Lecture 2

Electric Fields

(Continuous charge distribution)

Chapter 22
15 April 2013

FOONG See Kit

Concept of Field

- Coulomb force is an "action at a distance" -- a non-contact force. So, how is it transmitted?
- **Faraday's Field**: *An electric field ("influence") extends outward from every charge and permeates all of space.*
- Introduced by Faraday (1791 -- 1867) around beginning of 19th century.

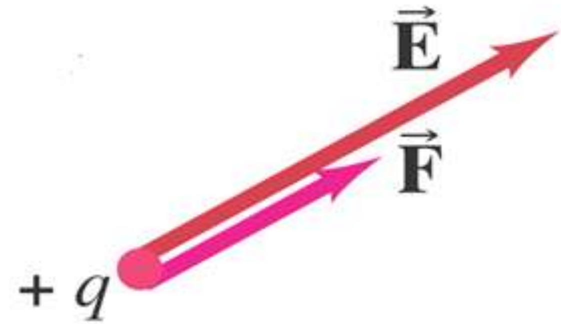
Note: Benjamin Franklin (1706-1790),
Coulomb (1736-1806), Coulomb's law (1785)

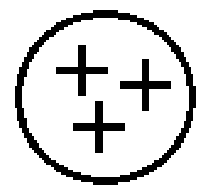
Concept of Field

- If a charge is immersed in the electric field, an interaction between charge and field results in the charge experiencing a force. *Electric field is the means by which one charge exerts a force on another.*
- *Modern understanding*: Photons are the force carriers. (The theory is called quantum electrodynamics (QED). Developed by physicists P. Dirac, R. Feynman, J. Schwinger, and S. Tomonaga from 1928 to 1940s.)

The Electric Field

The electric field is measured through the electric force on a small test charge *q*, divided by its charge, thus electric field equals electric force on the charge per unit charge:





$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

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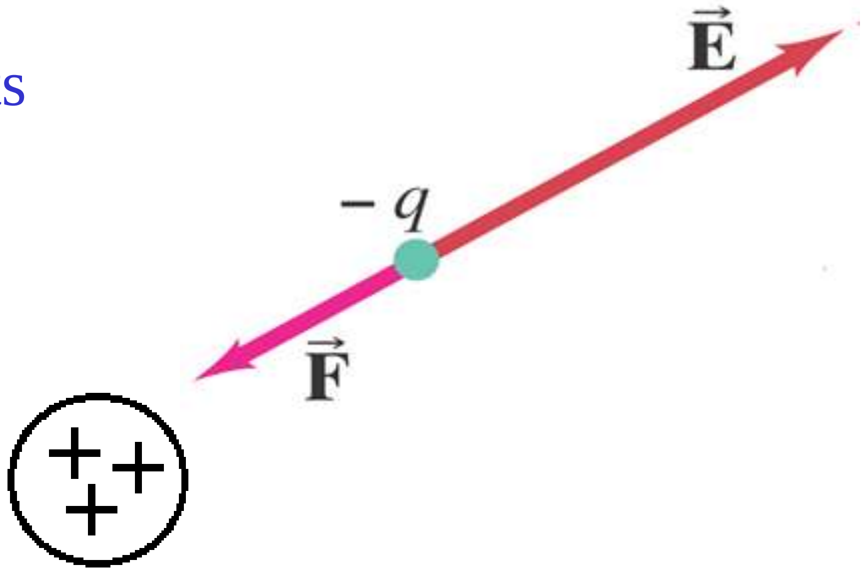
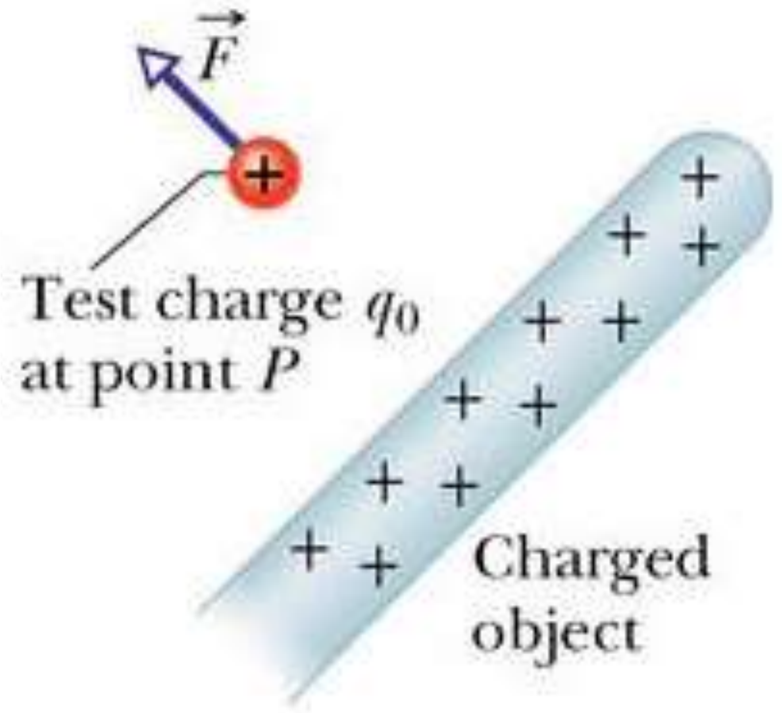


