Social Security, Economic Growth, and Social Welfare in an Overlapping Generations Model with Idiosyncratic TFP Shock and Heterogeneous Workers

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Abstract

This paper develops an overlapping generations model of endogenous growth by incorporating an idiosyncratic productivity shock and heterogeneous individual labor productivity. The idiosyncratic shock generates ex-post inequalities, whereas workers' heterogeneity generates ex-ante inequalities. Social security programs might improve social welfare by providing insurance for risks not covered by private annuities and redistribution for inequalities. The equilibrium growth rate achieved under a pay-as-you-go pension system is lower than the growth rate achieved under the fully funded pension systems because the pay-as-you-go pension system hinders capital accumulation. However, a pay-as-you-go pension with additional benefits for savings enhances capital accumulation by incentivizing people to save. If the degree of relative risk aversion is sufficiently low, then the equilibrium growth rate under the modified unfunded pension system exceeds that under the funded pension system. In terms of social welfare within the Rawlsian welfare function, if people are highly risk-averse and therefore strongly inequality-averse, a pay-as-you-go system with no savings credit outperforms a fully funded system. By contrast, pay-as-you-go with savings credit is preferred if people have low risk aversion leading to weak inequality aversion.

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1. Introduction

Economically developed countries provide their own social security programs to safeguard the incomes of elderly persons during old age in case of insufficient savings. Most of these countries' social security programs are based on pay-as-you-go systems, but a few countries manage their social security systems entirely or partially through funded systems.[1] Retirement benefits in pay-as-you-go systems are financed by contributions levied from people in the current working generations. Because they do not directly depend on pensioners' paid contributions, pay-as-you-go systems have intergenerational redistribution effects. By contrast, a fully funded system offers retirement benefits directly proportional to the pensioners' earnings and contributions. Therefore, such a system has no intergenerational redistribution effects.

Against this background, social security has been studied as a core issue in public finance, public economics, and macroeconomics.[2] Specifically addressing the issues of savings and old-age income, previous research has shown that social security impedes capital accumulation and induces early retirement (e.g., Feldstein, 1974, 1977; Kotlikoff, 1979). The findings show that a fully funded system is preferable to a pay-as-you-go system in a dynamically efficient economy, with the former being superior to the latter when considering economic growth. However, the importance of social security is based on the presence of uncertainty because it affects the sharing of risks and the optimality of allocation under various shocks and improves social welfare (e.g., Enders and Lapan, 1982; Gordon and Varian, 1988; Thøgersen, 1998; Demange and Laroque, 1999; Wagener, 2004; Gottardi and Kubler, 2011).

Recent studies of the literature on social security have addressed idiosyncratic shocks and individual heterogeneity to elucidate the redistributive effects of social security (e.g., Conesa and Krueger, 1999; Harenberg and Ludwig, 2015, 2019; Bagchi, 2019).[3] Individual earnings are affected by idiosyncratic shocks, resulting in income disparities. To account for idiosyncratic shocks, we must examine the effects of social security not only on intergenerational inequality but also on intragenerational inequality. One study is related to the elaborate research developed by Harenberg and Ludwig (2015). They showed that when markets are insufficient, pay-as-you-go social security can provide partial insurance against idiosyncratic and aggregate risks. Their study clarifies insurance against risks and crowding-out of capital due to distortionary taxes as

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[1] For example, the pension system in Singapore, Central Provident Fund (CPF), is a social security system funded by both employers' and employees' contributions. Superannuation in Australia and the premium pension in Sweden are funded parts of the earnings-related pension, although these two countries have pay-as-you-go pension systems, which work as the intragenerational redistribution devices.

[2] Issues related to social security have been discussed widely, for example, intergenerational risk sharing (e.g., Smith, 1982; Bohn, 2001; Gollier, 2008), adverse selection (e.g., Abel, 1986), and optimal social security (e.g., Samuelson, 1975; Sheshinski and Weiss, 1981).

[3] Bagchi (2019) specifically examined the presence of differential mortality. Regarding this, Kelly (2021) demonstrated that the assumption of mortality homogeneity biases the equilibrium growth rate and welfare analysis. Harenberg and Ludwig (2015, 2019) treated the aggregate and idiosyncratic earning shocks.

key determinants of the welfare effects of social security.

In reality, social security programs are specifically operated depending on each country's economic circumstances, even though most are based on pay-as-you-go systems. For instance, some countries provide fringe benefits to encourage savings for retirement (e.g., tax deductions, credits, and allowances).[4] In the United Kingdom (UK), the *Savings Credit* of social security is an extra payment for people who have saved up money for retirement. These facts naturally cast some doubt on the negative effects of social security on capital accumulation if it is operated as a pay-as-you-go system with additive benefits to induce retirement savings.

Furthermore, two views of public pensions exist concerning its redistributive effects within pay-as-you-go systems: Beveridgean and Bismarckian schemes.[5] The former has a weak link between individuals' contributions and pension benefits, which exhibits large intragenerational redistribution (e.g., Australia, Ireland, Netherlands, and UK). Meanwhile, the latter exhibits less intergenerational redistribution because individuals' contributions are tightly linked to their retirement benefits (e.g., France, Germany, and Italy). Conde-Ruiz and Profeta (2007) investigated which scheme is chosen under majority voting using an overlapping generations (OLG) model with three income classes of low, medium, and high-income individuals.[6] Without uncertainty, they showed that low-income individuals prefer a small Beveridgean system, whereas middle-income individuals favor a large Bismarckian system. Meanwhile, high-income individuals wish for a fully funded system. Therefore, depending on the density of income classes, either scheme could be chosen politically.

Integrating findings obtained from earlier studies, one finds that the analysis of the effects of social security on economic growth and intragenerational and intergenerational redistribution is fundamentally important to evaluate the welfare effects of social security under risks.[7] We should pay special attention to idiosyncratic shocks in total factor productivity (TFP) and heterogeneity in individual labor productivity. One of the reasons for social security is to compensate for the inefficiency of incomplete markets. Ex-post personal income differences are generated by realistic risks such as idiosyncratic TFP shocks and incomplete capital markets. Furthermore, because of worker heterogeneity, exogenous factors, such as residential areas, parental characteristics, or race, may be important determinants of an individual's lifetime income (Chetty and Hendren, 2018a, 2018b; Chetty et al., 2020). Social security allows for the provision of

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[4] *Savings Credit* is one type of fringe benefit to attract people to save more. According to the European Commission's high-level group of experts on pensions, some countries (e.g., Germany, Croatia, and Italy) apply tax exemptions and incentives to encourage personal savings. The tax deduction functions similarly to savings credit here.

[5] Disney (2004) provided details of the Beveridgean and Bismarckian schemes.

[6] Numerous studies have addressed this issue (e.g., Casamatta et al., 2000; Cremer and Pestieau, 2000; Cremer et al., 2007; Glasso and Profeta, 2007).

[7] Empirical studies have elucidated a significant relationship between economic growth and shocks by postulating risks (e.g., Kormendi and Meguire, 1985; Grier and Tullock, 1989; Ramey and Ramey, 1995; Furceri and Karras, 2007; Imbs, 2007, Alouini and Hubert, 2019). Emphasizing income risks by productivity shocks is important for examining the social security effects. Cottle Hunt and Caliendo (2022) provided an excellent survey on social security and risks.

insurance as a form of redistribution for the ex-ante risk of drawing a low wage with low labor productivity.

To address this issue, the present study develops an OLG model of endogenous growth with social security under ex-post TFP shock and ex-ante heterogeneous workers. According to previous research, the welfare effects of social security under risk are composed of insurance/redistribution benefits for ex-post/ex-ante inequality and general equilibrium effects through capital accumulation (Harenberg and Ludwig, 2015, 2019; Cottle Hunt and Caliendo, 2022). The former effects of social security are determined by the features of social security programs. Furthermore, the latter have long-run effects with positive equilibrium growth rates under various types of social security programs. These two critical factors are incorporated into the dynamic analysis.

Specifically, this paper considers the following social security programs, which are related to existing social security programs: the fully funded (FF), pay-as-you-go (PG), and modified unfunded (MU) systems. PG pension has been examined as a pension program widely adopted in the real world, whereas FF pension has been considered as an alternative to PG pension for the discussion about social security privatization during the 1990s and early 2000 (Cottle Hunt and Caliendo, 2022). However, the unfunded pensions have numerous versions. For instance, the MU pension in the model is a PG system with Savings Credit in the UK, which are fringe benefits anticipated as stimulating incentives to save money for old-age consumption. This type of MU pension has a positive growth effect linked to a long-run welfare effect. Therefore, in addition to the FF and PG systems, the MU pension must be examined.

The main findings of this paper are as follows. First, I demonstrate that the equilibrium growth rate under the MU system might be larger than that under the FF system, depending on the relative risk aversion. A savings credit offers people an incentive to save more. With low relative risk aversion, people might prefer saving because it increases their future returns. They also benefit from large returns with risks. Hence, MU pension is superior in terms of growth enhancement. However, people with high relative risk aversion wish to avoid risky behavior. The growth rate under the MU system becomes the second-lowest after PG. These growth effects of social security represent the long-run welfare effects in general equilibrium.

Second, I derive the welfare effects of various pension systems. Under the Rawlsian welfare function, the social welfare level under the FF pension system is the lowest because it provides no benefit to the poorest people. Hence, the FF system has no short-run and long-run welfare effects from insurance benefits and economic growth. By contrast, the PG system with an optimal interior social security tax rate generates a higher welfare level than the FF system because of the insurance benefits and economic growth. Furthermore, a PG pension is superior to an unfunded pension with fringe benefits (MU) at the welfare level, except for small relative risk aversion.

When the degree of relative risk aversion is sufficiently small, the fringe benefits strengthen the positive welfare effects of public pensions more than the negative ones. Therefore, there exists an optimal level of savings credit.

Third, numerical analyses confirm the theoretical findings and quantitative implications. The results provide illustrative examples of the theoretical conclusions and extended results for the other form of the social welfare function. In particular, people who are highly risk-averse will prefer the PG pension system regardless of the type of social welfare function and distribution of labor endowments. Under the Benthamite welfare function, the distribution of labor endowments, ex-ante inequality, affects the optimal social security tax under the PG system. The thick population of the low-income class tends to increase the optimal tax rate.

