

# Lagrangian trajectory of small particles in superfluid He II

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**Abstract** Small tracer particles in HeII are visualized and their motions are analyzed. Lagrangian velocity distribution is computed by analyzing the visualized particle images through PTV (Particle Tracing Velocimetry) technique. We studied how the particle sizes affect the statistics of particle motions.

**Keywords** Lagrangian velocity · particle size · PTV measurement

## 1 Introduction

Turbulence of Helium (superfluid) is an interesting research area and it is important both in fundamental science and applications [1–3]. Superfluid exhibits the behavior similar to that found in a classical fluid, but it may take forms that are unknown in classical fluid once the quantized vortices are generated and mutual friction appears.

In the recent decade, new techniques have been applied to visualize the flow of superfluid helium and measure the local velocity fluctuation. Among them the convenient ones are particle image velocimetry (PIV) and particle tracing velocimetry (PTV) techniques, which are potential tool to measure local velocity and promises to provide a deeper understanding of complex superfluid motions. Readers who are not familiar with these techniques are recommended to refer [4]. Both PIV and PTV are standard techniques developed in classical fluid research, but their application to superfluid has just started, and several problems remain. The most difficult one is the production of appropriate imaging particles that can be spread over the liquid and track the flow field. There were several early challenges to the measurement of particle velocity in

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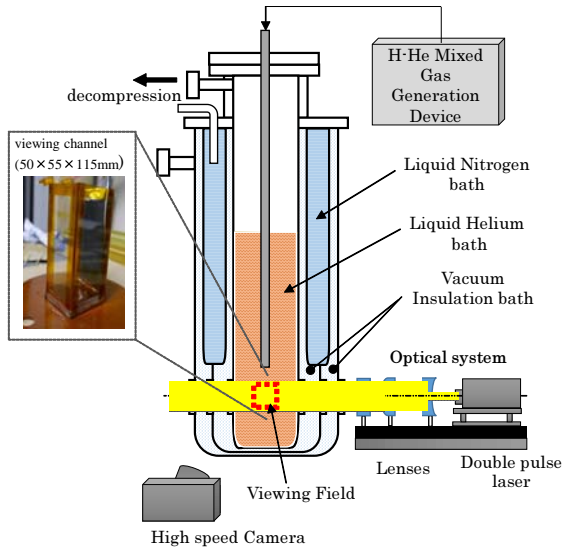
superfluid turbulence. One is the use of commercial polystyrene microspheres [5] and the other is the condensation and dispersion of solid hydrogen particulates from the gas phase [6, 7]. These studies revealed the new features of superfluid. Flow visualization techniques were also studied by Charles University in Prague [8, 9].

However, the particle sizes were too large to visualize the quantized vortex motions accurately. The particle size is on the order of microns and the vortex core diameter is estimated in angstroms. The novel approach was undertaken using Helium metastable molecules (excimers [10]) as tracers in superfluid. Triplet-state  $\text{He}_2^*$  molecules, with a radiative lifetime in liquid helium of about 13 [sec], can be imaged using laser-induced-fluorescence. Helium excimers have advantages as tracer particles: (i) the molecule size is several angstroms, (ii) they are neutral particles in superfluid, and (iii) they do not aggregate. These characteristics can make the excimers trace flow of the normal-fluid component in superfluid above 1 [K], and be trapped on quantum vortex lines below 0.5 [K]. This technique was developed by Gao et al. [11], and interesting results have been reported.

In the present study, small tracer particles in HeII are visualized and their Lagrangian motions are analyzed. The particles are made of solid hydrogen whose diameter is on the order of microns, which is the same method developed by Murakami et al. [6] and Bewley et al. [7]. In a counterflow, changing the heat flux and bath temperature, Lagrangian velocity and accelerations are computed by analyzing the visualized images through the PTV technique. Next, we report how the particle sizes affect the statistics of particle motions. The probability density functions of acceleration show nearly a stretched exponential shape. They are not affected by particle sizes if normalized by their standard deviation. However, it was found that the velocity distribution is affected by the particle sizes. In this paper, we report the result of velocity statistics.

