THE EFFECTS OF TEMPERATURE AND ION ON THE REFRACTIVE INDEX OF VLF RADIO WAVES IN THE LOWER IONOSPHERE

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The role of the lower ionosphere in the propagation of VLF radio waves have been studied extensively by many workers.

However, the greater part of them have discussed it only on the basis of classical magnetoionic theory, thus it seems to be very important to discuss the role of the ionosphere taking into account the existence of ions and the effect of temperature.

It is the purpose of this paper to discuss the effects of temperature and ion on the refractive index of VLF radio waves, which is a fundamental quantity to the study of various problems in the lower ionosphere.

The refractive index which takes the effects of temperature and ion into account is given as follows,

$$\mu^2 = \mu_i^2 - \cfrac{X_e}{U_e - \cfrac{1}{2}\cfrac{Y_{Te}^2}{U_e - \cfrac{X_e}{\mu_i^2}} \pm \left[\cfrac{1}{4}\cfrac{Y_{Te}^4}{\left(U_e - \cfrac{X_e}{\mu_i^2}\right)^2} + Y_{Le}^2\right]^{\frac{1}{2}}} \quad\cdots\cdots\cdots\cdots\cdots(1)$$

and

$$\mu_i^2 = 1 - \frac{X_i}{U_i} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where, $X = ne^2/\varepsilon_0 m\omega^2$, $\quad U = 1 - j(\nu/\omega) + k^2(\gamma\kappa T/m\omega^2)$, $\quad Y_T = (e\mu_0 H_0/m\omega)\sin\theta$, $Y_L = (e\mu_0 H_0/m\omega)\cos\theta$ and subscripts e,i stand for electron and ion, respectively. And n, m, e, T, ν, are density, mass, electronic charge, temperature, and collision frequency, respectively. ε_0, μ_0, κ, γ are electric permittivity in free space, magnetic permittivity in free space, Boltzman's constant, and ratio of the specific heat at constant pressure to that at constant volume. Lastly, H_0, ω, k, θ are magnitude of the earth's magnetic field, angular wave frequency, angular wave number and angle between H_0 and wave normal.

The derivation of these equations are given in the appendix.

As it is implied in Eq. (2), the effect of earth's magnetic field on the ions can be neglected, because the gyro-frequency of the ions is below a few hundred cycles and this is much less than the collision frequency of ion and the frequency of VLF radio waves.

It is evident that Eq. (1) and Eq. (2) are the same expressions as those of the classical magnetoionic theory except that the parameter U is a function of the temperature. This means that the effect of temperature is controlled only by the following inequality,

$$1-j\frac{\nu}{\omega} \gtreqless k^2 \frac{\gamma\kappa T}{m\omega^2} = \mu^2\left(\frac{\gamma\kappa T}{mc^2}\right) \quad\text{..(3)}$$

where, c is light velocity.

If the left side of Eq. (3) is much larger than the right side, the effect of the temperature can be neglected and Eq. (1) and Eq. (2) can be treated as the classical magnetoionic theory, but if the left side of Eq. (3) is comparable with or less than the right side, The temperaure effect cannot be neglected and the classical magnetoionic theory is invalid.

Similarly, the effect of ion can be examined with the follwing inequality,

$$1 \gtreqless \frac{X_i}{U_i} = \frac{\dfrac{n_i e^2}{\varepsilon_0 m_i \omega^2}}{1-j\dfrac{\nu_i}{\omega}-k^2\left(\dfrac{\gamma\kappa T_i}{m_i\omega^2}\right)} \quad\text{..(4)}$$

If the right side of Eq. (4) is much less than unity, the effect of ion can be neglected, but if not, the effect of the ion can not be neglected and the problem must be solved taking the ion effect in consideration.

Now we shall show the results of the numerical calculations. Figs. 1 (a), (b), (c), are the distributions of density, collision frequency and temperature against the altitude which are used in our calculations. The density and the collision frequency profiles we use are the results of theoretical calculations given by Cole and Pierce (1965). The distribution of the temperature is quoted from, U. S. Standard Atomosphere Supplement (1966). Firstly, we shall examine the effect of temperature. From Fig. 1 (c), the temperature for altitudes from 40(km) to 90(km) is found to be in the range

$$T_e = T_i = 180 \sim 270 (°K)$$

Then, the right side of Eq. (3) for electron becames to

$$\mu^2\left(\frac{\gamma\kappa T_e}{m_e c^2}\right) = (0.91 \sim 1.4)\times 10^{-7}\mu^2 \quad\text{..(5)}$$

similarly, for ions we gets

$$\mu^2\left(\frac{\gamma\kappa T_i}{m_i c^2}\right) = (1.7 \sim 2.6)\times 10^{-12}\mu^2 \quad\text{...(6)}$$

