# Allometric scaling of leaf mass based on the pipe model theory for woody plant species 

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#### Abstract

There is an exceptional case in which Shinozaki's pipe model predictions should not explain that the allometric scaling exponent between foliage mass and stem diameter at the crown base is bigger than 2 empirically. This study proposes an allometric scaling model with components $f$ and $b$, which are the scaling exponent between the sapwood area and stem cross-sectional area and foliage mass and the stem cross-sectional area at crown base, respectively. The scaling exponent $f$ of sapwood area vs. stem diameter at the crown base has no effect on leaf mass vs. stem diameter at the crown base, while the scaling exponent $b$ has effects on leaf mass vs. stem diameter at the crown base. Because the value of $b$ is greater than unity, this scaling model predicts that the diameters of conduits at the crown base widen with tree size, indicating tip-base widening of conduits. Because the proportion of sapwood area remains constant (or $f=0$ ) across sizes, the assumption of Shinozaki's pipe model theory appears to apply to the case $f=0$.


Keywords allometric scaling, conduit diameter, leaf mass, pipe model theory, stem diameter at crown base, sapwood area

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## Introduction

According to the classical pipe model theory developed by Shinozaki et al. (1964a, b), the leaf mass of individual trees is closely related to the stem diameter at the crown base, which is therefore a useful metric for estimating the leaf mass. Based on the pipe model theory, the allometric scaling exponent between leaf mass and stem diameter at the crown base is theoretically 2 . However, there is an exceptional case when the allometric scaling exponent is greater than 2 , which occurs when the proportionality of leaf mass to the square of the stem diameter at the crown base tends to segregate canopy trees from suppressed trees (Shinozaki et al. 1964b). Therefore, the present models have a limited (Kobayashi and Maskawa 1973) ability to express the mechanism of the allometric scaling between leaf mass and stem diameter at the crown base.

Several subsequent models have been developed by distinguishing between active and disused pipes, and by considering sapwood area equivalent to active pipe area, since the foliage mass should be closely related to the sapwood area (as reviewed by Lehnebach et al. 2018). As a result of these approaches, the allometric scaling exponent becomes unity in the leaf masssapwood area relationship, in which the pipe model ratio, or the ratio of foliage mass to the sapwood ratio at the crown base, which was proposed in lieu of the specific pipe length (Shinozaki et al. 1964a), should be constant (e.g. Whitehead et al. 1984, Shelburne et al. 1993, Mencuccini and Grace 1995, Mäkelä et al. 1995, Vanninen et al. 1996, Meadows and Hodges 2002).

The pipe model theory is one of the most important approaches for understanding the hydraulic architecture of trees. However, it has major limitations that suggest that the pipe structure as proposed by Shinozaki et al. (1964a, b) cannot be found in an actual tree. The most critical assumption relies on the fact that the pipes of the basic elements of the plant vascular system are cylindrical, i.e., the conduit diameter does not change axially. However, the main property of the hydraulic system of trees is to have tip-base widening of the conduits (e.g., West et al. 1999; Anfodillo et al. 2006; Petit and Anfodillo 2009; Savage et al. 2010; Olson et al. 2014, 2018, 2021; Koçillari et al. 2021). Therefore, this universal, but ineludible, pattern has a profound effect on the relationship between foliage mass and stem diameter.

Recently, Ogawa (2015) examined the allometric scaling exponent between leaf mass and stem diameter at the crown base among several forest trees, and reported values greater than 2 for all species. Ogawa (2015) applied the stratified clipping method (Monsi and Saeki 1953) to conduct a mathematical analysis of the mechanism that causes the scaling exponent to exceed 2 , and found that the scaling exponent is controlled by the size dependence of the specific pipe length, which is related to competition among forest trees and can be used as an index of the degree of suppression by larger trees.

However, it is not known how the pipe model ratio (cf. Lehnebach et al. 2018) is related to the allometric scaling exponent between leaf mass and stem diameter at the crown base in classical pipe model theory (cf. Shinozaki et al. 1964a, b). That is, if the pipe model ratio is constant, under what conditions does the classical pipe model theory hold? Because the pipe model ratio is related to the stem sapwood area, while the classical pipe model theory is related to the stem cross-sectional area, the proportion of sapwood area might explain why the allometric scaling exponent exceeds 2 by linking the pipe model ratio and classical pipe model theory. In addition to the sapwood area proportion, tip-to-base xylem conduit widening should be considered for the exceptional cases in which the pipe model predictions should not explain the correlation between leaf mass and diameter of the stem using experimental data.

