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A comparative interpolation method of WIM and a cubic spline function to longitudinal height data during adolescence in boys

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The purpose of this paper is to show that a Wavelet Interpolation Method is most effective for local approximation with judging from comparing with a cubic sline function and to analyze the characteristics of height growth velocity curve. The growth distance curve is interpolated to a longitudinal height growth distance data (age 6 to 17 years old) by the Meyer Wavelet. We present a Wavelet Interpolation Method by describing a growth velocity curve and identifying the peak age, using computer simulation. The statistic of the peak age is used to classify the maturity rate. Then We attempt to describe the mean growth distance and velocity curve respectively based upon classification of maturity type to examine the appearances of a mid-growth spurt and an after-growth spurt.

1. Introduction

We consider the problem of analyzing the height growth pattern which is based upon longitudinal height data. There is a considerable literature on this problem. Tanner[16) estimated the Peak Height Velocity (PHV) age by his drawing method. However, Tanner's method was vague, because his method could not determine the peak age precisely, based upon yearly time series data.

Since Deming[3) used a Gomperz function in 1957, a great effort has been directed to fitting the growth distance curve to mathematical functions. [See, for example, Joossens[7), Preece[13), Largo[8), Berkey[2) and Marubini.[10)] In Japan, Matsuura[11) investigated the appearance of a mid-growth spurt and an after-growth spurt by fitting the polynomial. In either case, however, it is difficult to identify the peak age or PHV age with accuracy, because not much is known about the relative performance of exact and asymptotic methods in all those methods.

Since wavelet beses are most effective for local approximation, we[4) propose the Wavelet Interpolation Method to analyze this problem. In this method, the growth distance curve F(t) and the growth velocity curve f(t), as the derivative of F(t) at t, are assumed to be smooth and $L^2(R)$ functions. These functions can be approximated by Wavelet series. As a result, based upon time series (longitudinal) data of height growth, we describe the growth velocity curve (t, f(t)) approximately by finite wavelet series. For the above reasons, the peak and the peak age are identified by computer simulation.

In section 2.3, we discuss a comparison of a wavelet interpolation method and the spline fitting method for six given mathematical functions. The main result is given in section 3, where the peak and the peak age of human height growth are identified and we propose the criterion of maturity rate in height growth to evaluate in accordance with PHV age. Further, we examine to classify the height growth pattern.

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2. Methods

Investigations of human growth is based on data of repeated measurement over time. In the growth curve model, fitting growth curves to mathematical functions has been used to analyze such data.

We propose an alternative analysis based on the Wavelet analysis. Wavelet or Wavelet transforms are a tool for decomposing functions in various applications, see Meyer[12], Mallat[9]. We apply the Wavelet analysis to interpolate the growth distance curve to each individual. This method leads us to describe the growth velocity curve.

2.1 Data

The data used for this study were gleaned from the Health Examination Records in Nagoya City during the period from 1972 to 1983. These longitudinal growth data consisted of height, body weight, chest girth and sitting height. We consider only the longitudinal height data of 98 Japanese boys, measured annually from 6 to 17 years of age (Table 1).

2.2 Wavelet Interpolation Method

Assume that a growth distance curve F(t) and a growth velocity curve f(t), f(t) = dF(t)/dt, are smooth and $L^2(R)$ functions where

$$L^2(R) = \{g(t) : \int_{-\infty}^{\infty} g(t)^2 dt < \infty\}$$

Then we have the following Wavelet expansion:

$$F(t) = \sum_{j,k} \alpha_{j,k}\, \psi(2^j t - k)$$

where j, k an integers.

