Economic Growth, Equilibrium Welfare, and Public Goods Provision with Intergenerational Altruism

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Abstract

This study examines the government policy of public goods provision and its effects on economic growth and welfare under intergenerational altruism. The study considers an endogenous growth model with altruistic overlapping generations. The preferences of the current youth exhibit a future bias, and thus, democratically elected governments are subject to this future bias. The optimal rule for the supply of public goods under future bias differs from the original Samuelson rule. Unlike the standard growth model without any bias, under the optimal rule, the equilibrium growth rate is not independent of government size. Future bias gives young generations the dynamic incentives to invest more. With future bias, the intergenerational redistributive effects of public goods stimulate such incentives under certain conditions. Hence, the government size affects the economic growth through intertemporal changes in their resource allocations. Moreover, the growth effect of the government size provides nontrivial outcomes of welfare analysis. Our numerical analyses show the growth and welfare superiority of the democratic governments to the nonbiased social planner.

Keywords: Intergenerational altruism; Future bias; Economic growth; Public goods
JEL classification: D71; D72; H40; O40

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1 Introduction

This study examines the government policy of public good provision, economic growth, and social welfare under intergenerational altruism. The formal analysis of public goods originated from Samuelson (1954), who derived the optimal rule of public goods supply. In the static model, the Samuelson rule requires that the sum of marginal rates of substitutions (MRS) between the public and private goods must be equal to the marginal rate of transformation (MRT) between the two goods.[1] Numerous studies have examined the optimal rule, and the dynamically extended Samuelson rule is also along with its original one under certain conditions (e.g., Myles 1997).[2] The optimal provision of public goods will not affect the capital accumulation and therefore economic growth if the government implements its public goods policy throughout the time period (e.g., Turnovsky 1996; Tamai 2010).[3]

In reality, the government policy is democratically determined by the people who have finite lifetime. Intergenerational conflicts occur and influence the provision of public goods. For example, existing generations have a concern about the health care and welfare programs that they can enjoy. At the same time, they will not care about these programs in the past and far future without altruism to the ancestors and descendants. Diamond (1965) proposed the overlapping generations model, which has the potential inefficiency in a dynamic sense. With public goods, the overlapping generations economy on the growth path that deviated from the modified golden rule path will be inefficient if only the Samuelson rule is satisfied (Batina 1990). However, these issues may be solved through intergenerational transfers such as bequests. With the intergenerational altruism, operative bequests connect the resources of different generations and enable them to have infinite lifetime like infinitely-lived households. Therefore, intergenerational altruism may internalize such conflicts and neutralize their effects on dynamic resource allocation although each generation finitely lives (e.g., Barro 1974).[4]

On the other hand, numerous theoretical studies showed that intergenerational altruism generates *time inconsistency* and some kinds of *bias* related to time preferences (e.g., Strotz 1956; Phelps and Pollak 1968; Gonzalez et al. 2018).[5] Then, democratic governments elected by the biased people have also the same bias and face *self-control problems* for fiscal policy (e.g., Krusell et al. 2002). Some experimental studies found evidence for *future bias* (Sayman and Öncüler 2009; Takeuchi 2011, 2012). The future bias indicates that people tend to undervalue the current benefit and leads to *reverse time inconsistency*; they tend to postpone taking a reward until the future. For instance, Gonzalez et al. (2018) theoretically proved that the intergenerationally altruistic preferences exhibit future bias in an overlapping generations model (Kimball 1987; Hori and Kanaya 1989). They also showed that future bias creates incentives to legislate and sustain a pay-as-you-go pension system. The reason is that the future-biased people prefer to receive transfer benefits in the future rather than do in the present.

These theoretical and experimental findings suggest the importance of studying government policy

[1] Atkinson and Stern (1974), Christiansen (1981), and Boadway and Keen (1993) studied the optimal provision of public goods when the supply costs are financed by distorting taxation.

[2] Myles (1997) showed that one of the key elements is a degree of intergenerational altruism with durable public goods. In the model, a perfect depreciation leads to the original form of the Samuelson rule. Pirttila and Tuomala (2001) derived the modified Samuelson rule and optimal nonlinear income tax in an overlapping generations model with durable public goods. Then, Aronsson and Granlund (2011) investigated the provision of durable public goods in an overlapping generations model with two types of consumers under present bias. They showed the formula derived by Pirttila and Tuomala (2001), including the self-control problem.

[3] Turnovsky (1996) and Tamai (2010) clarified the theoretical interaction between public goods and economic growth in endogenous growth models. They demonstrated that the government expenditure will not affect the equilibrium growth rate when the public good is optimally supplied by its cost-financing coming from non-distortionary taxes.

[4] Barro (1974) showed the debt neutrality, and Becker (1974) found a similar result, which is known as the *Rotten Kid Theorem*. Moreover, Andreoni (1989, 1990) demonstrated that the Ricardian equivalence does not hold under impure altruism (warm glow).

[5] Numerous studies analyzed the bequest behavior. In particular, Strotz (1956) pioneered the study, which was then developed by Phelps and Pollak (1968), Kohlberg (1976), Goldman (1979), Harris (1985), Ray (1987), Kimball (1987), and Hori and Kanaya (1989). These approaches have adopted specifications in intergenerational altruism. Some of them prove or suggest the existence of the time inconsistency and present bias. Then, Jackson and Yariv (2014) proved that utilitarian aggregation exhibits present bias if heterogeneity exists in the population. They showed that three-quarters of social planners exhibited present biases, and less than 2% were time consistent in laboratory experiments.

under future bias. Naturally, the present study focuses on public goods; for example, health care and welfare programs will work as effective redistributive policy instruments for democratic governments. For instance, to prevent the spread of diseases, health care programs should be nonrival and nonexcludable.[6] This case benefits all existing generations regardless of paying the supply cost. Moreover, in general, the retired (old) generation obtains more benefits than does the working (young) generations because aging people are more liable to diseases. By contrast, private consumption allocation of rivalrous and excludable goods is defined as reconciliation of intergenerational interests, leading to intergenerational inequality. The current young generation may want a low level of health care to consume more private goods, but they may worry about the low level of health care when they get old. In practice, once health care and welfare programs are provided, we wish to support the system. However, we sometimes tend to favor economic expansion at the expense of health care and social welfare expenditures. Hence, as a political solution, democratic governments may continue to designate high or low levels of health care through a self-enforcing commitment mechanism. This case will decrease the loss from misallocation involved with future bias and may affect intertemporal resource allocation.

This study aims to examine the optimal provision of public goods under intergenerational altruism that emerges future bias to understand why democratic governments continue to choose high or low levels of public goods supply in reality. Moreover, we investigate how this provision affects the long-term economic performance, such as economic growth and welfare. This study is helpful to theoretically explain the coexistence of a high-benefit welfare system and high economic growth or, inversely, the trade-off between both. To address these issues, the present study incorporates public goods into an endogenous growth model with altruistic overlapping generations. In this study, one-period lived governments are obliged to supply public goods to maximize their democratic objectives. Such democratic governments are delegates of the coexisting generations, such as the young and the old, and therefore, they are future-biased.

With the future-biased governments, the present study derives the extended Samuelson rule from the dynamic model. Results show that the sum of MRS between the public and private goods must be weighted by the preference parameters. Such parameters represent the elasticity of marginal utility (the degree of inequality aversion), the taste for public goods (the efficiency of the public sector), and the degree of altruism for its ancestors and descendants. The rule under future bias is sub-optimal because it differs from the optimal rules, which should be satisfied in the near future. The weight for the old in the present time is lower than that in the future. Hence, the current consumption allocation (income distribution) becomes more disadvantageous to the old than does the desired allocation in the future. As public goods are means to improve the income distribution, the current government determines the supply level of public goods depending on the inequality aversion and the current and anticipated income distributions that the current government faces. Moreover, the current government expects that the future governments will also do the same. Undesirable allocations from the current government's perspective will be chosen in the future by the future governments. Hence, the government strategically chooses the investment level to compensate for the misallocation of consumption resources.

Considering that the old are more weighted than are the young in the democratic decision, the current income distribution is more equal than is the desired income distribution in the future. If the society is strongly inequality averse, satisfying that the elasticity of intertemporal substitution (EIS) is less than unity,[7] then the current government prefers low public goods provision today but high in the future. Moreover, the current government anticipates that the future governments have the same preferences, and therefore, they will choose low public goods provision in the future. For given allocation, the resource misallocation lowers future welfare (*income effect*) and transferring the resource to the future decreases the return to investment (*substitution effect*). In the strongly inequality-averse society, the latter effect dominates because people are reluctant to accept swings in

[6] Several other examples exist, such as public education. Public education provides the basic skill to work and enhance the quality of life. Furthermore, its effects vary according to generations.

[7] Numerous studies have been examined the EIS and reported the varied values. Havranek et al. (2015) noted that the mean elasticity is 0.5 in 169 published studies. The values larger than unity are also sometime set, as for example, a value of 2 is used (e.g., Ai 2010; Barro 2009; Colacito and Croce, 2011).

intertemporal/intergenerational consumption. The current government strategically chooses to invest more for future generations to compensate for the future low public goods provision. Then, improving the efficiency of the public sector enhances economic growth because it enables the current government to save resources for providing public goods and obtain the resource for more investment. Naturally, these outcomes are reversed in the weakly inequality-averse society.

If the old are less weighted than are the young in a democratic decision, then the anticipated future income distribution could be more or less equal than the current distribution. In particular, focusing on one of the insightful cases, the results indicate the coexistence of a high-benefit welfare system and high economic growth. If the old are more weighted than the young in the future, with smaller weight for the old today, then the current young's consumption share overweighs that of the current old. However, the future old's consumption share overweighs that of the future young with larger weight for the old in the future. This situation means that the current income distribution is nearly even and more equal than the anticipated future distribution. If people are willing to allow the dispersion of consumption, then the non-rivalrous public goods are more weighted relative to the rivalrous private goods in a more equal distribution case. Hence, the current government chooses low public goods provision today compared with in the future but may be higher than the first-best. The future public goods allocation chosen by the future government is a bit large for the current government. The presence of public goods more weakens the substitution effect and more strengthens income effect. More the important public goods cause more investment. Consequently, the government size and the log-run growth rate are positively correlated.

Our theoretical findings imply the coexistence of a high-benefit welfare system and high economic growth, similar to the trade-off between a high-benefit welfare system and economic growth. In particular, the relationships between economic growth and government size explain the controversial empirical findings.[8] A high-benefit welfare system does not necessarily impede the dynamic performance of the economy although it has been suspected. The high-benefit welfare system can be viewed as a result of a self-enforcing commitment mechanism to increase future benefits under future bias. The overall implication of the presence of public goods under future bias has also nontrivial consequences of the welfare properties. The present study numerically shows that the welfare levels in the administration of the democratic governments are superior to that of the planned economy with a non-biased planner. To study the details, the remainder of this paper is organized as follows. Section 2 describes the settings of our theoretical framework. Section 3 derives the equilibrium policy determined by future-biased authorities and characterizes the equilibrium outcome. Finally, Section 4 concludes the study.