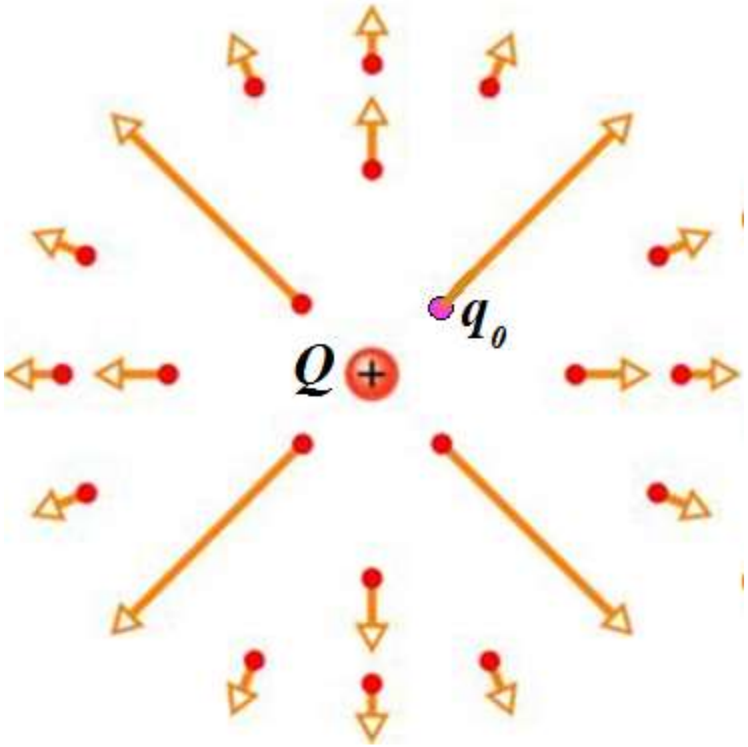
Test Charge

The test charge q_0 used must be small so that it will not change appreciably the distribution of charge on charged object.



Electric Field of a point charge ***Q***

$$E = \frac{F}{q_0}$$



Electric force experienced

by test charge q_0 :

$$F = \frac{kQq_0}{r^2}$$

$$\therefore E = \frac{kQq_0 / r^2}{q_0} = \frac{kQ}{r^2}$$

22.2 The Electric Field:

Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

Calculation of Force on Charge

- If ***E*** at a location is given, then the force experienced by *q* at that location is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

- (Assumption: *Either* the value of ***E*** around *q* is not affected by the size of *q*, *or* it is the value the field settled down to *after q* is introduced)

Superposition principle for electric fields

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots$$

If we place a positive test charge q_0 near n point charges q_1, q_2, . . . , q_n, then, the net force, $\mathbf{F_o}$, from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

The net electric field at the position of the test charge is

$$\begin{aligned}\vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.\end{aligned}$$

Example, The net electric field due to three charges:

Figure 22-7a shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

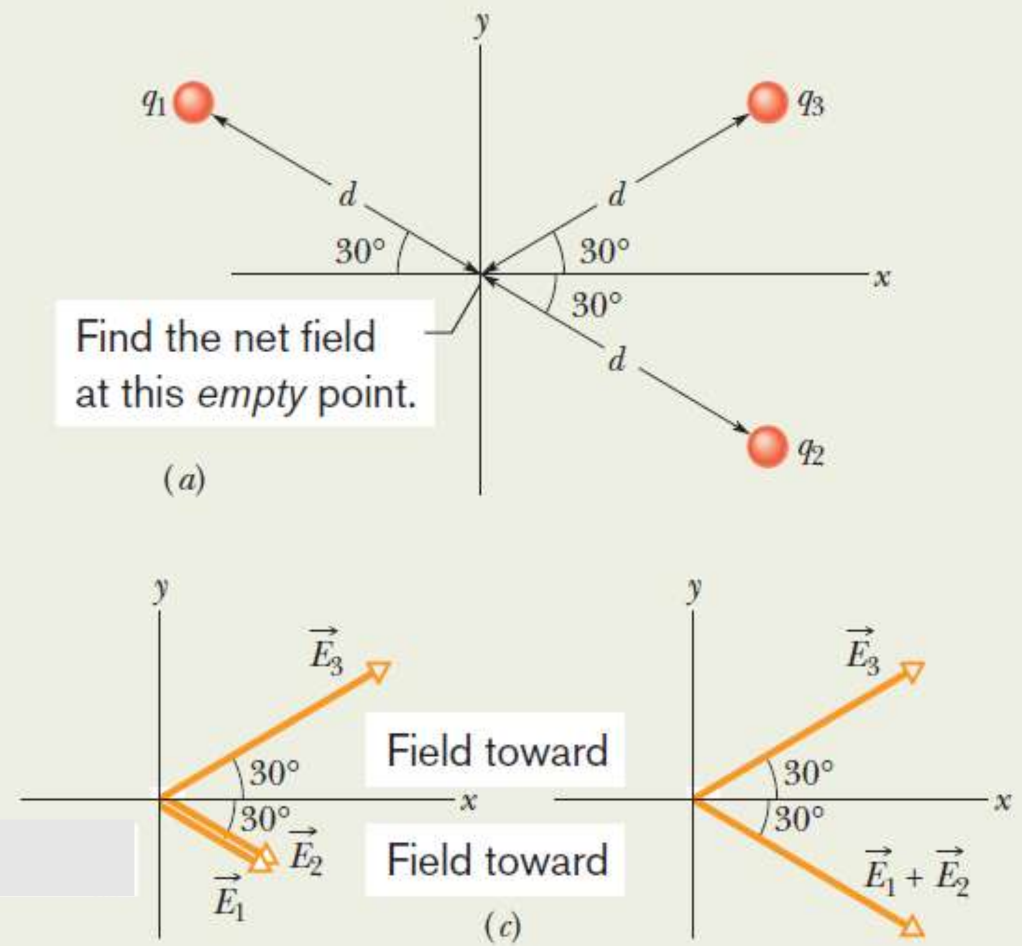


Fig. 22-7 (a) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (b) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (c) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

From the symmetry of Fig. 22-7c, note that the equal y components of our two vectors cancel and the equal x components add.