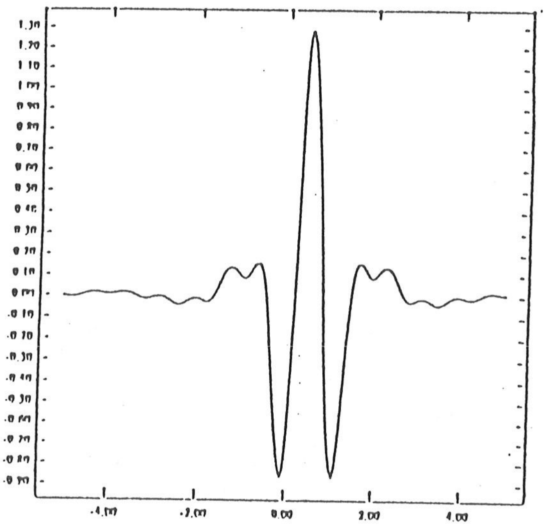

Fig 1 Yves. Meyer Wavelet

Table 1 The longitudinal height data (cm)
Time of Observation
Time of Observation

Individual	6	7	8	9	10	11	12	13	14	15	16	17
1	118.1	123.4	132.7	133.0	140.4	150.4	159.0	163.3	166.7	167.5	169.0	169.0
2	·	·	·	·	·	·	·	·	·	·	·	·
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	119.5	125.2	129.0	133.5	137.6	141.8	149.2	159.0	163.5	165.6	166.1	166.5

$\psi(t)$ is Meyer Wavelet that has a localised oscillatory form as Fig 1 $\{\psi(2^j t-k) : j, k; an\ integer\}$ are an orthogonal basis for $L^2(R)$. So, this Wavelet expansion is becoming the extension of Fourier series which had a characteristic of localitization.[14]

The algorithums to a Wavelet Interpolation Method (WIM) can be stated as follows:

1. Time series (longitudinal) data $\{(t_i, y_i) : i = 1, 2, \cdots\cdots, n\}$ are given.
2. Construct the Meyer Wavelet $\psi(x)$ which satisfies the following condition.

   $$|\psi(t)| \leqq \varepsilon \quad \text{for} \quad t<0 \quad \text{or} \quad t>1.$$

3. Take n pairs of integer (j, k) which satisfy

   $$|\psi(2^j t_i - k)| \geqq \varepsilon : i = 1, 2, \cdots\cdots, n.$$

4. Determine Wavelet coefficients $\{a_{j,k}; j, k\}$ from the time series data in Table 1 and values of a Wavelet funcion $\psi(2^j t_i - k)$, by solving such simultaneous linear equations such that

   $$y_1 = \sum_{j,k}^{n} \alpha_{j,k} \psi(2^j t_1 - k)$$

   $$y_n = \sum_{j,k}^{n} \alpha_{j,k} \psi(2^j t_n - k)$$

5. Substitute $\{a_{j,k}; j, k\}$ for the following equations and describe a grapf of $y = F_n(t)$ and $y = f_n(t)$.

   $$F_n(t) = \sum_{j,k}^{n} \alpha_{j,k} \psi(2^j t - k)$$

   $$f_n(t) = \sum_{j,k}^{n} 2^j \alpha_{j,k} \psi(2^j t - k)$$

6. Determine the extremal values of y = fn(t) by computer simulation.
7. Investigate the other local maximal points of y = fn(t).
8. Examine the characteristics of time series model from the data of extremal values by the statistical analysis.

2.3 Comparison of a WIM and the spline fitting method

A simulation study was performed to compare WIM with the cubic spline fitting method[1]. We use the following six functions on [0, 5],

(1) $F(t) = \dfrac{t^4}{4} - 2t^3 + \dfrac{11t^2}{3} - 6t$

(2) $F(t) = \dfrac{t^3}{3} - \dfrac{7t^2}{2} + 10t$

(3) $F(t) = t^2 e^{-t}$

(4) $F(t) = \dfrac{1}{1+e^t}$

(5) $F(t) = \dfrac{1}{1+25t^2}$

(6) $F(t) = cost + cos2t$

Assume that the values of a funciton $F(t)$ are given at each of 11 times $t_j = \dfrac{j}{2}$, $j = 0, 1, \cdots, 10$. Then for each methods, we can find its approximation functions $Fn(t)$ and $fn(t)$, where f is a first derivative function for F. For a good uniform approximation to a given funciton (F or f), it seems reasonable to expect the error to be fairly uniformly distributed through the interval [0, 5]. Thus, we divide an interval [0, 5] into 50 equal parts by the partition $(0, \frac{1}{10}, \frac{2}{10}, \cdots, 4+\frac{9}{10}, 5)$. We consider the mean of errors on times $t_k = \dfrac{k-1}{10}$, $k = 0, 1, 2, \ldots, 50$.

