CALCULATION OF WAVEFORMS RADIATING FROM RETURN STROKES

Akira IWATA

Abstract

The velocity of luminosity progress along a vertical lightning channel is assumed for the first, and for the subsequent, return stroke to be expressed with a double exponential formula, whose parameter values are introduced afresh for the use of a digital computor taking the final channel height into account.

The waveforms of atmospherics radiating from return strokes are numerically calculated. It is shown that the waveforms calculated respectively for the first, and for the subsequent, stroke are slightly different with each other at 100 km propagation distance from their origin, and mainly consist of radiation field component at VLF band. Below 1kHz the effect of electro-static, and of induction, field component can not be neglected at this propagation distance.

1. Introduction

It is well known that VLF atmospherics mainly radiate from return strokes involved in a lightning discharge. Thus the original waveform of atmospherics is determined, assuming current waveform, velocity of lightning luminosity progress along a vertical lightning channel (Pierce, 1960; Bhattacharya, 1963; Hill, 1966; Jones, 1969). Up to the present, the velocity of lightning luminosity progress has been taken to be expressed exclusively with a single-exponential formula given by Bruce and Golde (1941), and the lightning current has been assumed everywhere to be uniform along the stroke channel.

But this leads to a paradox that the electric charge runs up the channel with an infinite drift-velocity. To overcome this Dennis and Pierce (1964) gave a charge distribution along the channel by assuming a finite drift-velocity of electric charge, thus calculated the VLF atmospherics radiating from lightning flash. It is the purpose of this paper to determine the empirical formula giving the velocity of lightning luminosity progress and to calculate the electromagnetic waveform radiating from a model return stroke and thus to obtain the frequency spectrum of it.

2. The Return Stroke Current

The empirical formula of current waveform of a return stroke was first given by Bruce and Golde (1941) as follows,

$$I = I_0 \left(e^{-\alpha t} - e^{-\beta t}\right) \quad \cdots\cdots (1)$$

where the parameter values they gave are $I_0 = 3\times10^4$ A, $\alpha = 4.4\times10^4\ \mathrm{sec}^{-1}$, $\beta = 4.6\times10^5\ \mathrm{sec}^{-1}$. Whereas Ishikawa (1961) derived, from his studies on atmospheric waveforms, the values of these parameters as : $I_0 = 3\times10^4$ A, $\alpha = 4\times10^4\ \mathrm{sec}^{-1}$, $\beta = 2\times10^5\ \mathrm{sec}^{-1}$. The formula is thought to be applicable to the current variation being observed at the root of a return stroke.

3. The Tip Velocity of Progressive Return Stroke

The empirical formula for the progressive velocity of return stroke luminosity was first given by Bruce and Golde (1941) as follows,

$$V = V_0\, e^{-\gamma t} \quad \cdots\cdots (2)$$

where $V_0 = 8\times10^7$ m/sec, $\gamma = 3\times10^4\ \mathrm{sec}^{-1}$. Srivastava (1966) gave another type of the formula refering to the time table given by Schonland (1956),

$$V = V_0 \left(e^{-at} - e^{-bt}\right) \quad \cdots\cdots (3)$$

where $V_0 = 3\times10^8$ m/sec, $a = 6\times10^4\ \mathrm{sec}^{-1}$, $b = 7\times10^5\ \mathrm{sec}^{-1}$. Eq. (3) along with these parameter values gives the velocity very large compared with the experimental statistical mean velocity, 5×10^7 m/sec (Hatakeyama and Kawano, 1955). The final

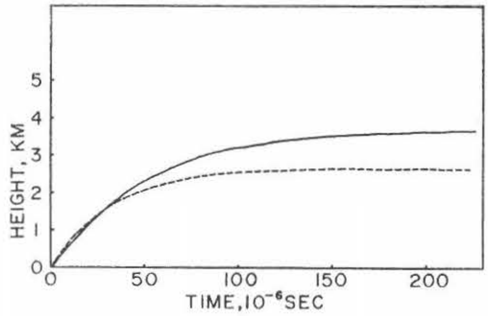

Fig. 1 Stroke altitude versus time curves for the first stroke. Rigid line represents Eq. (3), broken line Eq. (2).

altitude being reached is assumed to be 4.7 km, which is too high compared with the experimental evidences (Hatakeyama and Kawano, 1955; Schonland, 1956). Thus another set of the values are so determined in a digital computer calculation that, they satisfy the following condition. First, the time taken for a return stroke to reach the cloud base (about 2 km high above the ground) is 40 μsec. Second, the final lightning channel altitude is 3.7 km which was given by Schonland (1956). Third, below the cloud base, the variation of progressive velocity with time is similar to that given by Bruce and Golde (1941).