## 2 Experimental setup

On the successful application of tracer particles, there have been two challenges to visualization in superfluid turbulence. One is the use of commercial polystyrene microspheres ( $d \simeq 1 \mu\text{m}$ ) and the other is the condensation and dispersion of solid hydrogen particulates from the gas phase. In the present study, we tried the second approach. Inside the dewar, the schematic view is shown in Fig. 1, the rectangular channel (cross section is  $A = 50 \times 55 \text{ [mm}^2\text{]}$  and the height is  $H = 115 \text{ [mm]}$ ) made of glass is placed. The bath temperature  $T_B$  is varied from 1.9 to 2.1 [K]. The plate heater is located at the bottom, and the heat flux  $q$  is set at 200, 400, 600, and 800 [ $\text{W/m}^2$ ]. Then the thermal counterflow is generated inside the channel. A high-speed camera ( $1024 \times 1024$  pixels, 12 bit) is used for visualizing the area  $9 \times 9 \text{ [mm}^2\text{]}$  at 250 [fps]. A continuous laser (wave length 532 [nm], Diode Pumped Solid State laser) is used to generate laser sheet with a thickness of about 1 [mm]. A helium and hydrogen



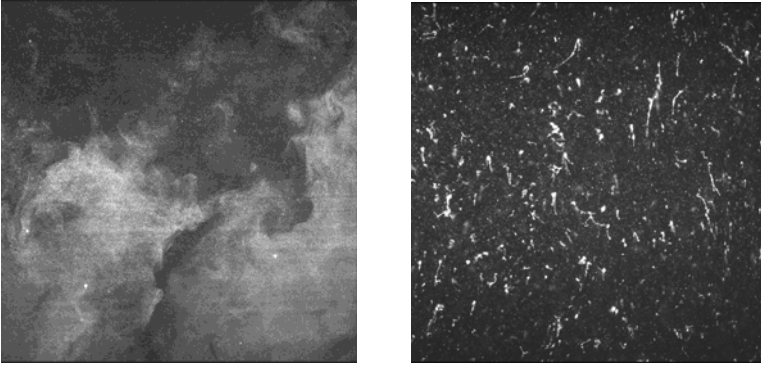
**Fig. 1** Schematic view of experimental apparatus (Color figure online).

mixing chamber is designed to change the mixing ratio and the spouting pressure, and hydrogen particles are generated in the liquid helium. After several test experiments, we adopted the condition that the mixing ratio is  $\text{He}:\text{H}_2 = 40:1$  and spouting pressure is 20k [Pa]. The injection is done just above the  $\lambda$  point and then decreases the bath temperature.

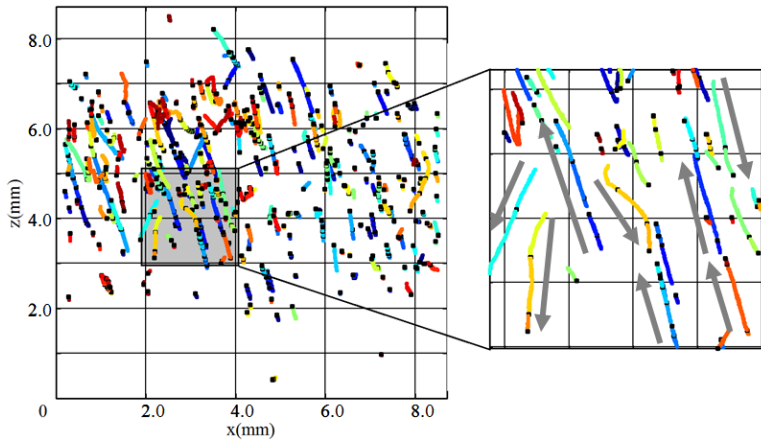
### 3 Results and discussion

Figure 2 shows visualized hydrogen particles. The visualized area is  $8 \times 8$  [mm<sup>2</sup>] near the injection nozzle. Small particles are generated after the injections. Below the  $\lambda$  point, particles are trapped by the tangle of quantized vortices. These visualizations are similar to the previous results reported by Bewley et al. [7]. We also observed the vortex reconnections [12]. From the observation of visualized images, we found that tracer particles trapped by quantized vortices move down and the particles forced by Stokes drag move upward. However, they cannot be classified completely into these two groups. Some particles trapped by a vortex are affected by Stokes drag caused by normal fluid and some transported by normal fluid are affected by mutual friction.