Thus, the net electric field at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30° = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

22.5 The Electric Field due to an Electric Dipole:

Magnitudes of electric field of +q and –q at P are:

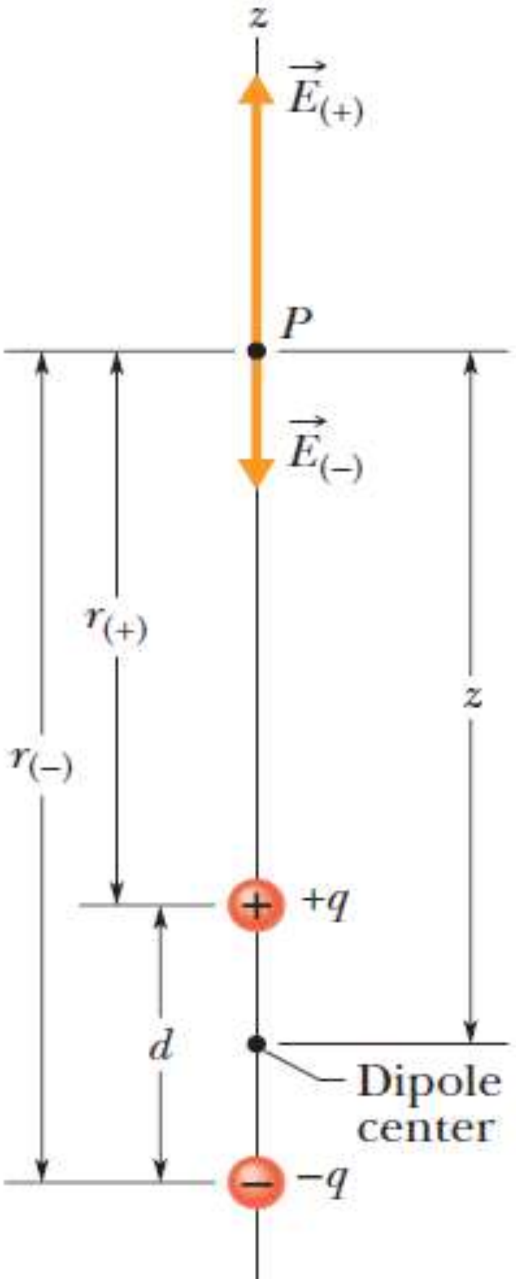
$$E_+ = k\frac{q}{r_+^2} \qquad E_- = k\frac{q}{r_-^2}$$

$$E = E_+ - E_-$$

$$r_+ = z - \frac{d}{2} = z\left(1 - \frac{d}{2z}\right) = z(1-\alpha),$$

$$r_- = z + \frac{d}{2} = z(1+\alpha)$$

$$E = E_+ - E_- = kq\left(\frac{1}{r_+^2} - \frac{1}{r_-^2}\right)$$



22.5 The Electric Field due to an Electric Dipole:

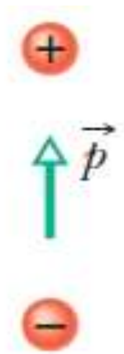
$$E = E_+ - E_- = kq\left(\frac{1}{r_+^2} - \frac{1}{r_-^2}\right), \qquad r_+ = z(1-\alpha), \qquad r_- = z(1+\alpha)$$

$$\begin{aligned}
E &= kq\left(\frac{1}{z^2(1-\alpha)^2} - \frac{1}{z^2(1+\alpha)^2}\right) = \frac{kq}{z^2}\left(\frac{1}{(1-\alpha)^2} - \frac{1}{(1+\alpha)^2}\right) \\
&= \frac{kq}{z^2}\left(\frac{(1+\alpha)^2 - (1-\alpha)^2}{(1-\alpha)^2(1+\alpha)^2}\right) = \frac{kq}{z^2}\left(\frac{(1+2\alpha+\alpha^2) - (1-2\alpha+\alpha^2)}{[(1-\alpha)(1+\alpha)]^2}\right) \\
&= \frac{kq}{z^2}\left(\frac{4\alpha}{[(1-\alpha^2)]^2}\right) \approx 4\frac{kq}{z^2}\alpha = 4\frac{kq}{z^2}\left(\frac{d}{2z}\right) = 2\frac{kqd}{z^3} = 2k\frac{p}{z^3}
\end{aligned}$$

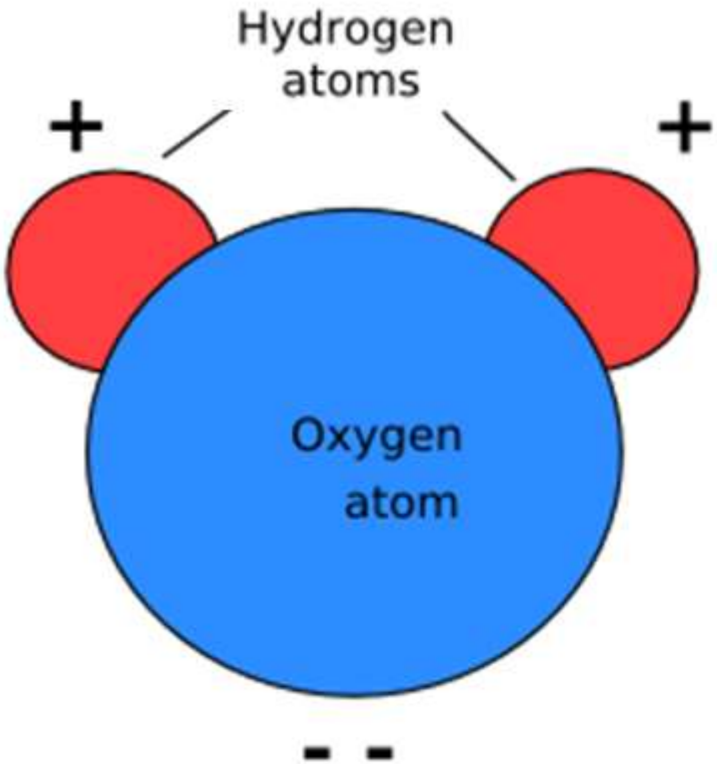
<u>Electric dipole moment vector</u>

Direction: points from the negative charge to the positive charge.