2 The basic model

The study considers a closed economy with altruistic overlapping generations who live for two periods and a linear production function with respect to capital. The population of each generation is normalized to unity. In the first (young) period, each generation earns income and receives bequests from the parent generation. Disposable income in the young period is allocated among the private consumptions in the first and second periods, that is, c_t^y and c_{t+1}^o, with an operative bequest. The production technology is formulated as[9]

$$y_t = Ak_t.$$

Hence, the budget equations in the young and old periods with the operative bequests yield

$$k_{t+1} - k_t = y_t - h_t - c_t^y - c_t^o, \tag{1}$$

where h_t is the lump-sum tax, c_t^y is the private consumption in the young period of generation born at t, and c_t^o is the private consumption in the old period of generation born at $t-1$.

[8] See Bergh and Henrekson (2011) for a general review on the empirical relationship between government size and economic growth.

[9] Alternatively, the production function with the knowledge spillover a la Romer (1986) can be assumed. However, such modification does not change our main results.

Each generation has two-sided altruism for the ancestors and descendants and benefits from own consumption on private and public goods. Assuming the time additive separability, the utility function is

$$U_t = u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \mu U_{t-1} + \lambda U_{t+1},$$

where g_t is the public good in period t, β is the weight of the period-t generation's utility in the old period ($\beta > 0$), μ is the degree of altruism for their ancestors ($\mu > 0$), and λ is the degree of altruism for their descendants ($\lambda > 0$). In addition, $u(c_t^i, g_t)$ is specified as

$$u(c_t^j, g_t) = \begin{cases} \frac{(c_t^j)^{1-\sigma} + \gamma g_t^{1-\sigma}}{1-\sigma} \text{ for } \sigma \neq 1 \text{ and } \sigma > 0, \\ \log c_t^j + \gamma \log g_t \text{ for } \sigma = 1, \end{cases}$$

where $\gamma > 0$ and $j = y, o$.

Kimball (1987) and Hori and Kanaya (1989) showed that $\mu + \lambda < 1$ leads to

$$\begin{aligned} U_t = & \sum_{s=1}^{\infty} \theta^s \left[u(c_{t-s}^y, g_{t-s}) + \beta u(c_{t-s+1}^o, g_{t-s+1}) \right] \\ & + u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \beta u(c_{t+s+1}^o, g_{t+s+1}) \right], \end{aligned} \tag{2}$$

where

$$\begin{aligned} \theta &= \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\lambda} \in (0, 1), \\ \delta &= \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\mu} \in (0, 1). \end{aligned}$$

The total differentiation of θ and δ yields

$$\frac{\partial \theta}{\partial \lambda} < 0, \frac{\partial \theta}{\partial \mu} > 0, \frac{\partial \delta}{\partial \lambda} > 0, \text{ and } \frac{\partial \delta}{\partial \mu} < 0.$$

The discount factor of the ancestors θ (of the descendants δ) is positively associated with the weight for the parents' utility μ (for the kids' utility λ) while the discount factor of the descendants δ (of the ancestors) is negatively associated with the weight for the parents' utility μ (for the kids' utility λ).

During the period t, there are two different generations: young and old. Omitting the term of dead ancestors, Eq. (2) leads to the utility function for the young generation during the period t as

$$\begin{aligned} U_t &\simeq \theta \beta u(c_t^o, g_t) + u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \beta u(c_{t+s+1}^o, g_{t+s+1}) \right] \\ &= -\left(\delta^{-1} - \theta \right) \beta u(c_t^o, g_t) + \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right]. \end{aligned} \tag{3}$$

Similarly, the utility function for the old generation in the period t becomes

$$\begin{aligned} U_{t-1} &\simeq \beta u(c_t^o, g_t) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s-1}^y, g_{t+s-1}) + \beta u(c_{t+s}^o, g_{t+s}) \right] \\ &= \delta \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right]. \end{aligned} \tag{4}$$

Following Gonzalez et al. (2018), we assume that a sequence of a one-period government exists, which is responsible for supplying a public good. The government imposes a lump-sum tax to finance the government expenditure for providing public goods. We assume that the MRT between private and public goods is equal to unity. The government's budget equation is

$$g_t = h_t \tag{5}$$

Eqs. (1) and (5) lead to the following resource constraint:

$$Ak_t = c_t^y + c_t^o + g_t + k_{t+1} - k_t = c_t + g_t + i_t, \tag{6}$$

where

$$\begin{aligned} c_t &\equiv c_t^y + c_t^o, \\ i_t &\equiv k_{t+1} - k_t. \end{aligned}$$

The period-t government has the objective function of

$$W_t = U_{t-1} + \eta U_t, \tag{7}$$

where $\eta > 0$.[10] Eq. (7) can be explained as a probabilistic voting model (Lindbeck and Weibull, 1987; Grossman and Helpman, 1998). Each generation t disregards dead ancestors by the period t. Hence, using Eqs. (3) and (4), Eq. (7) can be reduced to

$$\begin{aligned} W_t &\simeq \delta \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right] \\ &\quad + \eta \left\{ \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right] - \left(\delta^{-1} - \theta \right) \beta u(c_t^o, g_t) \right\} \\ &= (\delta + \eta) \left\{ u(c_t^y, g_t) + \psi u(c_t^o, g_t) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \phi u(c_{t+s}^o, g_{t+s}) \right] \right\}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \psi &\equiv \frac{(1 + \eta\theta)\beta}{\delta + \eta}, \\ \phi &\equiv \frac{\beta}{\delta}. \end{aligned}$$

In Eq. (8), ψ is the weight for the old during the period t, and ϕ is the weight for the old in periods after the period t. The period-t government faces the different weights of the old to the young. The weight parameters (ψ and ϕ) depend on discount factors (θ and δ), β, and η:

Lemma 1. *(i) $\psi < \phi$, (ii) $\psi \gtreqless 1 \Leftrightarrow \beta \gtreqless \frac{\delta+\eta}{1+\eta\theta}$, and (iii) $\phi \gtreqless 1 \Leftrightarrow \beta \gtreqless \delta$.*

Part (i) of Lemma 1 shows that the weight of the old to the young in the current period is less than that in the future. Hence, the intergenerational altruism leads to future-biased preferences with time inconsistency (see Krusell et al. 2002; Gonzalez et al. 2018). We can easily verify this notion. From the restrictions of parameters, we have $\delta\theta < 1$. Hence, the difference between ψ and ϕ is

$$\psi - \phi = \frac{(1 + \eta\theta)\beta}{\delta + \eta} - \frac{\beta}{\delta} = \left[\frac{\delta\theta - 1}{(\delta + \eta)\delta} \right] \beta\eta < 0.$$

Parts (ii) and (iii) of Lemma 1 demonstrate that the weights could be larger or less than unity depending on the deep parameters. Lemma 1 also implies that the weight has three cases: (a) $1 < \psi <$

[10] Eq. (7) is functionally equivalent to the populational welfare function presented by Hori (1997). See also Aoki and Nishimura (2017) for the formulation.

ϕ, (b) $\psi < 1 < \phi$, and (c) $\psi < \phi < 1$. Case (a) means that the period-t government attaches weight to the old in the current and future periods. Then, Case (b) indicates that the government cares about the current young but not too much care about the future young. Finally, Case (c) shows that the government attaches weight to the young in the current and future periods, contrary to Case (a).

If $\beta = 1$, then the people equally evaluate their youth and their old-age utilities.[11] Then, $\phi > 1$ holds. However, if $\beta < 1$, then ϕ could be either of larger or less than unity. The partial derivatives of ϕ and ψ with respect to β and η are

$$\begin{aligned} \frac{\partial \phi}{\partial \beta} &= \delta^{-1} > 0, \\ \frac{\partial \psi}{\partial \beta} &= \frac{1+\eta\theta}{\delta+\eta} > 0, \\ \frac{\partial \psi}{\partial \eta} &= -\frac{(1-\delta\theta)\,\beta}{(\delta+\eta)^2} < 0. \end{aligned}$$

An increase in β increases ψ and ϕ in the straightforward ways. Moreover, an increase in η decreases ψ because the current democratic government more discounts the utility of the current old in compared with the future old by increasing the young's political weight. Therefore, Cases (a)–(c) are all possible depending on the parameter values.

Table 1 reports the calculated values of θ, δ, ψ and ϕ, corresponding to the given values of μ and λ with $\beta = 1$. The discount factors are above the corresponding parameters of altruism to ancestors or descendants. Smaller μ and larger λ lead to smaller ψ. Moreover, the values of ψ are less than unity in some cases. Considering that β tends to be less than unity in reality, ψ is more likely to be less than unity under strong altruism to descendants relative to ancestors. Moreover, smaller η may generate $\psi > 1$. ψ decrease with η, and η tends to be less than unity because of the less voter turnout rate of the young to the old. Thus, ψ possibly takes a value larger than unity (as in the case of $\eta = 0.6$ in Table 1) if people are more altruistic to their ancestors than to their descendants.

3 Equilibrium government policy

This section considers that the policymaking by the period-t government can achieve the desired period-t allocation. Furthermore, the study focuses on the growth and welfare properties of the planning economy. The planning problem is based on the setting developed by Krusell et al. (2002) and Gonzalez et al. (2018). The governments directly choose the allocation of the aggregate resources between consumption and investment and of the aggregate consumption between the young and the old during the period t. As mentioned earlier, such governments can be regarded as democratically elected planners. In the present study, this economy is referred to as *a democratically planned economy*.

The static problem to choose the consumption allocation in period t is formulated as follows:[12]

$$\max_{0 \le \pi \le 1} \left[u(\pi c, g) + \psi u((1-\pi)\, c, g) \right],$$

where

$$\pi \equiv \frac{c^y}{c}.$$

The subscript t is omitted from the notation hereafter (i.e., c stands for c_t). Furthermore, the prime is used to represent the variables one period later; c' is used for c_{t+1}. By solving the optimization problem, we obtain

$$\pi^* \equiv \frac{1}{1+\psi^{\frac{1}{\sigma}}}.$$

[11] $\beta = 1$ corresponds to Gonzalez et al.'s (2018) model if $\gamma = 0$.

[12] See Hori (1997) and Aoki and Nishimura (2017) for solving the maximization problem.

The young's share of private consumption decreases with the relative utility weight of the old period.