The remainder of this paper is organized as follows. Section 2 presents a description of the basic setup of our model. Section 3 examines the equilibrium properties of economies with the funded system and provides equilibrium analyses of economies with the unfunded system. Section 4 investigates the relationship between risk, economic growth, and social welfare under different pension systems. Furthermore, Section 5 discusses the policy implications of the study based on the related literature by addressing the viewpoints of the welfare states. Finally, Section 6 concludes the paper.

2. The model

Consider a closed economy with a homogeneous good. The economy is in discrete time, and the time is indexed by subscript t. Firms exist as a continuum. The production technology is assumed to be Turnovsky (2000) and Kenc's (2004) specification, which includes the external effects of capital on labor productivity. The present study specifically assumes Romer's (1986) positive externality of aggregate capital. This positive externality is an important source of endogenous growth with a positive long-run growth rate. Assume that the production externality is caused by the interaction between the aggregate capital and labor ratio. The production function is formulated as

$$y_t = X_t k_t^{\alpha} (K_t/L_t)^{1-\alpha} l_t^{1-\alpha},$$

where y_t is the firm output, X_t is the firm productivity, k_t is the capital input, K_t is the aggregate capital stock, l_t is the firm's labor input, and L_t is the aggregate labor input.

As will be shown later, there is no uncertainty in the predetermined stock level of capital. In equilibrium, firm's capital input level coincides with the mean of the capital–labor ratio. This serves as the microeconomic foundation for each firm's production function:[8]

$$y_t = X_t k_t. \tag{1}$$

Based on the production function (1), this paper does not explicitly treat labor input in the production, despite the fact that labor is an input in the production process. Furthermore, the paper assumes that the capital is fully depreciated during one period.

TFP X_t is a probabilistic variable with a probabilistic density function $f(x_t)$; it is independent and identically distributed over time. Each firm faces different productivity shocks even though identical stochastic processes generate them. Furthermore, this paper considers the incompleteness of asset markets, as analyzed by Harenberg and Ludwig (2015). As a result, if there is no insurance to cover the risks, investors face idiosyncratic TFP shocks that generate disparity in private investment. In other words, the ex-post inequalities are caused by idiosyncratic TFP shocks. Instead of private insurance, social security programs considered later may provide insurance to cover the risks caused by idiosyncratic TFP shocks.

Workers receive labor rewards based on their relative contribution to the average worker, as demonstrated by Turnovsky (2000) and Kenc (2004). Assuming that capital and labor shares are constant over time, we have

$$r_t k_t = \alpha y_t,$$

$$w_t = (1-\alpha) y_t,$$

where r_t denotes the interest rate, w_t signifies the wage payment of each firm, and α represents the capital distribution rate ($0 < \alpha < 1$). Inserting Equation (1) into the above equations yields

$$1 + r_t = \alpha X_t, \tag{2}$$

$$w_t = (1-\alpha) X_t k_t. \tag{3}$$

According to Equations (2) and (3), these factor prices are stochastic variables affected by the idiosyncratic TFP shocks.

Individuals live in two periods: young (working) and old (retirement). This paper considers a stationary population, similar to Harenberg and Ludwig (2015). Hence, the population of each generation is normalized to unity. Several studies have found that exogenous factors, such as residential areas, parental characteristics, or race, can influence an individual's lifetime earnings (Chetty and Hendren, 2018a, 2018b; Chetty et al., 2020). This is represented by the heterogeneity of workers' labor productivities. Hence, the present study considers the following labor productivity for employees: During the young period, the generation born at period t supply Z_t unit of labor, where Z_t represents a probabilistic variable with probabilistic density function $g(z_t)$ that is independently and identically distributed over time. The young generation receives

[8] Bruce and Turnovsky (2013) examined the relation between social security, economic growth, and welfare under lifetime uncertainty using a continuous-time overlapping generations model with a production function similar to ours.

rewards for labor, depending on h_t, which is $h_t \equiv Z_t/\bar{Z}_t$, and $\bar{Z}_t$ is defined as $\bar{Z}_t \equiv \int_0^\infty z_t g(z_t) dz_t$. Therefore, h_t can be interpreted as worker's productivity, which is also a probabilistic variable following $g(z_t)$.

Meanwhile, labor income $w_t h_t$ is allocated to purchasing private goods, paying social security tax, and savings. In the retired period, individuals live off savings and public pension benefits. Then, the budget equations for period-*t* generation in the two periods are

$$c_t^y + s_t + \tau_t = w_t h_t, \tag{4}$$

$$c_{t+1}^o = (1 + r_{t+1}) s_t + b_{t+1}, \tag{5}$$

where c_t^y denotes private consumption in the young period, s_t represents saving, τ_t stands for the social security tax, c_{t+1}^o expresses private consumption in the old period, and b_{t+1} stands for the pension benefit.

The lifetime utility function for the period *t* generation is

$$U_t = \frac{\left(c_t^y\right)^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{t+1}^o)^{1-\theta} - 1}{1-\theta} \text{ for } \theta > 0, \theta \neq 1,$$

$$U_t = \log c_t^y + \frac{1}{1+\rho} \log c_{t+1}^o \text{ for } \theta = 1,$$

where U_t represents the utility level, ρ is the discount rate ($\rho > 0$), and θ denotes the relative risk aversion ($\theta > 0$). Each individual decides how much to save to maximize expected lifetime utility subject to Equations (4) and (5) as well as information about future returns on savings, wage rates, and social security programs (e.g., social security tax rates and future pension benefits).

A heterogeneity of worker's labor productivity is exogenous to each individual because their decisions are made after they draw some productivities from a probabilistic distribution. Individual decision-making is unaffected by ex-ante inequalities caused by heterogeneous worker labor productivity. In contrast, individual ex-post income inequalities at an individual level are generated by idiosyncratic TFP shocks because of the incompleteness of financial markets. Depending on their risk coverage, social security programs may have an impact on individuals' lifetime consumption and saving decisions. As a result, even though its shocks are perfectly mutualized at the macroeconomic level, idiosyncratic TFP shocks affect economic growth and social welfare in macroeconomic equilibrium. With the pension system ($\tau_t \geq 0$ and $b_{t+1} \geq 0$), the first-order condition becomes

$$\frac{dE[U_t]}{ds_t} = -(w_t h_t - s_t - \tau_t)^{-\theta} + \frac{1}{1+\rho} E\left[R_{t+1}(R_{t+1} s_t + b_{t+1})^{-\theta}\right] = 0, \tag{6}$$

where $R_{t+1} \equiv 1 + r_{t+1}$.

To solve the optimization problem using the first-order condition, each individual must anticipate their savings and pension benefits returns using their information about TFP shocks and public pension systems. Given that incompleteness of financial market exists, the households have no choice but to expect future returns relying on the probabilistic density function of TFP shocks, which is common knowledge and information provided by the government for social security programs. Especially, the household's expectation on social security benefits is important to decide the saving rate.[9] Therefore, depending on the risks, the social security taxes, and the retirement benefits, Equation (6) provides the individual saving function, which will be discussed in the next section.

The pension system is operated publicly. As aforementioned, this paper considers FF, PG, and MU systems representing the existing pension schemes. Regardless of the operation methods, the aggregate tax revenue and retirement benefits in period t are

$$T_t = \iint \tau_t f(x_t) g(z_t) dx_t dz_t, \tag{7}$$

$$B_t = \iint b_t f(x_t) g(z_t) dx_t dz_t. \tag{8}$$

Equations (7) and (8) imply that the shocks are mutualized at the macroeconomic level.

To consider the capital market equilibrium condition, one must set up the details of the pension system. Depending on the pension system, the public pension budget is varied: For the funded pension system (FF), the social security tax revenue in period t must be equal to the aggregate retirement benefits in the next period. Because we have $T_t = B_{t+1}$ for FF, the capital market equilibrium condition becomes

$$K_{t+1} = S_t + T_t = \iint s_t f(x_t) g(z_t) dx_t dz_t + \iint \tau_t f(x_t) g(z_t) dx_t dz_t, \tag{9}$$

where S_t denotes aggregate saving.

By contrast, unfunded pension systems based on a PG principle require that the tax revenue in period t is equal to the aggregate retirement benefits in period t. Therefore, $T_t = B_t$ holds. The capital market equilibrium condition under the unfunded pension system is

$$K_{t+1} = S_t = \iint s_t f(x_t) g(z_t) dx_t dz_t. \tag{10}$$

Going forward, I assume that X_t and Z_t follow lognormal distributions, respectively, such that

$$f(x_t) = \frac{1}{\sqrt{2\pi}\sigma_x x_t} \exp\left(-\frac{(\log x_t - \mu_x)^2}{2\sigma_x^2}\right),$$
$$g(z_t) = \frac{1}{\sqrt{2\pi}\sigma_z z_t} \exp\left(-\frac{(\log z_t - \mu_z)^2}{2\sigma_z^2}\right).$$

[9] Further analysis of this issue is developed in Section 5.

Lognormality of shocks is used widely in the literature on risk.[10]

Finally, the labor market equilibrium condition requires that the demand for labor is equal to the labor supply. Consequently, we have

$$L_t = \int_0^\infty z_t g(z_t)\, dz_t = \exp\left(\mu_z + \frac{\sigma_z^2}{2}\right) = \bar{Z}.$$

In those equations, $\bar{Z}$ is the aggregate labor supply, which is equal to the mean of labor supply by the normalization of population. Capital and labor market equilibrium conditions derive the resource constraints of this economy. With production technology (1) and the budget constraints of social security programs, the aggregate economic variables of K_t, Y_t, B_t, and S_t grow at the identical (equilibrium) growth rate. Hereinafter, the present study specifically examines the case in which the equilibrium growth rate is positive.

3. Social security programs

This section explains in detail the three pension systems and characterizes their equilibrium with social security programs. In particular, Section 3.1 specifically examine FF pension systems. Sections 3.2 and 3.3 present an examination of unfunded pension systems.

3.1. Fully funded pension

Fully funded pensions can have two patterns of *expected returns* for individuals: *risky* and *risk-free*. Given that private investment (saving) involves a risky return on investment, individuals will choose different options concerning risks.

Fully funded pension with the announcement of the expected returns (as a benchmark: BM). The Central Provident Funds (CPF) are a publicly run, fully funded pension system in Singapore. For insured individuals, the CPF guarantees a rate of return of at least 2.5%. The point of this type is that the fund announces the pension's expected returns. This type of social security program has the potential to provide perfect insurance against idiosyncratic TFP shocks. Therefore, the pension is risk-free for insured individuals, although there is heterogeneity of their earnings.