Figure 3 shows the Lagrangian trajectories obtained by the image processing of visualized images. The trajectories are originally three-dimensional, but they are visualized inside a two-dimensional sheet in this measurement. Different colors indicate the different trajectories, and the end points where we can track the particles are marked by black circles. When the black circles are located below the starting points, we assume that particles are usually trapped by vortex tangles. But they are sometimes affected by normal fluid locally



**Fig. 2** Hydrogen tracer particles in liquid Helium. Visualized area is  $8 \times 8$  [mm<sup>2</sup>] near the injection nozzle. Temperature is above the  $\lambda$  point in left image and below the  $\lambda$  point in image at right.



**Fig. 3** Trajectory of tracer particles at  $400$  [W/m<sup>2</sup>] and  $T_B = 2.09$  [K]. Different colors show the different particles. Black dots indicate end points of trajectories and arrows show directions of motion (Color figure online).

and move upward. On the other hand, when the particles move upward, they assumed to be flowed by normal fluid for the most part. And their motions are sometimes disturbed with quantized vortices. Inset shows such trajectories indicated by arrows.

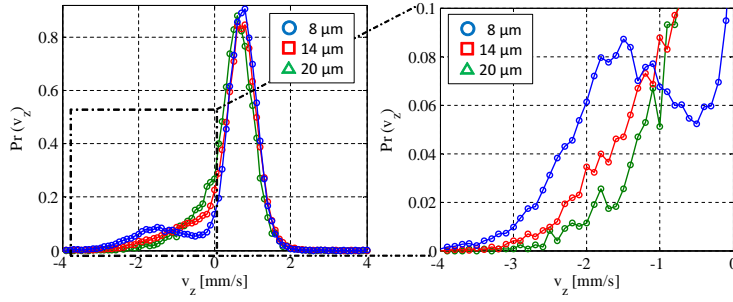
Particle sizes are classified into three groups. They are expressed by A ( $5.5 \leq d \leq 10.5$  [ $\mu\text{m}$ ],  $8$  [ $\mu\text{m}$ ] on average), B ( $11.9 \leq d \leq 16.4$  [ $\mu\text{m}$ ],  $14.2$  [ $\mu\text{m}$ ] on average), and C ( $17.8 \leq d \leq 22.2$  [ $\mu\text{m}$ ],  $20$  [ $\mu\text{m}$ ] on average). We adopted the Mie scattering method for evaluating the particle sizes [4]. For the first step, the polystyrene particle, which has a similar density with water and the average diameter of  $4$  [ $\mu\text{m}$ ] is dispersed in the water. They are visualized by the laser system and recorded by high-speed camera. The visualization system and measurement conditions (focal length, fps, shutter speed, resolution) are

matched with those of a helium experiment. We found that Mie scattering correction works well and particle sizes can be estimated adequately. From these experiences, the particle sizes in a Helium experiment are also evaluated. However, it should be noted that the particle shape is not spherical, so particle size estimation by this method involves some uncertainty.