Notably, the weight for the old, ψ, in π^* is powered by the EIS (i.e., σ^{-1}). The inverse of EIS can be interpreted as the degree of inequality aversion in consumption between the young and the old. That is, it measures the effect of a 1% increase in the consumption dispersion on a σ% decrease in the ratio of marginal utilities (i.e., the elasticity of marginal utility or the welfare cost of the inequality). If σ is smaller, then the utility decreases more slowly as consumption increases. This situation means that people are more willing to allow intergenerational consumption dispersion. When $\sigma = 1$, the marginal utility decreases at 1% as consumption increases at 1%. Therefore, the society with $\sigma < 1$ has a weak inequality aversion relative to the society with $\sigma > 1$.

If $\sigma^{-1} \to 0$, then the young's and the old's consumption share are fifty-fifty. No inequality exists in consumption because the young's consumption complements that of the old's and vice versa in the society. By contrast, if $\sigma^{-1} \to \infty$, two possible cases arise ($\psi < 1$ and $\psi > 1$). When people are more (less) altruistic to their descendants than to their ancestors and discount their utilities in their old-age, ψ tends to be smaller (larger) than unity. As $\sigma^{-1} \to \infty$, in the society, the young (old) consume all private consumption resources because their consumption is perfectly substitutable for the old's consumption, and the weight for the old (young) is smaller than that for the young (small). In each case with $\sigma^{-1} \to \infty$, there is no welfare cost of consumption inequality.

The reported values of the EIS by empirical studies vary and are typically in the range of 0.1 to 10. For calibrations, the value of 0.5 (e.g., Trabandt and Uhlig 2011; Jin 2012; Rudebusch and Swanson 2012) or the value of 2 (Barro 2009; Ai 2010; Colacito and Croce 2012) is used. Notably, the corresponding values of σ are $\sigma = 2$ for the former and $\sigma = 0.5$ for the latter. Havranek et al. (2015) found that households in countries with higher income per capita and higher stock market participation show larger EIS values. Hence, the EIS in the developed countries is supposed to be larger than that in the developing countries. Whether the EIS is larger than unity or not will be one of the keys to understanding the significance of public goods in this model.

The following optimization problem represents the decision-making of the period-t government:[13]

$$V_0(k) = \max_{k',g} \left\{ q(\pi,\psi) \frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\psi) \frac{g^{1-\sigma}}{1-\sigma} + \delta V(k') \right\}, \tag{9}$$

with

$$V(k) = q(\hat{\pi},\phi) \frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\phi) \frac{g^{1-\sigma}}{1-\sigma} + \delta V(k'), \tag{10}$$

where $\hat{\pi}$ is the anticipated future consumption share,

$$c = (A+1)k - g - k', q(\pi,z) \equiv \pi^{1-\sigma} + z(1-\pi)^{1-\sigma}, \text{ and } \Gamma(z) \equiv (1+z)\gamma \text{ for } z = \phi,\psi.$$

$q(\pi,z)$ and $\Gamma(z)$ respectively denote the aggregate weight of the utility from private and public goods consumption for the given utility weight of the old relative to the young z. The parameters ψ and ϕ are important to determine whether the period-t government has a present or future bias. Gonzalez et al. (2018) showed that the period-t government has a future bias, and $\psi < \phi$ holds (Lemma 1).

The first-order conditions are

$$g : -q(\pi,\psi) c^{-\sigma} + \Gamma(\psi) g^{-\sigma} = 0, \tag{11}$$

$$k' : -q(\pi,\psi) c^{-\sigma} + \delta \frac{\partial V(k')}{\partial k'} = 0. \tag{12}$$

Eq. (11) leads to

$$\frac{\Gamma(\psi) g^{-\sigma}}{q(\pi,\psi) c^{-\sigma}} = 1 \Leftrightarrow \left[\frac{q(\pi,\psi)}{\Gamma(\psi)}\right]^{\frac{1}{\sigma}} \frac{g}{c} = 1 \Leftrightarrow \frac{g}{c} = \chi, \tag{13}$$

[13] The value function (9) is a reduced form: $W(k) = (\eta+\delta) V_0(k)$.

where

$$\chi \equiv \left[\frac{\Gamma(\psi)}{q(\pi,\psi)}\right]^{\frac{1}{\sigma}}.$$

Eq. (13) corresponds to the extended Samuelson rule; that is, it optimally determines the allocation between private and public good during the period t with ψ.[14] The basic premise of Eq. (13) is equating the weighted sum of MRS and MRT between public and private goods, which is equal to unity (leftmost of Eq. (13)). The weight in Eq. (13) depends on the parameters related to the intertemporal concerns for ancestors and descendants, inequality aversion, the taste for public goods, and others through the key parameter ψ. χ includes such preference parameters and indicates the demand for public goods relative to private goods. Then, $\chi^{-\sigma}$ measures all the impacts of the preference parameters on the cost side and can be interpreted as the effective marginal cost of public goods provision. Generalizing that the MRT between private and public goods is unity, γ measures the efficiency of the public goods sector because γ^{-1} units of private goods are needed to produce one unit of public goods.

We now turn to the effects of public goods on the dynamic performance of the economy. Eq. (11), $V_0(k)$, and $V(k)$ imply that the utility weights in the next periods differ from those in the current period. Hence, the extended Samuelson rule (13) will not be the best for the future periods because of the difference between ψ and ϕ. Specifically, if the government could choose future public goods provision, then it would choose a different allocation, satisfying $\Gamma(\phi)\, g^{-\sigma} = q(\widehat{\pi},\phi)\, c^{-\sigma}$. Although the future provision of the public goods is determined by the future governments, each future government faces the same decision-making for the current government in the administration. Therefore, Eq. (13) is sub-optimally carried over to the future governments. Moreover, the deviation between ψ and ϕ brings about welfare loss by consumption misallocation. This deviation should be compensated by more or less investment, depending on ψ, ϕ, γ, and σ. This economic response influences the economic growth rate through such an investment change. For the formal analysis of the details, we introduce the following definition:

Definition 1. *A Markov strategy of the period-t government is a triplet of $\{c_t(k_t), i_t(k_t), g_t(k_t)\}$. A Markov perfect equilibrium is a set of sequences $\{c_t(k_t), i_t(k_t), g_t(k_t)\}_{t=0}^{\infty}$, satisfying Eqs. (6), (9)–(12), and $\{c_t(k_t), i_t(k_t), g_t(k_t)\} = \{c(k_t), i(k_t), g(k_t)\}\ \forall t$.*

To derive the equilibrium government policy, we assume that the period-t government has a linear strategy and anticipates future government's policy as follows:

$$k' = \begin{cases} \kappa k \text{ for period } t, \\ \widehat{\kappa} k \text{ for the periods after period } t. \end{cases} \tag{14}$$

The definition of investment function and Eq. (14) derive

$$i(k) \equiv \begin{cases} (\kappa - 1)\, k \text{ for period } t, \\ (\widehat{\kappa} - 1)\, k \text{ for the periods after period } t. \end{cases}$$

Using Eqs. (10), (12), (13) and (14), we obtain

$$\kappa = \frac{1 + A}{1 + \left[\frac{1+\chi}{\frac{q(\widehat{\pi},\phi)}{q(\pi,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi} \frac{\delta^{-1} - \widehat{\kappa}^{1-\sigma}}{(A+1-\widehat{\kappa})^{1-\sigma}}\right]^{\frac{1}{\sigma}}} \equiv B(\pi, \widehat{\pi}, \widehat{\kappa}). \tag{15}$$

Eq. (15) is the best response of the period-t government for future governments.[15]

Following the standard growth models, we impose the following assumption.

[14] Myles (1997) derived a similar condition by considering a durable public good and showed that the degree of intertemporal concern (i.e., discounting based on intergenerational altruism) and the long-lived nature of the public goods weight the marginal benefit of public goods. He showed that the Samuelson rule is independent of the discounting rate under perfect depreciation. By contrast, the degree of intertemporal concern for ancestors and descendants is reflected in Eq. (13).

[15] If $\chi = 0$ (i.e., $\gamma = 0$), then Eq. (15) is identical to that derived by Gonzalez et al. (2018).

Assumption 1. $(1+A)^{-1} < \delta < (1+A)^{\sigma-1}$.

Regarding existence and uniqueness of a Markov perfect equilibrium, Eqs. (6), (12), (13), (14), and (15) with $\pi = \hat{\pi} = \pi^*$ and $\kappa = \widehat{\kappa}$ provide the following proposition (see Appendix A for the proof of Proposition 1):

Proposition 1. *In a democratically planned economy, there exists a unique Markov perfect equilibrium in linear strategies with*

$$
\begin{aligned}
i^*(k) &= (\kappa^* - 1)\, k > 0, \\
c^*(k) &= \left(\frac{A + 1 - \kappa^*}{1 + \chi^*}\right) k > 0, \\
g^*(k) &= \left(\frac{A + 1 - \kappa^*}{1 + \chi^*}\right) \chi^* k > 0,
\end{aligned}
$$

where

$$
\chi^* \equiv \left[\frac{\Gamma(\psi)}{q(\pi^*, \psi)}\right]^{\frac{1}{\sigma}},
$$

and κ^* is given by the following:

$$
\frac{\kappa^\sigma}{\delta} = \left\{\frac{q(\pi^*, \phi)}{q(\pi^*, \psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)} \chi^*\right\} \frac{A + 1 - \kappa}{1 + \chi^*} + \kappa \equiv D(\kappa).
$$

The investment and private and public goods consumption are the functions with respect to capital if κ is given. The last equation in Proposition 1 corresponds to the consumption Euler equation in the equilibrium and determines the equilibrium value of κ. If $\phi = \psi$, then the right-hand side of the consumption Euler equation becomes $D(\kappa) = 1 + A$. Hence, $D(\kappa)$ captures the deviation from the gross rate of return on investment $(1 + A)$ by a future bias. The left-hand side of the consumption Euler equation represents the MRS of future consumption for the current consumption.

Figure 1 illustrates the degree of the deviation, $\delta D(\kappa)$, and the MRS between the current and future consumption, κ^σ, in the consumption Euler equation. κ^σ-curves have different shapes depending on σ. The EIS, σ^{-1}, measures the sensitivity of the growth rate of consumption to the rate of return on investment. Given that $\sigma^{-1} > 1$ for $0 < \sigma < 1$ ($\sigma^{-1} < 1$ for $\sigma > 1$), an increase in the rate of return on investment more (less) increases the consumption growth rate; the κ^σ-curve is concave (convex) downward. The future bias increases the benefit of investment (i.e., increasing future consumption) by compensating the sub-optimality of future private and public goods consumption allocations. Hence, $\delta D(\kappa)$ depends on investment κ. The utility weight of the first period in the planning horizon is less than that of the period afterward. On the one hand, the higher weight of future utility motivates the agent to invest. On the other hand, the current cost of investment (reducing consumption) is not sufficiently compensated by higher future consumption. Therefore, $\delta D(\kappa)$ is decreasing in κ, which indicates the downward curve in Figure 1. As κ^σ is the upward curve in each panel of Figure 1, these two graphs reveal that there exists a unique intersection point E.

