

quantum toroidal \mathfrak{gl}_1 algebra $U_{q_1,q_2}(\mathfrak{gl}_1)$ and then solve the equations. Subsequently, motivated by the AGT correspondence, we show how to interpret the solution of the MacMahon KZ equations as a generalization of Nekrasov factor, which is a fundamental quantity in five-dimensional supersymmetric gauge theory. These MacMahon KZ equations and the generalized Nekrasov factor are believed to be related to various objects in quantum field and string theory.

After that, we proceed to investigate the quasi-Hopf twist of quantum toroidal \mathfrak{gl}_1

algebra $U_{q_1,q_2,p}(\mathfrak{gl}_1)$. We uncover the hidden relation between the quasi-Hopf twisted \$ R \$-matrix and the elliptic Nekrasov factor, which is considered as a fundamental building block of six-dimensional supersymmetric gauge theory. Then,

we derive KZ equations of $U_{q_1,q_2,p}(\mathfrak{gl}_1)$ corresponding to the Fock representation, and show that the solution of the KZ equation can be written in terms of the product of elliptic Nekrasov factors.