By computer simulation, these statistics are illustrated in Tables 2, 3. It follows that WIM generally provides an improved approximation over the cubic spline fitting method as to the first derivative function of $F(t)$.

Table 2 Comparison of WIM and Spline fitting method for F(t)

	(1)	(2)	(3)	(4)	(5)	(6)
W I M	0.049	0.060	0.001	0.0003	0.007	0.009
Spline	0.037	0.01	0.002	0.005	0.004	0.009

The mean of errors for each functions F

Table 3 Comparison of WIM and Spline fitting method for f(t)

	(1)	(2)	(3)	(4)	(5)	(6)
W I M	0.336	0.364	0.006	0.002	0.041	0.056
Spline	3.452	6.734	0.396	0.236	0.280	1.493

The mean of errors for each functions f

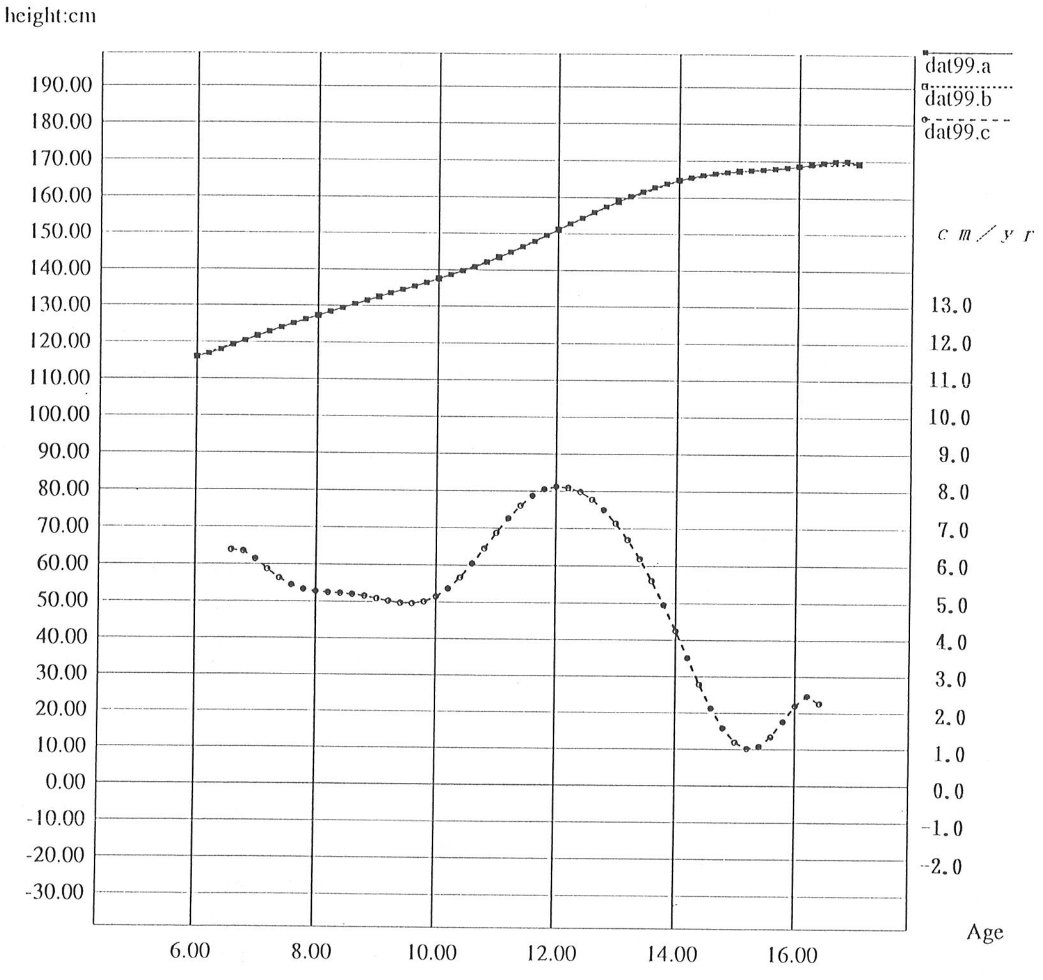

Fig 2 Example graph of growth distance and velocity curve approximated by Wavelet Interpolation Method