To verify the properties of the dynamic equilibrium shown in Proposition 1, we consider the planning economy that the period-t government can commit to all allocations from period $t+1$ onward as a benchmark case (Gonzalez et al., 2018). The equilibrium in the planned economy with a nonbiased dictator (e.g., the elderly) can be derived from a standard dynamic optimization problem.[16] This benchmark economy is referred to as *an elderly planned economy.* The problem of choosing the sharing rule is rewritten as

$$
\max_{0 \le \pi \le 1} \left[u(\pi c, g) + \phi u((1 - \pi)\, c, g)\right].
$$

[16] The derivations of key equations in the planned economy are derived from the results of Proposition 1 by substituting ϕ for ψ.

Hence, the sharing rule $\pi^{\dagger}$ and the optimal growth factor become

$$\pi^{\dagger} = \frac{1}{1+\phi^{\frac{1}{\sigma}}} \text{ and } \kappa^{\dagger} = \delta^{\frac{1}{\sigma}} \left(1+A\right)^{\frac{1}{\sigma}}.$$

Furthermore, the consumption and investment functions are

$$i^{\dagger}(k) = \left(\kappa^{\dagger}-1\right)k, c^{\dagger}(k) = \left(\frac{A+1-\kappa^{\dagger}}{1+\chi^{\dagger}}\right)k, \text{ and } g^{\dagger}(k) = \left(\frac{A+1-\kappa^{\dagger}}{1+\chi^{\dagger}}\right)\chi^{\dagger}k,$$

where

$$\chi^{\dagger} \equiv \left[\frac{\Gamma(\phi)}{q(\pi^{\dagger},\phi)}\right]^{\frac{1}{\sigma}}.$$

Comparisons between the democratically planned and the elderly planned cases characterize the properties of the Markov equilibrium derived in Proposition 1. First, the welfare weights take an identical function form, and the differences are only the utility weights for the old. The following lemma is useful to the comparisons between two Markov equilibria with different utility weights for the old (see Appendix B for the proof of Lemma 2):

Lemma 2. *Let be $\pi(z) = \frac{1}{1+z^{\frac{1}{\sigma}}}$, where $z = \psi, \phi$. Then,*

$$(i)\ \pi(z) \gtreqless \frac{1}{2} \Leftrightarrow z \lesseqgtr 1, \frac{z}{\pi(z)}\frac{d\pi(z)}{dz} = -\frac{1-\pi(z)}{\sigma} < 0,$$

$$(ii)\ q(\pi(z),z) = \left(1+z^{\frac{1}{\sigma}}\right)^{\sigma}, \frac{z}{q(\pi(z),z)}\frac{dq(\pi(z),z)}{dz} = \frac{\left(\frac{1-\pi(z)}{\pi(z)}\right)^{1-\sigma}}{1+\left(\frac{1-\pi(z)}{\pi(z)}\right)^{1-\sigma}z} > 0,$$

$$(iii)\ \Gamma(z) = (1+z)\gamma, \frac{z}{\Gamma(z)}\frac{d\Gamma(z)}{dz} = \frac{z}{1+z} > 0.$$

Part (i) of Lemma 2 shows that the young's share of private consumption, $\pi(z)$, decreases with the utility weight for the old, z. This result is straightforward but has an implication on the inequality matters. If $\pi > 0.5$ (i.e., $z < 1$), then the inequality decreases with an increase in z. However, if $\pi < 0.5$ (i.e., $z > 1$), then the inequality increases with an increase in z.

Part (ii) of Lemma 2 denotes the effect of an increase in the utility weight for the old, z, on the welfare weight of the utility from private consumption, $q(\pi(z),z)$. Given that the private goods are rivalrous, the private consumption allocation affects $q(\pi(z),z)$. Moreover, if the society is weakly inequality averse (i.e., $\sigma < 1$), then the welfare cost of the inequality expansion is small. Hence, the government more values the utility weight that has larger consumption share than the other. For $\pi > 0.5$, an increase in z has smaller impact on $q(\pi(z),z)$ than has that for $\pi < 0.5$ because an increase in z decreases the inequality for $\pi > 0.5$. If the society is strongly inequality averse (i.e., $\sigma > 1$), in a response to an increase in z, then the old's utility from private goods for $\pi > 0.5$ is more weighted than that for $\pi < 0.5$. The reason is that an increase in z decreases the inequality for $\pi > 0.5$. Therefore, the evaluation of the weight in the strongly inequality-averse society is a reversal to that in the weakly inequality-averse society.

Part (iii) of Lemma 2 indicates the effect of an increase in z on the welfare weight of the utility from public goods, $\Gamma(z)$. $\Gamma(z)$ is a simple sum of the young's and the old's utility from public goods owing to its non-rivalrous nature. Naturally, this sum is independent of the private consumption allocation. Therefore, the impact of an increase in z on $\Gamma(z)$ could be larger or smaller than that on $q(\pi(z),z)$, depending on the private consumption allocation and inequality aversion.

Using Lemma 2, we obtain another lemma (see Appendix B for the proof of Lemma 3):

Lemma 3. $\frac{d\chi}{dz} \gtreqless 0 \Leftrightarrow z^{\frac{\sigma-1}{\sigma}} \gtreqless 1 \Leftrightarrow (1-\sigma)\left(\pi(z)-\frac{1}{2}\right) \gtreqless 0.$

The intuition of Lemma 3 can be explained through Parts (i)–(iii) of Lemma 2 and its interpretation. The key factors are the degree of inequality aversion (the elasticity of marginal utility/the inverse of the EIS) and the private consumption allocation. χ is the ratio of $\Gamma(z)$ to $q(\pi(z), z)$ powered by the inverse of the degree of inequality aversion (i.e., the EIS). Hence, the percentage change of χ in response to a change in z breaks down into the percentage change of $\Gamma(z)$ in response to a change in z and the percentage change of $q(\pi(z), z)$ in response to a change in z:

$$\frac{z}{\chi}\frac{d\chi}{dz} = \left[\frac{z}{\Gamma(z)}\frac{d\Gamma(z)}{dz} - \frac{z}{q(\pi(z), z)}\frac{dq(\pi(z), z)}{dz}\right]\sigma^{-1}.$$

As shown in Part (iii) of Lemma 2, a 1% increase in z causes a $z/(1+z)$% increase in the aggregate weight of the utility from public goods, $\Gamma(z)$. Given that the public goods are non-rivalrous, how much share z has in total of $\Gamma(z)$ is important for the sensitivity of a change in $\Gamma(z)$ to a change in z. By contrast, $q(\pi(z), z)$ depends on the private consumption allocation by its rivalrous nature. Considering $\sigma < 1$, the society has a relatively small welfare cost of inequality. People are willing to allow the dispersion of consumption. The old's utility from private goods is less weighted for $\pi > 0.5$ and more weighted for $\pi < 0.5$ because an increase in z decreases the inequality for $\pi > 0.5$ while it increases the inequality for $\pi < 0.5$. Therefore, as derived in Part (ii) of Lemma 2, if $\pi > 0.5$ ($\pi < 0.5$), a 1% increase less (more) increases $q(\pi(z), z)$ than $z/(1+z)$%. Therefore, χ is increasing (decreasing) in z if $\sigma < 1$ and $\pi > 0.5$ ($\pi < 0.5$). This relationship between χ and z illustrates the inverted-U curve in the top diagram of Figure 2. This notion implies that, for more equal distribution (closer to $z = 1$), the public goods are more needed because the public goods are more weighted, thereby having a large welfare impact. Naturally, if $\sigma > 1$, then it will derive the opposite results. The situation is illustrated in the bottom graph of Figure 2. For more unequal distribution (far away from $z = 1$), the public goods are more desired.

Lemmas 1, 2, and 3 and Proposition 1 derive the following results concerning private and public consumptions (see Appendix B for the proof of Proposition 2):

Proposition 2. *(i) The sharing rules of private consumptions in the two planned economies complies with*

$$\begin{aligned}
(a)\ \pi^{\dagger} &< \pi^{*} < \frac{1}{2}\ for\ 1 < \psi < \phi,\\
(b)\ \pi^{\dagger} &< \frac{1}{2} < \pi^{*}\ for\ \psi < 1 < \phi,\\
(c)\ \frac{1}{2} &< \pi^{\dagger} < \pi^{*}\ for\ \psi < \phi < 1.
\end{aligned}$$

(ii) The ratios of public goods consumption to private goods consumption in the two planned economies satisfy

$$\chi^{*} \gtreqless \chi^{\dagger} \Leftrightarrow (\sigma - 1)\left(\pi^{\dagger} + \pi^{*} - 1\right) \gtreqless 0.$$

Part (i) of Proposition 2 is similar to that of Gonzalez et al. (2018) because a future bias induces a higher share of the young consumption to aggregate consumption. In particular, Part (i)–(a) is identical to that of Gonzalez et al. (2018). In this case, for $1 < \psi < \phi$, the elderly planner (government) chooses a more unequal income distribution than does the democratic government. On the contrary, Part (i)–(c) demonstrates that the elderly government chooses a more equal income distribution than does the democratic government if the utility weights for the old are less than unity ($\psi < \phi < 1$). Part (i)–(b) is the intermediate case between (i)–(a) and (i)–(c). Table 2 reports the calculated values based on the parameters used in Table 1 with $\sigma = 0.5$ and $\sigma = 2$, except for the value of β. For the example of $\phi < 1$, we set $\beta = 0.8$ here. As shown in Table 2, we observe $\pi^{\dagger} < 0.5 < \pi^{*}$ for $\psi < 1 < \phi$ and smaller inequality than that of (a) or (c).

According to the income distribution, Cases (a)–(c) are characterized by $\pi^{*} + \pi^{\dagger}$. Cases (a) and (b) in the case of $\left|\pi^{\dagger} - 0.5\right| > |\pi^{*} - 0.5|$ satisfy $\pi^{*} + \pi^{\dagger} < 1$. Hence, when $\pi^{*} + \pi^{\dagger} < 1$, the democratic government has a more equal income distribution than has the elderly government. Case (b) in the case of $\left|\pi^{\dagger}-0.5\right| < |\pi^{*}-0.5|$ and Case (c) have $\pi^{*}+\pi^{\dagger} > 1$. The democratic government faces a more unequal distribution than does the elderly government. These facts indicate that the democratic government will use public goods as redistributive instruments for compensating the inequality under future bias, depending on the degree of inequality aversion. Indeed, Part (ii) of Proposition 2 demonstrates that the relationship between public goods provision and inequality.

