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報 告 番 号	※ 第 号

主 論 文 の 要 旨

論 文 題 目　Wiener amalgam spaces and nonlinear evolution equations　（ウィナーアマルガム空間と非線形発展方程式）

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論 文 内 容 の 要 旨

1 Introduction

In the past few years, the appeal of modulation spaces and Wiener amalgam spaces has risen significantly for researchers in the field of PDE. This is due to the capability of these spaces to treat both local and global beheaviour of functions separately. Unimodular Fourier multiplier operators $e^{i|D|^\alpha}$, which are generally unbounded on L^p and Besov spaces are bounded on modulation spaces and Wiener amalgam spaces with zero to a small loss of regularity [1, 3, 5]. Research papers dedicated to establish basic properties of these spaces are constantly being published. In this thesis, we give new results on boundedness of unimodular Fourier multipliers on Wiener amalgam spaces. We also determined optimal inclusion relations between L^p-Sobolev and Wiener amalgam spaces, which enables us to describe the mapping properties of unimodular Fourier operators $e^{i|D|^\alpha}$ between L^p-Sobolev and Wiener amalgam spaces. Moreover, some Littlewood-Paley type inequalities were derived from the inclusion.

2 Main Results

Theorem 2.1. *Let $\alpha \geq 2$ and μ be a real-valued homogeneous function on $\mathbb{R}^n$ of degree α which belongs to $C^\infty(\mathbb{R}^n \backslash \{0\})$. Let $1 \leq p, q \leq \infty$ and $s \in \mathbb{R}$. Then Fourier multiplier operator $e^{i\mu(D)}$ is bounded from $W_s^{p,q}(\mathbb{R}^n)$ to $W^{p,q}(\mathbb{R}^n)$ whenever*

$$s > n(\alpha - 2)|1/p - 1/2| + n|1/p - 1/q|.$$

In the following theorem we prove optimality of the threshold in Theorem 2.1 for certain values of p and q.

Theorem 2.2. *Let $\alpha \geq 2$ and μ be a real-valued homogeneous function on $\mathbb{R}^n$ of degree α which belongs to $C^\infty(\mathbb{R}^n \backslash \{0\})$. Suppose there exist a point $\xi_0 \neq 0$ at which the Hessian determinant of μ is not zero. Let $max(1/q, 1/2) \leq 1/p, min(1/q, 1/2) \geq 1/p$ and $s \in \mathbb{R}$ and suppose the Fourier multiplier operator $e^{i\mu(D)}$ is bounded from $W_s^{p,q}(\mathbb{R}^n)$ to $W^{p,q}(\mathbb{R}^n)$. Then*

$$s \geq n(\alpha - 2)|1/p - 1/2| + n|1/p - 1/q|.$$

We first need to define the following indices in order to state our results on inclusion relations between L^p-Sobolev and Wiener amalgam spaces. For $(1/p, 1/q) \in [0,1] \times [0,1]$ we define the indices $\tau_1(p,q)$ and $\tau_2(p,q)$ as follows:

$$\tau_1(p,q) = \begin{cases} 0 & \text{if } (1/p,1/q) \in I_1^* : \min(1/p', 1/2) \geq 1/q \\ 1/p + 1/q - 1 & \text{if } (1/p,1/q) \in I_2^* : \min(1/q, 1/2) \geq 1/p' \\ 1/q - 1/2 & \text{if } (1/p,1/q) \in I_3^* : \min(1/p', 1/q) \geq 1/2 \end{cases}$$

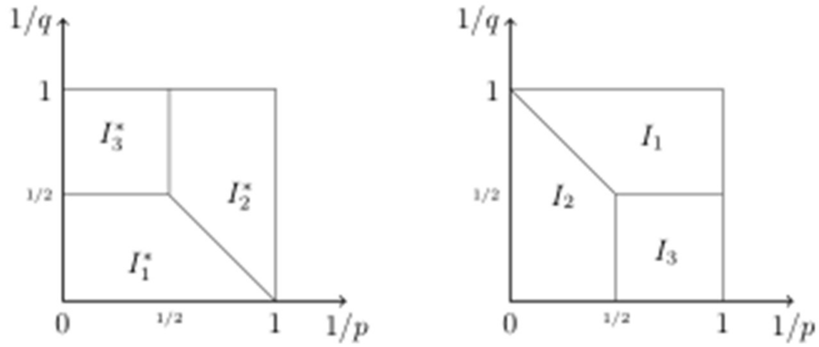

Figure 1: The index sets

$$\tau_2(p,q) = \begin{cases} 0 & \text{if } (1/p,1/q) \in I_1 : \max(1/p', 1/2) \leq 1/q \\ 1/p + 1/q - 1 & \text{if } (1/p,1/q) \in I_2 : \max(1/q, 1/2) \leq 1/p' \\ 1/q - 1/2 & \text{if } (1/p,1/q) \in I_3 : \max(1/p', 1/q) \leq 1/2 \end{cases}$$

where $1/p + 1/p' = 1 = 1/q + 1/q'$. See Figure 1 for a visualization.

