof of Lemma 1):

Lemma 1 *(i) If the governments choose a unit tax,*

$$T_1 = T_2 = bK - \sigma, \textit{ where } \sigma \equiv \frac{\theta}{1-\theta}\mu.$$

(ii) If the governments choose an ad valorem tax,

$$t_1 = t_2 = \frac{bK - \sigma}{a}.$$

(iii) If region i government chooses a unit tax and the other opts for an ad valorem tax,

$$T_i = \frac{bK - \sigma}{2} + \frac{H}{4} \textit{ and } t_j = \frac{H}{2(a - bK)}, i, j = 1,2 \textit{ and } i \neq j,$$

where $H \equiv 6a - bK - \sigma - \sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}$.

The parameter, σ, denotes a degree of employment externality weighted by the government's objective parameter. When $\sigma = 0$, the results from (i) through (iii) in Lemma 1 go back to those derived by Lockwood (2004) and Akai et al. (2011). Considering that μ represents the sign of cross-derivatives of K and L, $\sigma = 0$ implies that no employment externality exists. If $\theta = 0$, the same situation occurs. Without the government's interest in regional employment, the employment externality does not arise. The employment level does not affect the government's objective function in each case. Therefore, this case is identical to the model presented by Akai et al. (2011).

Lemma 1 also indicates the possibility of capital subsidy. In reality, positive lump-sum taxes must be available for financing negative capital taxes. However, negative government revenue can be considered because we assume the linear preference with respect to employment and government revenue/expenditure. Furthermore, capital subsidies aiming to stimulate employment can be observed in the real economy. Based on Lemma 1, the tax rates in cases (i) and (ii) are negative for $\sigma > bK$. This result implies that the governments are motivated to use capital subsidies when positive externalities are sufficiently large, which is shown by the other literature on tax competition and unemployment (e.g., Ogawa et al., 2006; Tamai and Myles, 2021). If the lump-sum tax can be limitedly used for ensuring non-negative tax revenue, the unit or ad valorem tax could be negative. Furthermore, the output must be positive. To ensure the positive output,

$\sigma < a$ is required.[8] Hence, we impose the following assumption:

Assumption 2 $a > \sigma$.

Furthermore, we need the following assumption to ensure positive capital demand.

Assumption 3 $a > 2bK$.

When $\sigma = 0$, $2a > 3bK$ is sufficient to ensure positive capital demand.[9] However, if $\sigma > 0$, higher productivity is needed because of the employment externality.

We now move on to the comparison between the equilibrium tax rates derived in Lemma 1. Hence, the tax rates must be converted to the comparable forms using $t_i f_K^i = T_i$. Following Akai et al. (2011), let t_i^{mn} be the region i's *effective* ad valorem tax rate in the Nash equilibrium when the region i government chooses tax instrument m $(m = U, A)$, and the other chooses tax instrument n $(n = U, A)$. Lemma 1 and $t_i f_K^i = T_i$ $(i = 1,2)$ yield the following equations (see Appendix C for the derivation of the following equations):

$$t_i^{UU} = \frac{2(bK - \sigma)}{2a - bK}, \tag{9a}$$

$$t_i^{AA} = \frac{bK - \sigma}{a}, \tag{9b}$$

$$t_i^{UA} = \frac{[2(bK - \sigma) + H][4(a - bK) - H]}{4a[4(a - bK) - H] - 2[2(bK - \sigma) + H](a - bK)}, \tag{9c}$$

$$t_i^{AU} = \frac{H}{2(a - bK)}. \tag{9d}$$

A comparison among Eqs. (9a)–(9d) makes the following proposition (see Appendix D for the proof of Proposition 1):

Proposition 1 *There exists $\hat{\sigma} \in (0, bK)$, satisfying $t_i^{UA} = t_i^{AA}$. Then, $t_i^{UU} > t_i^{AU} > t_i^{UA} > t_i^{AA}$ holds if $\sigma < \hat{\sigma}$, whereas $t_i^{UU} > t_i^{AU} > t_i^{AA} \geq t_i^{UA}$ if $\hat{\sigma} \leq \sigma < bK$. When $\sigma > bK$, the orders of tax rates become $t_i^{UU} < t_i^{AU} < t_i^{UA} < t_i^{AA}$.*

The degree of employment externality affects the magnitude relation of the tax rates under different scenarios. If the employment externality effect is not too strong, the lowest tax rate within four scenarios is the tax rate when both governments choose an ad valorem tax. This

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[8] $bK > \sigma$ requires that the employment externality is sufficiently small to ensure positive values of G_i. This condition and Assumption 1 lead to $a > \sigma$.

[9] See Appendix C. For instance, Eq. (C1) is positive if Assumption 3 holds.

corresponds to the case shown by Akai et al. (2011). However, if the employment externality effect is sufficiently large, the smallest tax rate changes from the tax rate when both governments choose an ad valorem tax to that when the region i's government chooses a unit tax and the other chooses an ad valorem tax. When the governments have preferences for employment, the capital-attracting effects of cuts of unit and ad valorem taxes are more evaluated by the governments than pure Leviathans. In particular, if the employment externality effect is sufficiently large or the governments have a strong preference for employment, the capital-attracting effect of decreasing ad valorem tax relative to that of unit tax is strengthened. Therefore, the government sets the lowest rate of ad valorem tax for $\hat{\sigma} < \sigma$, facing the other unit tax as its tax instrument. If $\sigma > bK$, the tax rates are all negative. Hence, the magnitude relationship must be inversed in the case of $\sigma < bK$.

The region *i* government's payoff is given as follows (see Appendix E for the proof of Lemma 2):

Lemma 2. *Let be* $\lambda \equiv (1-\theta)^{-1}\theta c$. *(i) If the governments choose a unit tax,*

$$V_i^{UU} = (1-\theta)\left(\lambda + \frac{bK^2}{2}\right).$$

(ii) If the governments choose an ad valorem tax,

$$V_i^{AA} = (1-\theta)\left[\lambda + \frac{(2a - bK + \sigma)bK^2}{4a}\right].$$

(iii) If region i government chooses a unit tax and the other opts for an ad valorem tax,

$$V_i^{UA} = (1-\theta)\left\{\lambda + \left(\frac{bK+\sigma}{2} + \frac{H}{4}\right)\frac{[2(bK-\sigma)+H](a-bK)}{2b[4(a-bK)-H]}\right\},$$

$$V_i^{AU} = (1-\theta)\left\{\lambda + \left[\sigma + \frac{(a-b\Gamma)H}{2(a-bK)}\right]\Gamma\right\}, \text{ where } \Gamma \equiv \frac{(a-bK)(6bK-H+2\sigma)-2bKH}{2b[4(a-bK)-H]}.$$

Based on the result of Lemma 1, the employment externality changes the magnitude relation of the payoff chosen by the governments, depending on its size. Table 1 represents the payoffs. Regarding this point, Lemma 2 derives the following result (see Appendix F for the proof of Proposition 2):

Proposition 2. *There exist* $\bar{\sigma}$ *and* $\tilde{\sigma}$, *satisfying* $V_i^{UA} \gtreqless V_i^{AA} \Leftrightarrow \sigma \lesseqgtr \bar{\sigma}$ and $V_i^{UU} \gtreqless V_i^{AU} \Leftrightarrow \sigma \lesseqgtr \tilde{\sigma}$, *respectively. (i) If* $0 \le \sigma < \min(\bar{\sigma}, \tilde{\sigma})$, *each government chooses a unit tax in the Nash equilibrium. (ii) If* $\min(\bar{\sigma}, \tilde{\sigma}) < \sigma < \max(\bar{\sigma}, \tilde{\sigma})$, *two possible cases exist:* $\bar{\sigma} < \sigma < \tilde{\sigma}$ *and* $\tilde{\sigma} < \sigma < \bar{\sigma}$. *When,* $\bar{\sigma} < \sigma < \tilde{\sigma}$, *two Nash equilibria exist such that one government selects the same tax instrument as the other. When* $\tilde{\sigma} < \sigma < \bar{\sigma}$, *two Nash equilibria exist such that one government* *chooses a unit tax and the other opts for an ad valorem tax. (iii) If* $\sigma > \max(\bar{\sigma}, \tilde{\sigma})$*, each government chooses an ad valorem tax in the Nash equilibrium.*