The stratified clipping method is a time- and labor-intensive approach, especially for trees, for which relevant data are very limited. Therefore, to clarify the factors that control the allometric scaling exponent of leaf mass in the pipe model, the present study provides a simple allometric scaling model of leaf mass based on the leaf mass and stem diameter at the crown base, considering the hydraulic vessel system, such as the proportion of sapwood area, and the conduit diameter.

All data used in this study were collected from four stands of hinoki cypress (Chamaecyparis obtusa (Sieb. et Zucc.) Endl.) located in Aichi Prefecture, central Japan (Table 1, cf. Hagihara et al. 1993), as follows:

1) Nagoya. This stand was established in an experimental forest of Nagoya University on flat terrain at 50 m a.s.l.. No artificial management has been performed since the plantation was established. Since the canopy was fully closed, undergrowth vegetation was nearly absent.
2) Inabu. This stand is located at Nagoya University Forest on a $35^{\circ}$ north-facing slope at an elevation of 500 m a.s.l.. Artificial management was performed only immediately after the plantation was established. Undergrowth vegetation was nearly absent.
3) Hourai. This stand is located at the Aichi Prefectural Forest Research Institute on an $18.5^{\circ}$ northwest-facing slope at 435 m a.s.l. No artificial management was performed in the stand within the 20 years preceding the survey. Undergrowth vegetation was nearly absent.
4) Dando. This stand is located in a national forest administered by the Shinshiro District Forest Office and Nagoya Regional Forest Office on a $25^{\circ}$ west-facing slope at 900 m a.s.l. Thinning was performed 4 years prior to the survey. Ground vegetation consisted of a dense population of suzudake (Sasamorpha borealis [Hack.] Nakai) that was ca. 2 m tall.

## Measurement of tree size and mass

After the sample trees were felled, the tree height, crown base height, stem diameter at a height of 1.3 m above the ground (DBH), and stem diameter at the crown base $\left(D_{\mathrm{B}}\right)$ were measured.

Following the tree size measurements, the stratified clipping method (Fig. 1) was applied to the felled trees (Monsi and Saeki 1953). The trees were stratified into several vertical strata defined at $0-0.3,0.3-1.3 \mathrm{~m}$, and then 1.0 m intervals from ground level to the top of the crown. The stems, branches, and leaves from each stratum were clipped separately from all layers. The stems, branches, and leaves in each stratum were weighed. The roots were washed and weighed following excavation.

Samples were collected from each stratum to estimate the dry/fresh mass ratio for each organ, including the roots. The samples were dried in ventilated ovens at $85^{\circ} \mathrm{C}$ for a few days, desiccated at room temperature, and then weighed. Fresh mass was converted to dry mass to obtain stem, branch, leaf, and root values per tree using their respective ratios.

## Measurement of sapwood area at the crown base

Measurements of sapwood area at the crown base (Adu-Bredu and Hagihara 1996) were performed using 12-year-old C. obtusa trees (Table 1) as follows: 13 trees were harvested, and stem discs were removed from the crown base (thickness, 1.5-4.0 cm). Sapwood was
differentiated from heartwood by color and staining with ferric chloride solution (e.g., Ryan 1989).

The disc circumference was measured using a steel measuring tape, and the heartwood circumference was measured using a Digital Curvimeter (S-880; Uchida, Tokyo, Japan). The sapwood cross-sectional area was calculated as the difference between the stem and heartwood cross-sectional areas, and the proportion of sapwood area was determined as the ratio of the sapwood cross-sectional area to the stem cross-sectional area.

## Model descriptions

Allometric scaling of leaf mass: Assuming that the pipe model ratio is constant at the crown base, i.e., an allometric scaling exponent of unity between the leaf mass and the sapwood area at the crown base, the leaf mass is not proportional to the sapwood area at the crown base as long as the proportion of the sapwood area is greater in larger trees than in smaller trees. Under this condition, the following model supports $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry, with a scaling exponent that tends to be greater than unity in several woody species (Ogawa 2015).