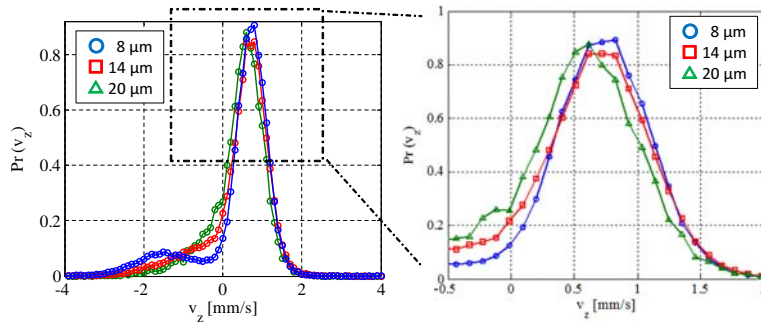
Figure 4 shows the probability density function of velocity in a vertical component. We calculate the velocity from the Lagrangian positions by means of forward difference. Checking the tail part of PDF, it is confirmed that the sampling frequency is small enough to calculate the small velocity fluctuations. A positive value is the upward velocity moved by normal fluid and a negative one is downward motion trapped by a quantized vortex. When the particle size is small (class A, 8 [ $\mu\text{m}$ ] on average), the distribution shows the local peak around  $v_z = -1.5$  [mm/sec]. But in the case of large size (class C, 20 [ $\mu\text{m}$ ] on average), this peak disappears. Because the larger particles get a larger viscous drag, they cannot be captured by the quantized vortices. In Fig. 5, positive velocity distribution is shown. It has also the local peak at  $v_z = 0.7$  [mm/sec]. This peak location is less dependent on particle size. That is, upward motion is not affected significantly by quantized vortices.

However, it is difficult to distinguish exactly the motions caused by normal fluid from the effect trapped by the tangle of quantized vortices. Upward motions seem to show the smooth trajectory, but careful monitoring of the local motion reveals it is not smooth. This may be affected by quantized vortices. It cannot be analyzed accurately under the present space and time-resolutions. In a statistical sense, we assume that the upward particle motions are dominated by normal fluid and downward ones are attributed to the quantized vortices. Neutrally buoyant particles much smaller than the smallest eddy scale of the turbulent flow are expected to behave as tracers, with the same dynamics as fluid particles. On the contrary particles with mismatch density and/or with size comparable to turbulent eddies, will not follow the flow exactly. We refer to these particles as inertia. Bourgoin et al. [13] reported the experimental investigation of inertial particle dynamics in turbulent air flow, over a wide range of sizes and densities. They found that particle velocity probability density function is nearly Gaussian. And the ratio of particle mean velocity  $v_{mean}$  and its root mean square  $v_{rms}$ ,  $v_{rms}/v_{mean}$ , is independent of particle properties and does not present any trend either with particle size or with particle density. Moreover,  $v_{rms}/v_{mean}$  is identical to the turbulence level of carrier flow itself. Therefore, the velocity statistics are not affected by particle inertia.

Even in classical turbulence, how the coupling between particle dynamics and fluid dynamics is influenced by size and density effects remains an open question. Several inertial effects may be interpreted qualitatively in terms of particle interaction with turbulent eddies. In quantum turbulence, also the quantized vortices have an important effect on the particle motions. This effect may be understood by analyzing the statistical property of particle motions in detail. We will report the Lagrangian acceleration property of inertial particles in the next step.



**Fig. 4** Probability density function of velocity fluctuation of vertical component at 400 [W/m<sup>2</sup>] and  $T_B = 2.09$  [K]. Different colors show the different particle sizes. Negative velocity (downward direction) is magnified in the right figure, and it is found that negative velocity is affected by the particle sizes (Color online).



**Fig. 5** Probability density function of velocity fluctuation of vertical component at 400 [W/m<sup>2</sup>] and  $T_B = 2.09$  [K]. Different colors show the different particle sizes. Positive velocity (upward direction) is magnified in the right figure, and it is found that positive velocity is not strongly affected by the particle sizes (Color online).

## 4 Summary

Using the PTV technique, we visualized the Lagrangian trajectories of small particles in counterflow of He II and calculate the Lagrangian velocity distribution. We then investigated the effect of particle size on the velocity fluctuations and found that upward particle velocity followed by normal fluid is less dependent on particle size. However, the downward velocity trapped by the quantized vortices is affected by the particle sizes. This may be because the large particles cannot be captured by the vortex due to the viscous drag, and they do not show negative velocity.

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