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SHORT NOTES

CALUCULATION OF WAVEFORMS RADIATING FROM RETURN STROKES (2)

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1. Introduction

The waveforms radiating from return strokes have theoretically been studied by many researchers till to the present, i. e., by Bruce and Golde (1941), Bhattacharya (1963), Bhattacharya and Rao (1964), Croom (1964), and Rao and Bhattacharya (1966) for the first return stroke, and by Pierce (1960), Dennis and Pierce (1964), Hill (1966), and Uman (1969) for the subsequent stroke. But the waveforms they derived on the subsequent stroke were merely the extension of the expression for the first stroke and the velocities of the subsequent stroke were assumed to be constant. Therefor the difference of the waveforms for the first stroke from those for the subsequent stroke were inevitably to be very large. It is a well known fact that the statistically frequent number of return strokes per one flash amounts to be from three to four as investigated by Schonland (1956) and Kitagawa et al. (1962). So the subsequent stroke is expected to be two times more frequently recorded than the first stroke. The author adopted in the previous paper the new formula representing the velocity of the subsequent stroke while the waveforms of discharge current were assumed to be the same for the first, as for the subsequent stroke (Iwata, 1970). It is the purpose of this paper to calculate the waveforms radiating from the subsequent stroke, and to compare the waveforms between the first and the subsequent stroke.

2. The Current Waveform

It is generally accepted that the current waveform of return strokes in a lightning discharge to ground can be represented by an experimental formula as follows:

$$I = I_0 \left(e^{-\alpha t} - e^{-\beta t} \right) \quad \cdots\cdots (1)$$

Berger (1967) directly measured the current waveform of return stroke which hit their lightning observation tower built on the top of Monte San Salvatore. Depending on his measurements, Dennis and Pierce (1964) gave the values of the parameters, $I_0=3\times10^4$ amperes, $\alpha=2\times10^4$ sec^{-1} and $\beta=2\times10^5$ sec^{-1} for the first return stroke, and $I_0=1\times10^4$ amperes, $\alpha=1.4\times10^4$ sec^{-1} and $\beta=6\times10^6$ sec^{-1} for the subsequent return stroke, while the author used Ishikawa's (1961) values in the previous paper, where $I_0=3\times10^4$ amperes, $\alpha=4\times10^4$ sec^{-1} and $\beta=2\times10^5$ sec^{-1} (Iwata, 1970).

3. The Velocity of Return Stroke

The velocity of return stroke can approximately be represented by a double-exponential formula as follows:

$$V = V_0 \left(e^{-at} - e^{-bt} \right) \quad \cdots\cdots (2)$$

This type of the formula was first given by Srivastava (1966) and the values of parameters given by him were $V_0=3\times10^8$ m/sec, $a=6\times10^4$ sec^{-1} and $b=7\times10^5$ sec^{-1}. Considering the statistical facts on the experiment in regard to return strokes, the author used new values of the parameters, $V_0=7.6\times10^7$ m/sec, $a=2\times10^4$ sec^{-1} and $b=8.9\times10^5$ sec^{-1} for the first return stroke, and $V_0=7.09\times10^7$ m/sec, $a=1.37\times10^4$ sec^{-1} and $b=4\times10^5$ sec^{-1} for the subsequent stroke (Iwata, 1970).

4. The Waveform of Return Stroke

The electric field E produced by the time variation of a vertical dipole on the ground with moment M at a distance d from the observer can be given by the following expression,

$$E = \frac{1}{4\pi\varepsilon} \left(\frac{M}{d^3} + \frac{1}{cd^2} \frac{dM}{dt} + \frac{1}{c^2 d} \frac{d^2 M}{dt^2} \right) \quad \cdots\cdots (3)$$

where c and ε are the light velocity and the permittivity of the atmosphere respectively. The three terms in Eq. (3) are respectively the electro-static field Es, the magnetic field Em and the radiation field Er. Assuming that the electric charges are uniformly distributed from the base to the top of a stroke channel, the electric moment M can be expressed as follows:

$$M = 2\left(\frac{1}{2}\int_0^t V dt\right)\left(\int_0^t I\, dt\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (4)$$

Now it is very easy to compute the electric field E from Eq. (1), (2), (3), and (4), (Iwata, 1970).

Fig. 1 shows the electric field E and the radiation field Er radiating from a typical first return stroke, at a distance of 30 km from the observer. The values of the parameters adopted here are $I_0=3\times10^4$ amperes, $\alpha=2\times10^4$ sec^{-1}, $\beta=2\times10^5$ sec^{-1}, $V_0=7.6\times10^7$ m/sec, $a=2\times10^4$ sec^{-1}, and $b=8.9\times10^5$ sec^{-1}.