According to traditional pipe model theory (cf. Lehnebach et al. 2018), the stem crosssectional area at the crown base and the sapwood area are denoted as $A_{\mathrm{B}}\left(=\frac{\pi}{4} D_{B}^{2}\right)$ and $A_{\mathrm{S}}\left(\mathrm{cm}^{2}\right)$, respectively, where the leaf mass $m_{\mathrm{L}}(\mathrm{kg})$ is assumed to be proportional to $A \mathrm{~s}$ as follows:

$$
\begin{equation*}
m_{L}=a A_{s} . \tag{1}
\end{equation*}
$$

where $a$ is a constant equivalent to the pipe model ratio at the crown base. This model has more generality in the $m_{\mathrm{L}}-A_{\mathrm{s}}$ relation of Eq. (1), so that the leaf mass $m_{\mathrm{L}}\left(\mathrm{cm}^{2}\right)$ scales as $A \mathrm{~s}^{b}$,

$$
\begin{equation*}
m_{L}=a A_{S}^{b} . \tag{2}
\end{equation*}
$$

If the relationship between the proportion of sapwood area at the crown base $\frac{A_{S}}{A_{B}}$ (dimensionless) and $A_{\mathrm{B}}\left(\mathrm{cm}^{2}\right)$ satisfies the following scaling relation,

$$
\begin{equation*}
\frac{A_{S}}{A_{B}}=e A_{B}^{f} \tag{3}
\end{equation*}
$$

then the relationship between $m_{\mathrm{L}}(\mathrm{kg})$ and $A_{\mathrm{B}}\left(\mathrm{cm}^{2}\right)$ is derived as

$$
\begin{equation*}
m_{L}=a e^{b} A_{B}^{(1+f) b} \tag{4}
\end{equation*}
$$

Therefore, the scaling exponent of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry is controlled by the values of coefficients $b$ and $f$ in Eqs. (2) and (3).

Specifically, if $f=0$ means that sapwood fraction remains constant across sizes in Eq. (2), then the $m_{\mathrm{L}}-A_{\mathrm{B}}$ allometry in Eq. (5) is simplified as the following scaling equation with the exponent of $b$ :

$$
\begin{equation*}
m_{L}=a e^{b} A_{B}^{b} \tag{5}
\end{equation*}
$$

Therefore, the scaling exponent of Eq. (5) is determined by the scaling exponent of $m_{\mathrm{L}}-A_{\mathrm{S}}$ allometry of Eq. (2) in the special case $f=0$. If the scaling exponent of $m_{\mathrm{L}}-A_{\mathrm{S}}$ allometry (Eq. 2 ) is larger than unity, the scaling exponent of $m_{\mathrm{L}}-A_{\mathrm{B}}$ allometry (Eq. 4 or 5) is larger than unity.

The case when the scaling exponent of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry becomes unity is generally $b=$ $\frac{1}{1+f}$ in Eq. (4). Therefore, Eq. (5) indicates that even if $f=0$, that does not necessarily mean only $b=1$ or the proportionality of $m_{\mathrm{L}}$ and $D_{\mathrm{B}}{ }^{2}$ from Eqs. (4). Equation (4) suggests that the leaf mass per unit sapwood area at the crown base $m_{\mathrm{L}} / A_{\mathrm{S}}\left(\mathrm{kg} \mathrm{cm}^{-2}\right)$ depends on the sapwood area $A_{\mathrm{S}}\left(\mathrm{cm}^{2}\right)$ as follows:

$$
\begin{equation*}
\frac{m_{L}}{A_{S}}=a A_{S}^{b-1} \tag{6}
\end{equation*}
$$

If the number of conduits per unit sapwood area is assumed to be constant, $m_{\mathrm{L}} / A_{\mathrm{S}}$ is regarded as an index of conduit diameter. That is, the larger $m_{\mathrm{L}} / A_{\mathrm{S}}$ is, the larger the conduit diameter is, and vice versa. Based on Eq. (6), Eq. (1) corresponds to the case when the conduit diameter is
constant irrespective of tree size because the value of $b$ is unity. In addition, the conduit diameter increases in larger trees compared to smaller trees if the value of $b$ is greater than unity.

## Regression analysis

Bivariate relationships, i.e., the $\frac{A_{S}}{A_{B}}-D_{\mathrm{B}}^{2}$ (Eq. 3), $m_{\mathrm{L}}-A_{\mathrm{S}}$ (Eq. 2), and $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ (Eq. 4) relationships, were analyzed using standardized major axis (SMA) regression (Warton et al. 2006) and ordinary least squares (OLS) regression, using the smatr package of R (v. 4.1.2, R Core Development Team, 2021). Significant differences among power (i.e., scaling) exponents were based on 95\% confidence intervals (CIs).