The intuition of Proposition 2 can be explained as follows. First, we focus on the result (i) where $0 \leq \sigma < \min(\bar{\sigma}, \tilde{\sigma})$. In other words, the employment externality is sufficiently small, or the government's preference is weak enough to hold Akai et al.'s (2011) scenario. Taking $\sigma = 0$ as an example, the intuition for the result (i) is along with that of Lockwood (2004) and Akai et al. (2011). Suppose that a small increase in public goods expenditure occurs in region 1. In this case, financed by increased unit tax, it will cause a capital outflow given by Δ from region 1. If region 2 adopts a unit tax, Δ unit of capital is employed in region 2, and thus, region 2's government revenue is increased by $T_2\Delta$. However, if region 2 uses ad valorem tax, it brings about a side effect that region 2 wishes to lower the tax rate because region 2's tax revenue is given by $t_2 f_k^2 \Delta$ and f_k^2 decreases with Δ. Therefore, in the first stage, both governments are willing to choose a unit tax rather than an ad valorem tax.

In the case of the result in (iii) where $\sigma > \max(\bar{\sigma}, \tilde{\sigma})$, the mechanism is contrary to the result in (i). As same as the case in (i), suppose that region 1's government increases a unit tax to finance a small increase in public goods expenditure. If region 2 uses an ad valorem tax, it means that region 2's government has an incentive to reduce the tax rate because of the side effect mentioned above. A decrease in the tax rate to attract more capital leads to a more increase in employment. With large σ, creating employment will cause large benefits relative to costs facing large elasticity of capital to the tax rate. This can be applied interactively to both regions. Hence, $V_i^{UU} < V_i^{AU}$ and $V_i^{UA} < V_i^{AA}$ hold; one region has no incentive to choose a unit tax if the other uses an ad valorem tax. Therefore, both governments will select an ad valorem tax as the tax instrument in the first stage.

We now turn to the result (ii) where $\min(\bar{\sigma}, \tilde{\sigma}) < \sigma < \max(\bar{\sigma}, \tilde{\sigma})$. This case is an intermediate between (i) and (iii), and the payoffs exhibit complicated magnitude relation for different values of σ. Thus, the regions may strategically choose different tax instruments depending on σ, even if the regions are symmetric.

We consider the case where $\bar{\sigma} < \sigma < \tilde{\sigma}$. Suppose that region 2 chooses unit tax. This choice is the signal to region 1 that region 2 has no intention to bring intense tax competition. If region 1 chooses an ad valorem tax as the tax instrument in the first stage for creating more employment. Nevertheless, it may cause fierce tax-cutting competition. Region 1 does not obtain benefits enough to cover the costs of intense tax competition because σ is not sufficiently large (i.e., $V_1^{UU} > V_1^{AU}$ for $\sigma < \tilde{\sigma}$). Therefore, the best strategy for region 1 is to choose a unit tax the same as region 2. The same logic can be applied to region 2. Next, suppose that region 2 uses an ad valorem tax for $\bar{\sigma} < \sigma < \tilde{\sigma}$. This choice would be the signal that region 2 intends to cause an intense tax competition for region 1. If region 1's government chooses unit tax to avoid intense tax competition, region 1 faces capital outflow to region 2. As a result, region 1 does not add benefits from such a choice compared when choosing a unit tax ($V_1^{UA} < V_1^{AA}$ for $\sigma > \bar{\sigma}$). Consequently, both regions choose an ad valorem tax in the first stage.

Finally, the case where $\tilde{\sigma} < \sigma < \bar{\sigma}$ can be explained as follows. Suppose that region 2 selects unit tax. If region 1, in turn, chooses an ad valorem tax, lowering the tax rate is required to keep the tax revenue, and then, the government expects to obtain additional employment benefits. By contrast, when region 1 uses a unit tax instead of an ad valorem tax, there is no opportunity for extra benefits from creating employment. Therefore, region 1 chooses an ad valorem tax in response to region 2's choice of unit tax ($V_1^{UU} < V_1^{AU}$ for $\sigma > \tilde{\sigma}$). We now consider the case that region 2 chooses an ad valorem tax. If region 1 uses an ad valorem tax, region 1 must countervail tax-cutting by region 2 to prevent additional capital flight. On the contrary, by using a unit tax, region 1 has no necessity to cope with such an unfavorable effect. Hence, region 1 selects a unit tax when region 2 uses an ad valorem tax ($V_1^{UA} > V_1^{AA}$ for $\sigma < \bar{\sigma}$). These results show that the best strategy for one region is choosing the same tax instrument in response to that chosen by the other region.

To close the analysis in this section, we provide numerical examples of critical values. Given that many parameters interact with each other and tremendous numbers of their combinations, focusing on all the cases is difficult. Hence, we fix the values of a, b, and K as $a = 3$, $b = 1$, and $K = 1$, respectively. Figure 1 illustrates the contour plot related to $\bar{\sigma}$ with respect to μ and θ, whereas Figure 2 shows the contour plot related to $\tilde{\sigma}$ with respect to μ and θ. Given that the level line of 0 is isoquant of the critical value, the area below (above) the 0-level line means $\sigma < \bar{\sigma}$ ($\sigma > \bar{\sigma}$) in Figure 1 or $\sigma < \tilde{\sigma}$ ($\sigma > \tilde{\sigma}$) in Figure 2. Both figures imply that higher values of θ and μ tend to generate the result (ii) or (iii) in Proposition 2.

4. Further analyses

In the previous sections, we assume that positive employment externalities exist and two regions are symmetric. However, in the real world, negative employment externalities may appear, and there are asymmetries in regions. Hence, we discuss the possibility of capital subsidy (negative tax rate on capital) and examine the effects of negative employment externalities and the equilibrium properties in the economy with asymmetric regions.

Negative employment externalities. A degree of employment externality can be measured using parameter μ. A negative employment externality implies that μ is negative. Then, $\sigma < 0$ holds by its definition. In reality, the value of μ is plausible to be negative, leading to a negative cross derivative with respect to capital and labor. A certain kind of improvement in production may damage workers, such as the unskilled, in the short term.[10] The task model developed by Autor et al. (2003) precisely elucidates such a situation. The production function used in this study is related to Autor et al.'s (2003) production technology. Therefore, $\sigma < 0$ should be included. If σ is sufficiently small in the absolute value, the qualitative effects are the same as those shown in the previous sections.

Asymmetric regions. We consider the equilibrium tax instruments chosen by the governments in asymmetric regions. For the reason explained in Section 3, we set a_i, b_i, c_i, θ_i, and K as $a_i = 3$, $b_i = 1$, $c_i = 1$, $\theta_i = 0.5$, and $K = 1$, respectively. Three cases are considered hereafter.

First, we consider positive employment externalities in both regions. Focusing on two subcases, Table 2 shows the payoffs in cases of $(\mu_1, \mu_2) = (1.5, 0.5)$ and $(\mu_1, \mu_2) = (2, 1.5)$. Based on Table 2, regions 1 and 2, respectively, use an ad valorem tax and a unit tax if $(\mu_1, \mu_2) = (1.5, 0.5)$, whereas both governments choose an ad valorem tax if $(\mu_1, \mu_2) = (2, 1.5)$. These results are examples of the results (ii) and (iii) in Proposition 2 for asymmetric regions.