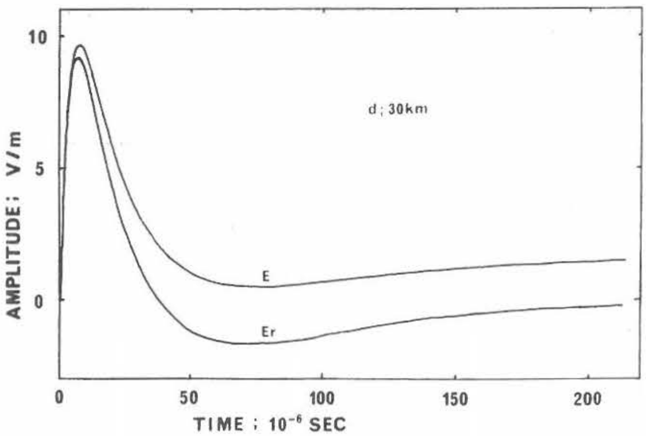

Fig. 1. The electric field E and the radiation field Er radiating from the first return stroke at 30 km distance.

Fig. 2 also shows the electric field E and the radiation field Er radiated from a typical subsequent return stroke at the same distance. The values of parameters used here are $I_0=1\times10^4$ amperes, $\alpha=1.4\times10^4$ sec^{-1}, $\beta=6\times10^6$ sec^{-1}, $V_0=7.09\times10^7$ m/sec, $a=1.37\times10^4$ sec^{-1}, and $b=4\times10^5$ sec^{-1}.

It is found that, at 30 km distance, the electric fields radiating from return strokes are affected by about 20 percent of their peak amplitudes by the combined effect of the electro-static and magnetic fields at the time moment 20 micro-seconds after the first appearance of the waveforms, and do not cross zero volt field lines. The peak amplitude of the electric field radiating from the first stroke is found to be about two times larger than that from the subsequent stroke. The positive peak amplitude of the electric field is found to be reached at about 8 micro-seconds (for the first stroke) and 5 micro-seconds (for the subsequent stroke) after the waveform initiation.

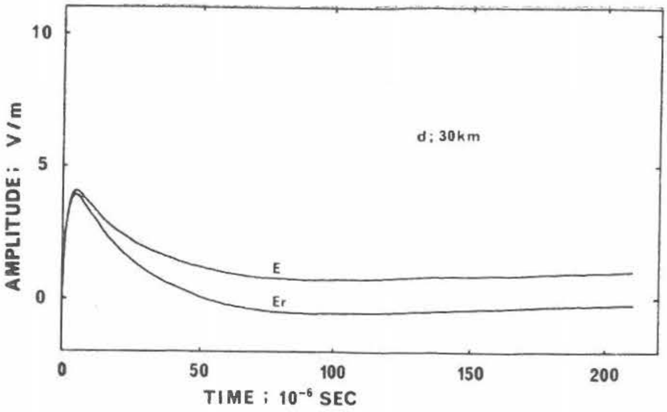

Fig. 2. The electric field E and the radiation Er radiating from the subsequent return stroke at 30 km distance.

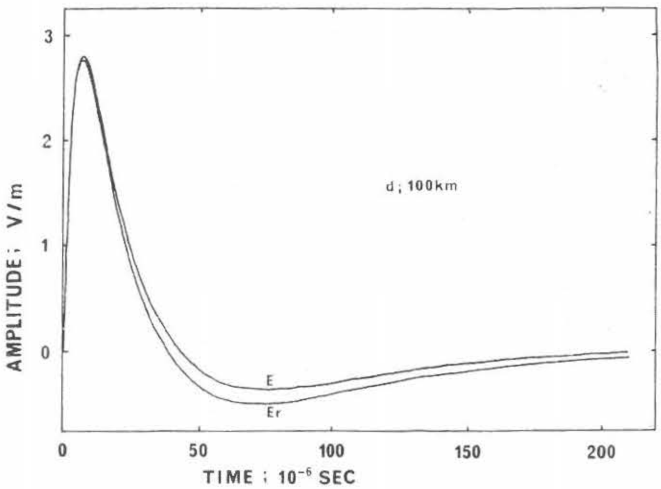

Fig. 3. The electric field E and the radiation field Er radiating from the first return stroke at 100 km distance.

The negative peak amplitude of the electric field is found to be reached at about 77 micro-seconds and 92 micro-seconds for the first and subsequent return stroke, respectively. The pulse widths where the electric fields cross over half of the peak amplitude values are 21 micro-seconds and 26 micro-seconds for the first and the subsequent stroke, respectively.

