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A wavelet interpolation method for a time series analysis in the growth and development study

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This paper proposes a new approach to the analysis of time series. From time series data, we determine coefficients of Wavelet expansion by solving the simultaneous linear equations. By means of these coefficients, the approximation curve of the first derivative function for a time series curve is described by computer simulation and the characteristics of the time series are examined from point of view of the extremal value for the approximation curve by statistical analysis. We applied this method to the height growth model of boys.

1. Introduction

There is a Fourier series method for analyzing time series data. Especially, a time — frequency analysis (= a spectrum analysis) is used in the time series analysis. Let F(t) be a function depending on a time. F(t) in $L^2(R)$ can be represented by its Fourier series,

$$(1\text{-}1)\quad F(t) = \sum_{-\infty}^{\infty} c_n e^{in\pi t}$$

Fourier series are used to analyze periodic functions or distributions, but the information concerning the time-localization cannot be read off easily from the Fourier transform or the Fourier coefficients $\{c_n: n=0, 1, \cdots\}$. Recently, mathematicians, scientists, and engineers have been active in seeking new methods for representing functions. Wavelets, which have a localized oscillatory form, was proposed for the analysis of seismic data by J. Morlet[9]. Yves. Meyer constructed Meyer Wavelet $\phi(t)$ which had a compactly supported Fourier transform and showed that $\{\psi(2^j t - k)$, j, k is an integer$\}$ were an orthogonal basis for $L^2(R)$[8]. Therefore we have

$$(1\text{-}2)\quad F(t) = \sum_{j,k} a_{j,k}\, \psi(2^j t - k)$$

where j, k an integer.

This expansion is called Wavelet series. The localization is preserved from Multiresolution Analysis(1)[4]. So, this Wavelet expansion with a localitization is a generalization of Shauder basis and is becoming an extension of Fourier series. If F(t) and f(t), a first derivative function of F, are $L^2(R)$ functions, then we have

$$(1\text{-}3)\quad f(t) = \sum_{j,k} a_{j,k}\, 2^j \psi'(2^j t - k)$$

$$\text{where, } \psi' = \frac{d\psi}{dt}$$

We consider the following interpolation problem by Meyer Wavelet series.

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Interpolation problem.

When time series data (the sample size of time is n) are given, interpolate F(t) and describe the graph of y=f(t), where n is a small number.

In section 2, the procedures of the Wavelet Interpolation Method are described. In section 3, we show that the method is capable of determining the peak point by computer simulation. So, applying this method to the human biology, we can determine the peak height velocity age (PHV) for each boy.

2. Method

It is necessary to solve the interpolation problem that the Wavelet function is a smooth function. So, we construct Y. Meyer's Analysing Wavelet function $\phi(t)$ that has compactly supported the Fourier transform[8),10)]. (Therefore ϕ is an infinitely differentiable function.)

We explain a Wavelet interpolation method to solve the interpolation problem from a time series data.

(B-1) A time series data $\{(t_i, y_i): i=1, 2, \cdots, n\}$ is given.

(B-2) Construct the Meyer Wavelet $\psi(x)$ and $\psi'(t)$

(B-3) Transform the Meyer Wavelet to be filled the following condition such that

$$\text{if } t<0 \text{ or } t>1, \text{ then } |\psi(t)| \leqq \varepsilon.$$

(B-4) Take n pairs of integer (j, k) such that satisfy

$$|\psi(2^j t_i - k)| \geqq \varepsilon$$

(B-5) Determine Wavelet coefficients $\{a_{j,k}; j, k\}$ from the time series data and Wavelet function $\psi(2^j t_i - k)$ by solving the following simultaneous linear equations such that

$$y_i = \sum_{j,k} a_{j,k}\, \psi(2^j t_i - k) : i = 1, 2, \cdots, n$$

(B-6) Substitute $\{a_{j,k}; j, k\}$ for the following equations and describe graphs of $y=F_n(t)$ and $y=f_n(t)$.

$$F_n(t) = \sum_{j,k}^{n} \alpha_{j,k}\, \psi(2^j t - k)$$

$$f_n(t) = \sum_{j,k}^{n} 2^j \alpha_{j,k}\, \psi(2^j t - k)$$

(B-7) Determine local maximum or minimum points of time series curve $f_n(t)$ by computer simulation.