We now move on to the case of positive employment externality in one region and negative employment externality in the other. The payoffs are calculated as shown in Table 3 for two subcases where $(\mu_1, \mu_2) = (1.5, -0.5)$ and $(\mu_1, \mu_2) = (0.5, -1.5)$. In this case, the governments select unit taxes as their tax instruments. This finding corresponds to the result in (i) of Proposition 2.

Finally, considering negative employment externalities in both regions, the payoffs are given in Table 4. For $(\mu_1, \mu_2) = (-0.5, -1.5)$ and $(\mu_1, \mu_2) = (-1.5, -2)$, the governments choose unit taxes. These numerical examples imply that all the cases of Proposition 2 are realized depending on the values of key parameters.

5. Conclusion

This paper examined the government's choice of tax instruments under unemployment. The government is assumed to seek larger employment and tax revenue. If positive employment externalities are sufficiently small, the governments choose unit taxes as their tax instruments. By contrast, if positive employment externalities are sufficiently large, ad valorem taxes are chosen as their tax instruments. Furthermore, suppose positive employment externalities are intermediate between the former two cases. In that case, one government chooses a unit tax, and the other opts for an ad valorem tax, or one government selects the same tax instrument that is chosen by the

[10] Frey and Osborne (2017) estimated that approximately 47% of the total US employment is in the high-risk category that jobs could be automated relatively soon.

other. Therefore, this study demonstrates that all combinations are enabled based on the degree of employment externality and the government's preference for seeking employment.

The mechanisms of our main findings are explicated along with the existing literature. Lockwood (2004) showed that an ad valorem tax gives governments an incentive to lower their tax rate because the rate of return on capital, that is, the marginal product of capital, decreases with capital inflow. Thus, the government facing a decrease in the rate of return must reduce the tax rate to keep the tax revenue. With positive employment externalities, this unfavorable side effect could be turned into a favorable one that generates employment benefits if the regional government has a strong interest in creating employment. Hence, if the government's preference for employment is sufficiently weak, both governments choose unit taxes, as shown by Lockwood (2004) and Akai et al. (2011). However, the governments with sufficiently strong preferences for employment use ad valorem taxes, similar to Aiura and Ogawa (2013) and Hoffmann and Runkel (2016), who allowed asymmetricity of regions. Most surprisingly, multiple equilibria can occur in intermediate cases. The robustness of our results is ensured by extensive analyses.

Finally, we would like to mention the future direction of this research. One possible extension is to investigate the relationship between leadership and competition in tax instruments and tax levels, as presented by Kempf and Rota-Graziosi (2010) and Ogawa (2013). This paper focused on choices of tax instruments in a noncooperative game with governments taking the Nash behavior. However, endogenizing leadership changes equilibrium choices of tax instruments by affecting the payoffs. Another plausible extension is to examine endogenizing the government's objectives as developed by Pal and Sharma (2013) and Kawachi et al. (2019). In this study, we considered that the governments care about regional employment and their budgets. Based on our findings, employment externalities are expected to affect the payoff structure in strategic delegation games. Hence, focusing on endogenizing the government's objectives, the extended analysis would elucidate why some governments aim to seek employment, whereas others act as Leviathans. This paper provides an analytical basis for these future extensions.

Appendix

A. Effects of tax policy on capital

Unit tax. Eqs. (1), (3), and (7) yield the following system:

$$a_1 - b_1 K_1 - T_1 = a_2 - b_2 K_2 - T_2,$$
$$K_1 + K_2 = K.$$

Solving the above system with respect to K_i and applying $a_1 = a_2 = a$ and $b_1 = b_2 = b$ to the solution, we obtain

$$K_i = \frac{bK - T_i + T_j}{2b}. \tag{A1}$$

The first-order and second-order partial derivatives with respect to T_1 are

$$\frac{\partial K_i}{\partial T_i} = -\frac{1}{2b} < 0 \text{ and } \frac{\partial^2 K_i}{\partial T_i^2} = 0. \tag{A2}$$

Ad valorem tax. Using a similar way to derive the effect of a change in unit tax on capital, Eqs. (1), (3), and (7) give

$$K_i = \frac{(1 - t_j)bK + (t_j - t_i)a}{(2 - t_i - t_j)b}. \tag{A3}$$

The first-order and second-order partial derivatives with respect to t_1 are derived as

$$\frac{\partial K_i}{\partial t_i} = \frac{-(2 - t_i - t_j)a + [(1 - t_j)bK + (t_j - t_i)a]}{(2 - t_i - t_j)^2 b} = -\frac{a - bK_i}{(2 - t_i - t_j)b} < 0, \tag{A4}$$

$$\frac{\partial^2 K_i}{\partial t_i^2} = \frac{b(2 - t_i - t_j)\frac{\partial K_i}{\partial t_i} - (a - bK_i)}{(2 - t_i - t_j)^2 b} = -\frac{2(a - bK_i)}{(2 - t_i - t_j)^2 b} = \frac{2}{2 - t_i - t_j}\frac{\partial K_i}{\partial t_i} < 0.$$

Unit vs. Ad Valorem. Without loss of generality, we suppose that region 1 chooses a unit tax and region 2 opts for an ad valorem tax. Eqs. (1), (3), and (7) yield $a_1 - b_1 K_1 - T_1 = (1 - t_2)(a_2 - b_2 K_2)$. Solving this with respect to K_i, we obtain

$$K_1 = \frac{(1 - t_2)bK + at_2 - T_1}{(2 - t_2)b} \text{ and } K_2 = \frac{bK - at_2 + T_1}{(2 - t_2)b}. \tag{A5}$$

Hence, the partial derivatives are

$$\frac{\partial K_1}{\partial T_1} = -\frac{1}{(2 - t_2)b} \; and \; \frac{\partial^2 K_1}{\partial T_1^2} = 0, \tag{A6}$$

$$\frac{\partial K_2}{\partial t_2} = -\frac{a - bK_2}{(2 - t_2)b} < 0 \; and \; \frac{\partial^2 K_2}{\partial T_2^2} = -\frac{a - bK_2}{(2 - t_2)^2 b} = \frac{1}{2 - t_2}\frac{\partial K_2}{\partial t_2} < 0. \tag{A7}$$

B. Proof of Lemma 1

Unit vs. Unit. In the second stage, the unit tax level must satisfy

$$\frac{\partial V_i}{\partial T_i} = \theta \frac{\partial L_i}{\partial T_i} + (1-\theta)\frac{\partial G_i}{\partial T_i} = \theta\mu \frac{\partial K_i}{\partial T_i} + (1-\theta)\left(K_i + T_i \frac{\partial K_i}{\partial T_i}\right) = 0, \tag{B1}$$

$$\frac{\partial^2 V_i}{\partial T_i^2} = \theta\mu \frac{\partial^2 K_i}{\partial T_i^2} + (1-\theta)\left(2\frac{\partial K_i}{\partial T_i} + T_i \frac{\partial^2 V_i}{\partial T_i^2}\right) = 2(1-\theta)\frac{\partial K_i}{\partial T_i} < 0.$$

Solving Eq. (B1) yields

$$K_i = -T_i \frac{\partial K_i}{\partial T_i} - \sigma \frac{\partial K_i}{\partial T_i}. \tag{B2}$$

Inserting Eqs. (A1) and (A2) into Eq. (B2) and applying $T_1 = T_2$, we obtain

$$T_1 = T_2 = bK - \sigma \text{ and } K_1 = K_2 = \frac{K}{2}. \tag{B3}$$

Ad Valorem vs. Ad Valorem. The optimality conditions in the second stage are

$$\frac{\partial V_i}{\partial t_i} = \theta \frac{\partial L_i}{\partial t_i} + (1-\theta)\frac{\partial G_i}{\partial t_i} = \theta\mu \frac{\partial K_i}{\partial t_i} + (1-\theta)\left[(a - bK_i)K_i + (a - 2bK_i)t_i \frac{\partial K_i}{\partial t_i}\right] = 0, \tag{B4}$$

$$\frac{\partial^2 V_i}{\partial t_i^2} = \left[2(a - 2bK_i) + \frac{2(a - bK_i)t_i}{2 - t_i - t_j} + (a - 2bK_i)t_i \frac{2}{2 - t_i - t_j}\right]\frac{\partial K_i}{\partial t_i} < 0.$$