Our main results are the following theorems.

Theorem 2.3. *Let $1 \leq p, q \leq \infty$ and $s \in \mathbb{R}$. Then $L^p_s \hookrightarrow W^{p,q}$ if one of the following conditions is satisfied.*

1. $p > q, q < 2$ *and* $s > n\tau_1(p,q)$;
2. $p \neq 1$, *max*$(1/p, 1/2) \geq 1/q$ *and* $s \geq n\tau_1(p,q)$;
3. $p = 1, q = \infty$ *and* $s \geq n\tau_1(1,\infty)$;
4. $p = 1, q \neq \infty$ *and* $s > n\tau_1(1,q)$;

Conversely, if $L^p_s \hookrightarrow W^{p,q}$, then $s \geq n\tau_1(p,q)$.

Theorem 2.4. *Let $1 \leq p, q \leq \infty$ and $s \in \mathbb{R}$. Then $W^{p,q} \hookrightarrow L^p_s$ if one of the following conditions is satisfied.*

1. $p < q, q > 2$ *and* $s < n\tau_2(p,q)$;
2. $p \neq \infty$, *min*$(1/p, 1/2) \leq 1/q$ *and* $s \leq n\tau_2(p,q)$;
3. $p = \infty, q = 1$ *and* $s \leq n\tau_2(\infty, 1)$;

4. $p = \infty, q \neq 1$ *and* $s < n\tau_2(\infty, q)$;

Conversely, if $W^{p,q} \hookrightarrow L^p_s$, *then* $s \leq n\tau_2(p,q)$.

Corolarry 2.5. *Let* $1 \leq p \leq \infty, 1/p + 1/p' = 1$ *and* $s \in \mathbb{R}^n$. *Then*

$$M^{min(p',2),p} \hookrightarrow \mathcal{F}L^p \hookrightarrow M^{max(p',2),p}.$$

This corollary gives us an understanding of the inclusion relations between modulation spaces and Fourier Lebesgue spaces $\mathcal{F}L^p$ which is an improvement of [7, Prop. 1.7]

As an application of the results, we discuss Littlewood-Paley type estimates derived from our Sobolev-Wiener amalgam inclusions. Let $q = 2$ and $p \geq 2$ in Theorem 2.3, we arrived to a Littlewood-Paley type inequality analogous to the work of Rubio de Francia. We write

$$||(\sum_{k \in \mathbb{Z}^n} |\varphi(D-k)f|^2)^{1/2}||_{L^p} \leq C_p ||f||_{L^p}.$$

Lastly, in the final chapter of the thesis, we survey some recent progress on Wiener amalgam spaces and modulation spaces and their connection to nonlinear evolution equations, i.e., well-posedness results, Strichartz estimates, smoothing estimates, etc.

References

[1] Á. Bényi, K. Gröchenig, K. Okoudjou and L.G. Rogers, Unimodular Fourier multipliers for modulation spaces. Journal of Functional Analysis , **246** (2007), 366–384.

[2] J. Cunanan, M. Kobayashi and M. Sugimoto, Inclusion relations between L^p-Sobolev and Wiener amalgam spaces. J. Funct. Anal., **268** (1) (2015), 239–254.

[3] J. Cunanan and M. Sugimoto, Unimodular Fourier multipliers on Wiener amalgam spaces. Journal of Mathematical Applications and Analysis, **419** (2) (2014), 738–747.

[4] M. Kobayashi and M. Sugimoto, The inclusion relation between Sobolev and modulation spaces. Journal of Functional Analysis, **260** (11) (2011), 3189–3208.

[5] A. Miyachi, F. Nicola, S. Rivetti, A. Tabacco and N. Tomita, Estimates for unimodular Fourier multipliers on modulation spaces. Proceedings of the American Mathematical Society, **137** (11) (2009), 3869–3883.

[6] M. Sugimoto and N. Tomita, The dilation property of modulation spaces and their inclusion relation with Besov spaces. Journal of Functional Analysis, **248** (2007), 79–106.

[7] J.Toft, Continuity properties for modulation spaces, with applications to pseudo-differential calculus-I. Journal of Functional Analysis, **207** (2) (2004), 399-429.