Suppose that the democratic government faces a more equal income distribution than does the elderly government ($\pi^{*}+\pi^{\dagger} < 1$). If $\sigma < 1$, then the society is willing to allow consumption inequality because of the relatively small welfare cost of inequality. Facing a more equal distribution, the democratic government more values the welfare impact of public goods than does the elderly government (Lemmas 1 and 2). Therefore, the democratic government spends more on public goods than does the elderly government; $\chi^{*} > \chi^{\dagger}$ (Lemma 3). By contrast, if $\sigma > 1$, then the society is unwilling to allow consumption inequality because of the relatively large welfare cost of inequality. More equal distribution makes a lower value in the welfare impact of public goods (Lemmas 1 and 2). Hence, the democratic government spends less on public goods than does the elderly government; $\chi^{*} < \chi^{\dagger}$ (Lemma 3).

When the democratic government faces a more unequal distribution than does the elderly government ($\pi^{*}+\pi^{\dagger} > 1$), all things reverse. If $\sigma < 1$, for a more unequal distribution, the democratic government less values the welfare impact of public goods than does the elderly government (Lemmas 1 and 2). This induces that the democratic government spends less on public goods than does the elderly government; $\chi^{*} < \chi^{\dagger}$ (Lemma 3). However, if $\sigma > 1$, then the democratic government concerns the inequality (Lemmas 1 and 2) and therefore spends more on public goods than does the elderly planner; $\chi^{*} > \chi^{\dagger}$ (Lemma 3). Table 2 shows examples of Part (ii) of Proposition 2 and that a more unequal distribution leads to higher public goods provision, whereas a more equal distribution leads to higher public goods provision.

Proposition 2 demonstrates that the democratic government uses public goods to compensate for the consumption misallocation under future bias, resulting in inequality (Part (i) of Proposition 2). The redistribution affects the consumption resource allocation between private and public goods (Part (ii) of Proposition 2). Non-rival public goods conserve the consumption resource compared with rivalrous private goods. Thus, the presence of public goods and the efficiency of the public sector will also influence the resource allocation between aggregate consumption and investment depending on the degree of inequality aversion. Therefore, Propositions 1 and 2 imply that economic growth is not independent of public goods supply, although it has no relation to public goods with full commitment.

Furthermore, exploring the growth properties, the comparison between κ^{*} and $\kappa^{\dagger}$ and the partial derivative of κ^{*} with respect to γ lead to the following proposition (see Appendix C for the proof of Proposition 3):

Proposition 3. *(i)* $\kappa^{*} > \kappa^{\dagger}$. *(ii)* $\frac{\partial \kappa^{*}}{\partial \gamma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1 \Leftrightarrow (1-\sigma)\left(\pi(z)-\frac{1}{2}\right) \gtreqless 0.$

Part (i) of Proposition 3 shows that the growth rate in the democratically planned economy exceeds that in the elderly planned economy. Figure 1 explains Part (i) of Proposition 3. The equilibrium in the elderly planned economy is given by the point F, whereas the equilibrium in the democratically planned economy is the point E in Figure 1. As explained earlier, the degree of deviation, $D(\kappa)$, is larger than $(1+A)$ for any value of $\kappa \in [0, 1+A)$. As in the logic of Gonzalez et al. (2018), future bias implies that the future old-age transfers are insufficient. Therefore, the future growth is excessive from the viewpoint of the government at any given period.

Part (ii) of Proposition 3 implies that the efficiency of the public sector, γ, affects economic growth even if the public goods are *unproductive*. Without any bias, the optimal growth rate is independent of γ (e.g., Turnovsky 1996; Tamai 2010).[17] By contrast, γ affects $B(\pi, \widehat{\pi}, \widehat{\kappa})$ (or equivalently $D(\kappa)$ in equilibrium) through a change in χ if a future bias exists. The impact depends on the weight for the

[17] Turnovsky (1996) also showed that the financing methods of public goods supply affect the growth rate because of the distortionary taxes.

old (i.e., income distribution) and the degree of inequality aversion. A rise in γ decreases the effective marginal cost of public goods. This cost reduction changes the allocation between private and public goods. Consequently, χ increases. Depending on the income distribution, a rise in γ increases or decreases the return to investment, which induces more or less investment. In equilibrium, $B(\pi, \widehat{\pi}, \widehat{\kappa})$ with $\pi = \widehat{\pi} = \pi^*$ yields $D(\kappa)$. Hence, Figure 1 characterizes a movement of the equilibrium. In Figure 1, if γ increases (decreases) $D(\kappa)$, the curve $\delta D(\kappa)$ moves upward (downward).[18] The corresponding equilibrium point moves to the upper-right along with κ^σ.

Basically, the consumption misallocation lowers welfare for given aggregate levels of future consumption. The government will invest more to compensate for this loss for future generations. However, transferring the resources to future decreases the return to investment because of future misallocation of the transferred resources. The government strategically reduces investment to decrease misallocation by substituting consumption intertemporally. The former and latter effects are the *income* and *substitution effects*, respectively. Depending on the value of σ, the presence of public goods influences economic growth through affecting the income and substitution effects. Therefore, the economic intuition of Part (ii) of Proposition 3 is explained as follows.

Considering $\pi^* < 0.5$ ($\psi > 1$), the democratic government currently faces a more equal income distribution; $\pi^\dagger < \pi^* < 0.5$. For $\sigma > 1$ and the consumption allocation, the non-rivalrous public goods are more weighted relative to the rivalrous private goods (Lemma 2). Then, the democratic government chooses low public goods provision today and wishes high future public goods provision (Lemma 3 and Part (ii) of Proposition 2). Moreover, the current government anticipates the same choice by the future government and the future public goods allocation is inefficiently small from the perspective of the current government. This event causes two opposing effects: the income and substitution effects. For $\sigma > 1$, the former effect dominates the latter effect. The current government chooses to raise investment to compensate for the consumption misallocation. Then, a rise in γ strengthens the income effect relative to the substitution effect because the welfare impact of public goods is largely weighted. In the other words, it enables the current government to save resources for providing public goods and obtain the resource for more investment. Therefore, the more efficient public sector (i.e., the more important public goods) induces more investment.

If $\sigma < 1$, then the reversal mechanism works. In this case, the private goods are more weighted relative to the public goods. The current government chooses high public goods provision today for $\pi^* < 0.5$ and desires low public goods provision in the future (Part (ii) of Proposition 2). However, the current government expects that the future governments will choose the high public goods provision similar to what the current government does. For $\sigma < 1$, the substitution effect dominates the income effect. The government has an incentive to mitigate to the loss from raising investment. Then, a rise in γ weakens the income effect relative to the substitution effect because the welfare impact of public goods is not much weighted. As the more efficient public sector (i.e., the more important public goods) increases, investment loss also increases. Therefore, a rise in γ lowers the economic growth rate.

Turning to $\pi^* > 0.5$ ($\psi < 1$), the democratic government could face a more unequal or equal income distribution. If $\sigma > 1$, then the private goods are more weighted relative to the public goods. Facing a more unequal distribution ($\pi^* > \pi^\dagger > 0.5$; $\pi^* + \pi^\dagger > 1$), the democratic government chooses high public goods provision today and prefers low public goods provision in the future. Then, the future public goods allocations chosen by future governments are too large for the current government. This event causes the income and substitution effects mentioned previously. For $\sigma > 1$, the former effect dominates the latter effect. If $\pi^* > 0.5$, a rise in γ weakens the income effect relative to the substitution effect, and therefore an increase in γ lowers economic growth rate. The reason is that an increase in γ generates unwilling future high public goods provision from the current government's perspective. By contrast, if $\sigma < 1$, then the public goods are more weighted relative to the private goods. The current government chooses low public goods provision today and wishes for high public goods provision in the future. However, the current government anticipates that the future low public goods provision will be chosen by the future governments. When $\sigma < 1$, the substitution effect dominates the income

[18] Around the equilibrium, current and next-period investments are strategic complements ($\partial B/\partial\kappa > 0$) for $\sigma > 1$ while they are strategic substitutes ($\partial B/\partial\kappa < 0$) for $\sigma < 1$. Notably, we have $sgn(\partial B/\partial\gamma) = sgn[(1-\sigma)(\pi - 0.5)]$.

effect. In this case, a rise in γ strengthens the income effect relative to the substitution effect because it contributes to future high public goods provision. Therefore, an increase in γ has a positive effect on economic growth rate.

For more equal distribution, the desired future public goods provision could be larger than the first best because the supply levels may be distributed around nearly the level in case of the perfect equality. When $\pi^\dagger < \pi^* < 0.5$, the situation is the reversal of more unequal distribution where $\pi^* > \pi^\dagger > 0.5$. In this case, the outcomes are opposite to those in case of $\pi^* > \pi^\dagger > 0.5$. However, if $\pi^\dagger < 0.5 < \pi^*$, the young are more weighted than the old today, whereas the future young are less weighted than the future old. This situation can be observed when the weights are placed on the different sides of the top of the inverted-U-shaped curve or at the bottom of the U-shaped curve in Figure 2. In particular, the coexistence of a high-benefit welfare system and high growth are suggested if the society is weakly inequality averse, $\psi < 1 < \phi$, and $|\pi^* - 0.5| < \left|\pi^\dagger - 0.5\right|$.