Eq. (B4) leads to

$$(a - bK_i)K_i = -(a - 2bK_i)t_i \frac{dK_i}{dt_i} - \sigma \frac{dK_i}{dt_i}. \tag{B5}$$

With the symmetric region condition, Eqs. (A3), (A4), and (B5) provide

$$t_1 = t_2 = \frac{bK - \sigma}{a} \text{ and } K_1 = K_2 = \frac{K}{2}. \tag{B6}$$

Unit vs. Ad Valorem. The first-order conditions are given by Eqs. (B1) and (B4). The second-order conditions are

$$\frac{\partial^2 V_1}{\partial T_1^2} = \theta\mu \frac{\partial^2 K_1}{\partial T_1^2} + (1-\theta)\left(2\frac{\partial K_1}{\partial T_1} + T_i \frac{\partial^2 K_1}{\partial T_1^2}\right) = 2(1-\theta)\frac{\partial K_1}{\partial T_1} < 0,$$

$$\frac{\partial^2 V_2}{\partial t_2^2} = \theta\mu \frac{\partial^2 K_2}{\partial T_2^2} + (1-\theta)\left[2(a - 2bK_2)\frac{\partial K_2}{\partial t_2} - 2bt_2\left(\frac{\partial K_2}{\partial t_2}\right)^2 + (a - 2bK_2)t_2 \frac{\partial^2 K_2}{\partial t_2^2}\right] < 0,$$

where

$$\sigma \frac{\partial^2 K_2}{\partial T_2^2} + 2(a - 2bK_2)\frac{\partial K_2}{\partial t_2} - 2bt_2\left(\frac{\partial K_2}{\partial t_2}\right)^2 + (a - 2bK_2)t_2 \frac{\partial^2 K_2}{\partial t_2^2}$$

$$= \frac{[\sigma + (4 + t_2)a - 8bK_2]}{2 - t_2}\frac{\partial K_2}{\partial t_2}.$$

Eqs. (A5), (A6), (A7), (B1), and (B4) provide

$$K_1 = -T_1 \frac{\partial K_1}{\partial T_1} - \sigma \frac{\partial K_1}{\partial T_1} = \frac{T_1 + \sigma}{(2 - t_2)b}, \tag{B7}$$

$$(a - bK_2)K_2 = -(a - 2bK_2)t_2 \frac{\partial K_2}{\partial t_2} - \sigma \frac{\partial K_2}{\partial t_2} \Leftrightarrow K_2 = \frac{(a - 2bK_2)t_2 + \sigma}{(2 - t_2)b}. \tag{B8}$$

Solving Eqs. (B7) and (B8) with respect to T_1 and t_2 gives

$$T_1 = \frac{bK - \sigma}{2} + \frac{H}{4} \text{ and } t_2 = \frac{H}{2(a - bK)}, \tag{B9}$$

where

$$H \equiv 6a - bK - \sigma - \sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}.$$

Using Eqs. (B7), (B8), and (B9), we arrive at

$$K_1 = \frac{[2(bK - \sigma) + H](a - bK)}{2b[4(a - bK) - H]} \text{ and } K_2 = \frac{(a - bK)(6bK - H + 2\sigma) - 2bKH}{2b[4(a - bK) - H]}. \tag{B10}$$

C. Derivation of equations from (9a) to (9d)

Unit vs. Unit. Using Eqs. (1), (7), and (B3), we obtain

$$r = \frac{2a - 3bK + 2\sigma}{2}. \tag{C1}$$

By the definition of t_i^{mn}, Eq. (C1) leads to

$$t_i^{UU} = \frac{bK - \sigma}{a - \frac{bK}{2}} = \frac{2(bK - \sigma)}{2a - bK}.$$

Ad Valorem vs. Ad Valorem. Eqs. (1), (7), and (B6) provide

$$r = \frac{(2a - bK)(a - bK + \sigma)}{2a}. \tag{C2}$$

In a similar way to deriving Eq. (9a), we arrive at

$$t_i^{AA} = \frac{bK - \sigma}{a}.$$

Unit vs. Ad Valorem. Eqs. (1), (7), (B9), and (B10) engender

$$r = (1 - t_2)(a - bK_2) = \frac{[2(a - bK) - H]\{2[4a - (3bK + \sigma)] - H\}}{4[4(a - bK) - H]}. \tag{C3}$$

Using the definition of t_i^{mn}, we obtain

$$t_i^{UA} = \frac{\frac{bK - \sigma}{2} + \frac{H}{4}}{a - bK_i} = \frac{[2(bK - \sigma) + H][4(a - bK) - H]}{4a[4(a - bK) - H] - 2[2(bK - \sigma) + H](a - bK)}, t_i^{AU} = \frac{H}{2(a - bK)}.$$

D. Proof of Proposition 1

t_i^{UU} *vs.* t_i^{AA}: Eqs. (9a) and (9b) lead to

$$t_i^{UU} - t_i^{AA} = \frac{2(bK-\sigma)}{2a-bK} - \frac{bK-\sigma}{a} = \frac{(bK-\sigma)bK}{(2a-bK)a} \gtreqless 0 \Leftrightarrow bK \gtreqless \sigma. \tag{D1}$$

t_i^{AU} *vs.* t_i^{AA}: Using Eqs. (9b) and (9d), we obtain

$$t_i^{AU} - t_i^{AA} = \frac{H}{2(a-bK)} - \frac{bK-\sigma}{a} = \frac{aH - 2(bK-\sigma)(a-bK)}{2a(a-bK)}.$$

The numerator can be decomposed to

$$aH - 2(bK-\sigma)(a-bK) = M_1 - M_2,$$

where

$$M_1 = 6a^2 - 3abK + 2b^2K^2 + a\sigma - 2bK\sigma > 0,$$
$$M_2 = a\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2} > 0.$$

We have

$$M_1^2 - M_2^2 = 4bK(a-bK)(bK-\sigma)(2a-bK+\sigma) \gtreqless 0 \Leftrightarrow bK \gtreqless \sigma.$$

Therefore, we obtain

$$t_i^{AU} \gtreqless t_i^{AA} \Leftrightarrow bK \gtreqless \sigma. \tag{D2}$$

t_i^{UA} *vs.* t_i^{AA}: Eqs. (9b) and (9c) yield

$$t_i^{UA} - t_i^{AA} = \frac{[2(bK-\sigma)+H][4(a-bK)-H]}{4a[4(a-bK)-H] - 2[2(bK-\sigma)+H](a-bK)} - \frac{bK-\sigma}{a}.$$

The above equation can be transformed to

$$a(O_1 - O_2)(t_i^{UA} - t_i^{AA}) = (O_3 - O_4),$$

where

$$O_1 = (3a-bK)\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2} > 0,$$
$$O_2 = 10a^2 + abK - b^2K^2 - 5a\sigma + 3\sigma bK > 0,$$
$$O_3 = [4a^2 - (a-bK)(bK-\sigma)]\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2} > 0,$$
$$O_4 = 24a^3 - 18a^2bK + 13ab^2K^2 + b^3K^3 + \sigma[10a^2 - bK(4bK-3\sigma) - a(10bK+3\sigma)] > 0.$$

Taking the difference of the squared terms of O_i provides

$$O_1^2 - O_2^2 = 8(a-bK)\Psi > 0,$$
$$O_3^2 - O_4^2 = 8(a-bK)(bK-\sigma)\Phi \gtreqless 0 \Leftrightarrow (bK-\sigma)\Phi \gtreqless 0,$$

where

$$\Psi \equiv (2a-bK)[14a(a-bK) + 3b^2K^2] + \sigma[14a(a-bK) + (2bK+\sigma)bK + 2a(6a-\sigma)] > 0,$$

$$\Phi \equiv bK(2a - bK)(4a^2 - 5abK + 3b^2K^2) - [(24a^2 - \sigma^2)(a - bK) + 5b^2K^2(5a - bK) + (10a^2 - 13abK + b^2K^2)\sigma]\sigma.$$

The sign of Φ is crucial to determine the sign of $O_3^2 - O_4^2$. We can easily verify

$$\Phi = bK(2a - bK)(4a^2 - 5abK + 3b^2K^2) > 0 \text{ if } \sigma = 0.$$

If $\sigma = 0$, $O_3^2 - O_4^2 > 0$. Therefore, $t_i^{UA} > t_i^{AA}$ holds if $\sigma = 0$.