In the present analysis, Inabu had 36 individuals whereas the other sites have six individuals on average (Table 1). Therefore, a mixed-effects linear regression model was performed in the $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry (Eq. 4) to evaluate whether there is autocorrelation between data points from the same site, using the lmerTest package of $R$.

## Results

## Allometric scaling of leaf mass

The allometric relationship between leaf mass ( $m_{\mathrm{L}}$ ) and the square of the stem diameter at the crown base $\left(D_{\mathrm{B}}{ }^{2}\right)$ was approximated by a single regression line on $\log -\log$ coordinates, applying to all stands (Fig. 2). The scaling exponent was 1.273 ( $95 \%$ CI, 1.179-1.375) in SMA and 1.226 ( $95 \% \mathrm{CI}, 1.128-1.324$ ) in the OLS regression, both of which were significantly greater than unity. Performing a mixed-effects linear regression model, the scaling exponent was calculated to be 1.160 ( $95 \%$ CI, 1.006-1.314), which was significantly higher than unity, as with the SMA and OLS regression methods. Therefore, there is no profound autocorrelation between data points from the same site.

## Allometry of leaf mass to sapwood area at the crown base

The allometric relationship between leaf mass $m_{\mathrm{L}}$ and sapwood area at the crown base $A_{\mathrm{s}}$ satisfied Eq. (2) for 12-year-old C. obtusa (Adu-Bredu and Hagihara 1996) (Fig. 3). The value of scaling exponent $b$ is significantly higher than unity $(1.448,95 \% \mathrm{CI}=1.126-1.862)$ in the SMA
regression. Because the isometric relation between $w_{\mathrm{L}}$ and $A_{\mathrm{s}}$ given by Eq. (1) is not valid, the pipe model ratio at the crown base is not constant among trees. In constrast, the value of $b$ is not significantly different from unity $(1.337,95 \% \mathrm{CI}=0.970-1.706)$ in the OLS regression. This method (OLS) at least covers isometry for the allometry of leaf mass to sapwood area at the crown base.

## Relationship between the proportion of sapwood area at the crown base and the square of the

 stem diameter at the crown baseIn a C. obtusa plantation, 12-year-old trees (Adu-Bredu and Hagihara 1996) had a negative correlation between the proportion of sapwood area $\frac{A_{S}}{A_{B}}$ at the crown base and tree size $D_{\mathrm{B}}{ }^{2}$, as given by Eq. (3) (Fig. 4). However, the $p$-value of the linear regression between the logarithm of these two terms is too high $(0.892)$ in both the SMA and OLS regression methods. This indicates that the available data do not show that the scaling exponent is significantly different from 0 . Therefore, the proportion of sapwood area at the crown base is independent of tree size, although the proportion of sapwood area varied vertically within the tree (Longuetaud et al. 2006).

## Discussion

## Allometric scaling relation of leaf mass

The allometric scaling relationships of $m_{\mathrm{L}}$ to $D_{\mathrm{B}}{ }^{2}$ (Fig. 2) suggest that the amount of foliage per tree should be most closely correlated with the sum of the cross-sectional areas of the living pipes supporting them, which in turn is thought to be well approximated by the cross-sectional area of the trunk at the crown base. These relationships support fundamental concepts related to active and disused pipes within the pipe model theory (Shinozaki et al. 1964a, b); therefore, the stem diameter at the crown base $D_{\mathrm{B}}$ can serve as an overall basis for foliage amount estimation in any stand, regardless of the tree age or habitat.

In the present study, a scaling model is used (Eq. 4) to explore the underlying causes of an exponent that might increase above a value of 2 . According to studies of plant hydraulic adaptation (Savage et al. 2010; Olson et al. 2014, 2018, 2021; Kocillari et al. 2021), natural
selection operates on allometric scaling exponents between sapwood area and stem crosssectional area (Eq. 2) and foliage mass and stem cross-sectional area at the crown base (Eq. 4).