While, the left side of Ep. (3) for electron and for ion becomes always larger than unity. Now, the value of refractive index μ^2 given by the classical magnetoionic theory has a magnitude less than 10^3 for a quiet ionospheric condition and less than 5×10^5 for a disturbed condition in the VLF range. Let us apply these values to Eq. (5) and Eq. (6), then, we get the right side of them as follows, for quiet ionosphere

$$< (0.91 \sim 1.4) \times 10^{-4} \qquad \text{(for electron)}$$
$$< (1.7 \sim 2.6) \times 10^{-9} \qquad \text{(for ion)}$$

for disturved ionosphere

$$< (4.6 \sim 7.0) \times 10^{-2} \qquad \text{(for electron)}$$
$$< (0.85 \sim 1.3) \times 10^{-6} \qquad \text{(for ion)}$$

The results show that the values of the right side of Eq. (3) which expresses the effect of temperature are always smaller than unity. Hence, the following ineqality is always valid in the lower ionosphere,

$$1 - j\frac{\nu}{\omega} \gg \mu^2 \left(\frac{\gamma \kappa T}{mc^2} \right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

This shows that effect of temperature is negligible for both electron and ions for VLF radio waves in the lower ionosphere, even in a disturbed ionospheric condition.

In order to examine the effect of ion, we shall rewrite X_i/U_i in Eq. (2) into the following expression assuming the inequality (7),

$$\frac{X_i}{U_i} = \frac{\left(\frac{\omega_{ri}}{\nu_i} \right) + j \left(\frac{\omega_{ri}}{\omega} \right)}{\left(\frac{\omega}{\nu_i} \right)^2 + 1} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

where, $\omega_{ri} = n_i e^2 / \varepsilon_0 m_i \nu_i$ is the conductivity parameter of ions and Fig. 2 shows its altitude distribution curves of them in the quiet ionosphere which were calculated by using the values shown in Fig. 1 (a), (b). For the comparison, the condutivity parameter of the electron is shown in it too. It is evident from Fig. 1 (b) and Fig. 2 for altitudes from 40 (km) to 90 (km) that

$$\nu_i \geq 3 \times 10^4, \qquad \omega_{ri} \leq 1.8 \times 10^3$$

and the following inequality is valid for VLF radio waves ($\omega \geq 1.88 \times 10^4$),

$$\frac{\omega_{ri}}{\nu_i} \ll 1, \qquad \frac{\omega_{ri}}{\omega} \ll 1$$

therefore, Eq. (8) becomes to

$$\frac{X_i}{U_i} \ll 1 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

In other words, the effect of ion becomes negligibley small in the quiet ionosphere.

At the time of a solar flare, the density of ions will be increased. However, the distribution of ions at the time of ionospheric disturbance doesn't seen to be well established. But if it could be assumed that the increasing rate of ions is the same order as the increasing rate of electron, the value of ω_{ri} increases to the order of 10^5 in maximum for VLF radio waves, and so $(\omega_{ri}/\nu_i) > 1$ and $(\omega_{ri}/\omega) > 1$ are valid. Then from Eq. (8), we obtain