Formally, if the government releases the expected returns of public pension as $\bar{R}_{t+1}$, then the budget of the public pension must satisfy

$$b_{t+1} = \bar{R}_{t+1}\tau_t.$$

[10] Harenberg and Ludwig (2015) considered two aggregate shocks: productivity shock to the unit user costs of capital and one idiosyncratic shock related to old-age wage. They also assumed that the shocks follow stochastic processes with a log-normal distribution.

Accounting for the portfolio selection between private savings and public pension, Equations (4)–(6) yield $s_t = 0$ and the following equation (see Appendix A):

$$\tau_t = \frac{1}{1+\beta A^{\eta}} w_t h_t \equiv \xi_{BM} w_t h_t, \tag{11}$$

where

$$\beta \equiv \left(\frac{\alpha^{1-\theta}}{1+\rho}\right)^{-\frac{1}{\theta}}, \eta \equiv \frac{\theta-1}{\theta}, \log A \equiv \mu_x + \frac{\sigma_x^2}{2}.$$

Using Equations (9) and (11) with $s_t = 0$, we can obtain the following equilibrium growth rate under fully funded pension with a risk-free return:

$$\gamma_{BM} \equiv \frac{(1-\alpha)A}{1+\beta A^{\eta}} - 1. \tag{12}$$

Fully funded pension with the announcement of the distribution of the expected returns (as a usual fully funded pension: FF). Superannuation in Australia is one kind of compulsory saving. It is based on a defined contribution pension system. Defined contribution pensions such as Superannuation and 401K are equivalent to personal savings if the pension programs do not cover the risks. In reality, there exists a risk that the deterministic return cannot be provided similarly to the BM case. Hence, one can suppose that the government announces that the return of the public pension depends on the productivity X_t with the probabilistic density $f(x_t)$. Here, the public pension does not differ from private savings because no disparity exists among the returns:

$$b_{t+1} = R_{t+1}\tau_t.$$

As a result, even though this system ensures actuarial fairness, this pension program has no insurance effect on ex-post idiosyncratic TFP shocks.

If individuals can optimally choose their private and public (pension) savings, then from Equations (4)–(6), the total saving function is obtained as (see Appendix A)

$$s_t + \tau_t = \frac{1}{1+\beta A^{\eta}\lambda^{1-\theta}} w_t h_t \equiv \xi_{FF} w_t h_t, \tag{13}$$

where

$$\log \lambda = \frac{\sigma_x^2}{2}.$$

Equations (9) and (13) lead to the equilibrium growth rate under FF pension:

$$\gamma_{FF} \equiv \frac{(1-\alpha)A}{1+\beta A^{\eta}\lambda^{1-\theta}} - 1. \tag{14}$$

Suppose that A is sufficiently large to ensure a positive rate of economic growth.[11]

The relation between growth rates under fully funded pension. Comparison between Equations (12) and (14) leads to

[11] This assumption covers all cases of economic growth rates under the three pension systems considered here.

$$\gamma_{FF} \gtreqless \gamma_{BM} \Leftrightarrow \theta \gtreqless 1.$$

When $\theta = 1$, no difference exists between risky and risk-free returns. The saving rates are identical (i.e., $\xi_{BM} = \xi_{FF}$) because the income and substitution effects of the interest rate change on the youth consumption exactly offset. However, if $\theta \neq 1$, the difference in public pension programs affects the expected return on social security depending on uncovered idiosyncratic shock. Hence, the interest rate disparity represents the difference between BM and FF. The income (substitution) effect dominates the substitution (income) effect if $\theta > 1$ $(\theta < 1)$. Given that $E\left[R_{t+1}^{1-\theta}\right]$ takes a smaller (larger) value than $\bar{R}_{t+1}^{1-\theta}$ for $\theta > 1$ $(\theta < 1)$, ξ_{FF} is larger/(smaller) than ξ_{BM}: The total effect of the interest rate change on youth consumption under risky returns is smaller (larger) than that under risk-free returns when $\theta > 1$ $(\theta < 1)$. Therefore, the growth rate under the FF-regime is higher (lower) than that under the BM regime if $\theta > 1$ $(\theta < 1)$.

3.2. Pay-as-you-go pension

Based on the *pay-as-you-go principle* (PG), the social security tax and benefits in period *t* must be balanced within the existing generations. Then, we have

$$T_t = B_t.$$

In many countries, a public pension aims to ensure a certain pension replacement rate. Then, the present study assumes that it is fixed over time. Consequently, a pension benefit is

$$b_{t+1} = \psi \bar{w}_{t+1} \bar{h}_{t+1},$$

where ψ denotes the fixed replacement rate $(0 < \psi < 1)$,

$$\bar{h}_{t+1} = \bar{Z}^{-1} \int_0^\infty z_t g(z_t) dz_t = 1,$$

$$\bar{w}_{t+1} = (1-\alpha)\bar{k}_{t+1} \int_0^\infty x_{t+1} f(x_{t+1}) dx_{t+1}.$$

With Equation (5), PG system provides partial insurance on ex-post idiosyncratic TFP shocks.

Disney (2004) stated that public pension programs are classified as Beveridgean and Bismarckian schemes. With the fixed replacement rate, the PG pension represents a significant departure from actuarial fairness. The PG pension system described in this section is similar to the Beveridgean scheme, which is used in Australia, Ireland, and the UK. Consider the Bismarckian scheme, in which individuals' contributions are linked to their retirement benefits. To examine the intragenerational redistribution effects of public pension, this paper specifically examines Beveridgean schemes such as the UK pension system. Therefore, the PG pension system will reduce ex-ante inequality caused by labor productivity heterogeneity. With the PG, the capital market equilibrium condition is (10). In the average term, we have $\bar{k}_{t+1} = \bar{s}_t$.

Individuals anticipate the future pension benefit as $b_{t+1}^{e} = (1-\alpha)\psi X_{t+1}\bar{k}_{t+1}$ because they regard the average capital stock as given and the productivity generated from the stochastic process. Then, the saving function under PG is derived from Equations (4)–(6), b_{t+1}^{e}, and $\bar{k}_{t+1} = \bar{s}_t$ (Appendix B):

$$s_t = \frac{(1-\psi)w_t h_t - \beta\chi\psi A^{\eta}\lambda^{1-\theta}\bar{s}_t}{1+\beta A^{\eta}\lambda^{1-\theta}},$$

where

$$\chi \equiv \left(\frac{1-\alpha}{\alpha}\right).$$

Given that the replacement rate is fixed over time, a rise in the replacement rate has the combined effect of an increased tax rate and a benefit. This result is parallel to the negative effect of public pensions on capital accumulation in the deterministic and stochastic models (e.g., Feldstein, 1974; Hauenschild, 2002; Hillebrand, 2012).[12] In the present study, sustainable growth is generated by capital accumulation. Decreasing capital accumulation will negatively affect economic growth (e.g., Yakita, 2001). To observe this, the present study considers aggregate capital accumulation through aggregate saving.

Using the individual saving function, the average saving function under an unfunded pension is (Appendix B)

$$\bar{s}_t = \left[\frac{1-\psi}{1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}}\right]\bar{w}_t \equiv \xi_{PG}\bar{w}_t. \tag{15}$$

Note that $\xi_{PG} = \xi_{FF}$ holds when $\psi = 0$. The PG social security program has two negative effects on savings: the distortionary effect of income tax and saving adverse effects of retirement benefits. Particularly, the latter effect is affected by the risks and relative risk aversion. The effect of the interest factor under risk appears in the deflator of Equation (15) as well as in Equation (14), although it is strengthened by the presence of the saving adverse effect in the case of PG. Furthermore, larger θ engenders a strong saving adverse effect. The relative risk aversion determines the sensitivity of economic growth change in response to risks. The average saving rate decreases with a fixed replacement rate:

$$\frac{\partial\xi_{PG}}{\partial\psi} = -\frac{1+(1+\chi)\beta A^{\eta}\lambda^{1-\theta}}{[1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}]^2} < 0.$$

The economic intuition follows the result obtained for individual saving.

Using Equations (10) and (15), we obtain the economic growth rate under PG, such that

$$\gamma_{PG} \equiv \frac{(1-\psi)(1-\alpha)A}{1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}} - 1. \tag{16}$$

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[12] Hauenschild (2002) developed a general equilibrium model with stochastic production and social security to examine the existence, uniqueness, and stability of stochastic equilibrium.

Partial derivation of Equation (16) with respect to ϕ yields

$$\frac{\partial \gamma_{PG}}{\partial \psi} = -\frac{(1-\alpha)\left[A+(1+\chi)\beta A^{1+\eta}\lambda^{1-\theta}\right]}{[1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}]^2} < 0.$$

Because the capital accumulation depends on saving, the replacement rate influences the economic growth rate through saving. As demonstrated, an increase in the replacement rate has a negative effect on savings because of the distortionary effect of an income tax and saving adverse effect of retirement benefits. Therefore, the economic growth rate is negatively associated with the replacement rate. Alongside this result and Lemma 1, comparing Equations (13) and (16), one obtains

Lemma 1. $\gamma_{PG} \leq \gamma_{FF}$ *for* $\psi \geq 0$.

Irrespective of the value of θ, the economic growth rate under PG pension is dominated by that under the FF pension. When $\psi = 0$, $\gamma_{PG} = \gamma_{FF}$ holds. In principle, the pension benefit reduces private savings. Furthermore, the pension benefits of PG are derived from the next generation's social security tax, which is not used in the capital market. The growth rate under PG is decreasing in the replacement rate. This is the same result as the model without risk. However, the risks have a quantitative impact on savings and economic growth.

3.3. Pay-as-you-go pension with additive benefits to stimulate saving incentives

Earlier, this paper considered the PG pension designed conventionally. As demonstrated in the existing studies and this model, PG discourages saving for retirement. Some countries have adopted fringe benefits to encourage savings for retirement (e.g., tax deduction, credit, and allowance). For instance, *Savings Credit* of the public pension in the UK was an extra payment for people who had saved up money for retirement. To address such a pension system in reality, this paper considers *the modified unfunded pension* (MU), which gives individuals an incentive to saving.

Given that savings credit is positively associated with private savings relative to average saving, the pension benefit is formalized as

$$b_{t+1} = \left[\pi + (1-\pi)\frac{s_t}{\bar{s}_t}\right]\psi \bar{w}_{t+1}\bar{h}_{t+1},$$

where $(1-\pi)$ denotes the parameter related to savings credit ($0 < \pi < 1$). This can be interpreted as one way to introduce actuarial fairness. When $\pi = 1$, the modified PG pension coincides with the standard PG system follows the Beveridgean manner. In contrast, the modified

PG system corresponds to unfunded but fund-like pension with the returns as the economic growth rate if $\pi = 0$. Therefore, MU such as $\pi = 0$ is one representative of the Bismarckian scheme.[13] The pension budget is the same as $T_t = B_t$ at the aggregate level.