Magnitude: $p=qd$



An example of electric dipole:



22.6 The Electric Field due to a Line of Charge:

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a *charge density rather than as a total charge.* For a line of charge, for example, we would report the *linear charge density* (or charge per unit length) λ, whose SI unit is the coulomb per meter.

Table 22-2 shows the other charge densities we shall be using.

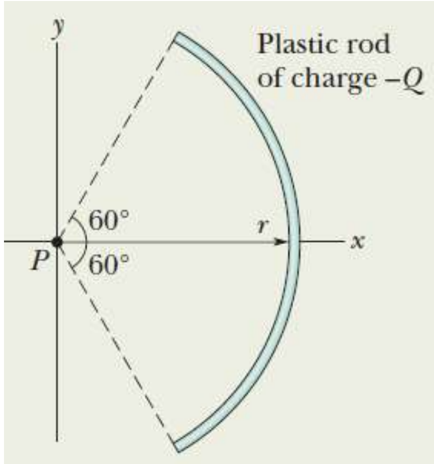
Table 22-2

Some Measures of Electric Charge

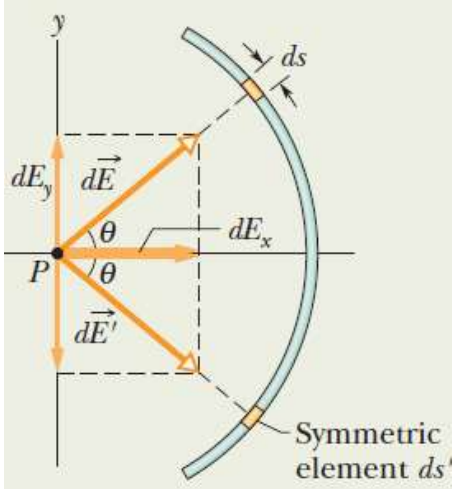
Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Example, Electric Field of a Charged Circular Rod

Figure 22-11*a* shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?



- Consider the element *ds* . Let *dq* be its charge. It has a symmetrically located (mirror image) element *ds'* .
- Let electric field vectors of *ds*, *ds'* be *dE*, *dE'*. $(dE = kdq/r^2)$
- Resolve *dE*, *dE'* into x and y components. Their y components cancel (equal magnitudes, opposite directions). Their x components add (equal magnitudes, same direction).



Line charge density λ (magnitude)

$$dq = \lambda\, ds.$$

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

$$dE = k\frac{dq}{r^2} = k\frac{\lambda ds}{r^2} \qquad dE_x = k\frac{\lambda ds}{r^2}\cos\theta = \frac{k\lambda}{r^2}\cos\theta\, ds$$

$$E = E_x = \frac{k\lambda}{r^2}\int \cos\theta\, ds = \frac{k\lambda}{r^2}\int \cos\theta\,(r d\theta) = \frac{k\lambda}{r}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos\theta\, d\theta$$

22.6 The Electric Field due to a Line Charge:

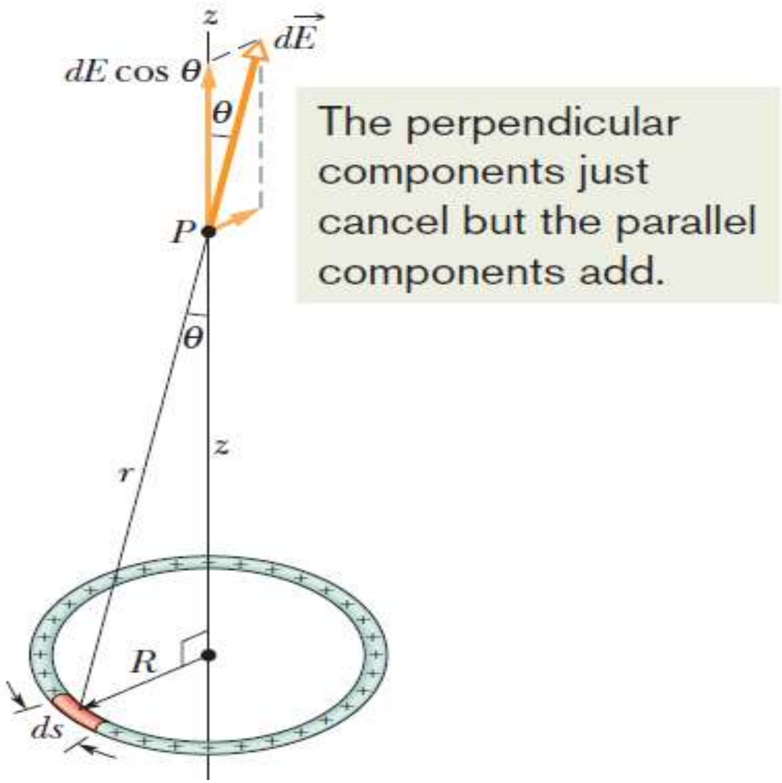


Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P. The component of $d\vec{E}$ along the central axis of the ring is $dE\cos\theta$.

We can mentally divide the ring into differential elements of charge that are so small that they are like point charges, and then we can apply the definition to each of them.

Next, we can add the electric fields set up at *P by all the differential elements*. The vector sum of the fields gives us the field set up at *P by the ring*.