(B-8) Examine the characters of time series model from the data of extremal values by statistical analysis.

3. Example

Example 1. Polynomial function.

We illustrate the above method by discussing the interpolation of a polynomial function. Let's consider the following function:

$$\text{(3-1)}\quad F(t) = \frac{t^3}{3} - \frac{7t^2}{2} + 10t$$

$$f(t) = (t-2)(t-5),$$

where f is a first derivative function for F.

Assume that the values of a function F(t) are given at each of 11 times (Data 1).

Data 1

t	0	1/2	1	3/2	2	5/2	3	7/2	4	9/2	5
F(t)	0	25/6	41/6	33/4	26/3	25/4	15/2	77/12	16/4	9/2	25/6

We solved the interpolation problem from the above data. As A result, we find that approximations functions F_n and f_n were identified almost on F and f from Figure 1.

Example 2. Application to analysis of the height growth.

There is a study of the height growth of boys and girls to elucidate the pattern of the growth from a time series data. If we assume that a growth distance curve is a smooth and $L^2(R)$ function, then the growth velocity curve y=f(t) can be interpolated from a longitudinal time series data of height (age 6 to 17), where data are in Table 1.

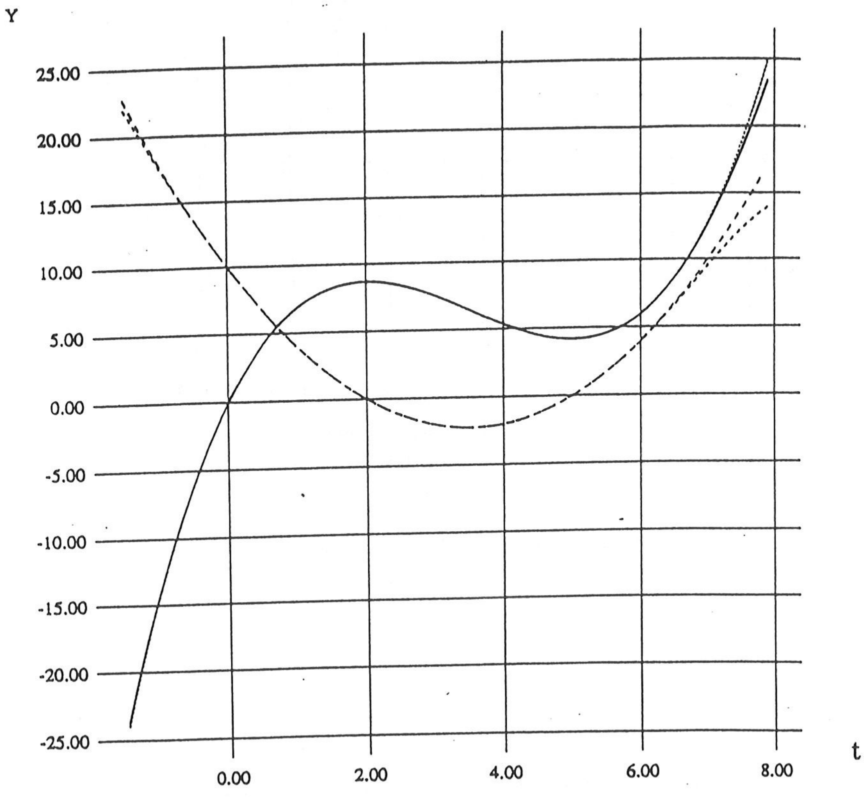

Figure 1. approximation of polynomial

———— the curve of Y=F(t)

.... the curve of Y=f(t)

