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報告番号	※ 第 号

主　論　文　の　要　旨

論文題目 On complements of complete Kähler domains
（完備ケーラー領域の補集合について）
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論　文　内　容　の　要　旨

Using complete Kähler metrics to describe Stein manifolds was first considered by H. Grauert. It is known that every Stein manifold carries a complete Kähler metric. A natural question is whether or not the converse holds, i.e., if M is non-compact and complete Kähler, is M necessarily Stein?

It was observed by Grauert that the answer is negative. Instead, for any closed analytic subvariety A of a Stein manifold M, he constructed a complete Kähler metric on $M \setminus A$. In other words, in order to guarantee holomorphic convexity, besides existence of complete Kähler metrics, some additional assumptions are necessary , which generally divide into two kinds of approaches: curvature assumptions on the complete Kähler metrics; boundary regularity assumptions on the domains under consideration.

Many notions in the classical potential theory can be generalized to several complex variables with subharmonic functions replaced by plurisubharmonic functions, e.g., pluripolar set, negligible sets, thin sets, etc. They are the important objects of study in the so-called pluripotential theory and play great roles in removable singularities and extension problems of analytic objects, etc. It is not difficult to see the following propositions: (A) analytic varieties are complete pluripolar and (B) Outside closed complete pluripolar sets one can construct complete Kähler metrics. In other words, closed complete pluripolar sets are contained in the complements of complete Kähler domains.

On complex-analyticity of real submanifolds as complements of complete Kähler domains, Ohsawa showed that for the two real codimensional case, merely C^1 regularity is sufficient. However, K. Diederich and J. E. Fornaess later considered the higher codimensional case and showed that C^ω-regularity is necessary and gave counterexamples on open manifolds. As our first new result, we generalize Diederich - Fornaess' examples to the compact case. More precisely, for any $k \in \mathbb{N}, k \geq 3$, we construct a compact C^∞ submanifold A of real codimension k in $\mathbb{P}^n$, such that A is not complex-analytic and $\mathbb{P}^n \setminus A$ admits a complete Kähler metric.

As in the classical potential theory, some results on the equivalence between locally and globally pluripolar or complete pluripolar sets were gotten by B. Josefson, E. Bedford and B. A. Taylor, M. Colţoiu. Inspired by the fact (B), we consider the problem: is it possible to patch up the potentials of complete Kähler metrics to obtain a global one? In other words, if a set is locally the complement of complete Kähler domains, is it globally the complement of some complete Kähler domain?

We answer this problem in the affirmative and prove the following:

Main Theorem. *Assume M is a Stein manifold and $A \subset M$ is a closed subset. If $M \setminus A$ locally admits complete Kähler metrics in the following sense:*

- *$\{U_i\}_{i\in\mathbb{N}}$ is a locally finite open covering of M with $U_i \Subset M, i \in \mathbb{N}$;*
- *on each U_i, there exists $\varphi_i \in \mathrm{PSH}(U_i) \cap C^\infty(U_i \setminus A)$ such that $\partial\bar{\partial}\varphi_i$ gives a complete Kähler metric on $U_i \setminus A$ along $A \cap U_i$,*

then there exists a complete Kähler metric on $M \setminus A$ induced by a globally defined plurisubharmonic function on M. Moreover, this potential can be chosen to be bounded from below and smooth outside A. In particular, if every local potential is continuous, the global potential is also continuous.

The idea of the proof is as follows: we divide the problem into two cases depending on whether the potentials are bounded. On one hand, if all potentials are bounded, we use cut-off functions to extend their domains of definition to the whole of M. However, some negativities may be brought in this process. In order to remove them, we need to compose the strictly plurisubharmonic exhaustion function of M with a suitably chosen increasing convex function. By adding this term to the sum of extended potentials, we can find a global potential. On the other hand, if on some open subset there exists a potential unbounded from below, we modify it to be bounded and reduce this case to the first one.