22.6 The Electric Field due to a Line Charge:

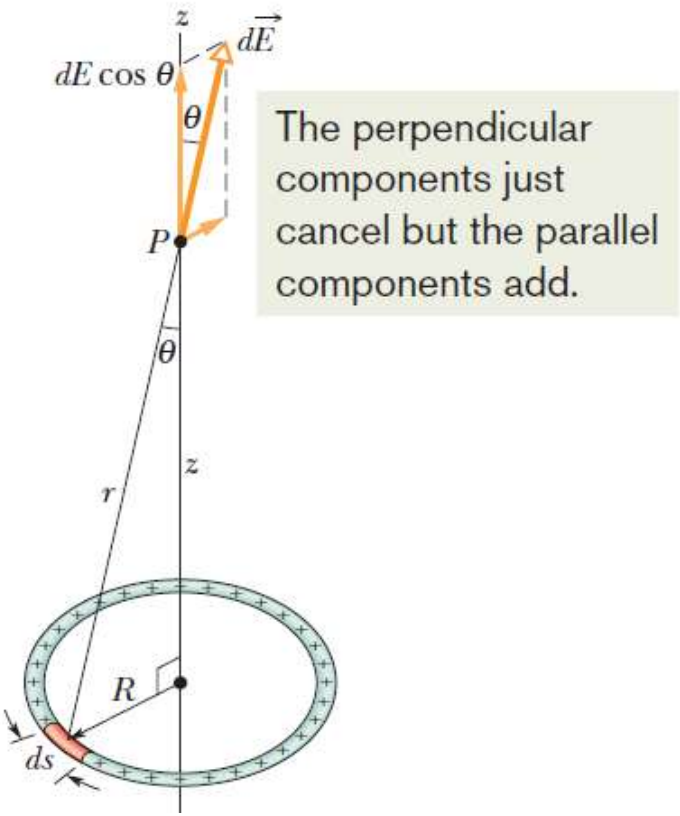


Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P. The component of $d\vec{E}$ along the central axis of the ring is $dE \cos\theta$.

Let ds be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds.$$

This differential charge sets up a differential electric field ***dE*** at point P, a distance r from the element.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$

All the ***dE*** vectors have components parallel and perpendicular to the central axis; the perpendicular components are identical in magnitude but point in different directions.

The parallel components are

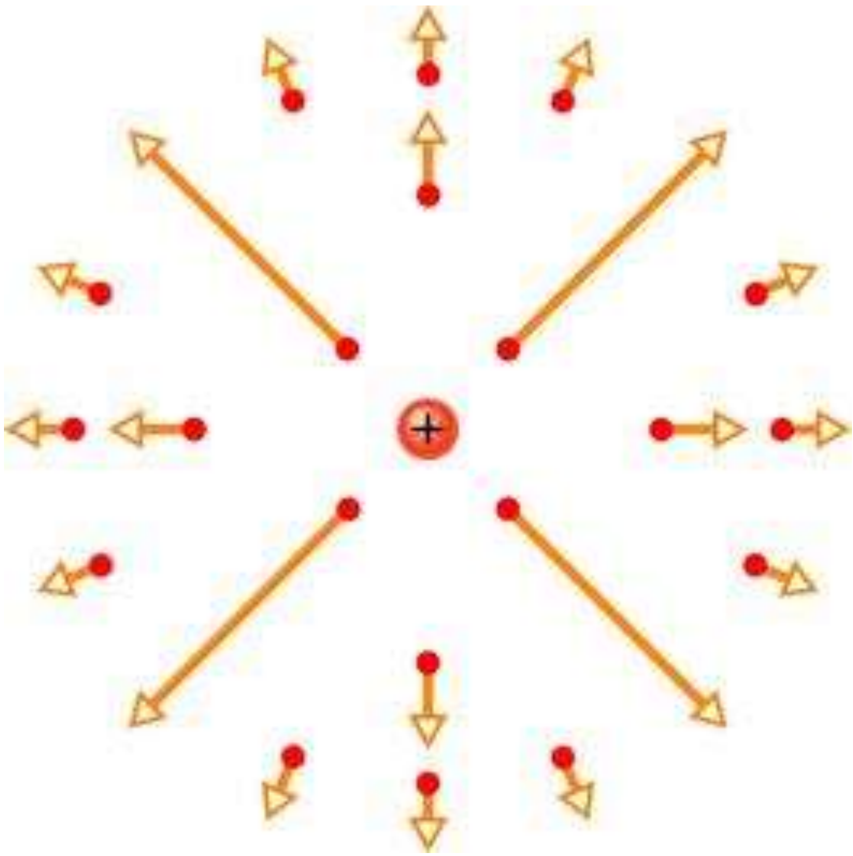
$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

Finally, for the entire ring,

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

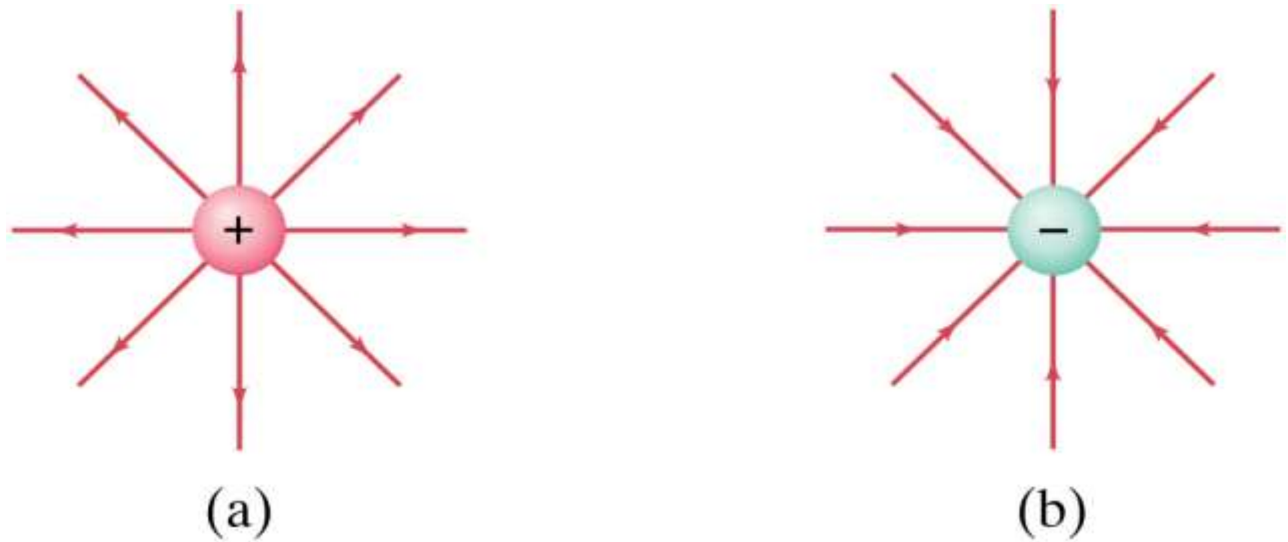
Representation of Electric Field



What's wrong with this representation?