- - - - - - the curve of $Y=F_{11}(t)$

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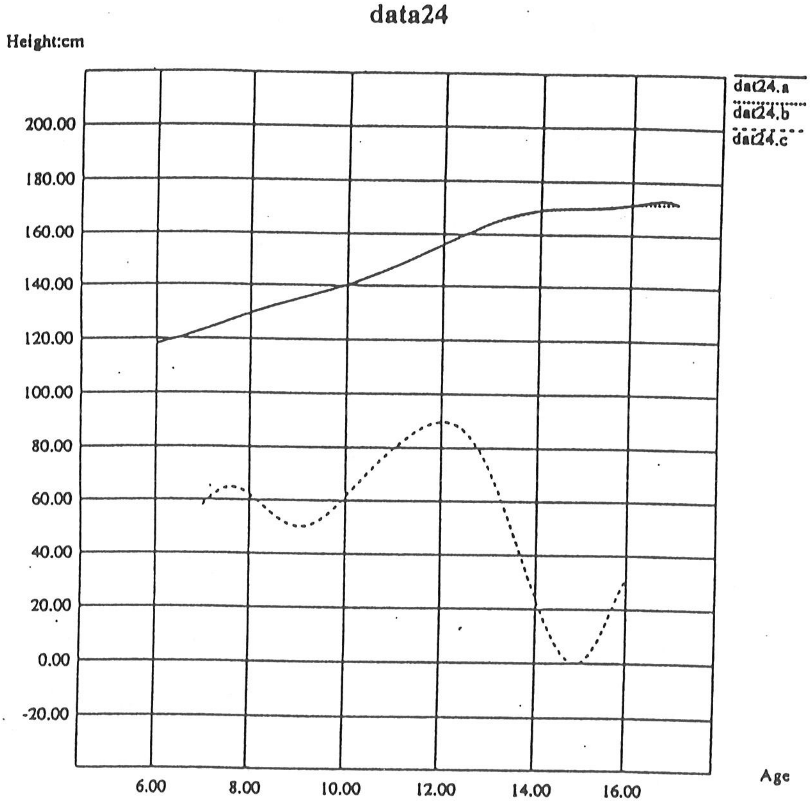

Figure 2. Height Growth Curves by Wavelet Interpolation Method

———— : the growth distance curve
-------- : the growth velocity curve

Table 1. the longitudinal height data (cm)
Time of Observation

Individual	6	7	8	9	10	11	12	13	14	15	16	17
1	118.1	123.4	132.7	133.0	140.4	150.4	159.0	163.3	166.7	167.5	169.0	169.0
2	.	.	.	.	.	.	.	.	.	.	.	.
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	119.5	125.2	129.0	133.5	137.6	141.8	149.2	159.0	163.5	165.6	166.1	166.5

Table 2. Mean and standard deviation of PHV age

mean	S D	Max	Min	Range
12.21	0.87	13.8	10.4	3.4

The data were gleaned from the Health Examination Records in Nagoya city during the period from 1972 to 1983. These longitudinal growth datas consisted of height, body weight, chest girth and sitting height. We consider only the longitudinal height data of 98 Japanese boys from 6 to 17 years of age, measured annually.

For each boy, we can identify his peak age and confirm the appearances of both mid-growth spurt and after-growth spurt clearly. Analysing these peak ages for boys (in Table 2), we examined to classify the pattern of the growth[3].

4. DISCUSSION

In the experiment science, there are many studies, for example Biology, Econometrics and so forth, where the measurement data is a time series data. If a number of time data size (n) is very large, then we can apply Fourier transform, Wavelet continuous transform and etc. However, when there is a few number, the Wavelet interpolation method is effective. Moreover, this method may be applied to the problems that must characterize some properties from time series data. In the case of the growth phenomena, it has been tried to fit suitable mathematical function as the growth curve, for example, there are polynomial functions[6], logistic curve[7], Gomperz function[2),5)], Spline function etc. The fault of these methods which fitted the above function is based on the flimsy grounds. But our method do not depend on the type of function and only assume the L^2-function property.

It is important to examine the character of time series model as the above example by specifying the extramal value of an individual time series curve.

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