Finally, the welfare properties are examined to make the following proposition (see Appendix D for the proof of Proposition 4):

Proposition 4. *The value functions in the two planned economies are*

$$\begin{aligned} W^* &= \Delta^* v(k, \kappa^*), \\ W^\dagger &= \Delta^\dagger v(k, \kappa^\dagger), \end{aligned}$$

where

$$\begin{aligned} v(k,\kappa) &\equiv \frac{(\delta+\eta)\,(A+1-\kappa)^{1-\sigma}\,k^{1-\sigma}}{(1-\delta\kappa^{1-\sigma})\,(1-\sigma)}, \\ \Delta^* &\equiv \left\{\left[1-\delta\,(\kappa^*)^{1-\sigma}\right] q\,(\pi^*,\psi) + \delta\,(\kappa^*)^{1-\sigma}\, q\,(\pi^*,\phi)\right\}\left(\frac{1}{1+\chi^*}\right)^{1-\sigma} \\ &\quad + \left\{\left[1-\delta\,(\kappa^*)^{1-\sigma}\right]\Gamma\,(\psi) + \delta\,(\kappa^*)^{1-\sigma}\,\Gamma\,(\phi)\right\}\left(\frac{\chi^*}{1+\chi^*}\right)^{1-\sigma}, \\ \Delta^\dagger &\equiv q\left(\pi^\dagger,\phi\right)\left(\frac{1}{1+\chi^\dagger}\right)^{1-\sigma} + \Gamma\,(\phi)\left(\frac{\chi^\dagger}{1+\chi^\dagger}\right)^{1-\sigma}. \end{aligned}$$

Then, the relationship between W^* and $W^\dagger$ satisfies

$$(1-\sigma)\left(\frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)} - 1\right) \gtreqless 0 \Leftrightarrow W^* \gtreqless W^\dagger.$$

In the present value of utility W, the coefficient Δ is integrated weights of private and public consumptions and $v(k,\kappa)$ is the utility level measured by a unit of integrated consumptions. For a given k, the value of κ to maximize $v(k,\kappa)$ is $\kappa = \kappa^\dagger$.[19] Hence, $v(k,\kappa^*) < v(k,\kappa^\dagger) < 0$ ($0 < v(k,\kappa^*) < v(k,\kappa^\dagger)$) holds for $\sigma > 1$ ($\sigma < 1$). Ignoring the difference between ϕ and ψ, $\kappa = \kappa^\dagger$ is the best solution of maximizing the welfare, which is similar to the standard AK growth model with geometrical discounting. In other words, the excess investment reduces the social welfare. However, the utility weights ϕ and ψ differ, so that Δ^* and $\Delta^\dagger$. Furthermore, κ^* affects the level of Δ^*. Given that the size relation of Δ^* and $\Delta^\dagger$ is ambiguous, comparing the welfare level is analytically hard. Alternatively, quantitative analysis provides evident numerical examples.

To illustrate realistic case, the parameters and the initial capital stock are specified as $\beta = 0.8$, $\eta = 0.8$, $\sigma = 2$, $A = 2.25$, and $k_0 = 1$. Figure 3 illustrates two curves W^* and $W^\dagger$ with respect to

[19] Partial differentiation of $v(k,\kappa)$ with respect to κ yields

$$\frac{\partial v}{\partial \kappa} = \frac{(A+1-\kappa)^{-\sigma}\left[(1+A)\,\delta\kappa^{-\sigma} - 1\right]}{(1-\delta\kappa^{1-\sigma})^2} \gtreqless 0 \Leftrightarrow \kappa \lesseqgtr \delta^{\frac{1}{\sigma}}\,(1+A)^{\frac{1}{\sigma}} = \kappa^\dagger.$$

$\gamma \in [0, 2]$ for $\mu = 0.4$ and $\lambda = 0.5$. The welfare in the democratically planned economy is larger than that in the elderly planned economy. Furthermore, the welfare difference $\left(W^{*} - W^{\dagger}\right)$ increases with γ. Setting $\gamma = 0.5$, the robustness to changes in μ and λ is examined. Figure 4 reveals that the welfare difference is positive on the domain of μ and λ. These results imply that the democratically planned economy exhibits the welfare dominance to the elderly planned economy.

4 Concluding remarks

This study examined the government policy of the public good provision and its effects on the economic growth and welfare in an endogenous growth model with altruistic overlapping generations. In the model, the democratically elected government is subject to future bias, which has been inherited from the existing individuals. The future bias influences the equilibrium government policy and economic performance of the equilibrium. Without any bias, the government policy of the public good provision does not affect the equilibrium resource allocation and therefore the economic growth. However, the economic growth is not independent of the government policy with future bias because the public goods and investment are strategic instruments to improve intertemporal/intergenerational resource misallocation. This growth effect of the government policy provides nontrivial outcomes in welfare analysis.

Our findings explain why democratic governments ongoingly choose the same high or low level of public goods supply. The governments do not have an incentive to repeal high- or low-benefit welfare system because the future governments have to solve the same problem for the governments before them. Furthermore, our results suggest not only the trade-off between high-benefit welfare system and economic growth but also the coexistence of high-benefit welfare system and high economic growth under certain conditions. If the government prefer to invest more for future generations to compensate for future consumption misallocation, then larger public sector (more efficient public sector) leads to higher economic growth because of the large welfare impact of public goods relative to private goods. In particular, our numerical analysis shows that the welfare in the democratically planned economy dominates that in the elderly planned economy.

Future directions of this research should be described. First, incorporating distortionary-tax financing with labor-leisure choice into our model is a natural way to extend our analysis. As mentioned earlier, some studies addressed similar issues with present bias. The extension of our analysis will provide a different policy insight, which is important to consider the policy with intergenerational conflicts. Second, considering the public good in the production and allocation between two public goods in the utility and production function is interesting. With this extension, as durable public goods, a public capital will be worthwhile to investigate. These analyses will lead to important policy implications under the democratic determination of policy and its effects on economic growth and welfare. The present study provides an analytical basis for these future studies.

Appendix

A. Proof of Proposition 1

Differentiating Eq. (10) with respect to k and using Eqs. (12) and (13) yield the following:

$$\begin{aligned}\frac{\partial V(k)}{\partial k} &= \left\{ q\left(\hat{\pi},\phi\right) + \Gamma\left(\phi\right)\left[\frac{\Gamma\left(\psi\right)}{q\left(\pi,\psi\right)}\right]^{\frac{1-\sigma}{\sigma}} \right\} c^{-\sigma}\frac{\partial c}{\partial k} + \delta\frac{\partial V(k')}{\partial k'}\frac{\partial k'}{\partial k} \\ &= \left\{ q\left(\hat{\pi},\phi\right) + \Gamma\left(\phi\right)\left[\frac{\Gamma\left(\psi\right)}{q\left(\pi,\psi\right)}\right]^{\frac{1-\sigma}{\sigma}} \right\} c^{-\sigma}\frac{1+A-\frac{\partial k'}{\partial k}}{1+\left[\frac{\Gamma(\psi)}{q(\pi,\psi)}\right]^{\frac{1}{\sigma}}} + q\left(\pi,\psi\right)c^{-\sigma}\frac{\partial k'}{\partial k}, \end{aligned} \qquad \text{(A1)}$$

where

$$\frac{\partial c}{\partial k} = \frac{A+1-\frac{\partial k'}{\partial k}}{1+\left[\frac{\Gamma(\psi)}{q(\pi^*,\psi)}\right]^{\frac{1}{\sigma}}}.$$

Inserting $\pi = \pi^* = \hat{\pi}$ and $\kappa = \widehat{\kappa}$ into Eq. (15) leads to

$$\frac{\kappa^\sigma}{\delta} = \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*}\left(A+1-\kappa\right)+\kappa. \qquad \text{(A2)}$$

The left-hand side of this equation, κ^σ/δ, monotonically increases with κ. Furthermore, we have $\kappa^\sigma/\delta = 0$ ($\kappa^\sigma/\delta \to \infty$) as $\kappa = 0$ ($\kappa \to \infty$). The right-hand side of the equation exhibits the following properties:

$$\begin{aligned} \frac{dD(\kappa)}{d\kappa} &= 1 - \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*} < 0, \\ \Gamma(0) &= \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*}\left(1+A\right) > 0, \\ \Gamma(1+A) &= 1+A. \end{aligned}$$

These results show that there exists a unique value of κ satisfies Eq. (A2).

Using $\kappa = \kappa^*$, Eq. (14) and the definition of investment function provide

$$i^*(k) = (\kappa^* - 1)k.$$

$\kappa^* > 1$ must hold to be $i^*(k) > 0$. By Assumption 1, we have $\delta\left(1+A\right) > 1$. Then, $\kappa^* > 1$ holds. Eqs. (6) and (13) yield

$$c^*(k) = \frac{Ak - i^*(k)}{1+\chi^*} \text{ and } g^*(k) = \frac{\chi^*\left[Ak - i^*(k)\right]}{1+\chi^*}.$$

To ensure positive consumptions, $Ak - i^*(k) > 0$ is needed; $\kappa^* < 1 + A$. We have

$$\kappa^* < 1 + A \Leftrightarrow \frac{\left(1+A\right)^\sigma}{\delta} > 1 + A \Leftrightarrow \delta < \left(1+A\right)^{\sigma-1}.$$

Assumption 1 shows that the above condition holds. Under Assumption 1, we have

$$\delta\left(\kappa^*\right)^{1-\sigma} < \delta\left(1+A\right)^{1-\sigma} < 1. \qquad \text{(A3)}$$

Using Eqs. (9) and (10), we obtain

$$\begin{aligned} V^* &= \Bigg\{ \frac{\left[1-\delta\left(\kappa^*\right)^{1-\sigma}\right] q\left(\pi^*, \psi\right)+\delta\left(\kappa^*\right)^{1-\sigma} q\left(\pi^*, \phi\right)}{1-\delta\left(\kappa^*\right)^{1-\sigma}}\left(\frac{1}{1+\chi^*}\right)^{1-\sigma} \\ &\quad +\frac{\left[1-\delta\left(\kappa^*\right)^{1-\sigma}\right] \Gamma(\psi)+\delta\left(\kappa^*\right)^{1-\sigma} \Gamma(\phi)}{1-\delta\left(\kappa^*\right)^{1-\sigma}}\left(\frac{\chi^*}{1+\chi^*}\right)^{1-\sigma}\Bigg\} \frac{\left(A+1-\kappa^*\right)^{1-\sigma} k^{1-\sigma}}{1-\sigma} \end{aligned} \tag{A4}$$

for $\sigma \neq 1$. Furthermore, the transversality condition holds if $\delta\left(\kappa^*\right)^{1-\sigma}<1$. Eq. (A3) is sufficient to ensure the bounded lifetime utility and transversality condition.

B. Proof of Lemmas 2 and 3 and Proposition 2

Proof of Lemma 2. Differentiating π with respect to z, we obtain

$$\frac{z}{\pi} \frac{d \pi}{d z}=-\frac{z^{\frac{1}{\sigma}}}{\left(1+z^{\frac{1}{\sigma}}\right) \sigma}=-\frac{1-\pi(z)}{\sigma}<0. \tag{A5}$$

$\pi(1)=0.5$ holds. Given that $\pi'(z)<0$, $\pi(z) \gtreqless 0.5 \Leftrightarrow z \lesseqgtr 1$. Inserting $\pi(z)$ into $q(\pi(z), z)$ yields

$$q(\pi(z), z)=\left(\frac{1}{1+z^{\frac{1}{\sigma}}}\right)^{1-\sigma}+z\left(\frac{z^{\frac{1}{\sigma}}}{1+z^{\frac{1}{\sigma}}}\right)^{1-\sigma}=\frac{1+z^{\frac{1}{\sigma}}}{\left(1+z^{\frac{1}{\sigma}}\right)^{1-\sigma}}=\left(1+z^{\frac{1}{\sigma}}\right)^{\sigma}.$$

Total differentiation of $q(\pi(z), z)$ and $\Gamma(z)$ give their elasticities.