Field Lines

The electric field can be represented by field lines which start on a positive charge and end on a negative charge. They have direction.

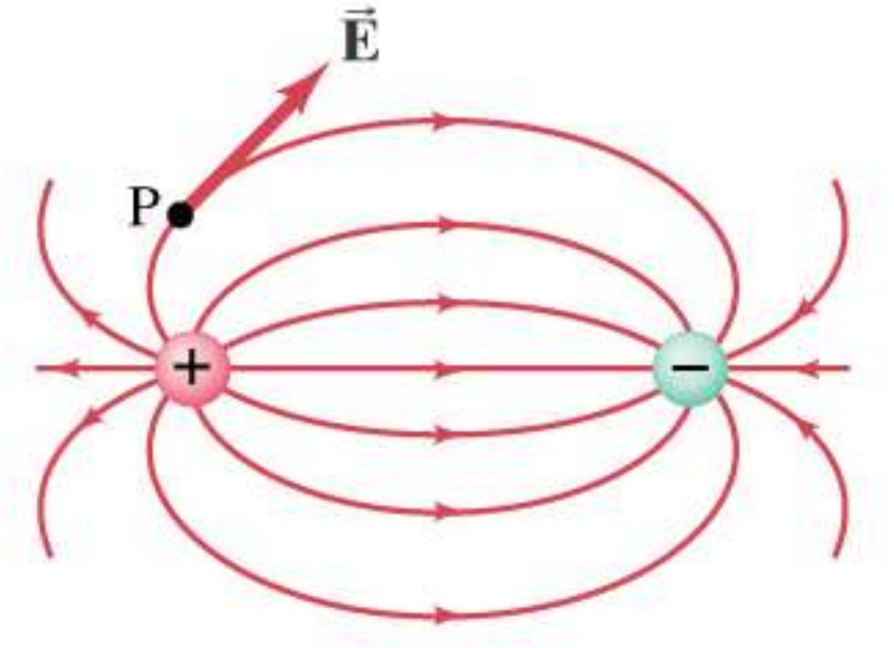


- The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.

- The electric field is stronger where the field lines are closer together.

Field Lines

Electric dipole: two equal charges, opposite in sign:



The field is tangent to field line, eg. electric field **E** at P.