The values obtained for Eq. (3) are $V_0=7.6\times10^7$ m/sec, $a=2\times10^4$ sec^{-1}, $b=8.9\times10^5$ sec^{-1}. Shown in Fig. 1 are the stroke height versus time curves, where the broken line represents Eq. (2) and the rigid line Eq. (3) with the modified values.

For the case of subsequent return stroke, the velocity is assumed to be contant in many papers, that is, $\gamma=0$ in Eq. (2). But this condition implies an infinite altitude of the final lightning channel ($H=V_0(1-\exp(-\gamma t))/\gamma$).

The waveform calculated for the subsequent stroke on the assumption of constant velocity is quite different from that of the first stroke, and it attains a maximum at the time origin $t=0$ (Dennis and Pierce, 1964). This contradicts with the observational understanding now generally accepted, thus we have a need for another assumption being made to overcome it, and to calculate the amplitude frequency spectrum (Hill, 1966). In regard to this our computer trial has shown that Eq. (3) is the better fit to the subsequent stroke. Another set of V_0, a, b, parameter values are taken to Eq. (3) using digital computer, so as the following conditions are being satisfied. First, the time which is necessary for the return stroke to reach the cloud base is 40 μsec. Second, final stroke channel altitude is 5 km. Third, the variation with time of stroke channel altitude roughly coincides with the experimental values which were given by Schonland (1956). The values obtained are, $V_0=7.09\times10^7$ m/sec, $a=1.37\times10^4$ sec^{-1}, $b=4\times10^5$ sec^{-1}.

Shown in Fig. 2 is the time variation of the stroke altitude, where the circles indicate Schonland's (1956) values. The variations of velocity with time are shown in Fig. 3, where curve 1 is for the first stroke, curve 2 for the subsequent stroke.

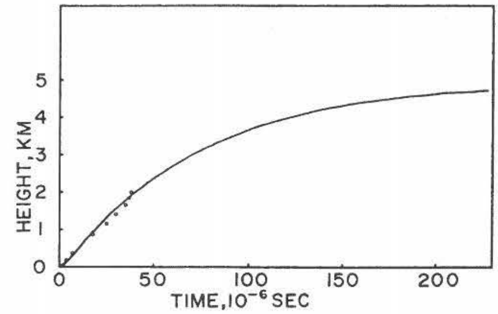

Fig. 2 Stroke altitude versus time curve for the subsequent stroke. The curve represents Eq. (3), where circules indicate the experimental values given by Shonland (1956).

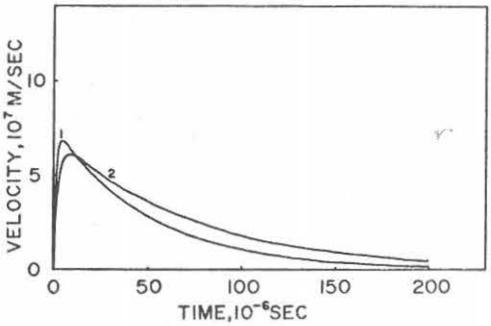

Fig. 3 Curves showing velocity variation with time given by Eq. (3). Curve 1 indicates the first stroke, where $a=2\times10^4 sec^{-1}$, $b=8.9\times10^5 sec^{-1}$, $V_0=7.6\times10^7$ m/sec. Curve 2 indicates subsequent stroke, where $a=1.37\times10^4 sec^{-1}$, $b=4\times10^5 sec^{-1}$, $V_0=7.09\times10^7$ m/sec.

4. Electromagnetic Radiation

It is well known that electric moment M placed on the ground perpendicular to it produces the following vertical electric field E, along the ground surface,

$$E=\frac{1}{4\pi\varepsilon}\left\{\frac{M}{d^3}+\frac{1}{cd^2}\frac{dM}{dt}+\frac{1}{c^2d}\frac{d^2M}{dt^2}\right\} \quad\cdots\cdots(4)$$

where d, c, and ε are distance from M to observation site on the ground, light velocity, and permittivity of the atmosphere respectively.