Fig. 3 shows the electric field E and the radiation field Er, radiating from a typical first return stroke at a distance of 100 km from the observer. The values of

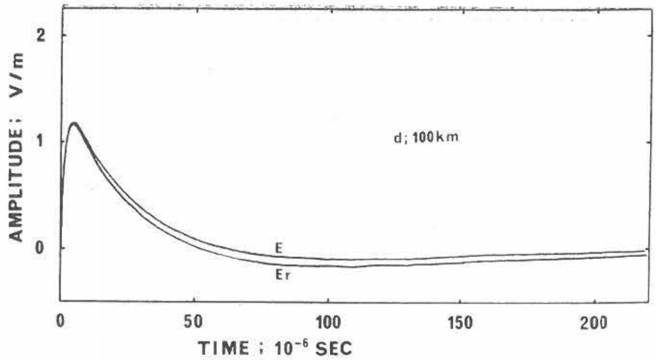

Fig. 4. The electric field E and the radiation field Er radiating from the subsequent return stroke at 100 km distance.

the parameters used here are the same as those for the case in Fig. 1.

Fig. 4 also shows the atmospherics waveforms from a typical subsequent stroke. The values of parameters used here are the same as those for the case in Fig. 2. At 100 km distance, the electric fields E are affected by several percent of their peak amplitudes by the combined effect of the electro-static and magnetic fields. If the height of the ionosphere is assumed to be 70 km, the time difference between the ground wave and the first sky wave is estimated to be about 240 micro-seconds at a distance of 100 km. Therefor the effect of the ionosphere can be neglected within the time length of 240 micro-seconds from the initiation of the waveform. The time moments on the waveforms where the electric fields reach the maximum are found to be 7.5 micro-seconds and 5 micro-seconds for the first and for the subsequent return stroke, respectively. The pulse widths with which the electric fields exceed half of their peak amplitude values are 19 micro-seconds and 21 micro-seconds for the first and for the subsequent return stroke, respectively. The time moments on the waveforms where the electric fields cross the zero volt field lines are found to be 42 micro-seconds and 62 micro-seconds for the first and subsequent return stroke, respectively, while the experimental data are reported to be from 30 to 80 micro-seconds (Taylor, 1963).

5. The Frequency Spectrum

The Fourier spectrum of the vertical electric field E(t) is given by the integral as follows:

$$E(\omega) = \int_0^{\infty} E(t)\, e^{-j\omega t}\, dt \quad \cdots\cdots (5)$$

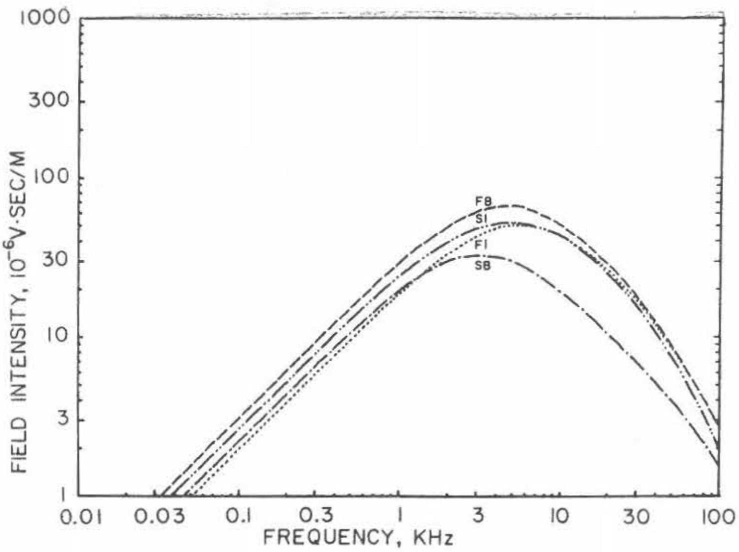

Fig. 5. The amplitude frequency spectra of the radiation field originating from the return strokes at 100 km distance. The curves FB, and FI respectively correspond to the first stroke where Berger's values of the parameters in the current waveform were adopted, and to the first stroke where we used Ishikawa's values; the curves SB and SI respectively correspond to the subsequent stroke where we used Berger's values, and to the subsequent stroke where we used Ishikawa's values.

where ω is the angular frequency $(2\pi f)$.

Now Eq. (5) can be written as :

$$E(\omega) = |E(\omega)|\, e^{-j\phi(\omega)} \quad \cdots\cdots\cdots\cdots \quad (6)$$

where $|E(\omega)|$ is the amplitude spectrum and $\phi(\omega)$ is the phase spectrum, as a function of frequency $(2\pi f)$.