Equations (4)–(6) engender the following (Appendix B):

$$s_t = \frac{(1-\psi)w_t h_t - \omega\beta\pi\chi\psi A^{\eta}\lambda^{1-\theta}\bar{s}_t}{1+(1+(1-\pi)\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}},$$

where $\omega \equiv (1+(1-\pi)\chi\psi)^{-\frac{1}{\theta}}$. $0 < \omega < 1$ holds. With Equation (5), MU system partly insures earnings against ex-post idiosyncratic TFP shocks and π determines its coverage. If $\pi = 0$, MU does not cover the ex-post shocks.

The average saving function becomes (Appendix B)

$$\bar{s}_t = \left[\frac{1-\psi}{1+(1+\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}}\right]\bar{w}_t \equiv \xi_{MU}\bar{w}_t. \tag{17}$$

Similar to our interpretation of Equation (17), the MU system has two negative effects on saving: a distortionary effect of income tax and a saving adverse effect of retirement benefits. The most attractive point is that the savings credit affects the savings adverse effect of retirement benefits; it will weaken the savings adverse effect. Using Equations (10) and (17), the economic growth rate under the MU pension system is

$$\gamma_{MU} \equiv \frac{(1-\psi)(1-\alpha)A}{1+(1+\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}} - 1. \tag{18}$$

We now characterize the effects of the unfunded pension with saving-induced fringe benefits on the economic growth rate. Partial differentiation of Equation (18) regarding ψ provides

$$\frac{\partial\gamma_{MU}}{\partial\psi} = -\frac{\left[1+(1+\chi)\omega\beta A^{\eta}\lambda^{1-\theta}\right] + (1-\psi)(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}\dfrac{\partial\omega}{\partial\psi}}{\left[1+(1+\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}\right]^2}(1-\alpha)A,$$

where

$$\frac{\partial\omega}{\partial\psi} = -\frac{(1+(1-\pi)\chi\psi)^{-\frac{1}{\theta}-1}(1-\pi)\chi}{\theta} < 0.$$

An unfunded pension with saving-induced fringe benefits might enhance economic growth because the growth effect of a rise in ψ through a change in ω is positive.

When $\psi = 0$, the marginal growth effect is

$$\left.\frac{\partial\gamma_{MU}}{\partial\psi}\right|_{\psi=0} = -\frac{\left[1+(1+\chi)\beta A^{\eta}\lambda^{1-\theta}\right] - \dfrac{(1-\pi)\chi\beta A^{\eta}\lambda^{1-\theta}}{\theta}}{(1+\beta A^{\eta}\lambda^{1-\theta})^2}(1-\alpha)A.$$

For a small θ, a rise in ψ increases the growth rate at $\psi = 0$. Contrarily, the effect of ψ on

[13] From this viewpoint, π can be interpreted as the Bismarckian factor, which is expressed by Conde-Ruiz and Profeta (2007). However, this specification is more specified for the existing pension program.

growth rate at $\psi = 1$ is negative, as

$$\left.\frac{\partial\gamma_{MU}}{\partial\psi}\right|_{\psi=1} = -\frac{1}{1+(1+\chi)\omega\beta A^{\eta}\lambda^{1-\theta}} < 0.$$

Therefore, the social security tax and economic growth rate have a hump-shaped (monotonically decreasing) relation if θ is sufficiently small (large). The reported values of θ by empirical studies are varied over the range of 0.2–10. The most widely accepted value of θ would be between 1 and 3 (Gandelman and Hernandez-Murillo, 2015). Gandelman and Hernandez-Murillo (2015) found that some economically developed countries have values smaller than 0.5 (e.g., Ireland, Japan, Korea, and the Netherlands).[14] Sufficiently small values of θ are plausible.

We next consider the effects of a change in π on economic growth. The partial derivative of Equation (18) with respect to π yields

$$\frac{\partial\gamma_{MU}}{\partial\pi} = -\frac{(1-\psi)(1-\alpha)\chi\psi\beta A^{1+\eta}\lambda^{1-\theta}}{[1+(1+\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}]^2}\frac{\partial\omega}{\partial\pi} < 0.$$

The equation above demonstrates that small (large) savings credits engender a low (high) economic growth rate.

Equations (13) and (18) show the properties of the equilibrium growth rate, and Lemma 1 provides the following (Appendix C provides the proof):

Proposition 1. *(i) When θ is sufficiently small, there exists $\hat{\psi}$ such that $0 < \hat{\psi} < 1$ and $\gamma_{MU} = \gamma_{FF}$. Then, $\gamma_{PG} < \gamma_{FF} < \gamma_{MU}$ if $\psi < \hat{\psi}$ while $\gamma_{PG} < \gamma_{MU} \leq \gamma_{FF}$ if $\psi \geq \hat{\psi}$. (ii) When θ is sufficiently large, $\gamma_{PG} < \gamma_{MU} < \gamma_{FF}$ holds for $\psi > 0$.*

Small θ denotes that the consumption between youth and old age are more substitutable. The income shocks have less effect on consumption-saving choices than large θ. Given that other economic conditions are unchanged, savings under small θ is less than under large θ. Therefore, the growth rate of an FF pension will be higher than that of a PG because the latter encourages people to consume more. When compared to an FF pension, an unfunded pension with saving-induced fringe benefits may accelerate economic growth by stimulating saving. For small savings under small,θ strengthening the savings incentive will positively affect economic growth. The MU pension system increases the economic growth rate over the level under full funding within the appropriate values of ψ.

[14] The values of θ are 0.35 in Ireland, 0.44 in Japan, 0.27 in Korea, and 0.10 in the Netherlands. $\theta = 2$ is rejected at the 10% level for all countries and $\theta = 1$ is also rejected at the 10% level in Korea.

4. Growth and welfare effects of risks through public pension

The tradeoff between economic growth and social justice is a central issue of public economics. If there are no transitional dynamics or economic agent heterogeneity with the production function (1), then the maximizing growth rate can be consistent with achieving social justice. This will not be the case with income shocks. To consider the tradeoff between economic growth and social justice, this section presents an examination of the growth and welfare effects of income risks through different public pension systems.

4.1. Risks and economic growth

Following conventional methods, risk and the degree of heterogeneity should be measured as variances. Parameters related to risks are μ_i and σ_i $(i = x, z)$. Both μ_i and σ_i affect the mean and variance of X_t or Z_t. The distribution of labor ability only influences individual saving and does not aggregate saving. The variance $Var[X_t]$ is zero if $\sigma_x = 0$. Here, this paper specifically examines the comparative statics of σ_x.

Taking a logarithmic differentiation of Equation (13) for σ_x the elasticity of the growth factor in the funded pension system for σ_x becomes

$$\epsilon_{FF} \equiv \frac{\sigma_x \partial \log(1+\gamma_{FF})}{\partial \sigma_x} = \left[1 - \frac{(\theta-1)^2}{\theta}\left(\frac{\beta A^\eta \lambda^{1-\theta}}{1+\beta A^\eta \lambda^{1-\theta}}\right)\right]\sigma_x^2.$$

Similarly, the logarithmic differentiations of Equations (16) and (18) for σ_x lead to

$$\epsilon_{PG} \equiv \frac{\sigma_x \partial \log(1+\gamma_{PG})}{\partial \sigma_x} = \left\{1 - \frac{(\theta-1)^2}{\theta}\left[\frac{(1+\chi\psi)\beta A^\eta \lambda^{1-\theta}}{1+(1+\chi\psi)\beta A^\eta \lambda^{1-\theta}}\right]\right\}\sigma_x^2,$$

$$\epsilon_{MU} \equiv \frac{\sigma_x \partial \log(1+\gamma_{MU})}{\partial \sigma_x} = \left\{1 - \frac{(\theta-1)^2}{\theta}\left[\frac{(1+\chi\psi)\omega\beta A^\eta \lambda^{1-\theta}}{1+(1+\chi\psi)\omega\beta A^\eta \lambda^{1-\theta}}\right]\right\}\sigma_x^2.$$

We now consider the marginal effects of increased risk on the growth effects from σ_x. Comparison between the elasticities results in the following.

Proposition 2. *(i)* $(1+\chi\psi)\omega < 1 \Leftrightarrow \epsilon_{PG} < \epsilon_{FF} < \epsilon_{MU}$, *while (ii)* $(1+\chi\psi)\omega > 1 \Leftrightarrow \epsilon_{MU} < \epsilon_{PG} < \epsilon_{FF}$.

A rise in σ_x increases both the average productivity and income risks. The overall effect on economic growth is varied, depending on the schemes and preference parameters. Specifically examining $\sigma_x = 0$ yields

$$(1+\chi\psi)\omega \gtreqless 1 \Leftrightarrow \theta \gtreqless \frac{\log(1+(1-\pi)\chi\psi)}{\log(1+\chi\psi)} \equiv \hat{\theta},$$

where $\hat{\theta} < 1$. For example, we consider $\theta > \hat{\theta}$. We obtain $\epsilon_{MU} < \epsilon_{PG} < \epsilon_{FF}$ from Proposition 2. Presuming that $\epsilon_{FF} < 0$ holds, then $|\epsilon_{MU}| > |\epsilon_{PG}| > |\epsilon_{FF}| > 0$ because of $\epsilon_{MU} < \epsilon_{PG} < \epsilon_{FF} < 0$. An unfunded pension with savings credits exhibits the highest sensitivity of economic growth to increased risk. However, if we assume $\epsilon_{MU} > 0$ and $|\epsilon_{FF}| > |\epsilon_{PG}| > |\epsilon_{MU}| > 0$ holds, the FF pension shows the highest sensitivity of economic growth to increased risk. The relation between risk and economic growth can be elucidated using numerical simulations.