Eq. (4) require the condition that M is negligibly small in length in comparison to distance d. The first term defines the electro-static field Es, the second the induction field Ei, the last the radiation field Er. Therefore we get next formulas.

$$Es=\frac{1}{4\pi\varepsilon d^3}M \quad\cdots\cdots(5)$$

$$Ei=\frac{1}{4\pi\varepsilon cd^2}\frac{dM}{dt} \quad\cdots\cdots(6)$$

$$Er=\frac{1}{4\pi\varepsilon c^2d}\frac{d^2M}{dt^2} \quad\cdots\cdots(7)$$

Assume that electric charge is uniformly distributed from ground to tip of a return stroke (Hill, 1966), then the moment M is expressed as,

$$M=2\left(\frac{1}{2}\int_0^t V\,dt\right)\left(\int_0^t I\,dt\right) \quad\cdots\cdots(8)$$

where 2 indicates the image effect produced by the earth as a perfect conductor, and 1/2 the effective height coefficient of the charge. Differentiating M we get,

$$\frac{dM}{dt}=V\int_0^t I\ dt+I\int_0^t V\ dt \quad \cdots\cdots (9)$$

$$\frac{d^2M}{dt^2}=\frac{dV}{dt}\int_0^t I\ dt+2VI+\frac{dI}{dt}\int_0^t V\ dt \quad \cdots\cdots (10)$$

From (5), (6), (7), and (8), (9), (10), we get,

$$Es=\frac{1}{4\pi\varepsilon d^3}\left(\int_0^t V\ dt\right)\left(\int_0^t I\ dt\right) \quad \cdots\cdots (11)$$

$$Ei=\frac{1}{4\pi\varepsilon c d^2}\left(V\int_0^t I\ dt+I\int_0^t V\ dt\right) \quad \cdots\cdots (12)$$

$$Er=\frac{1}{4\pi\varepsilon c^2 d}\left(\frac{dV}{dt}\int_0^t I\ dt+2VI+\frac{dI}{dt}\int_0^t V\ dt\right) \quad \cdots\cdots (13)$$

And from (1) and (3), we get,

$$\int_0^t V\ dt=V_0\left\{\frac{1}{a}(1-e^{-at})-\frac{1}{b}(1-e^{-bt})\right\} \quad \cdots\cdots (14)$$

$$\int_0^t I\ dt=I_0\left\{\frac{1}{\alpha}(1-e^{-\alpha t})-\frac{1}{\beta}(1-e^{-\beta t})\right\} \quad \cdots\cdots (15)$$

$$\frac{dV}{dt}=-V_0(a\ e^{-at}-b\ e^{-bt}) \quad \cdots\cdots (16)$$

$$\frac{dI}{dt}=-I_0(\alpha\ e^{-\alpha t}-\beta\ e^{-\beta t}) \quad \cdots\cdots (17)$$

From (11) and (14) (15), from (12) and (1) (3) (14) (15), and from (13) and (1) (3) (14) (15) (16) (17), we get the following formulas for Es, Ei, and Er, respectively.

$$Es=\frac{V_0I_0}{4\pi\varepsilon d^3}\left\{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)+\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{b}e^{-bt}-\frac{1}{a}e^{-at}\right)+\left(\frac{1}{a}-\frac{1}{b}\right)\right.$$

$$\left.\left(\frac{1}{\beta}e^{-\beta t}-\frac{1}{\alpha}e^{-\alpha t}\right)+\frac{1}{a\alpha}e^{-(a+\alpha)t}-\frac{1}{a\beta}e^{-(a+\beta)t}-\frac{1}{b\alpha}e^{-(b+\alpha)t}+\frac{1}{b\beta}e^{-(b+\beta)t}\right\} \quad \cdots\cdots (18)$$

$$\mathrm{Ei}=\frac{V_0I_0}{4\pi\varepsilon c d^2}\left\{\left(\frac{1}{a}-\frac{1}{b}\right)(e^{-\alpha t}-e^{-\beta t})+\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)(e^{-at}-e^{-bt})-\left(\frac{1}{a}+\frac{1}{\alpha}\right)e^{-(a+\alpha)t}\right.$$

$$\left.+\left(\frac{1}{a}+\frac{1}{\beta}\right)e^{-(a+\beta)t}+\left(\frac{1}{b}+\frac{1}{\alpha}\right)e^{-(b+\alpha)t}-\left(\frac{1}{b}+\frac{1}{\beta}\right)e^{-(b+\beta)t}\right\}\cdots\cdots(19)$$

$$\mathrm{Er}=\frac{V_0I_0}{4\pi\varepsilon c^2 d}\left\{\left(\frac{1}{a}-\frac{1}{b}\right)(\beta\, e^{-\beta t}-\alpha\, e^{-\alpha t})+\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)(b\, e^{-bt}-a\, e^{-at})+\frac{(a+\alpha)^2}{a\alpha}e^{-(a+\alpha)t}\right.$$

$$\left.-\frac{(a+\beta)^2}{a\beta}e^{-(a+\beta)t}-\frac{(b+\alpha)^2}{b\alpha}e^{-(b+\alpha)t}+\frac{(b+\beta)^2}{b\beta}e^{-(b+\beta)t}\right\}\cdots\cdots(20)$$

From Eq. (20) we get the radiation field shown in Fig. 4, which corresponds to the case of $d=100$ km, and I_0, α and β values given by Ishikawa (1961). Curve 1 shows the case of first stroke, curve 2 shows the case of subsequent stroke.