Because the scaling exponent of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry is controlled by the $f$ value of the coefficient in the case $b=1$ in Eq. (4) or Eq. (1), the scaling exponent of Eq. (4) is unity because the value of $f$ in Eq. (3) is regarded as zero (Fig. 4). Therefore, this case cannot be applied to the present result of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry (Fig. 2). We conclude that the pipe model ratio at the crown base is not constant irrespective of tree size, and varies among individual trees. In fact, the allometric scaling exponent between leaf mass $w_{\mathrm{L}}$ and sapwood area at the crown base $A_{\mathrm{s}}$ is significantly higher than unity in SMA regression (Fig. 2).

According to Shelburne et al. (1993) and Mäkelä and Albrektson (1992), the foliage area to sapwood area ratio is lower in suppressed trees than in dominant trees, indicating that the constant pipe model ratio given by Eq. (1) is not applicable to a wide range of tree sizes. Shinozaki et al. (1964b) stated that the proportionality of $w_{\mathrm{L}}$ to $D_{\mathrm{B}}{ }^{2}$ tends to segregate canopy trees from suppressed trees with less foliage. In suppressed trees, the scaling exponent $(1+f) b$ of the $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ relationship (Eq. 4) becomes greater than unity, resulting in a positive scaling with the specific pipe length $L(\mathrm{~m})$ (Shinozaki et al. 1963a), namely $L \propto\left(D_{B}^{2}\right)^{\xi}$, where $\xi$ is a positive scaling exponent. Thus, it is reasonable that $L$ is related to competition among forest trees and is used as an index of the degree of suppression (Ogawa 2015).

However, the present $w_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry with a scaling exponent greater than unity can be explained based on the more general scaling relation given by Eq. (4) and the sapwood area proportion relation given by Eq. (3). If the value of $f$ in Eq. (3) is zero because the correlation between the proportion of sapwood area $\frac{A_{S}}{A_{B}}$ at the crown base and tree size $D_{\mathrm{B}}{ }^{2}$ is not significant (Fig. 4), the value of the scaling exponent of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry equals the value of the scaling exponent of $m_{\mathrm{L}}-A_{\mathrm{S}}$ allometry. Therefore, the scaling exponent of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ allometry can be determined by factors such as the size dependency of the pipe model ratio (Eq. 4) and constancy of the sapwood area proportion (Eq. 3) at the crown base.

## Hydraulic architecture

From the perspective of the hydraulic architecture of trees, Eq. (1) is based on the assumption that the basic pipe elements of the plant vascular system are cylindrical, i.e., the conduit diameter does not change axially. However, the main property of the hydraulic system of trees is to have tip-base widening of the conduits theoretically (West et al. 1999; Koçillari et al. 2021) and experimentally (Anfodillo et al. 2006; Petit and Anfodillo 2009; Savage et al. 2010; Olson et al. 2014, 2018, 2021). Therefore, this universal, ineludible pattern is considered to have a profound effect on the relationship between foliage mass and stem diameter.

Based on SMA regression in Eq. (2), the conduit diameter is higher in larger trees than in smaller trees because the scaling exponent of $b$ is greater than unity (Eq. 7). In contrast, the OLS regression in Eq. (2) indicates that the conduit diameter is constant irrespective of tree size because the value of $b$ did not differ significantly from unity. However, considering the result of the scaling exponent between leaf mass and the square of stem diameter at the crown base (Fig. 2), it is reasonable to consider that the diameter of conduits widens with tree size.

Shinozaki's pipe model theory (1964a, b) assumes that plants do not have heartwood, or wood that is not conducting water to the foliage above the crown base. In the present study, the proportion of sapwood area $\frac{A_{S}}{A_{B}}$ is constant (or $f=0$ ) irrespective of tree size (Eq. 3) and in turn $\frac{A_{S}}{A_{B}}$ did not affect the allometric scaling relation in Eq (4). In this case ( $f=0$ in Eq. 4), the allometric scaling of $m_{\mathrm{L}}-D_{\mathrm{B}}{ }^{2}$ is operated only by the scaling exponent of $b$ in the allometry of leaf mass $m_{\mathrm{L}}$ to sapwood area $A_{\mathrm{S}}$ (Eq. 2). Therefore, the assumption of Shinozaki's pipe model theory seems to apply to the case $f=0$ in Eq. (4).