Proof of Lemma 3. Using Lemma 2, we have

$$\frac{d}{d z}\left[\frac{\Gamma(z)}{q(\pi, z)}\right]=\frac{\theta \pi(z)^{1-\sigma}}{q(\pi, z)^2}\left(1-z^{\frac{1-\sigma}{\sigma}}\right) \gtreqless 0 \Leftrightarrow z^{\frac{\sigma-1}{\sigma}} \gtreqless 1. \tag{A6}$$

Taking the logarithm on both sides of the last inequality in Eq. (A6) provides

$$z^{\frac{\sigma-1}{\sigma}} \gtreqless 1 \Leftrightarrow \log z^{\frac{\sigma-1}{\sigma}}=\frac{\sigma-1}{\sigma} \log z \gtreqless \log 1=0. \tag{A7}$$

Using $\pi(z) \gtreqless 0.5 \Leftrightarrow z \lesseqgtr 1$, we obtain

$$\pi(z) \gtreqless \frac{1}{2} \Leftrightarrow \pi(z)-\frac{1}{2} \gtreqless 0 \Leftrightarrow z \lesseqgtr 1 \Leftrightarrow \log z \lesseqgtr 0. \tag{A8}$$

Eqs. (A7) and (A8) lead to

$$z^{\frac{\sigma-1}{\sigma}} \gtreqless 1 \Leftrightarrow \frac{\sigma-1}{\sigma} \log z \gtreqless 0 \Leftrightarrow(1-\sigma)\left(\pi(z)-\frac{1}{2}\right) \gtreqless 0. \tag{A9}$$

Therefore, we arrive at

$$sgn \frac{d \chi}{d z}=sgn\left(z^{\frac{\sigma-1}{\sigma}}-1\right)=sgn(1-\sigma)\left(\pi(z)-\frac{1}{2}\right).$$

Proof of Proposition 2. (i) By $\pi'(z)<0$ and $\psi<\phi$ (Lemmas 1 and 2), $\pi^*=\pi(\psi)>\pi(\phi)=\pi^{\dagger}$ holds.

(ii) For $1 < \psi < \phi$ ($\pi < 0.5, \pi^* + \pi^\dagger < 1$), Eq. (A6) has a positive (negative) sign with $\sigma > 1$ ($\sigma < 1$). Hence, we obtain

$$\frac{g^*}{c^*} = \chi^* \lesseqgtr \chi^\dagger = \frac{g^\dagger}{c^\dagger} \Longleftrightarrow \sigma \gtreqless 1 \ for \ 1 < \psi < \phi \ (\pi^* + \pi^\dagger < 1). \tag{A10}$$

By contrast, Eq. (A6) is negative for $\psi < \phi < 1$ ($\pi > 0.5, \pi^* + \pi^\dagger > 1$). This result leads to

$$\frac{g^*}{c^*} = \chi^* \gtreqless \chi^\dagger = \frac{g^\dagger}{c^\dagger} \Longleftrightarrow \sigma \gtreqless 1 \ for \ \psi < \phi < 1 \ (\pi^* + \pi^\dagger > 1). \tag{A11}$$

We consider $\psi < 1 < \phi$ ($\pi^\dagger < 0.5 < \pi^*$). For $\sigma > 1$, the minimum value of χ is given at $\pi = 0.5$. Hence, if $\pi^\dagger$ (π^*) is closer to 0.5 than π^* ($\pi^\dagger$), we have $\chi^* > \chi^\dagger$ ($\chi^* < \chi^\dagger$). This can be described as

$$\left|\pi^\dagger - 0.5\right| \lesseqgtr \left|\pi^* - 0.5\right| \Leftrightarrow 0.5 - \pi^\dagger \lesseqgtr \pi^* - 0.5 \Leftrightarrow \pi^\dagger + \pi^* \gtreqless 1 \Leftrightarrow \chi^* \gtreqless \chi^\dagger \ for \ \sigma > 1. \tag{A12}$$

If $\sigma < 1$, the maximum value of χ is given at $\pi = 0.5$. When $\pi^\dagger$ (π^*) is closer to 0.5 than π^* ($\pi^\dagger$), then we have $\chi^* < \chi^\dagger$ ($\chi^* > \chi^\dagger$). Therefore, we obtain

$$\left|\pi^\dagger - 0.5\right| \lesseqgtr \left|\pi^* - 0.5\right| \Leftrightarrow 0.5 - \pi^\dagger \lesseqgtr \pi^* - 0.5 \Leftrightarrow \pi^\dagger + \pi^* \gtreqless 1 \Leftrightarrow \chi^* \lesseqgtr \chi^\dagger \ for \ \sigma < 1. \tag{A13}$$

Eqs. (A10)–(A13) are summarized as

$$\chi^* \gtreqless \chi^\dagger \Leftrightarrow (\sigma - 1)\left(\pi^\dagger + \pi^* - 1\right) \gtreqless 0.$$

C. Proof of Proposition 3

Partial differentiation of $q(\pi^*, z)$ with respect to z is

$$\frac{\partial q(\pi^*, z)}{\partial z} = (1 - \pi^*)^{1-\sigma} > 0.$$

This equation shows that $q(\pi^*, \phi) > q(\pi^*, \psi)$ holds. By the definition of $\Gamma(z)$, $\Gamma(\phi) > \Gamma(\psi)$ is obtained. These two inequalities yield

$$\frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1 + \chi^*} > 1.$$

Hence, we have

$$D(0) = \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1 + \chi^*}(1 + A) > 1 + A.$$

Considering $D'(\kappa) < 0$ and $D(1 + A) = 1 + A$, $D(\kappa) > 1 + A = (\kappa^o)^\sigma / \delta$ holds for $\kappa \in [0, 1 + A)$. Using $1 < \kappa^* < 1 + A$, we arrive at

$$D(\kappa^*) = \frac{(\kappa^*)^\sigma}{\delta} > 1 + A = \frac{(\kappa^o)^\sigma}{\delta} \Rightarrow \kappa^* > \kappa^o.$$

Partial differentiation of $D(\kappa)$ with respect to γ is

$$\begin{aligned}
\frac{\partial D(\kappa)}{\partial \gamma} &= \frac{\left[\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}\right](A + 1 - \kappa)}{(1 + \chi^*)^2}\frac{\partial \chi^*}{\partial \gamma} \\
&= \frac{\left[\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}\right](A + 1 - \kappa)\chi^*}{(1 + \chi^*)^2 \gamma\sigma} \\
&= \frac{(\phi - \psi)\left(1 - \psi^{\frac{1-\sigma}{\sigma}}\right)(A + 1 - \kappa)\chi^*}{(1 + \psi)\left(1 + \psi^{\frac{1}{\sigma}}\right)^\sigma (1 + \chi^*)^2 \gamma\sigma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.
\end{aligned}$$

Using this equation and (A9), we obtain

$$\frac{\partial \kappa^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow \frac{\partial D(\kappa)}{\partial \gamma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.$$

D. Proof of Proposition 4

The value function in the elderly planned economy is

$$V^\dagger = \frac{q\left(\pi^\dagger, \phi\right)\left(\frac{1}{1+\chi^\dagger}\right)^{1-\sigma} + \Gamma\left(\phi\right)\left(\frac{\chi^\dagger}{1+\chi^\dagger}\right)^{1-\sigma}}{1-\delta\left(\kappa^\dagger\right)^{1-\sigma}} \frac{\left(A+1-\kappa^\dagger\right)^{1-\sigma} k^{1-\sigma}}{1-\sigma}.$$

We have

$$W^* = (\delta+\eta)\, V^* \text{ and } W^\dagger = (\delta+\eta)\, V^\dagger.$$

Dividing W^* by $W^\dagger$ leads to

$$\frac{W^*}{W^\dagger} = \frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)}.$$

Notably, $v(k,\kappa) < 0$ ($W^* < 0$ and $W^\dagger < 0$) for $\sigma > 1$. Hence,

$$\frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)} \gtreqless 1 \Leftrightarrow \frac{W^*}{W^\dagger} \gtreqless 1 \Leftrightarrow W^* \lesseqgtr W^\dagger.$$

For $\sigma < 1$, we have

$$\frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)} \gtreqless 1 \Leftrightarrow \frac{W^*}{W^\dagger} \gtreqless 1 \Leftrightarrow W^* \gtreqless W^\dagger.$$