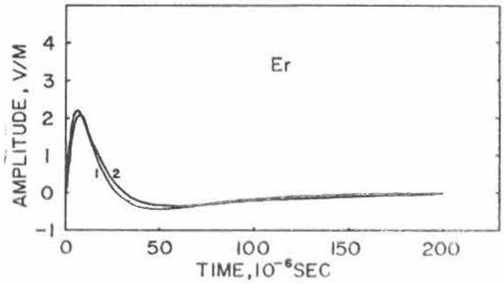

Fig. 4 Waveform of radiation field component of return stroke at a distance 100 km. Curve 1 indicates the first stroke, curve 2 subsequent stroke.

Total electric field E is also obtained from Eq. (4) and (18) (19) (20), as shown in Fig. 5 where curve 1 represents first stroke and curve 2 subsequent stroke. The two curves are found to differ from each other, but the differences they give are likely to be insignificant. The comparison of the two respective curves between Fig. 4 and Fig. 5, clearly shows that the differences are within a few percentages.

The frequency spectrum of the electric field $E(t)$ can be obtained by using the Fourier integral as follows,

$$E(\omega)=\int_{-\infty}^{\infty} E(t)\ e^{-j\omega t}\,dt\cdots\cdots(21)$$

From Eq. (21) we get the amplitude frequency spectrum $|E(\omega)|$ and the phase frequency spectrum $P(\omega)$ as follows,

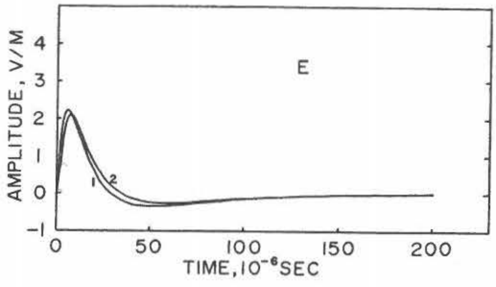

Fig. 5 Total atmospherics waveforms radiating from return strokes at a distance 100 km. Curve 1 is for the first stroke, curve 2 for the subsequent stroke.

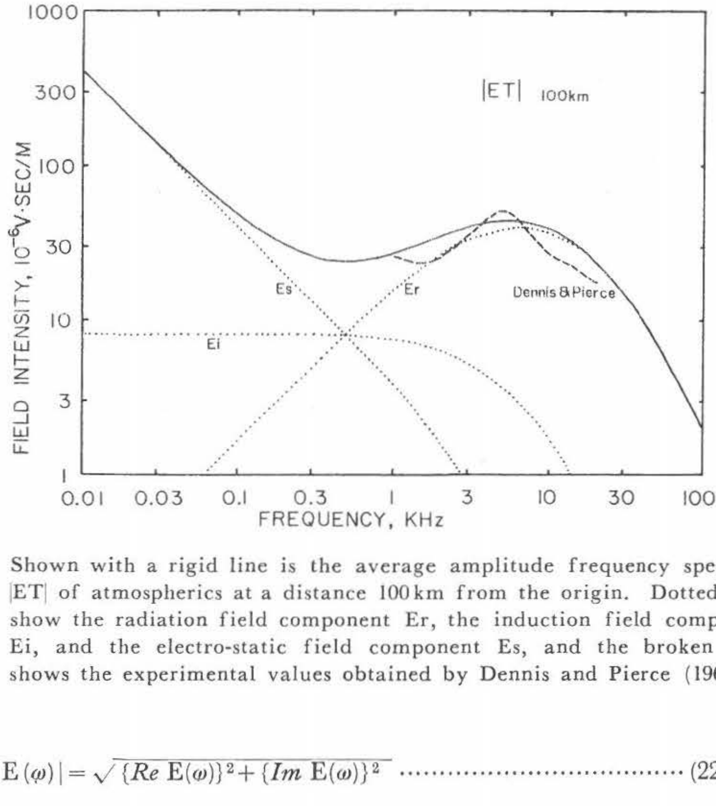

Fig. 6 Shown with a rigid line is the average amplitude frequency spectrum |ET| of atmospherics at a distance 100 km from the origin. Dotted lines show the radiation field component Er, the induction field component Ei, and the electro-static field component Es, and the broken lines shows the experimental values obtained by Dennis and Pierce (1964).