## Conclusions

The present scaling model (Eq. 5) can cover an exceptional case in which Shinozaki's pipe model predictions (1964a, b) should not explain that leaf mass vs. stem diameter at the crown base has an exponent greater than 2 . The scaling exponent $f$ of sapwood area vs. stem diameter at the crown base has no effect on leaf mass vs. stem diameter at the crown base, while the scaling exponent $b$ of leaf mass vs. sapwood area at the crown base affects leaf mass vs. stem diameter at the crown base. Because the value of $b$ is higher than unity, the present scaling model predicts
that the diameter of conduits at the crown base widens with tree size, indicating tip-base widening of xylem conduits.

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## Figure legends

Fig. 1 Diagram of the stratified clipping method (Monsi and Saeki 1953). In the practical experiment, the trees were stratified into several vertical strata defined at $0-0.3,0.3-1.3$, and subsequent 1.0 m intervals from ground level to the top of the crown.

Fig. 2 Allometric scaling relationships between the total leaf mass $m_{\mathrm{L}}$ and the square of the stem diameter at the crown base $D_{\mathrm{B}}{ }^{2}$ in 52 Chamaecyparis obtusa trees at four study sites. Circles, Nagoya; squares, Inabu; triangles, Hourai; crosses, Dando. The regression line in the diagram is the equation $y=0.0191 x^{1.273}$ (scaling exponent $95 \%$ confidence interval [CI], 1.179-1.375; $R^{2}=$ $0.927 ; P<0.001$ ) for the SMA regression method. The respective equations with the OLS regression method and linear mixed model are $y=0.0125 x^{1.206}$ (scaling exponent 95\% CI, 1.0911.333; $R^{2}=0.875 ; P<0.001$ ) and $y=0.0322 x^{1.160}$ (scaling exponent 95\% CI, 1.006-1.314; $P<$ 0.001).

Fig. 3 Allometry between the total leaf mass $m_{\mathrm{L}}$ and sapwood area at the crown base $A_{\mathrm{s}}$ in 12-year-old C. obtusa trees. The regression line in the diagram is an approximation of $y=$ $0.0218 x^{1.448}$ (scaling exponent $95 \%$ CI, $1.126-1.448 ; R^{2}=0.854 ; P<0.001$ ) for SMA regression method. The equation with the OLS regression method is $y=0.0301 x^{1.338}$ (scaling exponent $95 \% \mathrm{CI}, 0.970-1.706 ; R^{2}=0.854 ; P<0.001$ ).

Fig. 4 Relationship between the sapwood area proportion at the crown base $\frac{A_{S}}{A_{B}}$ and the square of the stem diameter at the crown base $D_{\mathrm{B}}{ }^{2}$ in 12-year-old C. obtusa. The regression line in the diagram is an approximation of $y=1.299 x^{-0.173}\left(R^{2}=0.00176 ; P=0.892\right)$ for the SMA regression method. The OLS regression equation is $y=0.723 x^{-0.00727}$ (scaling exponent 95\% CI, -$0.122-0.108 ; R^{2}=0.00176 ; P=0.892$ )

Fig. 1



Fig. 2


Fig. 3


Fig. 4

Table 1. Summary of the trees used for sapwood area measurement ( $\$$ ) and the allometric scaling relation (no symbols).

| Site | Age [yr] | Stand density [ $\mathrm{ha}^{-1}$ ] | Tree height [m] | Crown base height [m] | DBH [cm] | $D_{\text {B }}$ [cm] | $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nagoya\$ | 12 | 15000 | 3.3-5.7 | $\mathrm{n} / \mathrm{a}$ | 2.90-8.28 | 2.71-8.56 | 13 |
| Nagoya | 12-16 | 15000 | 4.67-7.55 | 2.10-4.55 | $4.77-7.35$ | 3.34-6.40 | 5 |
| Inabu | 16-25 | 6000-7500 | 4.73-12.1 | 1.25-6.00 | $3.47-13.4$ | 2.26-9.99 | 36 |
| Hourai | 26-27 | 2281 | 7.92-12.2 | 5.06-5.90 | $8.02-23.1$ | 3.66-16.6 | 6 |
| Dando | 58-59 | 2700 | 16.1-20.1 | 9.60-13.1 | 16.6-29.3 | 11.0-22.3 | 5 |
| $D_{B}$ : stem diameter at crown base. |  |  |  |  |  |  |  |
| $n$ : number of sample trees. |  |  |  |  |  |  |  |