We now consider $0 < \sigma < bK$. Then, we have $\text{sgn}(O_3^2 - O_4^2) = \text{sgn}\Phi$. The derivative of Φ with respect to σ gives

$$\frac{d\Phi}{d\sigma} = -\{3(8a^2 - \sigma^2)(a - bK) + b^2K^2[5(5a - bK) + 2\sigma] + 2a\sigma(10a - 13bK)\} < 0.$$

Furthermore, we have $\Phi = -16a^3bK$ if $\sigma = bK$. These properties show that a critical point $\hat{\sigma}$ exists such that $\Phi = 0$ and $\hat{\sigma} \in (0, bK)$ for $\sigma \in [0, bK]$. Therefore, we obtain

$$O_3^2 - O_4^2 > 0 \text{ for } \sigma \in [0, \hat{\sigma})$$

$O_3^2 - O_4^2 \leq 0$ for $\sigma \in [\hat{\sigma}, bK]$. Finally, we arrive at

$$t_i^{UA} \geq t_i^{AA} \text{ for } \sigma < \hat{\sigma} \text{ while } t_i^{UA} \leq t_i^{AA} \text{ for } \sigma \geq \hat{\sigma} \qquad \text{(D3)}$$

t_i^{UA} *vs.* t_i^{AU}: Eqs. (9c) and (9d) derive

$$t_i^{UA} - t_i^{AU} = \frac{[2(bK - \sigma) + H][4(a - bK) - H]}{4a[4(a - bK) - H] - 2[2(bK - \sigma) + H](a - bK)} - \frac{H}{2(a - bK)}.$$

This equation can be rewritten as

$$(a - bK)(O_1 - O_2)\left(t_i^{UA} - t_i^{AU}\right) = 2(N_1 - N_2),$$

where

$$N_1 = (30a^3 - 21a^2bK + 15ab^2K^2 + b^3K^3 - a^2\sigma - 6abK\sigma - 3b^2K^2\sigma + a\sigma^2) > 0,$$

$$N_2 = [5a^2 + b^2K^2 - a(bK + \sigma)]\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2} > 0.$$

As we have $O_1 > O_2$, $\text{sgn}\left(t_i^{UA} - t_i^{AU}\right) = \text{sgn}(N_1 - N_2)$ holds. Calculation shows

$$N_1^2 - N_2^2 = -4(a - bK)^2(bK - \sigma)\{(5a - 6bK)(2a - bK)bK + [30a(a - \sigma) + 2bK(a + bK)]\sigma\}.$$

Therefore, we obtain

$$t_i^{UA} \lesseqgtr t_i^{AU} \Leftrightarrow N_1^2 - N_2^2 \lesseqgtr 0 \Leftrightarrow bK \gtreqless \sigma. \qquad \text{(D4)}$$

t_i^{UU} *vs.* t_i^{AU}: Eqs. (9a) and (9d) lead to

$$t_i^{UU} - t_i^{AU} = \frac{2(bK - \sigma)}{2a - bK} - \frac{H}{2(a - bK)} = \frac{4(bK - \sigma)(a - bK) - (2a - bK)H}{2(2a - bK)(a - bK)}$$

$$= \frac{4(bK - \sigma)(a - bK) - (2a - bK)H}{2(2a - bK)(a - bK)}.$$

Using this equation, we obtain

$$2(2a-bK)(a-bK)\left(t_i^{UU}-t_i^{AU}\right)=Q_1-Q_2,$$

where

$$Q_1=(2a-bK)\sqrt{36a^2-36abK+25b^2K^2+12a\sigma-22bK\sigma+\sigma^2}>0,$$
$$Q_2=12a^2-12abK+5b^2K^2+2a\sigma-3bK\sigma>0.$$

After some calculations, we have

$$Q_1^2-Q_2^2=8bK(a-bK)(2a-\sigma)(bK-\sigma)\gtreqless 0 \Leftrightarrow bK\gtreqless\sigma.$$

Therefore, we arrive at

$$t_i^{UU}\gtreqless t_i^{AU}\Leftrightarrow bK\gtreqless\sigma. \tag{D5}$$

Comparison between tax rates. Eqs. (D1)–(D5) derive the following results for $\sigma<bK$:

$$t_i^{UU}>t_i^{AU}>t_i^{UA}>t_i^{AA}\text{ for }\sigma<\hat{\sigma},$$
$$t_i^{UU}>t_i^{AU}>t_i^{AA}\geq t_i^{UA}\text{ for }\sigma\geq\hat{\sigma}.$$

However, these orders are inverted in the case of $\sigma>bK$ because the tax rates are negative. Furthermore, if $\sigma>bK$, $\sigma>\hat{\sigma}$ holds. Consequently, we obtain

$$t_i^{UU}<t_i^{AU}<t_i^{UA}<t_i^{AA}\text{ for }\sigma>bK.$$

E. Proof of Lemma 2

Unit vs. Unit. Inserting Eqs. (4), (6), (9a), and (B3) into Eq. (8) yields

$$V_i^{UU}=\theta c+(1-\theta)\left[\frac{\sigma K}{2}+\frac{(bK-\sigma)K}{2}\right]=(1-\theta)\left(\lambda+\frac{bK^2}{2}\right).$$

Ad Valorem vs. Ad Valorem. Using (4), (6), (8), (9b), and (B6), we obtain

$$\begin{aligned}V_i^{AA}&=\theta c+(1-\theta)\left[\frac{\sigma K}{2}+\left(a-\frac{bK}{2}\right)\left(\frac{bK-\sigma}{a}\right)\frac{K}{2}\right]\\&=(1-\theta)\left[\lambda+\frac{(2a-bK)(bK-\sigma)K+2a\sigma K}{4a}\right]\\&=(1-\theta)\left[\lambda+\frac{(2a-bK+\sigma)bK^2}{4a}\right].\end{aligned}$$

Unit vs. Ad Valorem. Eqs. (4), (6), (8), (9c), and (B10) lead to

$$\begin{aligned}V_i^{UA}&=\theta c+(1-\theta)(\sigma K_i+T_iK_i)\\&=(1-\theta)\left\{\lambda+\left(\frac{bK+\sigma}{2}+\frac{H}{4}\right)\frac{[2(bK-\sigma)+H](a-bK)}{2b[4(a-bK)-H]}\right\}.\end{aligned}$$

Similarly, Eqs. (4), (6), (8), (9d), and (B10) provide

$$V_i^{AU}=\theta c+(1-\theta)[\sigma K_i+t_i(a-bK_i)K_i]=(1-\theta)\left\{\lambda+\left[\sigma+\frac{(a-b\Gamma)H}{2(a-bK)}\right]\Gamma\right\}.$$