4.2. Optimal pension system

Using Equations (4), (5), and (14), the indirect utility function under an FF pension with risky returns is

$$U_t^{FF} = \frac{\left((1-\psi_{FF})w_t h_t\right)^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{E\left[(R_{t+1}\psi_{FF}w_t h_t)^{1-\theta}\right] - 1}{1-\theta}.$$

Similarly to deriving W_t^{FF}, we obtain the indirect utility functions under PG and MU pension systems such that

$$U_t^{PG} = \frac{\left[\frac{(1-\psi)\beta A^{\eta}\lambda^{1-\theta}w_t h_t + \beta\chi\psi A^{\eta}\lambda^{1-\theta}\bar{s}_t}{1+\beta\lambda^{1-\theta}A^{\eta}}\right]^{1-\theta} - 1}{1-\theta}$$

$$+\frac{1}{1+\rho}\frac{E\left[\left(\frac{(1-\psi)w_t h_t - \beta\chi\psi A^{\eta}\lambda^{1-\theta}\bar{s}_t}{1+\beta A^{\eta}\lambda^{1-\theta}}R_{t+1} + \psi\bar{w}_{t+1}\bar{h}_{t+1}\right)^{1-\theta}\right] - 1}{1-\theta},$$

$$U_t^{MU} = \frac{\left\{\frac{[(1-\psi)w_t h_t](1+(1-\pi)\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta} + \omega\beta\pi\chi\psi A^{\eta}\lambda^{1-\theta}\bar{s}_t}{1+(1+(1-\pi)\chi\psi)\omega\beta A^{\eta}\lambda^{1-\theta}}\right\}^{1-\theta} - 1}{1-\theta}$$

$$+\frac{1}{1+\rho}\frac{E\left[\left(R_{t+1}s_t + \left[\pi + (1-\pi)\frac{s_t}{\bar{s}_t}\right]\psi\bar{w}_{t+1}\bar{h}_{t+1}\right)^{1-\theta}\right] - 1}{1-\theta}.$$

I assume that the social welfare function takes the form of the Rawlsian welfare function. Formally, the social welfare function under pension scheme *i* ($i = FF, PG, MU$) is defined as

$$W_t^i \equiv \sum_{T=t}^{\infty}\left(\frac{1}{1+\delta}\right)^T \min U_T^i,$$

where δ denotes the social discount rate. $\gamma_i < \delta$ ($i = FF, PG, MU$) is necessary to ensure that social welfare is bounded. Other welfare criteria, such as the Benthamite welfare function, can be presumed:

$$V_t^i \equiv \sum_{T=t}^{\infty} \left(\frac{1}{1+\delta}\right)^T \iint U_T^i \; f(x_T) g(z_T) dx_T dz_T.$$

However, deriving analytical results other than the Rawlsian welfare function is complicated. Therefore, this paper specifically examines the Rawlsian welfare function, although the Benthamite social welfare function is analyzed numerically.

We turn to analyzing the welfare effects of the unfunded pension systems. Within the unfunded pension systems, the social security tax rates affect the economic growth rates. Therefore, the increased pension replacement rate influences social welfare through short-term and long-term effects: the former involves insurance (for ex-post shocks) and redistribution (for ex-ante inequalities) effects; the latter is based on the negative growth effects of public pension. As previously stated, relative risk aversion exerts a negative growth effect because the adverse effects of retirement benefits are weakened or strengthened depending on θ. Therefore, the welfare effects of unfunded social security programs and the optimal system differ depending on θ. Taking these effects into account, we arrive at the following proposition (the proof is in Appendix D):

Proposition 3. *There exists an interior optimal social security tax rate of PG pension,* ψ^*. *(i) If* θ *is sufficiently small, then there might exist an interior optimal savings credit rate* π^*. *(ii) Contrarily, PG is optimal if* θ *is sufficiently large. In either case,* W_0^{MU} *is equal to or greater than* W_0^{PG} *for the optimal savings credit rate.*

People with no income do not benefit from economic growth at present because they cannot consume and save. For the social security tax, we have[15]

$$\operatorname{sgn}\frac{\partial W_0^{PG}}{\partial \psi} = \left[\frac{\beta\chi A^\eta \lambda^{1-\theta}\xi_{PG}}{1+\beta\lambda^{1-\theta}A^\eta}\right]^{1-\theta}\left(1+\frac{\psi}{\xi_{PG}}\frac{\partial \xi_{PG}}{\partial \psi}\right) + \frac{1}{1+\rho}E\left[\left(\frac{c_{t+1}^o}{w_t}\right)^{-\theta}(\Lambda+\Gamma)\right], \tag{19}$$

where

$$\Lambda \equiv 1+\gamma_{PG} - \frac{\beta\chi A^\eta \lambda^{1-\theta}\xi_{PG}}{1+\beta A^\eta \lambda^{1-\theta}} R_{t+1} - \frac{\beta\chi\psi A^\eta \lambda^{1-\theta}}{1+\beta A^\eta \lambda^{1-\theta}}\frac{\partial \xi_{PG}}{\partial \psi} R_{t+1},$$

$$\Gamma \equiv \psi \frac{\partial \gamma_{PG}}{\partial \psi}.$$

The first term on the RHS of (19) is the effect of redistribution for ex-ante inequalities caused by the increase in social security taxes. In a general equilibrium, the second term includes factor

[15] See Appendix D for the derivation of this equation. This formula is similar to that derived by Harenberg and Ludwig (2015, 2019). The present study differs from them concerning the long-run growth rate: it is positive in the study, whereas it is zero in their study.

price changes and growth effects, analogous to Harenberg and Ludwig (2015).[16] Term Λ represents the tradeoff between welfare gains due to the insurance effect of social security and welfare losses due to social security returns that are lower than the expected return on savings. Furthermore, term Γ denotes the growth effect.

For $\psi = 0$, Equation (19) can be reduced to

$$\operatorname{sgn}\left.\frac{\partial W_0^{PG}}{\partial \psi}\right|_{\psi=0} = \left[\frac{\beta\chi A^{\eta}\lambda^{1-\theta}\xi_{FF}}{1+\beta\lambda^{1-\theta}A^{\eta}}\right]^{1-\theta} + \frac{1+\gamma_{FF}}{1+\rho}E\left\{\left(\frac{c_{t+1}^{o}}{w_t}\right)^{-\theta}\left[1-\frac{\beta\chi A^{\eta}\lambda^{1-\theta}\xi_{FF}R_{t+1}}{(1+\beta A^{\eta}\lambda^{1-\theta})(1+\gamma_{FF})}\right]\right\} > 0,$$

where $\xi_{PG} = \xi_{FF}$ and $\gamma_{PG} = \gamma_{FF}$ hold for $\psi = 0$. Individuals are willing to receive pension benefits for $\psi = 0$ because the redistribution and insurance effects are large enough, and there is no negative growth effect Given that the negative welfare effects from negative growth effects increase in proportion to the social security tax rate, governments must be concerned about negative growth effects of PG because future pension benefits depend on the next young generations' income levels (ultimately economic growth rate). For a large value of ψ, the negative welfare effects of social security dominate its positive welfare effects. Therefore, the social security tax rate is optimally selected as an interior solution that depends on θ, σ_x, and so on.

Concerning the optimal savings credit rate, a government might choose either an optimal interior credit rate (MU) or no savings credit (PG). If $\pi = 0$, then there is no pension for the poorest people. Therefore, the government is incentivized to increase π and decrease savings credit because the current effect on the poorest people includes the redistributive insurance effect of PG if $\pi > 0$. For $\pi > 0$, the poorest person's utility depends on the economic growth rate. By the way similar to the social security tax, the welfare effect of savings credit can be decomposed into the redistribution effect for ex-ante inequalities, the effect of insuring ex-post inequalities, and the growth effect.

Starting from PG ($\pi = 1$) for $\psi > 0$, we have

$$\begin{aligned}\operatorname{sgn}\left.\frac{\partial W_0^{MU}}{\partial \pi}\right|_{\pi=1} &= \left(\frac{\beta\chi\psi A^{\eta}\lambda^{1-\theta}\xi_{PG}}{1+\beta A^{\eta}\lambda^{1-\theta}}\right)^{1-\theta} \\ &\quad\times\left\{1+\left[\frac{1}{1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}}+\frac{(\theta-1)\beta A^{\eta}\lambda^{1-\theta}}{1+\beta A^{\eta}\lambda^{1-\theta}}\right]\frac{\chi\psi}{\theta}\right\} \\ &\quad+\frac{1}{1+\rho}\psi^{1-\theta}(1+\gamma_{PG})^{1-\theta}\left[1-\frac{(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}}{1+(1+\chi\psi)\beta A^{\eta}\lambda^{1-\theta}}\frac{\chi\psi}{\theta}\right].\end{aligned} \tag{20}$$

In the RHS, the first term represents the redistribution effect of savings credit for ex-ante

[16] Note that the growth effect is non-zero in contrast to that of Harenberg and Ludwig (2015).

inequality, whereas the second term represents the insurance and general equilibrium effects related to elderly consumption.

Considering that an increase in π means a decrease in savings credit, the former one exhibits a negative welfare effect of increasing savings credit in sum. The latter can be either a positive or negative welfare effect of increasing the savings credit, depending on the relative risk aversion. If θ is sufficiently low, the government aims not to decrease the economic growth rate too much and might keep it high to set $\pi < 1$. Here, for the elderly utility level, the general equilibrium effect of increasing savings credit dominates the other. Hence, increasing savings credit has a positive welfare effect. However, if θ is sufficiently large, then no savings credit (i.e., $\pi = 1$) is preferred because the insurance and general equilibrium effects of increasing savings credit have a negative welfare effect in sum——high growth and risks with large savings credit negatively affect utility level.

4.3. Numerical analysis

This subsection provides a numerical representation of the qualitative results presented in the preceding subsections in order to elucidate the qualitative insights and to extend the welfare analysis using the Benthamite welfare function. The parameters are set as $\alpha = 0.3$ and $\rho = 0.5$ as a baseline case.[17] This paper also considers a low α scenario for example of presenting the theoretical results. Following the estimated result presented by Gandelman and Hernandez-Murillo (2015), the key parameter θ is considered at four different values: 0.01, 0.5, 1, and 1.5.[18] Furthermore, $\mu_z = 0$, $\sigma_z = (0.5, 1, 3, 5, 10)$, $\mu_x = 2$ and $\sigma_x = (0, 0.25, 0.5, 0.75, 1)$.

Economic growth, public pension, and risks. Actually, $\psi = 0.2$ and $\pi = 0.9$ are assumed for calculating the equilibrium growth rate under the PG and MU systems. The equilibrium growth rates in cases of a baseline and low α are shown respectively in Tables 1 and 2. A comparison of benchmark and FF in Table 1 provides a numerical example of the Section 3 results. Table 2 shows that the growth rate under MU is greater than the growth rate under PG in each case (Proposition 1). A comparison of results in Tables 1 and 2 within a baseline scenario shows that an FF pension has the highest growth rate. Therefore, FF is the best way to boost economic growth within plausible parameters.