22.3 Electric Field Lines:

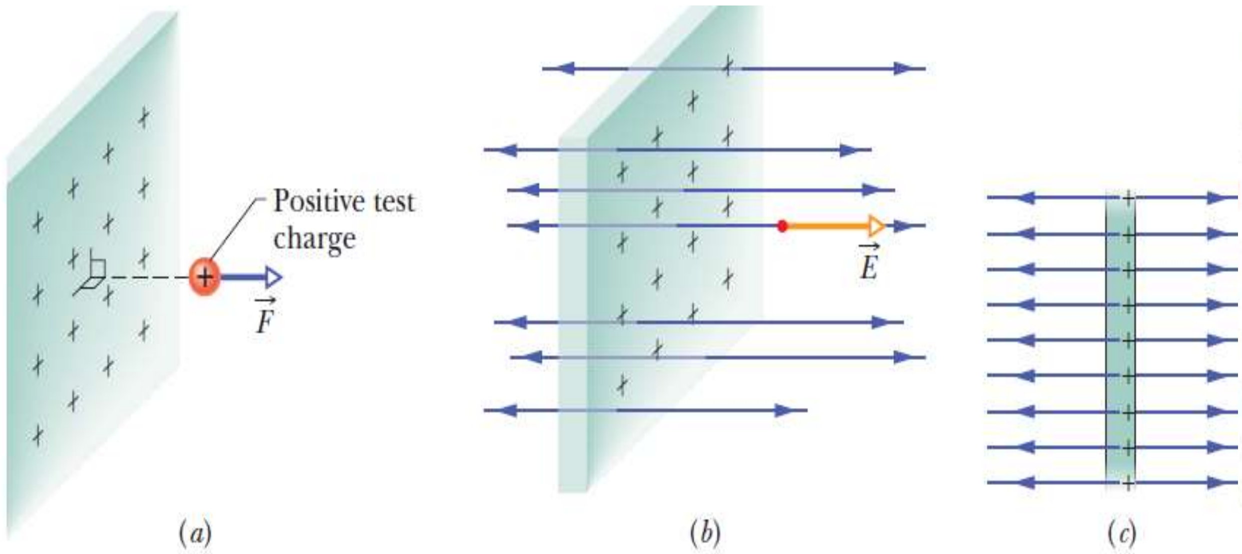


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

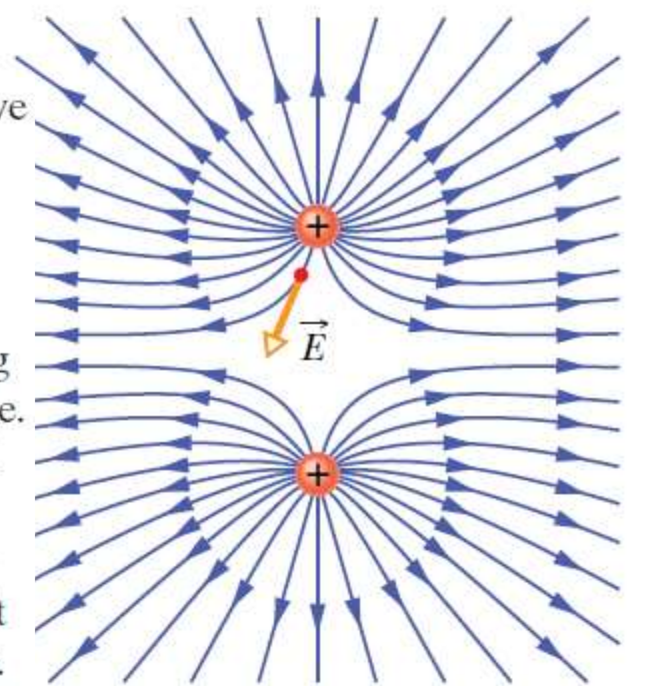
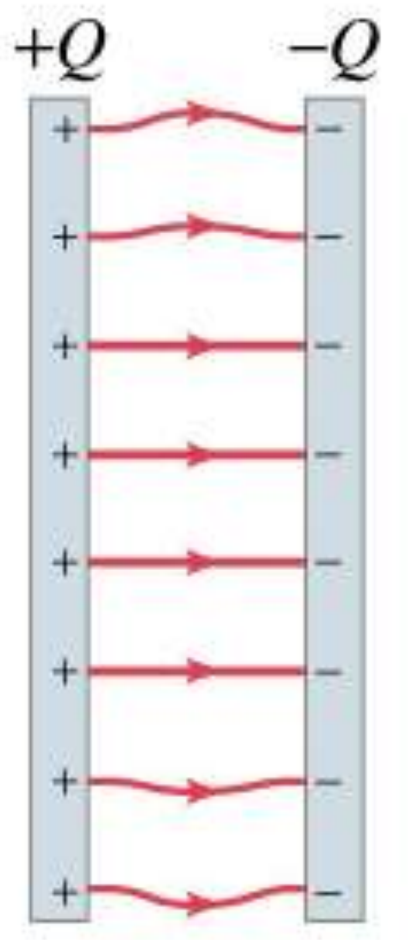


Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.

Field Lines



(d)

The electric field between two closely spaced, oppositely charged parallel plates is constant.

Field Lines

Summary of field lines:

1. Field lines indicate the direction of the field; the field is tangent to the line.

2. The magnitude of the field is proportional to the density of the lines.

(The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of ***E****.* Thus, ***E*** is large where field lines are close together and small where they are far apart.)

3. Field lines start on positive charges and end on negative charges; the number is proportional to the magnitude of the charge.

10^{23}

Conductors & Insulators

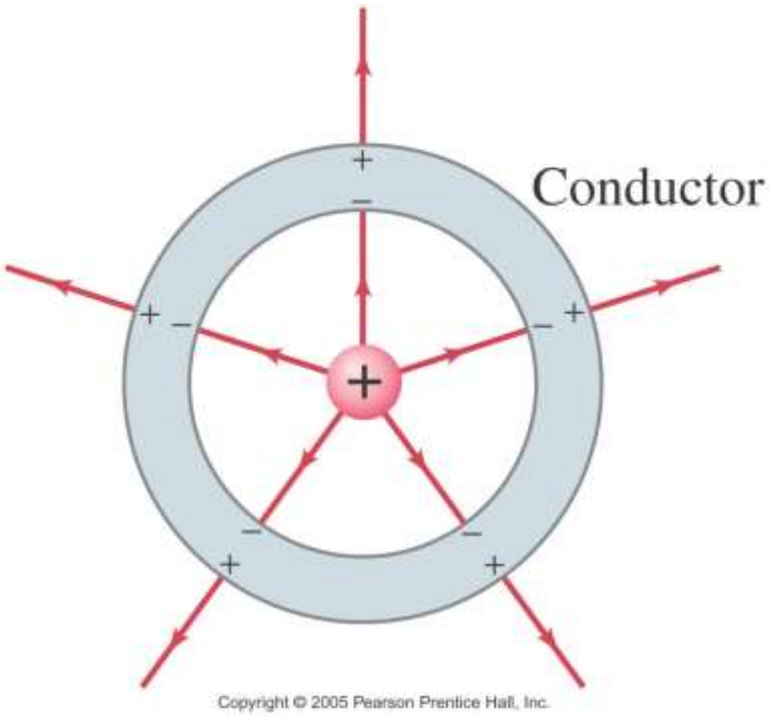
1. Conductor: Some electrons are loosely bound, can move about freely. About 10^{23} free (conduction) electrons per cm^3, or about one per atom. Eg. metals.
2. Insulator: Electrons are tightly bound. Not even 1 free electron per cm^3.
3. Semiconductor: $10^{10} - 10^{12}$ free electrons. per cm^3. Eg. Carbon, silicon.