$$\frac{X_i}{U_i} > 1 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$
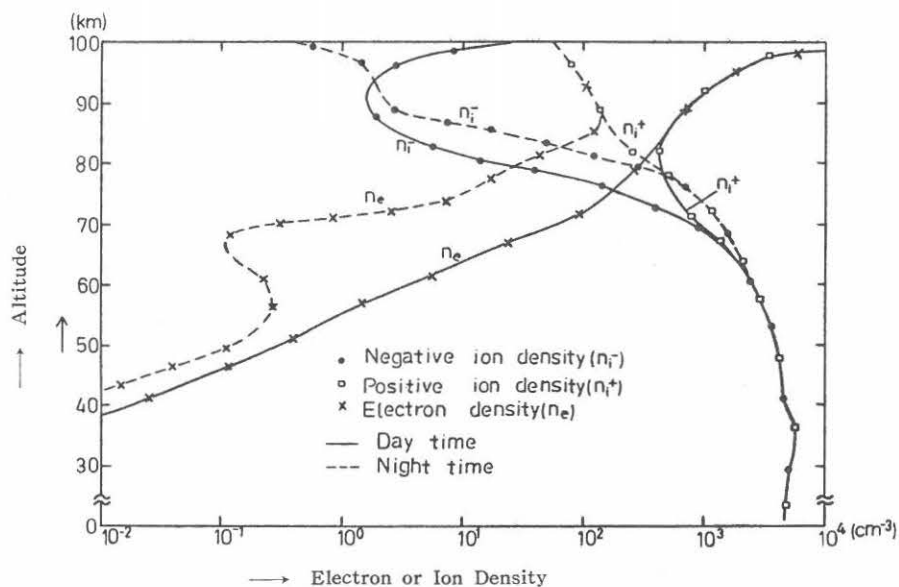
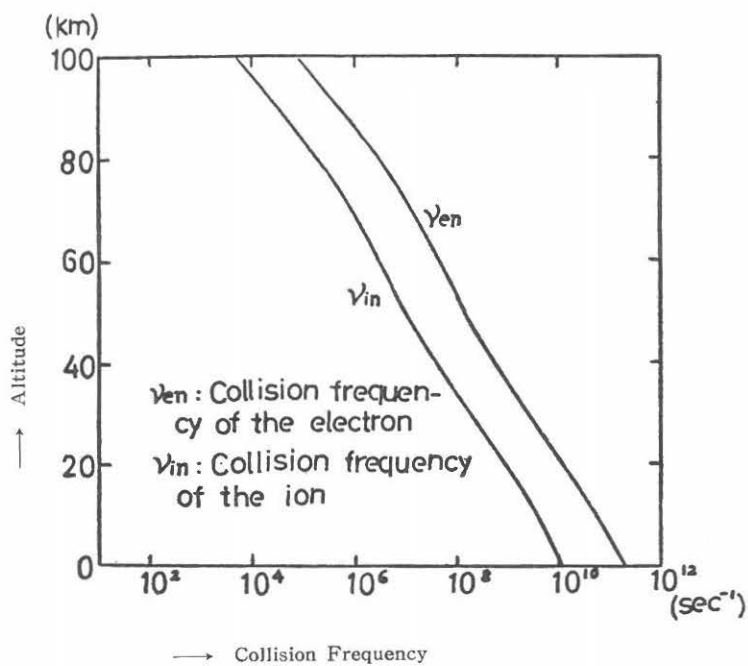Fig. 1. (a) Profiles of the electron and ion density.

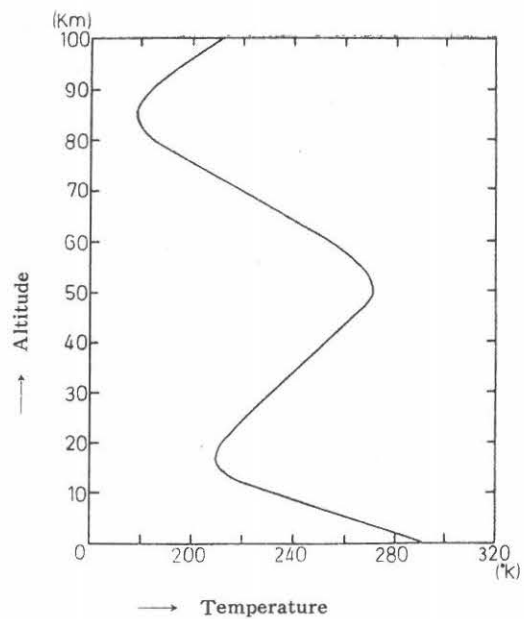Fig. 1 (b). Profiles of the collision frequency.

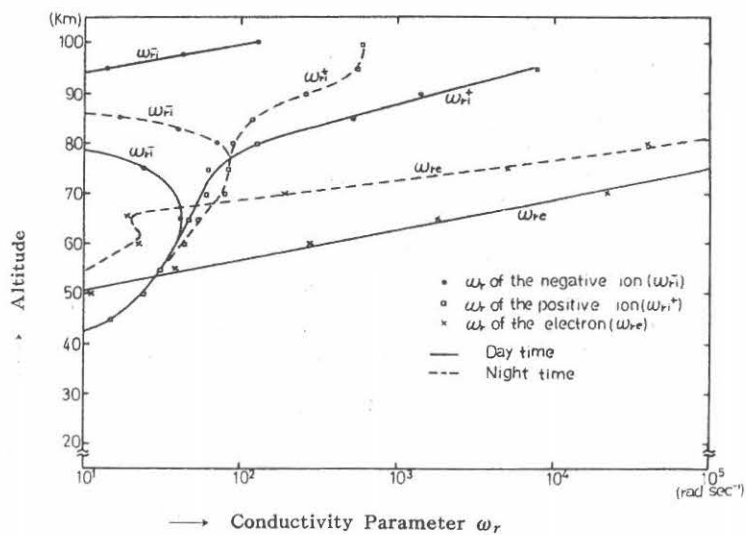Fig. 1 (c). Temperature profiles (Seasonal average at 30°N).