3. Example

The graph of the growth velocity curves $y = f(t)$ is described approximately by the Wavelet interpolation method. Fig 2 shows the mean growth and velocity curve of 98 individuals as an example. The maximam peak (PHV age) was described very clearly. So, this curve leads us to determine the PHV age by computing. The growth velocity curve was described and determined the PHV age for each individual in table 1 by computer simulation.

Summary statistics for the PHV age are presented in Table 4. Based upon this result, we examine to classify the maturity rate (in Table 4). Fig 3 – Fig 7 show the mean growth and velocity curves for each maturity type.

Fig 3 shows the mid-growth spurt in height, see Tanner[17]. By using the method of fitting the polynomial, Matsuura[11] defined the after-growth spurt as a growth-spurt which appeared after the PHV age. Fig 4 and Fig 7 show the existence of this spurt. These spurts can be analyzed by describing a growth velocity curve to each individual.

All computations have been performed on H-P UNIX Machine.

4. Discussion

In many studies of human growth, methods of analysis for repeated measure data often center on fitting the growth curves to mathematical functions (for example: linear equation, polynomial, logistic function, Spline function, etc.). In these methods, it is difficult to determine and to explain which function fit to those curves. Marbini[10] suggested that the use of polynomial to describe human growth curve should be avoided. However, by using Meyer Wavelet having characteristics of similarity, localization in time, and smoothness, it is possible to investigate the growth curve models by the simple and unified method. We need only assume that a growth distance curve and the growth velocity curve are smooth and $L^2(R)$ functions. This assumption is very natural and realistic. Although our method needs computer simulation for analyzing the growth curve, our computer program package would not be difficult in carrying out the computations based on time series data.

In this paper, by describing the growth velocity curve to individual approximately, we can determine PHV age to each individual. Therefore, it is possible to discuss the maturity rate in height growth (in Table 4).

Table 4 Mean and standard deviation of PHV age, and classifications of maturity type

Mean	12.21	early maturity type	$PHV \leq 10.8$
SD	0.87	somewhat early maturity type	$10.9 \leq PHV \leq 11.7$
Max	13.8	average type	$11.8 \leq PHV \leq 12.7$
Min	10.4	somewhat late maturity type	$12.8 \leq PHV \leq 13.5$
Range	3.4	late maturity type	$13.6 \leq PHV$