Electric Fields and Conductors

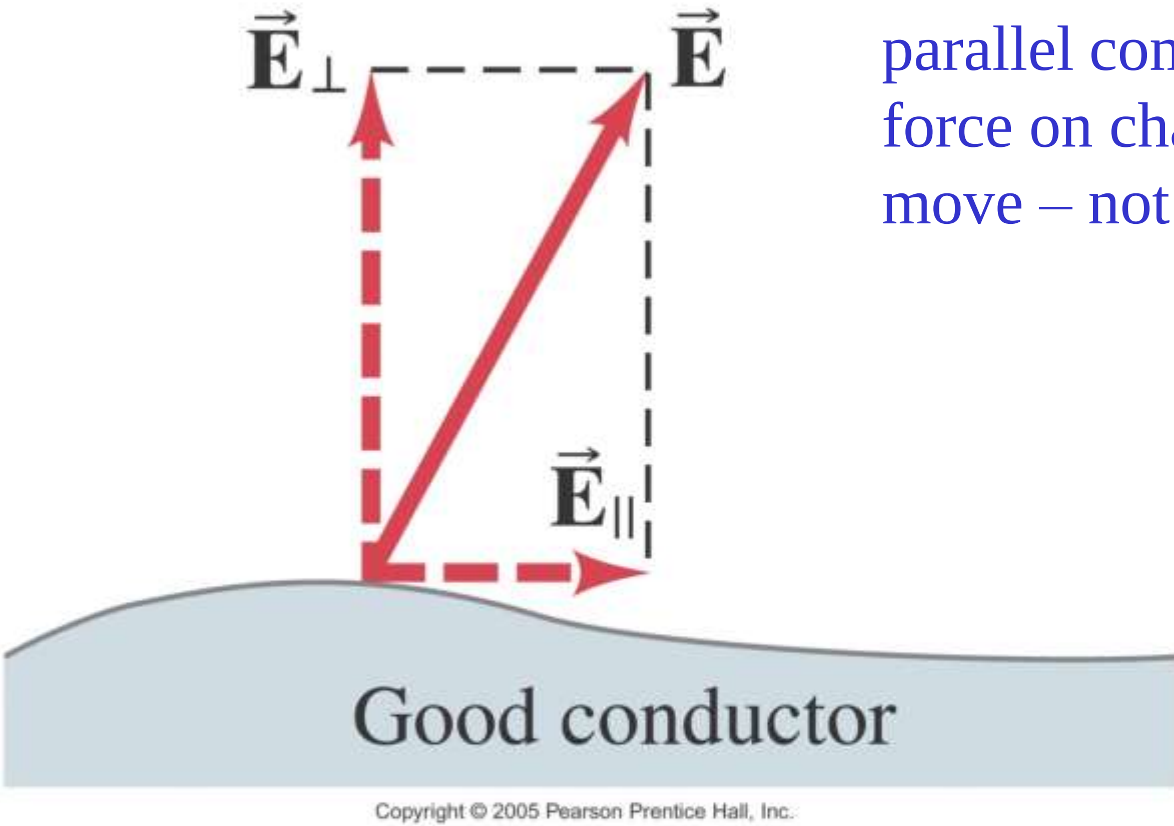
• The electric field (static case, no charge movement) inside a conductor is zero – if it were not, the charges would move.

• The net charge on a conductor is on its surface – to get as far from one another as possible.

• E field will “penetrate” a spherical conducting shell to get out



Electric Fields and Conductors



In static electricity, the electric field is perpendicular to the surface of a conductor – if it were not, then the parallel component results in a force on charges which would move – not static.

22.8: A Point Charge in an Electric Field:

Measuring the Elementary Charge

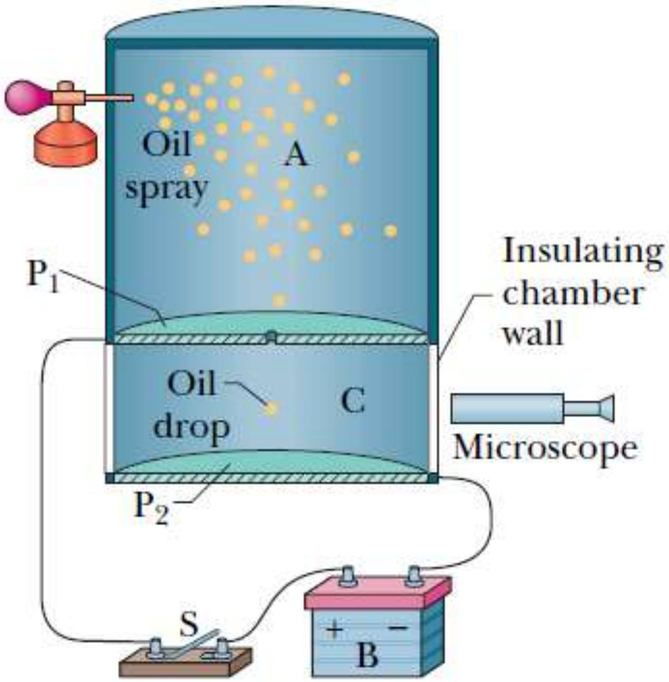


Fig. 22-14 The Millikan oil-drop apparatus for measuring the elementary charge e. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

Ink-Jet Printing

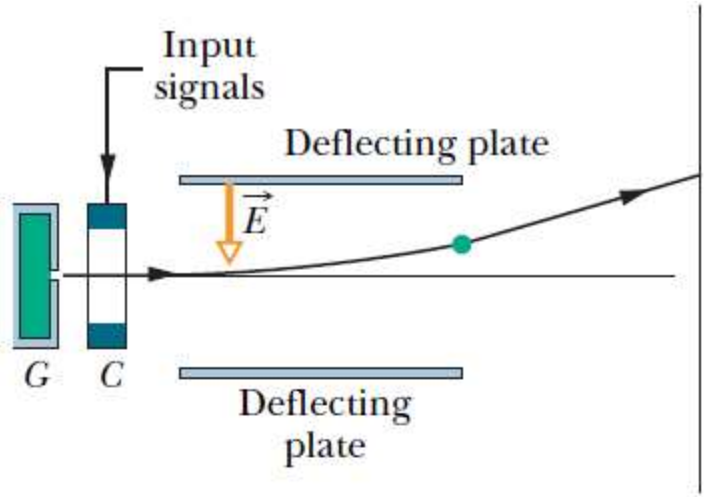


Fig. 22-15 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ where the drop lands on the paper.

Example, Motion of a Charged Particle in an Electric Field

Figure 22-17 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

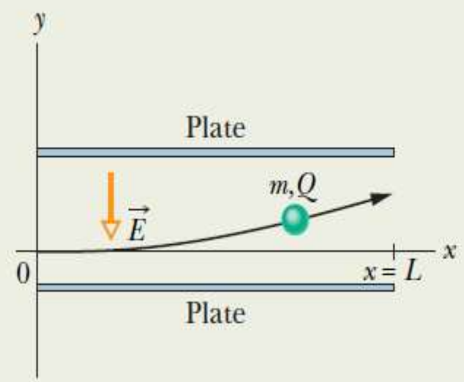


Fig. 22-17 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x, it accelerates upward with some constant acceleration a_y.

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \qquad (22\text{-}30)$$

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \qquad (22\text{-}31)$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y, we find

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2}$$

$$= 6.4 \times 10^{-4}\ \text{m}$$

$$= 0.64\ \text{mm}. \qquad \text{(Answer)}$$