Fig. 5 shows the amplitude frequency spectra of the radiation fields to be recorded at a distance of 100 km from the origin, and gives the cases of the first and of the subsequent return stroke, respectively. The parameter values in the return stroke velocity used here are taken from Iwata (1970). FB and SB curves indicate the amplitude spectra for the first and for the subsequent return stroke, respectively and the parameter values in the current waveforms are taken from Berger (1967). FI and SI curves indicate another amplitude spectra for the first and for the subsequent return stroke, respectively, and the parameter values in current waveforms are taken from Ishikawa (1961). The difference between the spectrum for the first, and for the subsequent, strokes which employed the Berger's values is found to be larger than that where we adopted Ishikawa's values. It is shown that the SB curve is quite different from the other three curves in the VLF frequency range and its peak frequency is found at about 3KHz. This is caused from the fact that the parameter β for the case of SB is much larger than for the others. The FB curve has the greatest intensity at any frequency range. On the other hand the SB curve has the smallest intensity in the frequency range larger than 1 KHz. This is due to the parameter I_0 being taken to be smaller than the others. As the number of strokes involved in a ground discharge is observed to be about three or four on average, the amplitude frequency spectrum $|ET|$ to be obtained from distant atmospherics may be represented by the following formula as given by Hill (1966),

$$|ET| = |EF|^{1/3} + |ES|^{2/3} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (7)$$

where $|EF|$ and $|ES|$ are the amplitude frequency spectrum of atmospherics originating from the first and the subsequent return stroke, respectively. Fig. 6 shows the average amplitude frequency spectra to be observed at a distance of 100 Km, which will be produced by a ground discharge involving one first stroke and two subsequent strokes. The two curves respectively correspond to Berger's values of the parameters in current waveform, and to Ishikawa's values. It is shown that Berger's curve shifts a little to the low frequency side compared with Ishikawa's.

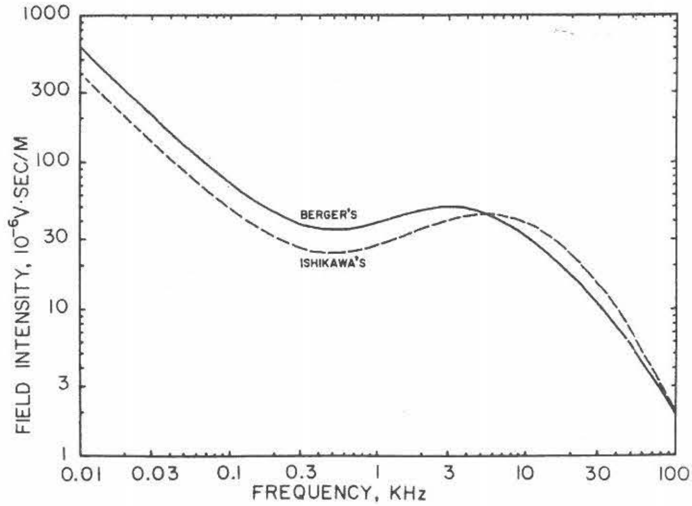

Fig. 6. The mean amplitude frequency spectrum of a model atmospherics at 100 km distance from the origin lightning discharge. The parameter values used in the current waveforms are taken from Ishikawa and from Berger, respectively.

6. Conclusions

(1) At 30 km distance of the lightning discharge, we are to measure the electric field E which is considerably affected by the electro-static field, as well as by the magnetic field. We can also distinguish between the electric fields radiating from the first return stroke and those from the subsequent stroke by the time lengths where the electric fields reach their positive and negative peaks.

(2) At a distance of 100 km from the lightning discharge, we can separate the ground wave from the sky wave, then avoid the effect of the ionosphere in the atmospherics waveforms in these distance ranges. It is found that the electro-static, and the magnetic, fields can affect only a few percent of the atmospherics waveforms, too. On the other hand, the effect of the electro-static and magnetic fields on the amplitude frequency spectrum is found to be not negligible below 1 KHz range.

(3) Though the pulse rise time of the subsequent stroke is shorter than that of the first stroke, the pulse width originating from the subsequent stroke is wider than that from the first stroke.

(4) The amplitude frequency spectrum of the atmospherics originating from a typical subsequent stroke is about half of that from the first stroke in the VLF frequency range.

(5) The spectral peak frequency of the subsequent stroke is lower than that of the first stroke. Thus the average amplitude frequency spectrum of the atmospherics shifts a little to the lower frequency compared to the previous result (Iwata 1970).

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