However, by adopting low α scenario, one can find several differences from a baseline result. For $\alpha = 0.3$, the MU system stimulates economic growth through savings credit, but its effect is

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[17] The capital share lies between 0.3 and 0.5. Therefore, the value around $\alpha = 0.3$ is frequently used for numerical analysis. For instance, Imrohoroglu et al. (1998) set $\alpha = 0.36$ to calibrate the effects of individual retirement accounts on capital accumulation.
[18] Based on the estimation of Gandelman and Hernandez-Murillo (2015), the density is quite low, out of the range of 0.5–1.5. Therefore, this paper specifically examines these three values as representatives.

too weak to outweigh that of the funded system within the values of $0.01 \leq \theta \leq 1.5$. However, when $\alpha = 0.2$ and $\theta = 0.01$, Tables 1 and 2 show that the growth rate under MU is larger than that under FF for $\sigma_x = 0$ or $\sigma_x = 0.25$.[19] For a small value of α and θ, the MU system generates an economic growth rate higher than the FF system even though the present study adopts a quite low value of θ as one of the examples. The recent trend for declining labor share may weaken the advantage of MU to FF in terms of economic growth.

Social welfare, public pension, and risks. Concerning economic growth, funded pension systems are superior to unfunded pension systems. However, under the Rawlsian welfare function, the FF pension system reaches about the lowest welfare level for the poorest workers because no income derives no saving for consumption in retirement. Therefore, we should quantitatively verify the order of welfare levels and the optimum social security. The labor supply shocks do not affect the optimal tax rates under the Rawlsian welfare function. Hereafter, the present study excludes the case where $\theta = 0.01$ as an extremely high-growth scenario.

$\delta = 1$ is set as the social discount rate. In Table 3, PG outcomes indicate that an optimal interior social security tax rate exists. For $\theta = 0.5$, the tax rate range of 4% and 15%. For $\theta = 1$ and $\theta = 1.5$, the tax rates are around 20%. Observation of PG outcomes indicates that larger σ_x engender a smaller tax rate. An increase in σ_x raises the expected productivity and its volatility. It increases the interest and economic growth rates. These effects diminish the need for raising the social security tax rate by increasing pension benefits.

In Table 3, MU can be superior to PG for $\theta = 0.5$, although PG is preferable (i.e., $\pi^* = 1$) for $\theta = 1$ and $\theta = 1.5$. When $\theta = 0.5$, the optimal percentage of savings credit is 78.9%–86.3% for the domain of σ_x (Proposition 3). Therefore, the unfunded system with savings credit is theoretically justified in certain cases. For small θ, high growth and interest rates by increased risks improve welfare. The poorest households seek to benefit from increasing interest and economic growth rates. Therefore, the savings credit is optimally selected.

Regarding the welfare level, Table 3 demonstrates that no savings credit ($\pi = 1$) is selected when $\theta = 1$ and $\theta = 1.5$; PG is in the first place. However, if $\theta = 0.5$, then the order is reversely changed. The MU system is a better choice by the intuition of Proposition 3.

Social welfare, public pension, and population density of income class. Based on computations, any class of specified welfare function gives an optimal social security tax rate and its welfare level. To verify the robustness of our guess related to the optimal social security tax rate, this paper considers the Benthamite welfare function V_t^i. Parameter σ_x is fixed at $\sigma_x = 0.5$, whereas σ_z is varied in $\sigma_z = (0.5, 1, 3, 5, 10)$. The density of each income class determines the optimal social security tax rate. Small σ_z indicates that the income distribution is not spread

[19] Examining the extreme case where $\pi = 0$ (the strongest growth effect), with $\alpha = 0.2$ and $\theta = 0.5$, the growth rate under MU dominates that under FF for $0 < \psi < 0.292$.

widely, whereas large σ_z indicates that the low-income people have a thick population.

The calibrated results are shown in Table 4. The outcomes of the Benthamite welfare function are similar to those of the Rawlsian welfare function in terms of social security tax rates and welfare level orders. The table demonstrates that the optimal social security tax rate is increasing in σ_z. A high density of low-income class engenders a high tax rate. Furthermore, high relative risk aversion tends to increase the social security tax rate similarly to that in Table 3.

5. Discussion

This section provides a discussion of the policy implications of the present study for existing *Welfare States* and further analyses of several topics such as the importance of household's expectation for future returns and endogenous fertility and longevity.

5.1. Policy implications for existing Welfare States

Theoretical and numerical analyses in the present study reveal that an unfunded pension is preferred to an FF pension if the government cares for poor people. Rich people naturally prefer to buy private annuities identical to an FF pension. However, for various political reasons, the governments in the real world must devote attention to low-income people. Therefore, the government operates the public pensions on a PG principle.

The results also indicate that the modified funded pension system has some advantages over a PG system and an FF pension system in response to risks. Although it provides a higher welfare level than others, this system loses the link between contributions and retirement benefits. In reality, it might be difficult to obtain public agreement for the modified funded system. Consequently, this paper concludes that the unfunded pension system might be a better choice in an economy with risks, which explains why unfunded pension systems are mainstream to provide social security programs.

Within unfunded pension systems lies the issue of intragenerational redistribution. As described earlier, real social security programs are classified as Beveridgean and Bismarckian pension systems. Using the political economic approach, some studies have tackled issues of which of them is politically chosen (e.g., Casamatta et al., 2000; Cremer and Pestieau, 2000; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007; Glasso and Profeta, 2007). They demonstrated that the Beveridgean system is politically supported by low-income people, although Bismarckian systems are favored by high-income people, and demonstrated that which system arises depends on the retirement timing, demographics, and so on.[20]

In the present study, qualitative and quantitative analyses demonstrate that the Beveridgean system such as PG is preferred if people tend to avoid risks strongly, whereas the Bismarckian system such as MU is favored when people tend to weakly avoid risks. In fact, this result is similar to that described earlier in the literature on Beveridgean and Bismarckian pension systems, because the relative risk aversion can be interpreted as the relative inequality aversion. However, the numerical results imply a difference in welfare implications in response to risks of different types.

The degree of savings credit, π, decreases with the variance of labor productivity directly related to earning, σ_z; a large variance of the worker's productivity engenders the Beveridgean. Furthermore, whether a small or large pension system is better depends on the labor-related risk and social welfare function. For a given firm's productivity, large inequality (i.e., large value of σ_z) meets a large PG pension. However, different patterns are visible for the firms' productivity shocks. A large variance of the firm's productivity, σ_x, engenders small PG pensions. Because the firm's productivity is linked to aggregate productivity, which determines the equilibrium economic growth rate, a large value of σ_x generates a high growth rate with high risks. It requires a reduction of the distortionary effect of a tax. The optimal social security tax rate decreases with σ_x.

These results imply that the desirable pension systems face different levels of risk and relative risk aversion. In reality, almost all developed countries have adopted PG pension systems between Beveridgean and Bismarckian schemes. Based on the analyses, if people are strongly risk or inequality averse, the society tends to prefer a large intragenerational redistribution within PG pension systems. The Bismarckian factor averages in 1988–2008 are 0.05 for Australia, 0563 for Germany, 0.341 for Ireland, 0.307 for the Netherlands, and 0.127 for the UK (Rivera-Rozo et al., 2018), whereas the corresponding values of the relative risk aversion are 1.17 for Australia, 0.77 for Germany, 0.35 for Ireland, 0.1 for Netherland, and 1.03 for the UK (Gandelman and Hernandez-Murillo, 2015). These tendencies support the result that large θ engenders large π. Therefore, the present model provides a plausible explanation of the existing Welfare States.[21]

[20] Kaganovich and Zilcha (2012) examined the fiscal sustainability of social security including the public education funding. They demonstrate that that the fully funded social security system generates political support for a higher education funding and therefore a higher economic growth rate than the pay-as-you-go system.

[21] Rivera-Rozo et al. (2018) found that the Bismarckian factor is affected by cultural factors (e.g., individualism) as the same as economic factors. Therefore, this paper cannot treat all the cases of the existing Welfare States. For example, the Bismarckian factor and relative risk aversion in United States are 0.489 and 1.39 respectively. To explain this, the present study must incorporate additional factors into the basic model.

5.2. Importance of household's expectation

If capital markets or social security program perfectly cover risks of all kinds, then all households purchase a perfect annuity similarly to BM in the model. This result depends on the households' expectations in their saving decisions. Therefore, if the capital market is incomplete, then how much risk is covered by social security programs is essential for a household's savings determination. The unfunded pension programs based on the pay-as-you-go principle might cover the productivity risks depending on a household's expectation. In earlier sections, this paper focuses on the situation in which the social security program provides partial insurance for the existing risks (e.g., ex-ante inequalities). When the households presume that there is no risk because of social security, its coverage is wider than that examined in Sections 2–4. If there is no difference in the equilibrium growth rate under different expectations, then the unfunded pension system would be more preferred by the households because of the insurance effects of social security.

We now consider the effects of the household's expectation, particularly addressing average savings. The household's expectations only affect the expected value of social security return. Hence, the expectation has no effect on analytical results on economic growth and social welfare. If the households believe that the social security perfectly covers productivity shocks, then they consider that the social security return is riskless. However, if not, they anticipate future retirement benefits based on $f(x_t)$. Figure 1 illustrates the average saving rates obtained under different expectations with respect to the pension returns, which are calculated using the baseline parameters of α ($\alpha = 0.3$ for panel (a) and $\alpha = 0.2$ for panel (b)), ρ, and θ (for $0.01 \le \theta \le 2$), $\mu_x = 2$, $\sigma_x = 0.5$, $\psi = 0.15$, and $\pi = 0.8$. Especially, ψ and π are set based on the values in Table 3. In Figure 1, a solid line curve represents the average saving rates based on the household's expectations in the preceding sections, whereas a dashed line curve shows the average saving rates under a riskless social security return.

The difference between saving rates under BM and FF observed in Figure 1 were explained in Section 3. Regarding PG and MU, the social security system partly covers risks because wage growth is insured. For each of PG and MU, the household's expectation used in previous sections positively affects the saving rate because people save more in preparation for future income risks. Hence, this effect of household's belief of partly covered risks is growth-enhancing, leading to higher welfare through the cumulative increase in future income. By contrast, uninsured future income risks cause intertemporal consumption misallocation, leading to negative welfare effects. The optimal tax rates will change even if the analytical results of comparative statics still hold.