Fig. 2. Profiles of the conductivity parameter ω_r

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Hence, if the condition of the above assumption is satisfied at the time of a solar flare, the effect of ions can not be neglected in treating of the refractive index of VLF radio waves in the lower ionosphere.

We conclude from the above discussions that the effect of temperature on the refractive index for VLF radio waves in the lower ionosphere seems to be negligible, even in the case of ionospheric disturbance, and we see that the classical magnetoionic theory is valid. The effect of the ion also seems to be negligible in the quiet ionosphere, but the effect seems not to be negligible in the ionospheric disturbance.

Reference

Cole, R.K. and Pierce, E.T. : Electrification in the Earth's Atmosphere for Altitudes between O and 100 kilo-meters. JGR. 70, 2735-2749 (1965).

Ratcliffe, J.A. : The magneto-ionic theory and its applications to the ionosphere, Cambridge at the university press, 165-171 (1962).

U.S.Standard Atmosphere Supplement, 14-15 (1966).

Appendix

The equations of motion and continuity for an electron, which are valid in the lower ionosphere, are written

$$m_e \left(\frac{\partial}{\partial t}+\nu_e\right) n_e v_{1e}+\gamma \kappa T_e \nabla n_{1e}=n_e e(E_1+v_{1e}\times B_0) \cdots\cdots(A-1)$$

$$n_e \nabla \cdot v_{1e} +\frac{\partial}{\partial t}n_{1e}=0 \cdots\cdots(A-2)$$

where, m_e, n_e, T_e, v_e, ν_e are mass, density, temperature, velocity, and collision frequency of electron, respectively. e, κ, γ are electronic charge, Boltzman's constant and ratio of the specific heat at constant pressure to that at constant volume. E_1, B_0 are electric field intensity and earth's magnetic flux density, and subscript 1 of n_e and v_e show their dynamic components.

Assuming the propagation in the form of exp $\{-j(kr-\omega t)\}$ and using the relation $v_{1e}=\partial r/\partial t=j\omega r$, these equations are expressed as follows

$$m_e(-\omega^2+j\omega\nu_e)n_e r -jk\cdot\gamma\kappa T_e n_{1e}=n_e e(E_1+j\omega r\times B_0) \cdots\cdots(A-3)$$

$$k n_e \omega r+j\omega n_{1e}=0 \cdots\cdots(A-4)$$

Eliminating n_{1e} from both equations, we obtain

$$m_e(-\omega^2+j\omega\nu_e)n_e r - k^2\gamma_\kappa T_e n_e r = n_e e(\mathbf{E}_1 + j\omega r \times \mathbf{B}_0) \quad\cdots\cdots\cdots\cdots\cdots\text{(A-5)}$$

This equation may be written

$$-\varepsilon_0 X_e \mathbf{E}_1 = \mathbf{P}_e U_e - j\mathbf{Y}_e \times \mathbf{P}_e \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(A-6)}$$

where, $\mathbf{P}_e = n_e e r$, $X_e = n_e e^2/\varepsilon_0 m_e \omega^2$, $\mathbf{Y}_e = e\mathbf{B}_0/m_e\omega$, $U_e = 1 - jZ_e + k^2(\gamma_\kappa T_e/m_e\omega^2)$, $Z_e = \nu_e/\omega$, and $\varepsilon_0 =$ electric permittivity.

From the equation of motion and continuity for an ion, we obtain, similarly

$$-\varepsilon_0 X_i \mathbf{E}_1 = \mathbf{P}_i U_i \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(A-7)}$$

where, the effect of the earth's magnetic field to the ion is neglected. Eqs. (A-6) and (A-7) have the same form as that of the classical magnetoionic theory except that the parameter U_e involves a temperature term. Therefore, we can calculate the refractive index by the use of Eqs. (A-6) and (A-7) in the same way as we do it in the classical magnetoionic theory. The details of the derivation of the refractive index using these equations are given elsewhere (for example, Ratcllife 1962).

And the final result is given as follows

$$\mu^2 = \mu_i^2 - \cfrac{X_e}{U_e - \cfrac{1}{2}\cfrac{Y_{Te}^2}{U_e - \cfrac{X_e}{\mu_i^2}} \pm \left[\cfrac{1}{4}\cfrac{Y_{Te}^4}{\left(U_e - \cfrac{X_e}{\mu_i^2}\right)^2} + Y_{Le}^2\right]^{\frac{1}{2}}} \quad\cdots\cdots\cdots\cdots\text{(A-8)}$$

$$\mu_i^2 = 1 - \frac{X_i}{U_i} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(A-9)}$$

where, Y_{Te}, Y_{Le} are transverse and longitudinal components of $\mathbf{Y}_e$, respectively.