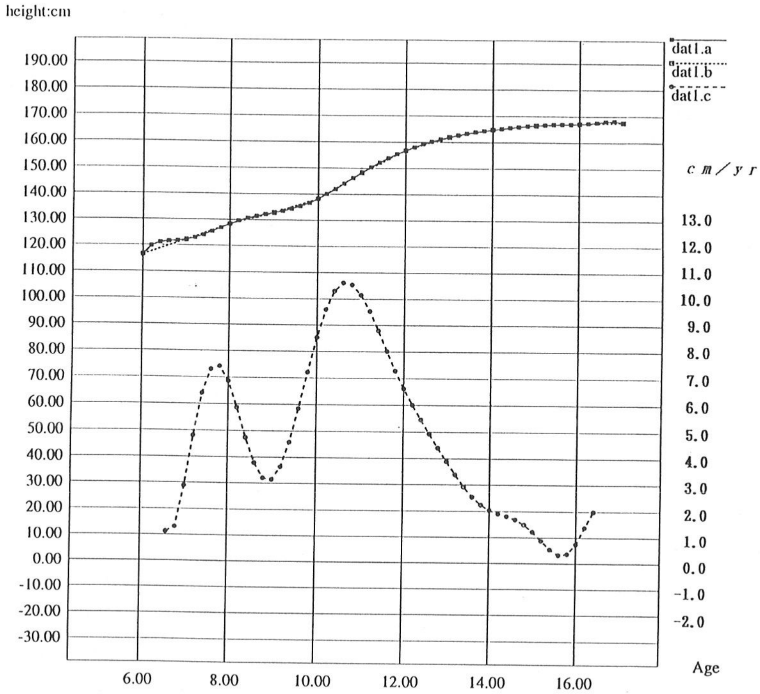

Fig 3 Mean growth and velocity curve in early maturity type

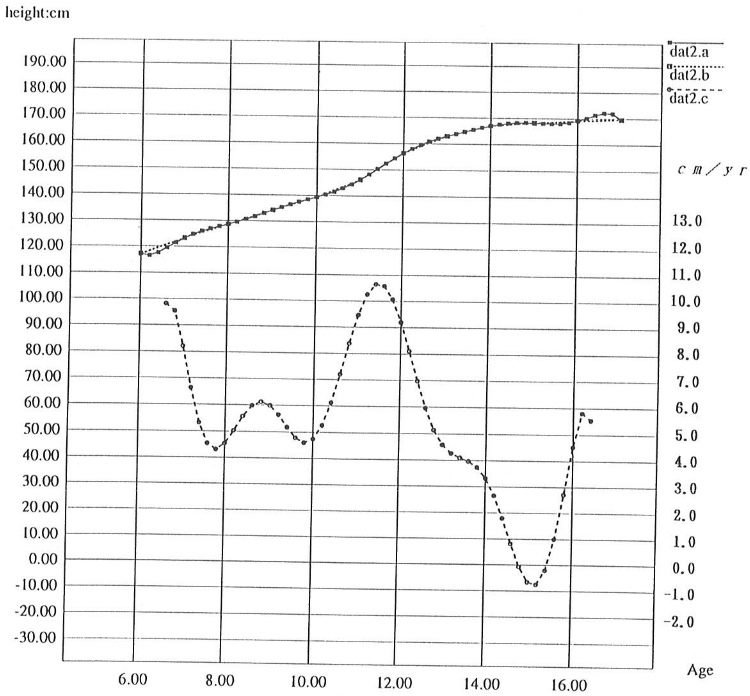

Fig 4 Mean growth and velocity curve in somewhat early maturity type

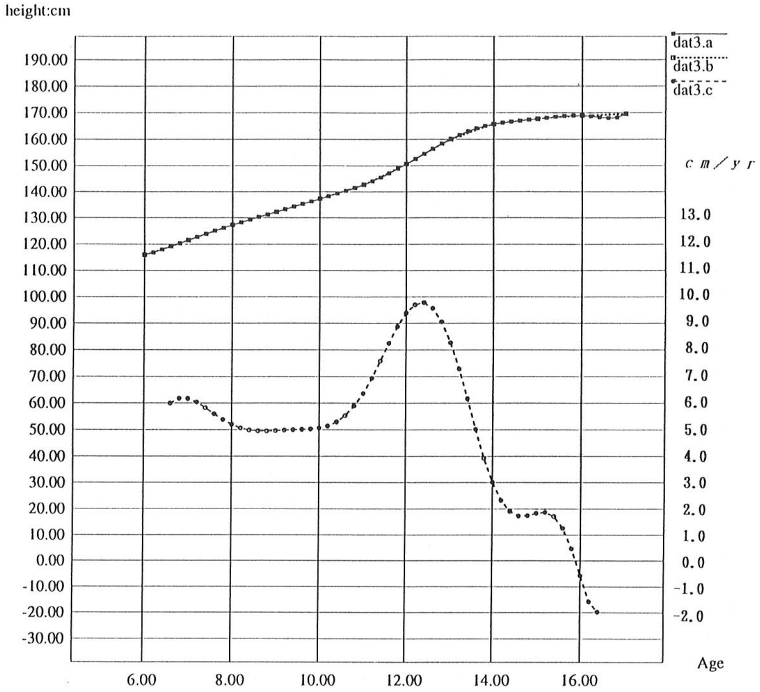

Fig 5 Mean growth and velocity curve in average type

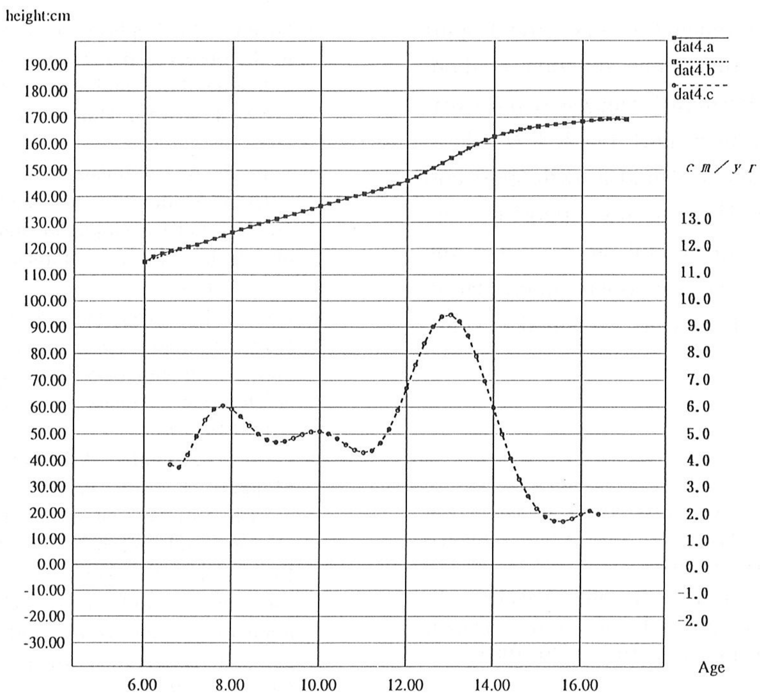

Fig 6 Mean growth and velocity curve in somewhat late maturity type

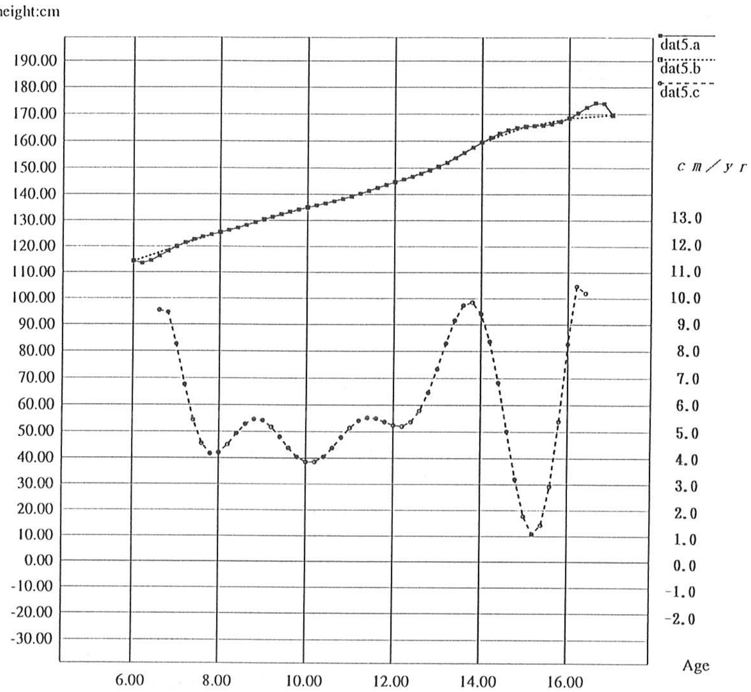

Fig 7 Mean growth and velocity curve in late maturity type

Tahara et al.[15] estimated the mid-growth spurt age in female by the Spline function method. Gasser[5), 6)] and Matsuura[11)] suggested the existence of the after-growth spurt. Fig 3, 4, 6, 7 have confirmed appearances of mid-growth spurt and after-growth spurt. These results support the fact that these spurts are useful in analyzing the human growth. The Wavelet interpolation method discussed above can be applied to other fields.

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