Table 5 also displays the specified saving rates shown in Figure 1 (a) (i.e., $\alpha = 0.3$). As explained above for Figure 1, the expectation "covered" means that households regard b_t as a deterministic variable (i.e., riskless), whereas "partially covered" implies that households treat b_t as a stochastic variable that is governed by $f(x_t)$. The latter case corresponds to the result presented in the previous sections. Table 5 shows that the saving rates under "covered" are less than those under "partially covered" within the unfunded pension systems. The related intuition is straightforward and explained above. Furthermore, Table 5 shows that the saving rate under MU are larger than those under PG. A large difference is apparent in the saving rates with different expectations. This difference can lower the growth effect of the MU system on social welfare.

Figure 2 shows the graphs of average saving rates under FF and MU systems with different values of savings credit, depending on the relative risk aversion. Panels (a) and (b) respectively corresponds to the case where $\alpha = 0.3$ and $\alpha = 0.2$. The top panels in each of (a) and (b) illustrate a solid line curve for the average saving rate with $\pi = 0.8$ and a dashed line curve for that with $\pi = 0.6$. These two curves imply that more savings credit engenders higher saving rate even when households expect that the social security perfectly covers the risks; savings credit has a positive growth effect. Furthermore, under this household's expectation, the middle and bottom images of panel (b) show that the saving rates under riskless MU with $\pi = 0.8$ dominate those under FF for θ around 0.1 and more if $\pi = 0.6$, whereas those of (a) demonstrate the saving rates under riskless MU are below those under FF. Proposition 1 holds in case of (b) $\alpha = 0.2$, but it does not in the case of (a) $\alpha = 0.3$. Therefore, the differences in the household' expectation in saving do not *qualitatively* affect the main analytical results in the present study.

5.3. Further analyses

This paper treats labor supply shocks to consider the heterogeneity of workers' earnings. Because the population of the economy is stationary, there is no demographic change. However, if the variance of exogenous labor supply is changed, the aggregate labor supply is also changed. Such a shock can be interpreted as shock to the working population. Therefore, the results are applicable to explain the effects of exogenous change in working populations on equilibrium outcomes through social security programs.

Demographic changes can be generated by changes in economic circumstances, such as income. Therefore, the demographic shocks are not independent of labor supply and are one determinant of fertility. Considering endogenous determination of the number of children, certain studies show that social security systems affect fertility rates (e.g., Cigno, 1993; Zhang, 1995; van Grozen et al., 2003; Sinn, 2004). Although this issue is fundamentally important to analyzing the demographic shocks concerning fertility and mortality rates, we can guess the feedback effects of social security on fertility using the theoretical findings presented in earlier studies of the literature.

Numerous researchers have examined the relation between population aging, social security, and economic growth using OLG models with endogenous fertility (e.g., Pecchenino and Pollard, 1997; Yakita, 2001; Hirazawa et al., 2010; Cipriani, 2014; Ono, 2017) or with both endogenous fertility and longevity (Pestieau et al., 2008; Fanti and Gori, 2014; Stauvermann and Kumar, 2016), which enables us to analyze population aging and its economic effect on (or through) social security.[22] However, demographic shocks have not been specifically examined in endogenous growth models, separate from fertility choice and health investment. The different demography in each generation causes asymmetric labor supply and intragenerational and intergenerational income inequality. Furthermore, productivity shocks are significant concerning economic growth. Recently, Fan et al. (2021) emphasized an inter-family risk pooling aspect of unfunded social security, although unrelated to economic growth. Focusing on economic growth, theoretical findings in the present study fill a gap in the research.

When the number of children is described as one of the consumption and normal goods, a decrease in income reduces the number of children. With large relative risk aversion, an increase in the variance of labor productivity will decrease the number of children. It negatively affects the revenue of social security taxes under PG pension systems and therefore might decrease social welfare. In contrast, the firms' productivity risks will affect fertility differently. An increase in the variance of the firms' productivity increases the economy-wide productivity and therefore raises average wages and interest rates with large volatilities of them. Depending on the relative risk aversion, these increases in wages, interest rates, and their volatilities affect the expected utility in the second period. However, the basic mechanism revealed by the present study will provide an analytical basis for the extension of endogenous fertility.

6. Conclusion

This paper develops an OLG model of endogenous growth by incorporating idiosyncratic TFP shocks and heterogeneous workers to examine the relationship between public pensions, economic growth, and social welfare. I consider three pension systems for comparison: FF, PG, and MU. PG provides a risk-pooling function, which involves insurance and redistribution effects, whereas FF has no such effects and MU additionally gives people incentives to save by fringe benefit. Particularly, the analysis of public pension under two different sources of inequalities is a characteristic of this study.

[22] Pestieau and Ponthiere (2016) provide a general review of the literature on this issue, including public policies on education and health.

First, I analyzed the relationship between economic growth and public pension. The equilibrium growth rate under the FF system is the highest among all pension programs if the degree of relative risk aversion is sufficiently large. In contrast, if the individuals have a sufficiently small relative risk aversion and the social security tax rate, MU attains the highest growth. The PG always generates the lowest growth. The results obtained for the FF and PG regimes complement those of earlier studies. The main contribution of this study is the role of fringe benefits in providing saving incentives, indicating that the equilibrium growth rate under MU might be higher than that under full funding, depending on the relative risk aversion. Because private saving behavior is affected by risks and fringe benefit, they can stimulate savings and economic growth.

Next, I investigated the relationship between social welfare and public pension. Specifically examining the Rawlsian welfare function, the social welfare level under FF is the lowest. FF does not differ from the private pension. The poorest people lacking income can neither save for retirement nor consume any goods. PG has its optimal social security tax rate because of its redistribution effect for ex-ante inequality and insurance effect for ex-post inequality. Its welfare level is always higher than that under FF and is superior to MU if the relative risk aversion is sufficiently large. However, MU might generate a higher welfare level than PG for sufficiently small relative risk aversion because MU stimulates economic growth more than PG.

This paper also conducted quantitative analysis with plausible values of parameters to elucidate the qualitative results and to provide realistic examples. Results demonstrate that the growth rate under FF is the highest. With highly relative risk aversion, the welfare under PG overweight that under MU and FF, irrespective of the types of social welfare functions and distribution of labor endowments. Furthermore, using the Benthamite welfare function, this study shows that the optimal social security tax under PG increases with the population of low-income class. These results elucidate the effects of social security on economic growth and welfare through ex-post idiosyncratic TFP shock and ex-ante heterogeneous workers. The calculated tax rates complement those in the study by De Menil et al. (2016), although the setting differs. This complementary nature implies that the demographic structure (especially income class) strongly influences optimal social security tax rates.

Lastly, I consider future research directions in this field. The Rawlsian social welfare function illuminates redistributive policy. However, other criteria must be considered. In terms of optimal social security, democratic determination of social security policy will provide new insights. Incorporating endogenous fertility and longevity into the model is also intriguing. These extensions, in essence, represent more realistic economic situations. We can draw some conclusions about extensions from the analyses. For example, substitution of a political determination of social security tax rate for the Rawlsian welfare function will weaken social needs for redistribution. Such an extension is likely to reduce the optimal social security tax rate while increasing the economic growth rate. Therefore, the present research provides an analytical foundation for these extensive analyses.

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Declarations

Conflict of interest I declare that I have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Tables

Table 1. Equilibrium growth rates within the funded system

Baseline	Benchmark				Fully funded			
σ_x	$\theta = 0.01$	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 0.01$	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$
0	4.172	1.567	1.069	0.910	4.172	1.567	1.069	0.910
0.25	4.337	1.690	1.135	0.957	4.337	1.669	1.135	0.977
0.5	4.861	2.092	1.344	1.107	4.861	2.000	1.344	1.192
0.75	5.852	2.880	2.741	1.382	5.852	2.641	1.741	1.605
1	7.528	4.278	2.411	1.825	7.528	3.763	2.411	2.316
Low α	Benchmark				Fully funded			
0	NA	1.343	1.365	1.372	NA	1.343	1.365	1.372
0.25	3.701	1.464	1.440	1.432	3.667	1.441	1.440	1.454
0.5	5.698	1.858	1.679	1.621	5.698	1.756	1.679	1.721
0.75	6.831	2.644	2.132	1.967	6.831	2.371	2.132	2.230
1	8.746	4.067	2.898	2.527	8.746	3.459	2.898	3.107

Note: The value of the cell "NA (Not applicable)" is negative.

Table 2. Equilibrium growth rates within the unfunded system

Baseline	Pay-as-you-go				Modified unfunded			
σ_x	$\theta = 0.01$	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 0.01$	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$
0	3.138	0.663	0.293	0.180	3.138	0.754	0.334	0.206
0.25	3.269	0.732	0.334	0.222	3.269	0.826	0.376	0.249
0.5	3.689	0.955	0.465	0.357	3.689	1.060	0.512	0.387
0.75	4.482	1.390	0.713	0.617	4.482	1.514	0.767	0.651
1	5.822	2.159	1.132	1.064	5.822	2.314	1.199	1.108
Low α	Pay-as-you-go				Modified unfunded			
0	NA	0.264	0.278	0.283	3.704	0.412	0.351	0.331
0.25	2.144	0.319	0.319	0.328	3.878	0.473	0.394	0.379
0.5	4.358	0.499	0.448	0.476	4.359	0.671	0.531	0.531
0.75	5.265	0.853	0.693	0.758	5.265	1.060	0.790	0.824
1	6.797	1.488	1.107	1.246	6.797	1.755	1.228	1.329

Note: The value of the cell "NA (Not applicable)" is negative.

