主 論 文 の 要 約

論文題目　**Data-driven Methods for Lyapunov Equations and Their Applications**
(リアプノフ方程式に対するデータ駆動型の手法とその応用)

氏　　名　**坂野　幾海**

論 文 内 容 の 要 約

Lyapunov equations are one of the fundamental equations for analysis and design of dynamical linear systems in control theory. In fact, the solutions to Lyapunov equations are strongly related to several properties of the systems. For example, a linear system is asymptotically stable if and only if a Lyapunov equation associated with the system has a unique solution and it is positive definite. Furthermore, the controllability Gramian and observability Gramian, which respectively quantifies a degree of controllability and observability, of the system can be computed by solving the corresponding Lyapunov equations.

If an exact model of a linear system is available, we can analyze the system by using several model-based methods including solving Lyapunov equations. However, in practice, it is not always possible to obtain a model due to insufficient measurement data. This motivates us to consider data-driven approaches for system analysis and control. Data-driven approaches are the strategies to solve a problem directly from a dataset on the dynamical system without identifying the system. They have the potential to obtain a solution even if a system model cannot be uniquely determined from the knowledge in the dataset. For this reason, data-driven approaches for system analysis and control have received much attention in recent years.

In this thesis, we focus on data-driven approaches for solving Lyapunov equations, which is useful for several types of analysis and design of unknown dynamical systems.

In some cases, the solution to a Lyapunov equation is uniquely determined from a dataset even if it does not contain sufficient information to identify the system; however, data-driven approaches for solving Lyapunov equations have not been fully studied.

In this background, this thesis develops several data-driven methods for Lyapunov equations and derives solutions to some problems as their applications. In particular, this thesis contains the following items: 1) solutions to two generalized classes of Lyapunov equations and computation of their sensitivities, 2) estimation and maximization of the controllability for network systems, 3) design of sparse event-triggered controllers, and 4) characterization of data informativity for Lyapunov equations.

In Chapter 1, the contributions of this thesis are summarized. Related works and open problems of our work are also explained in this chapter.

In Section 2, we propose data-driven solutions to generalized classes of Lyapunov equations and computation of their sensitivities. The Lyapunov equations considered in this work are useful when designing network structure or allocating actuators or sensors for a dynamical system. To solve Lyapunov equations from data on a system, data-driven Lyapunov equations is developed. Data-driven Lyapunov equations are constructed from state trajectory data of a dynamical system and are equivalent to Lyapunov equations under certain conditions. As the result, data-driven solutions to Lyapunov equations are reduced into solving the corresponding data-driven Lyapunov equations. Furthermore, the sensitivities of the Lyapunov equations are computed by solving two data-driven Lyapunov equations. The proposed methods are demonstrated by numerical examples to discuss their validity and the accuracy of the solutions. The results show that the proposed methods provide more accurate solutions than the combination of system identification and model-based method in some cases. Since the proposed methods are based on solving linear equation, we can employ the (unconstrained or constrained) least squares method to reduce the noise effect or to incorporate prior knowledge about the solution.

Section 3 presents data-driven methods for controllability estimation and maximization of network systems as applications of data-driven Lyapunov equations. The controllability is a fundamental property of linear systems that represents whether the state is reachable from any initial state to the desired state by some control input. Therefore, evaluation and improvement of a degree of the controllability are important in system control. We first show that a data-driven method for computing the controllability Gramians is obtained by simply solving a data-driven Lyapunov equation. We then present a model-free method for maximizing the controllability metric, defined by the trace of the controllability Gramians, with respect to input matrices. In particular, we consider two types of the maximization in terms of the candidate sets for input matrices: the case for diagonal matrices and general matrices. A key idea of the proposed method is characterization of optimal input matrices by the solution to Lyapunov equations. We also propose data-driven controllability maximization with respect to the network structure and the connection strengths between the nodes of the network system. By considering that the objective function (i.e., the trace of the controllability Gramian) is non-convex with respect to the design variables, the proposed method is constructed by a gradient-based maximization with data-driven computation of the gradient of the objective function based on data-driven Lyapunov equations. The matrix sequence generated by the proposed method is guaranteed to converge to a stationary point. The proposed methods are demonstrated by numerical examples for noise-free/noisy data. In addition, the estimation and maximization of finite-time controllability Gramians are also discussed, where the estimation and maximization are achieved under an assumption of the commutativity of two matrices.

In Section 4, we develop a model-free method for sparse event-triggered control as an application of data-driven Lyapunov equations. Sparse event-triggered control is a control strategy for intermittently-actuated systems, where the control input is intermittent and the time instants at which nonzero input is applied are determined in an event-triggered manner. In the proposed method, the event-triggered conditions, which determine the events, are adaptively constructed from online data on the state trajectory of the dynamical system. A key idea of the adaptive scheme is to estimate a quadratic Lyapunov function of a linear system associated with the closed-loop dynamics, which is derived by considering an online version of the result in Section 2. As the result, the control input is intermittent and the origin of the control system is globally exponentially stable under the assumption associated with the acquired data sequence.

In Section 5, we characterize the data informativity for Lyapunov equations, i.e., a condition for a dataset to contain sufficient information to uniquely determine the solution to a Lyapunov equation. To this end, we first clarify the relationship between solutions to Lyapunov equations and an invariant subspace of a matrix. Then, based on this result, we derive a necessary and sufficient condition for a dataset to be informative for the Lyapunov equations, where the condition is expressed as the possibility of matrix decomposition associated with the dataset, called data-basis decomposition, for a matrix that determines the Lyapunov equation. This result reveals that there exists a situation where the solution to the Lyapunov equation is uniquely determined from the dataset even if the dataset does not contain enough information for identifying the system. Furthermore, we also present a data-driven method for computing the unique solution to the Lyapunov equation under no assumptions except for the data informativity. In the proposed method, the solution to a continuous-time Lyapunov equation is obtained by solving a discrete-time Lyapunov equation constructed from the dataset.

Section 6 concludes this thesis. In particular, we provide a summary of the contributions on our work and future works.