$$|\mathrm{E}(\omega)| = \sqrt{\{Re\ \mathrm{E}(\omega)\}^2 + \{Im\ \mathrm{E}(\omega)\}^2} \quad \cdots\cdots (22)$$

$$\mathrm{P}(\omega) = \tan^{-1}\frac{Im\ \mathrm{E}(\omega)}{Re\ \mathrm{E}(\omega)} \quad \cdots\cdots (23)$$

Hill (1966) considered the number of return strokes per flash to be limited between three and four statistically, accordingly the average frequency spectrum of atmospherics is thought to be a weighted average of a first return stroke, and two subsequent strokes. The average electric amplitude frequency spectrum |ET| is then obtained as the sum of 1/3 times the first stroke spectrum and 2/3 times the subsequent stroke spectrum. The result of this calculation is shown in Fig. 6 with the rigid line. Dotted lines show the radiation componet spectrum Er, the induction component spectrum Ei, and the electro-static component spectrum Es, and the broken line shows the experimental result obtained by Dennis and Pierce (1964). The average phase spectrum PT similarly calculated as the average amplitude frequency spectrum, is shown in Fig. 7 with rigid line. Dotted lines show the radiation component spectrum PrT, the induction component spectrum PiT, and the electro-static component spectrum PsT respectively.

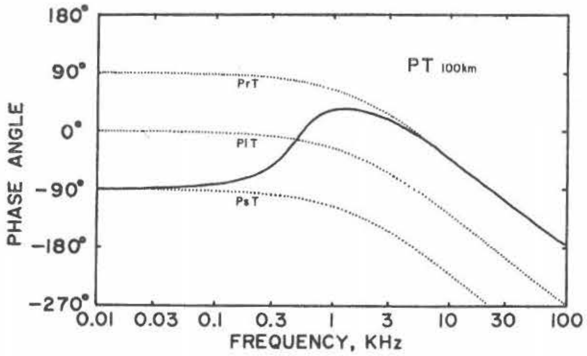

Fig. 7 Shown with a rigid line is the average phase frequency spectrum of atmospherics at a distance 100 km from the origin. Dotted lines show the radiation field component spectrum PrT, the induction field component spectrum PiT, and the electro-static field component spectrum PsT.

5. Conclusion

In VLF frequency range, the wave forms of atmospherics which radiate from return strokes are appreciably affected by the velocity of return stroke. Using the velocity formula of a double exponential type with new parameter values, the waveforms radiating from the first, and the subsequent, strokes are calculated and it is found that they are slightly different with each other at 100 km distance from their origin. Therefore the explanation to be made on the difference at present will be that the atmospherics mainly consist of radiation field component in VLF frequency range at this distance, but in the lower frequency range, we must consider the influence of induction, and of electro-static, field component as well.

Acknowledgement

I am very much indebted to Prof. H. Ishikawa and Dr. M. Takagi for their criticism to this paper.

References

Bhattacharya, H. : Amplitude and Phase Spectrum of Radio-Atmospherics, J. Atmosph. Terr. Phys., **25**, 445-550 (1963)

Bruce, C. E. R. and R. H. Golde : The Lightning Discharge, J. I. E. E., **88** (II), 487-505 (1941)

Dennis, A. S. and E. T. Pierce : The Return Stroke of the Lightning Flash to Earth as a Source of VLF Atmospherics, Radio Sci., **68D**, 777-794 (1964)

Hatakeyama, H. and M. Kawano : Atmospheric Electricity, Iwanami Library 201, 120 (1955) (in Japanese)

Hill, R. D. : Electromagnetic Radiation from the Return Stroke of a Lightning Discharge, J. Geophys. Res., **71**, 1963-1967 (1966)

Ishikawa, H. : Nature of Lightning Discharges as Origins of Atmospherics, Proc. Res. Inst. Atmospherics, Nagoya Univ., **8A**, 148 (1961)

Jones, D. L. : The Spectrum and Structure of Atmospherics, URSI 16th General Assembly, Invited Review Paper, Commission VIII, Session VIII-1 (1969)

Pierce, E. T. : Atmospherics from Lightning Flashes with Multiple Strokes, J. Geophys. Res., **65**, 1867-1871 (1960)

Schonland, B. F. J. : The Lightning Discharge, Encyclopaedia of Physics, **22**, 576-628, Springer-Verlag, Berlin, Göttingen, Heiderberg (1956)

Srivastava, K. M. L. : Return Stroke Velocity of a Lightning Discharge, J. Geophys. Res., **71**, 1283-1286 (1966)