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Figures

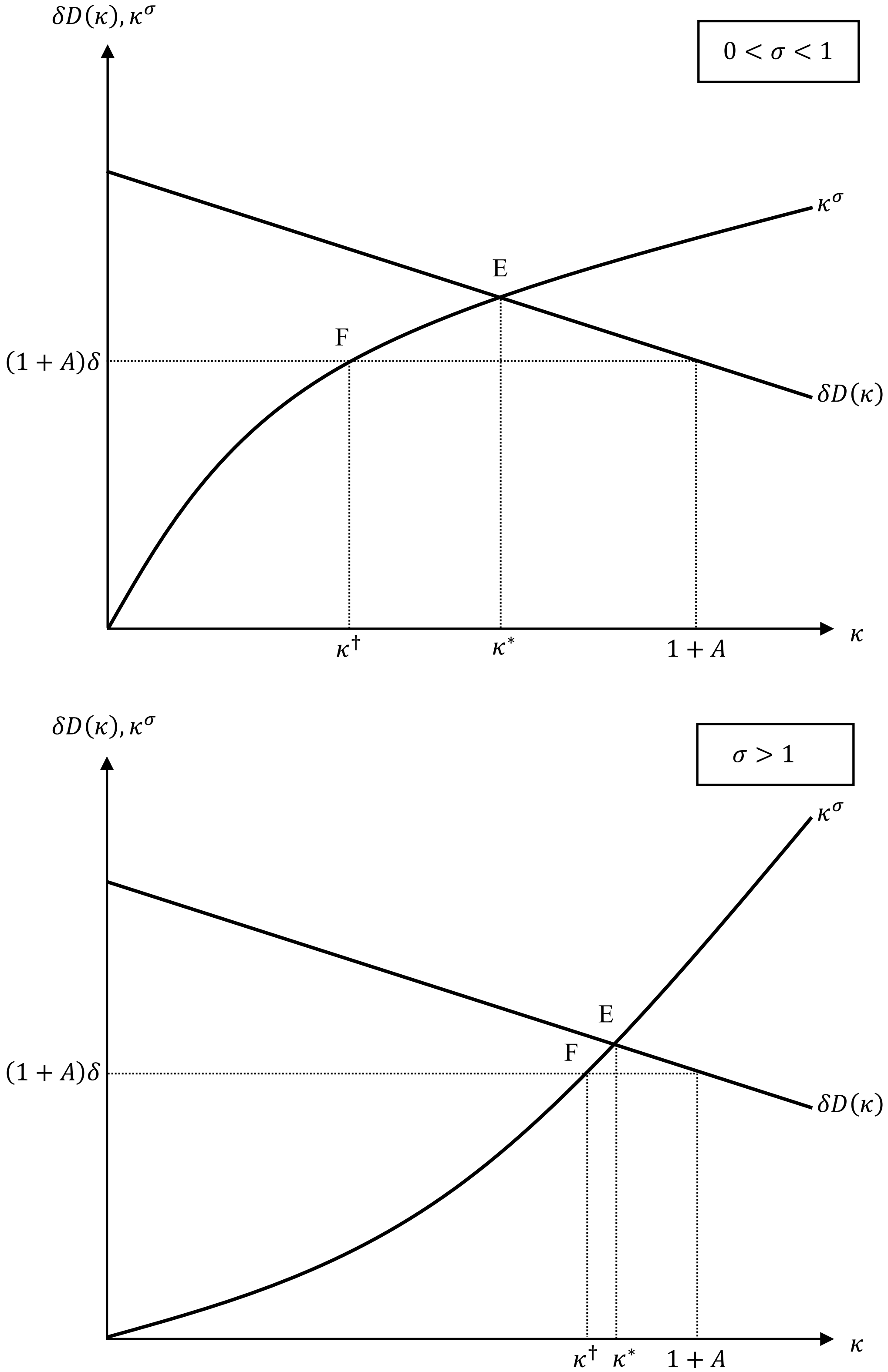

Figure 1. Existence and uniqueness of equilibrium

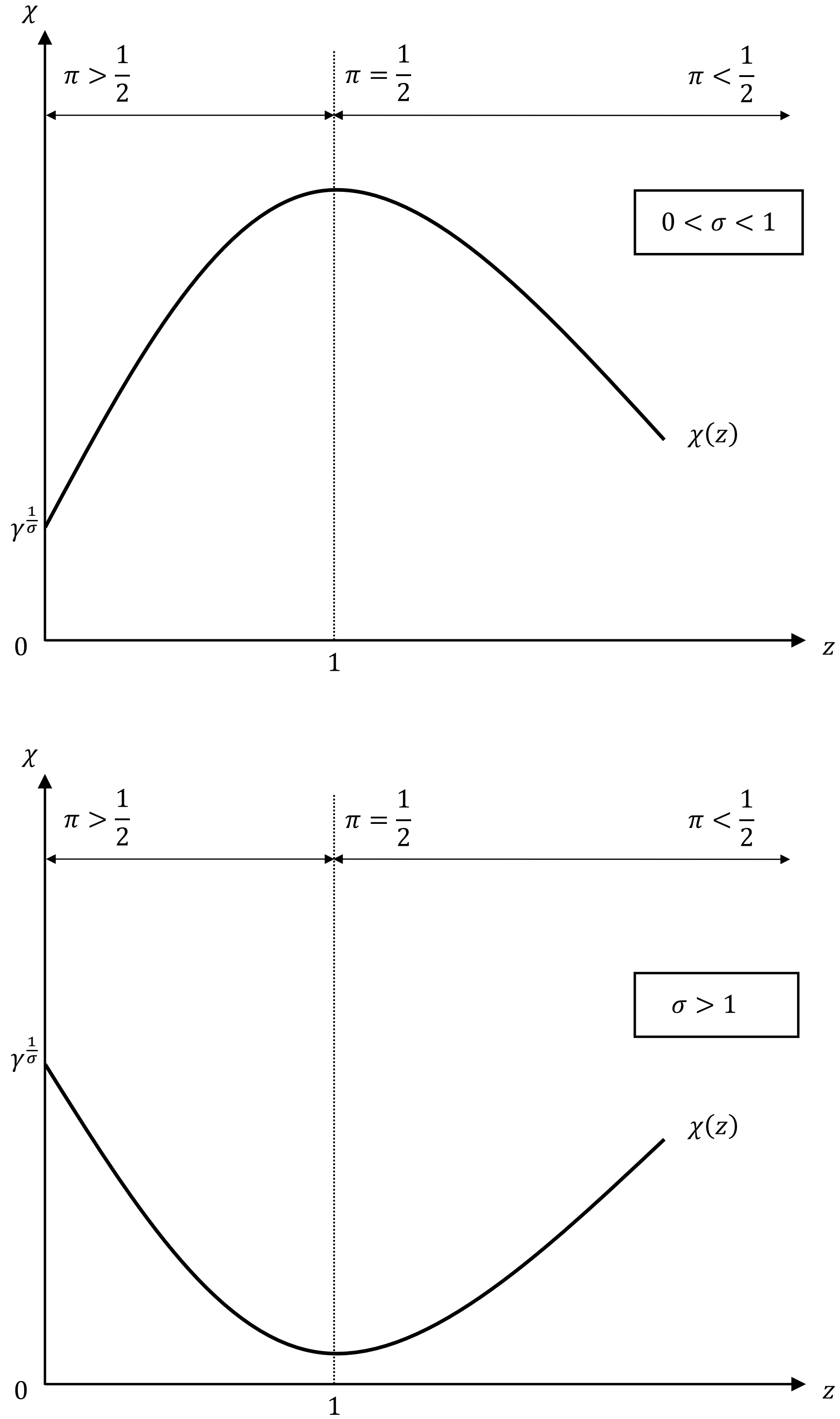

Figure 2. Public goods provision (relative to private goods) and the weight for the old

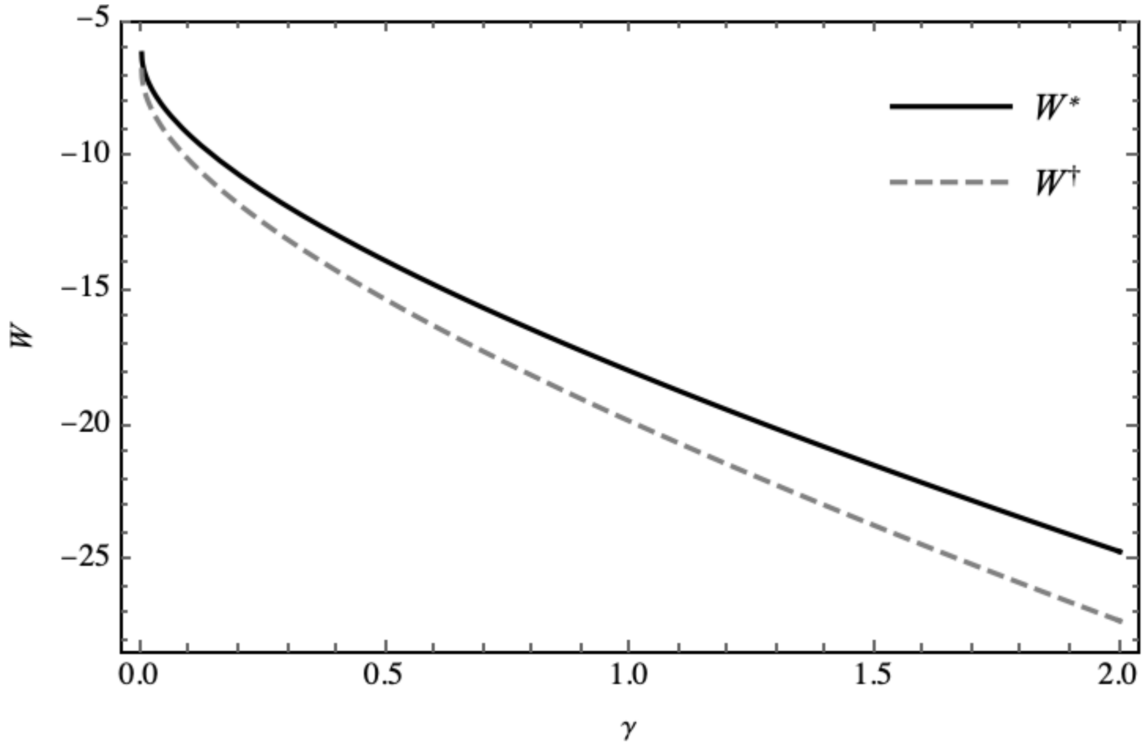

Figure 3. The efficiency of the public sector and the welfare level

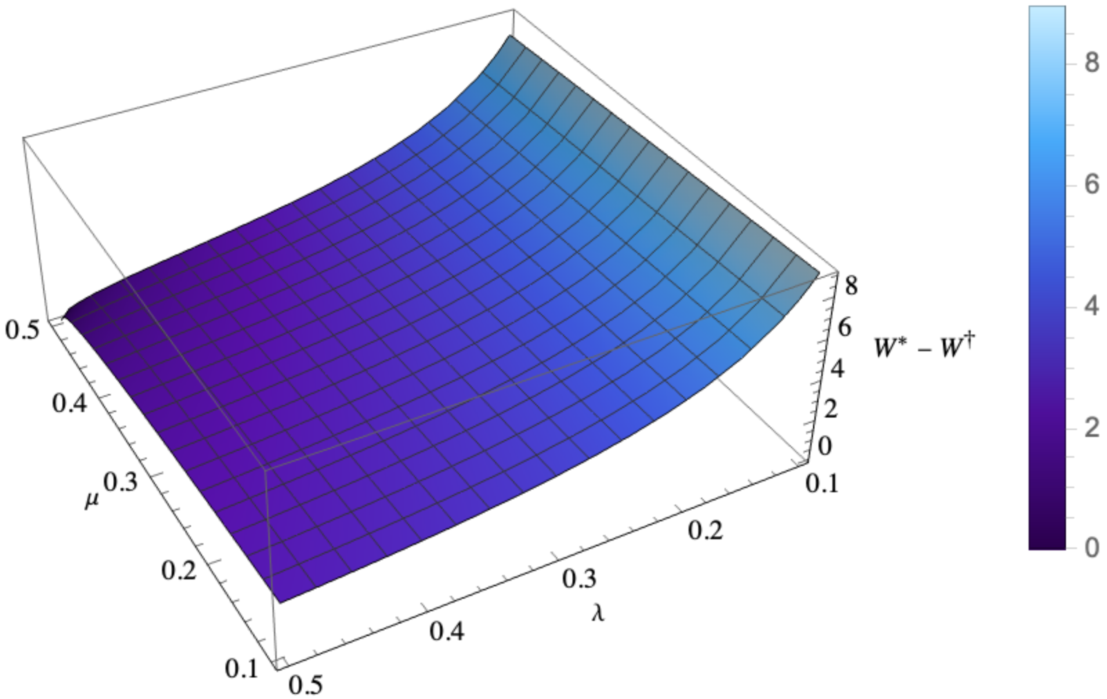

Figure 4. The degree of altruism and welfare difference