F. Proof of Proposition 2

V_i^{UA} *vs.* V_i^{AA}: Lemma 2 gives

$$\mathrm{sgn}\left(V_i^{UA}-V_i^{AA}\right)=\mathrm{sgn}\left\{\left(\frac{bK+\sigma}{2}+\frac{H}{4}\right)\frac{[2(bK-\sigma)+H](a-bK)}{2b[4(a-bK)-H]}-\frac{(2a-bK+\sigma)bK^2}{4a}\right\}.$$

The terms if the curly brackets on the right-hand side of the above equation become

$$\left(\frac{bK+\sigma}{2}+\frac{H}{4}\right)\frac{[2(bK-\sigma)+H](a-bK)}{2b[4(a-bK)-H]}-\frac{(2a-bK+\sigma)bK^2}{4a}$$
$$=\frac{[2(bK-\sigma)+H][2(bK+\sigma)+H](a-bK)}{8b[4(a-bK)-H]}-\frac{(2a-bK+\sigma)bK^2}{4a}$$
$$=\frac{J_1-J_2}{4ab[4(a-bK)-H]},$$

where

$$J_1=24a^3(a-2bK)+a^2(6a^2-\sigma^2)+9ab^2K^2(a-bK)+(20a^2-3b^2K^2)b^2K^2$$
$$+6a^2(a^2-2bK\sigma)+12ab^2K^2\sigma+4b^3K^3\sigma+bK\sigma^2(a-bK)>0,$$
$$J_2=[(a-bK)(5a^2+b^2K^2)+a^2(a-\sigma)+bK\sigma(a+bK)]$$
$$\times\sqrt{36a^2-36abK+25b^2K^2+12a\sigma-22bK\sigma+\sigma^2}>0.$$

Hence, we have

$$\mathrm{sgn}\left(V_i^{UA}-V_i^{AA}\right)=\mathrm{sgn}(J_1-J_2), \tag{F1}$$

Using J_1 and J_2 makes

$$J_1^2-J_2^2=4bK(a-bK)(2a-bK+\sigma)\Omega,$$

where

$$\Omega\equiv 24a^3b^2K^2-42a^2b^3K^3+23ab^4K^4-4b^5K^5$$
$$-[72a^4-60a^3bK-12a^2b^2K^2+17ab^3K^3-8b^4K^4-12a^3\sigma+10a^2bK\sigma$$
$$+7ab^2K^2\sigma+4b^3K^3\sigma-abK\sigma^2]\sigma.$$

Regarding Ω, we have

$$\Omega=b^2K^2(2a-bK)(12a^2-15abK+4b^2K^2)>0 \text{ for } \sigma=0,$$
$$\Omega=-32a^2b^2K^2(a-bK)(2a-bK+\sigma)(9a^2-12abK+5b^2K^2)<0 \text{ for } \sigma=bK.$$

Furthermore, Ω is monotonically decreasing in σ because of

$$\frac{d\Omega}{d\sigma}=12a^2[3a(a-2bK)+(a-bK)bK+a(3a-2\sigma)]+[17a-8(1-\sigma)]b^3K^3$$
$$+(20a+14bK-3\sigma)abK\sigma<0.$$

Hence, there exists $\bar{\sigma}\in(0,bK)$, which is $\Omega=0$ ($J_1^2-J_2^2=0\Leftrightarrow J_1=J_2$). Using this, we obtain

$$J_1 \gtreqless J_2 \Leftrightarrow \Omega \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \bar{\sigma}. \tag{F2}$$

Eqs. (F1) and (F2) derive

$$V_i^{UA} \gtreqless V_i^{AA} \Leftrightarrow \sigma \lesseqgtr \bar{\sigma}. \tag{F3}$$

V_i^{UU} *vs.* V_i^{AU}: Using Lemma 2, we have

$$\mathrm{sgn}\left(V_i^{UU} - V_i^{AU}\right) = \mathrm{sgn}\left\{\frac{bK^2}{2} - \left[\sigma + \frac{(a - b\Gamma)H}{2(a - bK)}\right]\Gamma\right\}.$$

The terms in the curly brackets on the right-hand side of the above equation are

$$\frac{bK^2}{2} - \left[\sigma + \frac{(a - b\Gamma)H}{2(a - bK)}\right]\Gamma$$

$$= \frac{I_1 - I_2}{\left[2a + 3bK - \sigma - \sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}\right]^2},$$

where

$$I_1 = (a - bK)\left[2a + 3bK - \sigma - \sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}\right]^2 b^2K^2 > 0,$$

$$\begin{aligned} I_2 = (a - bK)\Big[&6a^2 - abK + 5b^2K^2 - (3a - bK)\sigma \\ &- (a + bK)\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}\Big] \\ \times\Big[&6a^2 - abK + 5b^2K^2 + (7a - 4bK - \sigma)\sigma \\ &- (a + bK + \sigma)\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2}\Big]. \end{aligned}$$

Therefore, we obtain

$$\mathrm{sgn}\left(V_i^{UU} - V_i^{AU}\right) = \mathrm{sgn}(I_1 - I_2). \tag{F4}$$

In Eq. (F4), we have

$$I_1 - I_2 = 2(a - bK)(P_1 - P_2), \tag{F5}$$

where

$$\begin{aligned} P_1 = {} & (6a^3 + 5a^2bK + 2ab^2K^2 + 2b^3K^3 + 5a^2\sigma + 2b^2K^2\sigma - 2a\sigma^2) \\ & \times\sqrt{36a^2 - 36abK + 25b^2K^2 + 12a\sigma - 22bK\sigma + \sigma^2} > 0, \\ P_2 = {} & 36a^4 + 12a^3bK + 5a^2b^2K^2 + 14ab^3K^3 + 8b^4K^4 + 36a^3\sigma - 10a^2bK\sigma - 14ab^2K^2\sigma \\ & + 8b^3K^3\sigma - 7a^2\sigma^2 + 6abK\sigma^2 - 16b^2K^2\sigma^2 + 2a\sigma^3 > 0. \end{aligned}$$

To verify the sign of Eq. (F5), we consider the difference between squared terms of P_1. For each end of the domain of σ, their values are

$P_1^2 - P_2^2 = 12b^3K^3(a - bK)^2(2a - bK)^2(4a + 3bK) > 0$ for $\sigma = 0$,

$$P_1^2 - P_2^2 = -64b^2K^2(a-bK)^2(9a^4 + 12a^3bK - 4a^2b^2K^2 + 4ab^3K^3 - b^4K^4) < 0 \text{ for } \sigma = bK.$$

Furthermore, $P_1^2 - P_2^2$ is decreasing in σ because

$$\frac{d(P_1^2 - P_2^2)}{d\sigma} = -8(a-bK)^2\{bK(2a-bK)(36a^3 - 35ab^2K^2 - 2b^3K^3) + 10bK(24a^3 - 18a^2bK - 10ab^2K^2 - 3b^3K^3)\sigma + 6(18a^3 + 43a^2bK + 12ab^2K^2 - 11b^3K^3)\sigma^2 + [(144a - 35\sigma)a + 88abK + 126b^2K^2]\sigma^3\} < 0.$$

These properties of $P_1^2 - P_2^2$ show that the critical point $\tilde{\sigma}$ exists such that $P_1^2 - P_2^2 = 0$ and $0 \le \tilde{\sigma} < bK$. Given that $\sigma < \tilde{\sigma} \Rightarrow P_1^2 - P_2^2 > 0$ and $\sigma > \tilde{\sigma} \Rightarrow P_1^2 - P_2^2 < 0$, Eqs. (F4) and (F5) lead to

$$V_i^{UU} \gtreqless V_i^{AU} \Leftrightarrow I_1 \gtreqless I_2 \Leftrightarrow P_1 \gtreqless P_2 \Leftrightarrow P_1^2 - P_2^2 \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \tilde{\sigma}. \quad \text{(F6)}$$