Table 3. Optimal tax rates and social welfare levels (Rawlsian welfare function)

Pay-as-you-go

	$\theta = 0.5$		$\theta = 1$		$\theta = 1.5$	
σ_x	ψ^*	W^{PG}	ψ^*	W^{PG}	ψ^*	W^{PG}
0.00	14.5%	0.451	21.3%	−4.669	22.1%	−3.270
0.25	14.0%	0.827	21.3%	−4.461	22.0%	−3.178
0.50	12.3%	2.166	21.3%	−3.836	21.7%	−2.911
0.75	9.1%	5.538	21.3%	−2.794	21.3%	−2.496
1.00	3.4%	18.982	21.3%	−1.336	20.7%	−1.971

Modified unfunded

	$\theta = 0.5$			$\theta = 1$			$\theta = 1.5$		
σ_x	π^*	ψ^*	W^{MU}	π^*	ψ^*	W^{MU}	π^*	ψ^*	W^{MU}
0.00	86.3%	16.9%	0.470	100%	21.3%	−4.669	100%	22.1%	−3.270
0.25	86.0%	16.4%	0.847	100%	21.3%	−4.461	100%	22.0%	−3.178
0.50	85.0%	14.7%	2.190	100%	21.3%	−3.836	100%	21.7%	−2.911
0.75	82.9%	11.2%	5.571	100%	21.3%	−2.794	100%	21.3%	−2.496
1.00	78.9%	4.4%	19.022	100%	21.3%	−1.336	100%	20.7%	−1.971

Table 4. Optimal tax rates and social welfare levels (Benthamite welfare function)

Pay-as-you-go

	$\theta = 0.5$		$\theta = 1$		$\theta = 1.5$	
σ_z	ψ^*	W^{PG}	ψ^*	W^{PG}	ψ^*	W^{PG}
0.50	0.0%	35.162	0.0%	4.469	0.0%	3.906
1.00	0.0%	31.420	0.0%	3.219	10.5%	3.246
3.00	3.0%	8.922	17.3%	−2.246	24.9%	0.778
5.00	10.9%	2.831	20.9%	−3.682	26.8%	0.060
10.0	12.3%	2.166	21.3%	−3.836	26.9%	−0.017

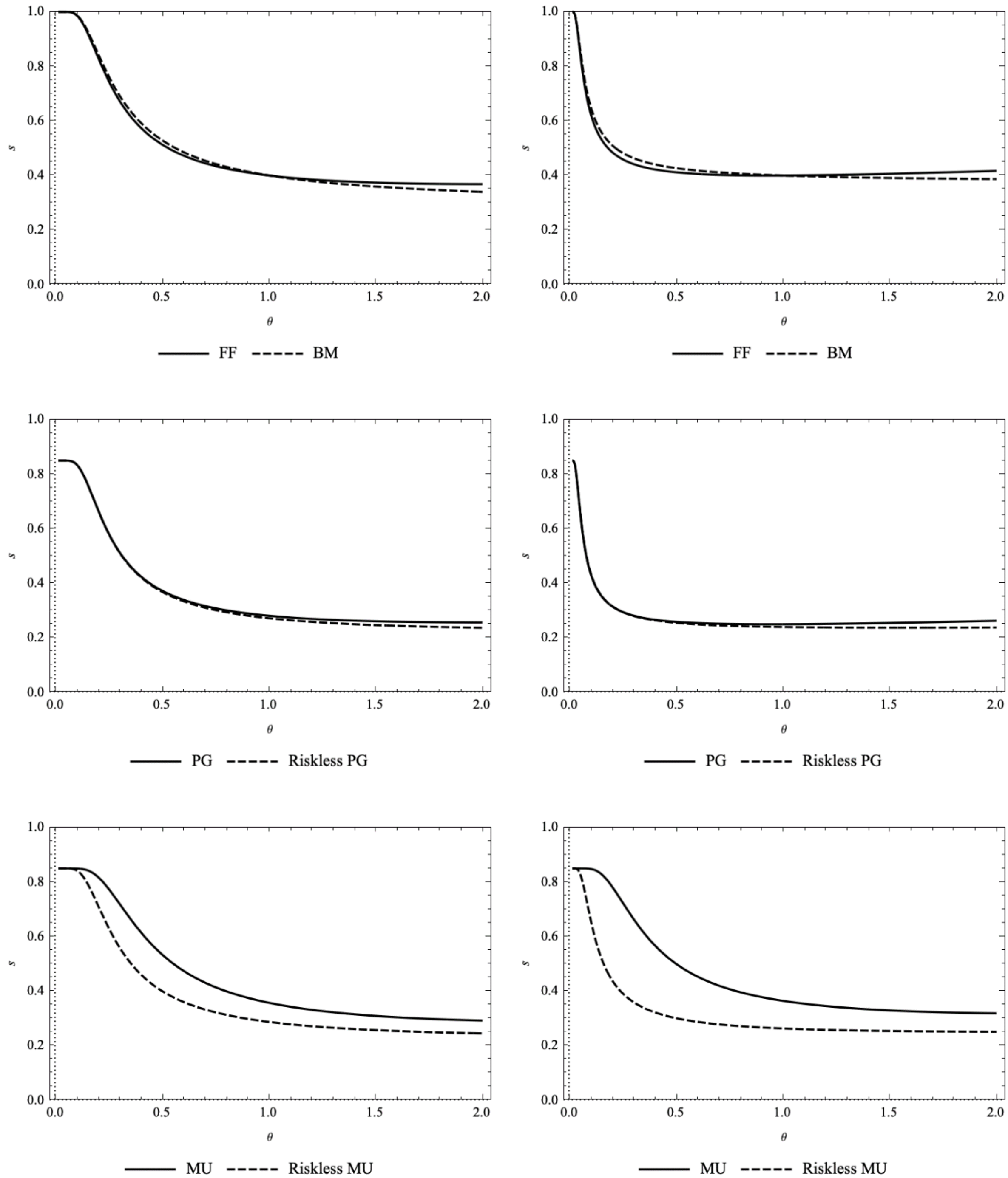
Modified unfunded

	$\theta = 0.5$			$\theta = 1$			$\theta = 1.5$		
σ_z	π^*	ψ^*	W^{MU}	π^*	ψ^*	W^{MU}	π^*	ψ^*	W^{MU}
0.50	0.0%	8.6%	37.018	3.4%	0.0%	4.469	100%	0.0%	3.906
1.00	0.0%	8.6%	33.109	8.9%	0.0%	3.219	100%	10.5%	3.246
3.00	29.5%	9.7%	9.327	100%	17.3%	−2.246	100%	24.9%	0.778
5.00	79.9%	13.8%	2.871	100%	20.9%	−3.682	100%	26.8%	0.060
10.0	85.0%	14.7%	2.190	100%	21.3%	−3.836	100%	26.9%	−0.017

Table 5. The difference in saving rates under different expectations concerning risks

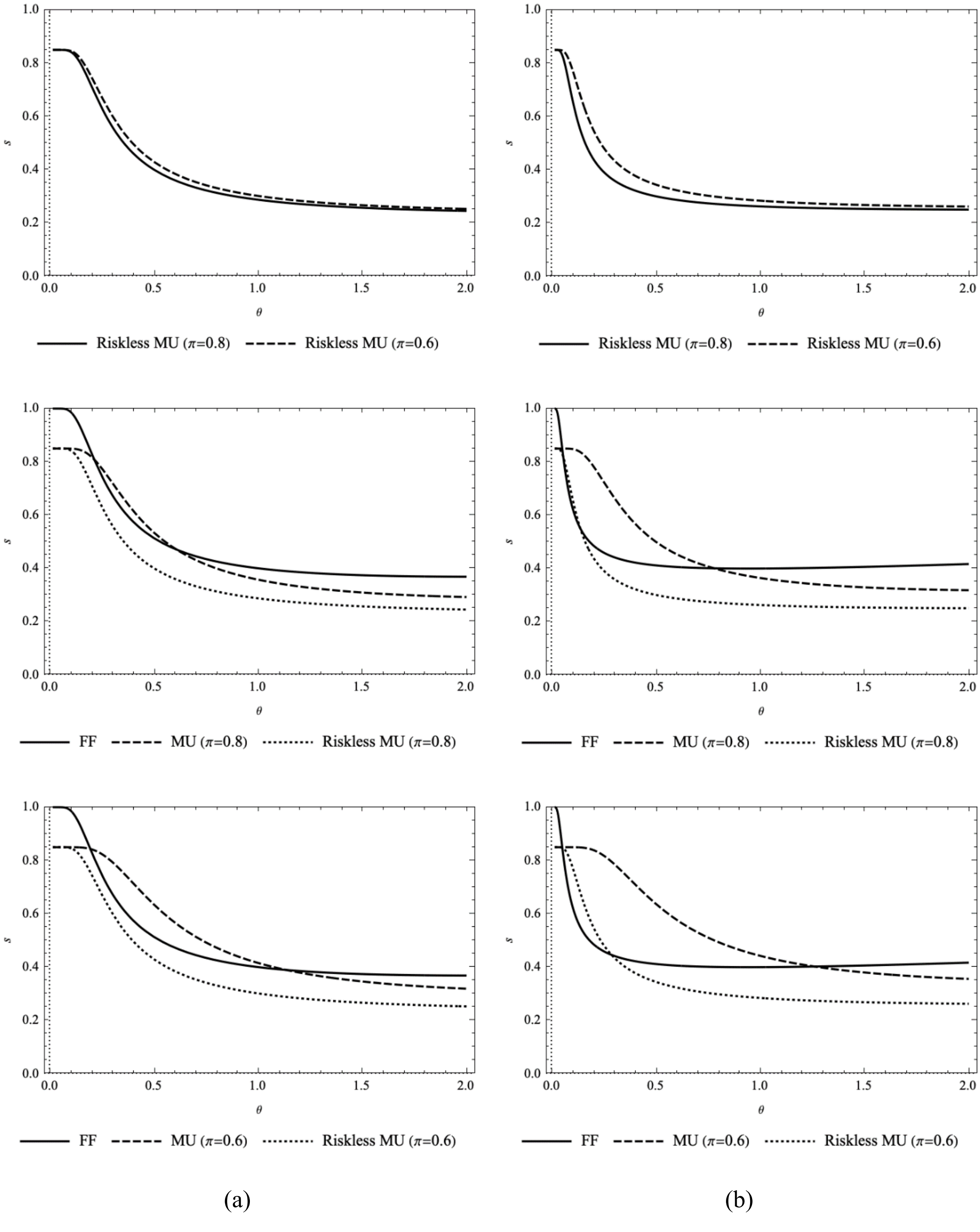
Expectation	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$
Fully funded			
Covered	52.7%	40.0%	36.0%
Partially covered	51.2%	40.0%	37.4%
Pay-as-you-go			
Covered	36.8%	27.2%	24.7%
Partially covered	37.2%	28.1%	26.1%
Modified unfunded			
Covered	39.9%	28.7%	25.7%
Partially covered	53.2%	35.7%	30.9%

Figures



(a) (b)

Figure 1. The relationship between average saving rate and relative risk aversion



(a) (b)

Figure 2. The effect of savings credit on average saving rate