We now consider three cases in turn: $0 \le \sigma < \min(\bar{\sigma}, \tilde{\sigma})$, $\min(\bar{\sigma}, \tilde{\sigma}) < \sigma < \max(\bar{\sigma}, \tilde{\sigma})$, and $\max(\bar{\sigma}, \tilde{\sigma}) < \sigma$. If $0 \le \sigma < \min(\bar{\sigma}, \tilde{\sigma})$, we derive $V_i^{UU} > V_i^{AU}$ and $V_i^{UA} > V_i^{AA}$ from Eqs. (F3) and (F6). Choosing a unit tax as a policy instrument is a dominant strategy. Hence, both governments choose a unit tax. If $\min(\bar{\sigma}, \tilde{\sigma}) < \sigma < \max(\bar{\sigma}, \tilde{\sigma})$, we obtain $V_i^{UU} > V_i^{AU}$ and $V_i^{UA} < V_i^{AA}$ for $\bar{\sigma} < \sigma < \tilde{\sigma}$, and $V_i^{UU} < V_i^{AU}$ and $V_i^{UA} > V_i^{AA}$ for $\tilde{\sigma} < \sigma < \bar{\sigma}$. For the case where $\bar{\sigma} < \sigma < \tilde{\sigma}$ ($\tilde{\sigma} < \sigma < \bar{\sigma}$), one government obtains more benefits by choosing the same tax instrument (the tax instrument other than that) chosen by the other government. Therefore, one government chooses a unit tax, and the other opts for an ad valorem tax. If $\max(\bar{\sigma}, \tilde{\sigma}) < \sigma$, then $V_i^{UU} < V_i^{AU}$ and $V_i^{UA} < V_i^{AA}$ hold. Considering that choosing an ad valorem tax is a dominant strategy, both governments use ad valorem taxes as tax instruments. Finally, these results hold for $\sigma > bK$ by the monotonicity and continuity of functions used for the proof in the case of $\sigma < bK$ if equilibrium values exist.

References

Aiura, H. and H. Ogawa (2013), Unit tax versus ad valorem tax: a tax competition model with cross-border shopping, *Journal of Public Economics*, 105 (C), 30–38.

Akai, N., Ogawa, H., and Y. Ogawa (2011), Endogenous choice on tax instruments in a tax competition model: unit tax versus ad valorem tax, *International Tax and Public Finance*,18 (5), 495–506.

Autor, D.H., F. Levy, and R.J. Murnane (2003), The skill content of recent technological change: an empirical exploration, *Quarterly Journal of Economics*, 118 (4), 1279–1333.

Baskaran, T., and M. Lopes da Fonseca (2013), The economics and empirics of tax competition: a survey, Discussion Papers, Center for European Governance and Economic Development Research, No. 163.

Bettendorf, L., Horst, A.V.D., and R.A. De Mooij (2009), Corporate tax policy and unemployment in Europe: an applied general equilibrium analysis, *World Economy*, 32 (9), 1319–1347.

Eichner, T. and T. Upmann (2012), Labor markets and capital tax competition, *International Tax and Public Finance*, 19 (2), 203–215.

Exbrayat, N., Gaigné, C., and S. Riou (2012), The effects of labour unions on international capital tax competition, *Canadian Journal of Economics*, 45 (4), 1480–1503.

Feld, L. and G. Kirchgassner (2003), The impact of corporate and personal income taxes on the location of firms and on employment: some panel evidence for the Swiss cantons, *Journal of Public Economics*, 87 (1), 129–155.

Feldmann, H. (2011), The unemployment puzzle of corporate taxation, *Public Finance Review*, 39 (6), 743–769.

Felix, R.A. (2009), Do state corporate income taxes reduce wages? *Economic Review*, Federal Reserve Bank of Kansas City, 94 (2), 77–102.

Frey, C.B. and M.A. Osborne (2017), The future of employment: how susceptible are jobs to computerisation? *Technological Forecasting and Social Change*, 114 (C), 254–280.

Gillet, H. and J. Pauser (2018), Public input provision in asymmetric regions with labor market imperfections, *German Economic Review*, 19 (4), 466–492.

Hoffmann, M. and M. Runkel (2016), A welfare comparison of ad valorem and unit tax regimes, *International Tax and Public Finance*, 23 (1), 140–157.

Kawachi, K., H. Ogawa, T. Susa (2019), Endogenizing government's objectives in tax competition with capital ownership, *International Tax and Public Finance*, 26 (3), 571–594.

Kempf, H. and G. Rota-Graziosi (2010), Endogenizing leadership in tax competition, *Journal of Public Economics*, 94 (9–10), 768–776.

Kikuchi, Y. and T. Tamai (2019), Tax competition, unemployment, and intergovernmental transfers, *International Tax and Public Finance*, 26 (4), 899–918.

Lockwood, B. (2004), Competition in unit vs. ad valorem taxes, *International Tax and Public Finance*, 11 (6), 763–772.

Lockwood, B and K. Wong (2000), Specific and ad valorem tariffs are not equivalent in trade wars, *Journal of International Economics*, 52 (1), 183–195.

Ogawa, H. (2013), Further analysis on leadership in tax competition: the role of capital ownership, *International Tax and Public Finance*, 20 (3), 474–484.

Ogawa, H. (2016), When ad valorem tax prevails in international tax competition, *International Review of Economics and Finance*, 46, 1–9.

Ogawa, H., Sato, Y., and T. Tamai (2006), A note on unemployment and capital tax competition. *Journal of Urban Economics*, 60 (2), 350–356.

Pal, R. and A. Sharma (2013), Endogenizing government's objectives in tax competition, *Regional Science and Urban Economics*, 43 (4), 570–578.

Sato, Y. (2009), Capital tax competition and search unemployment, *Papers in Regional Science*, 88 (4), 749–764.

Tamai, T. and G. Myles (2022), Unemployment, tax competition, and tax transfer policy, *Journal of Public Economic Theory*, 24 (3), 407–503.

Wilson, J.D. (1986), A theory of inter-regional tax competition, *Journal of Urban Economics*, 19 (3), 296–315.

Yellen, J.L. (1984), Efficiency wage models of unemployment, *American Economic Review*, 74 (2), 200–205.

Zirgulis, A. and T. Šarapovas (2017), Impact of corporate taxation on unemployment, *Journal of Business Economics and Management*, 18 (3), 412–426.

Zodrow, G.R. (2010), Capital mobility and capital tax competition, *National Tax Journal*, 63 (4), 865–902.

Zodrow, G.R. and P. Mieszkowski (1986), Pigou, Tiebout, property taxation, and the underprovision of local public goods, *Journal of Urban Economics*, 19 (3), 356–370.

Tables

Table 1. Payoff table

		Region 2	
		Unit	Ad Valorem
Region 1	Unit	V_1^{UU}, V_2^{UU}	V_1^{UA}, V_2^{AU}
	Ad Valorem	V_1^{AU}, V_2^{UA}	V_1^{AA}, V_2^{AA}

Table 2. Payoff table when $\mu_i > 0$

$\mu_1 = 1.5, \mu_2 = 0.5$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	0.9444, 0.6111	0.9400, 0.6069
	Ad Valorem	0.9495, 0.6123	0.9453, 0.6078

$\mu_1 = 2, \mu_2 = 1.5$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	0.8403, 0.6736	0.8525, 0.6930
	Ad Valorem	0.8661, 0.6817	0.8760, 0.6992

Table 3. Payoff table when $\mu_1 > 0$ and $\mu_2 < 0$

$\mu_1 = 1.5, \mu_2 = -0.5$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	1.1944, 0.5278	1.1596, 0.5165
	Ad Valorem	1.1910, 0.5274	1.1594, 0.5165

$\mu_1 = 0.5, \mu_2 = -1.5$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	1.1944, 0.5278	1.1067, 0.5063
	Ad Valorem	1.1683, 0.5246	1.0995, 0.5060

Table 4. Payoff table when $\mu_i < 0$

$\mu_1 = -0.5, \mu_2 = -1.5$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	0.9444, 0.6111	0.8673, 0.5588
	Ad Valorem	0.8849, 0.5914	0.8393, 0.5535

$\mu_1 = -1.5, \mu_2 = -2$		Region 2	
		Unit	Ad Valorem
Region 1	Unit	0.8403, 0.6736	0.7497, 0.5927
	Ad Valorem	0.7500, 0.6250	0.7081, 0.5781

Figures

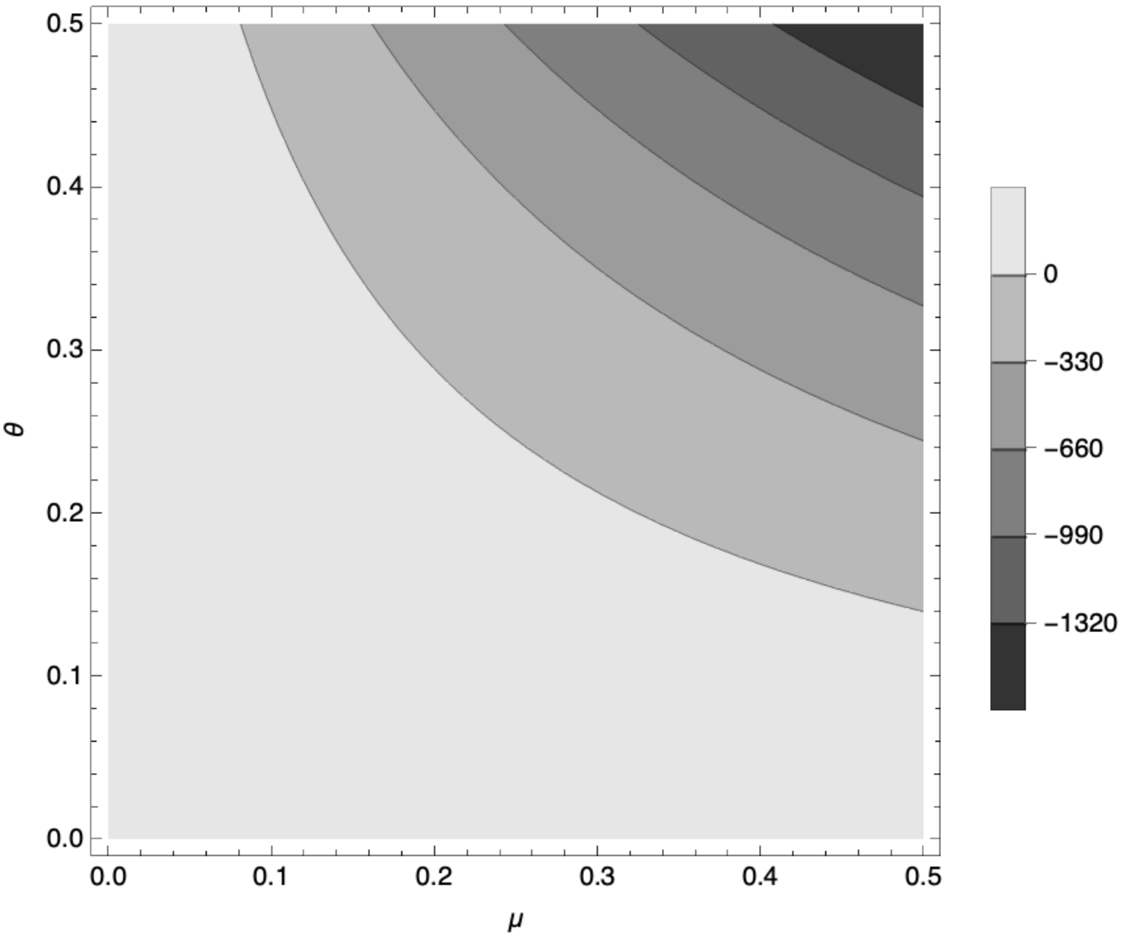

Figure 1. Contour plot related to $\bar{\sigma}$

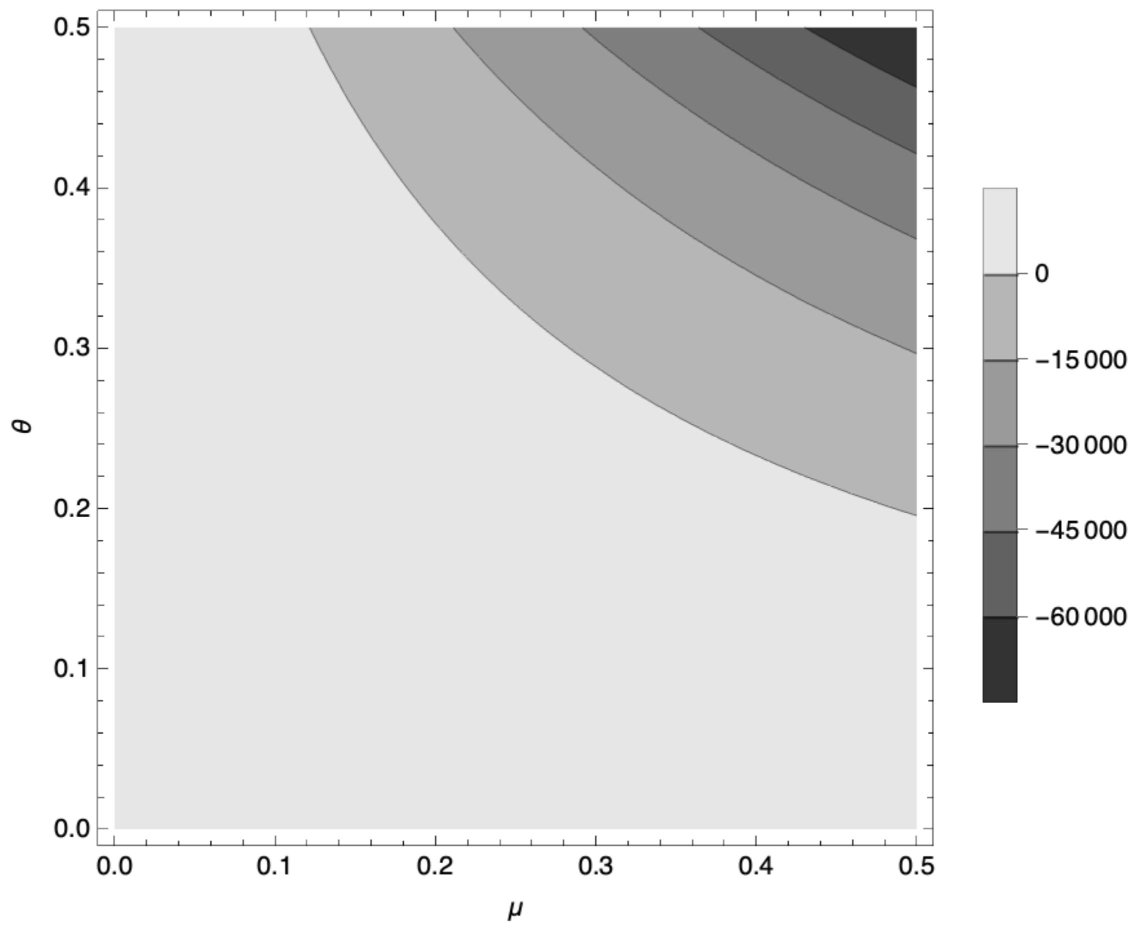

Figure 2. Contour plot related to $\widetilde{\sigma}$