

Lesson 9

Rational Function and Irrational Function

9A

- **Linear Rational Function**
- **Simple Rational Function**
- **Standard Form**
- **Inequality**

Linear Rational Function

Rational Function

$$y = \frac{g(x)}{h(x)}$$

where $g(x)$ and $h(x)$ are polynomials

Linear Rational Function

$$y = \frac{ax + b}{cx + d} \quad (ad - bc \neq 0, c \neq 0)$$

In the case of $ad - bc = 0$

$$\therefore \frac{a}{c} = \frac{b}{d} (\equiv k), \therefore a = ck, b = dk, \therefore y = \frac{ckx + dk}{cx + d} = k \quad : \text{Constant}$$

In the case of $c = 0$

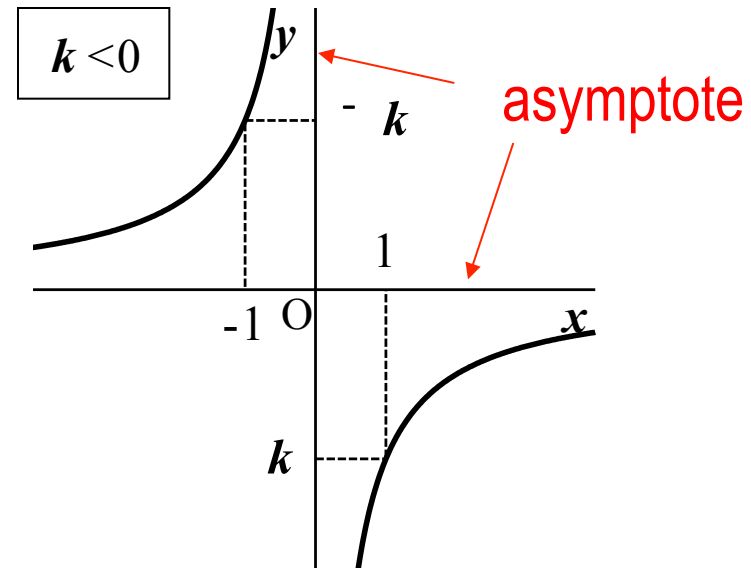
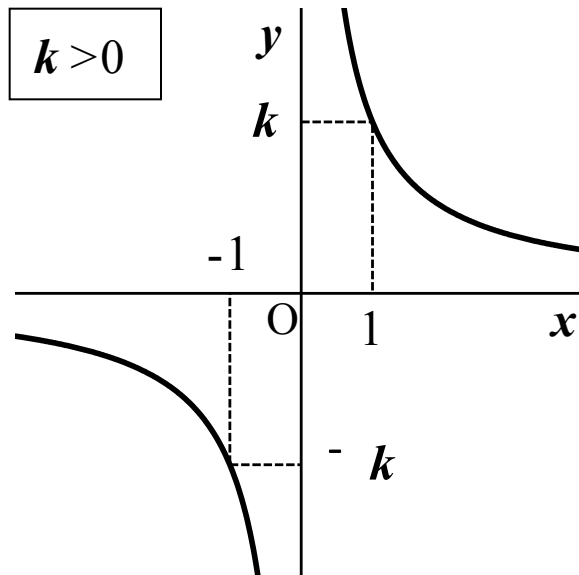
$$y = \frac{a}{d}x + \frac{b}{d} \quad : \text{Linear function}$$

The Simplest Rational Function

$$y = \frac{k}{x}$$

- Two variables are **inversely proportional**.

- The horizontal asymptote is $x = 0$ / the vertical asymptote is $y = 0$
- The graph is an **equilateral hyperbola**.

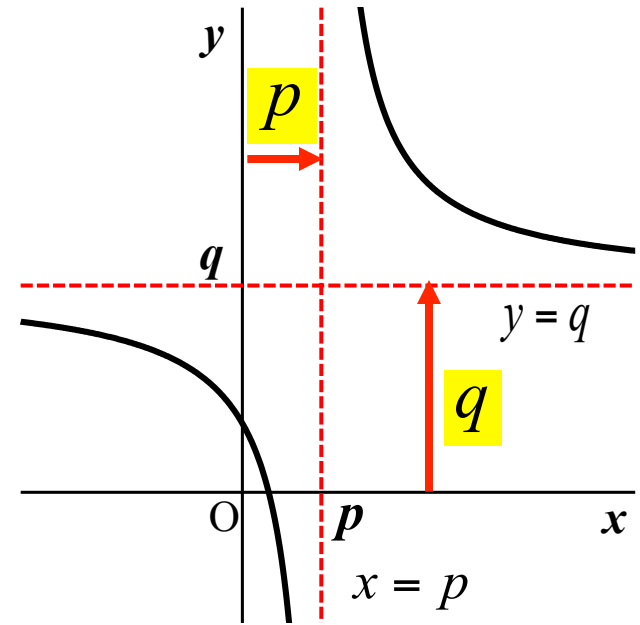


Function $y = (ax + b)/(cx + d)$

Change to the standard form

$$y = \frac{ax + b}{cx + d}$$

$$\rightarrow y = \frac{k}{x - p} + q$$



- The vertical asymptote is $x = p$
the horizontal asymptote is $y = q$.
- The domain is $x \neq p$; the range is $y \neq q$.

Example

Example 1. Illustrate the rational function $y = \frac{3x - 6}{x - 1}$, following each steps.

- (1) **[Step 1]** Convert the function to the standard form.
- (2) **[Step 2]** Find the horizontal and vertical asymptotes.
- (3) **[Step 3]** Find the x-intercept and y-intercept.
- (4) **[Step 4]** Illustrate the graph.

Ans.

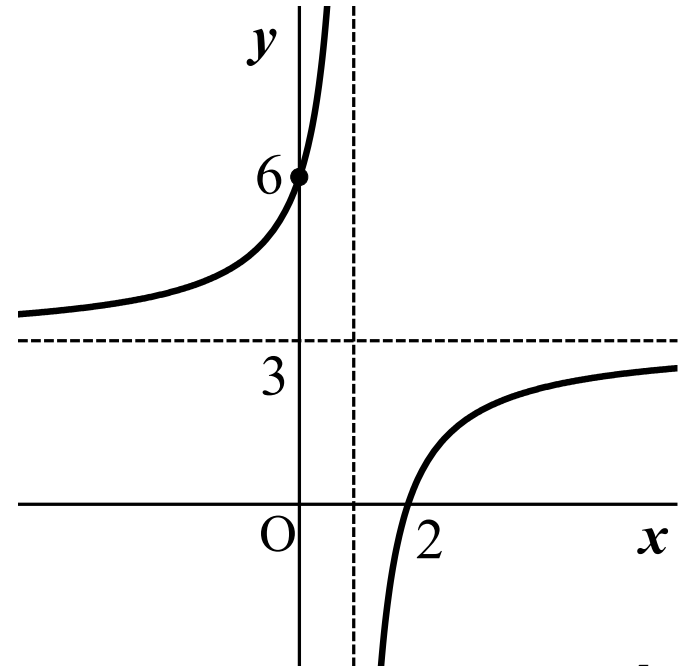
(1) By rearranging, it becomes

$$y = \frac{3x - 3 - 3}{x - 1} = 3 - \frac{3}{x - 1}$$

(2) The horizontal asymptote is $y = 3$
The vertical asymptote is $x = 1$

(3) By putting $y = 0$, the x-intercept is $x = 2$
By putting $x = 0$, the y-intercept is $y = 6$

(4) See the right figure



Inequality

Example 2. Solve the following inequality

$$\frac{3x - 6}{x - 1} \geq -x + 6$$

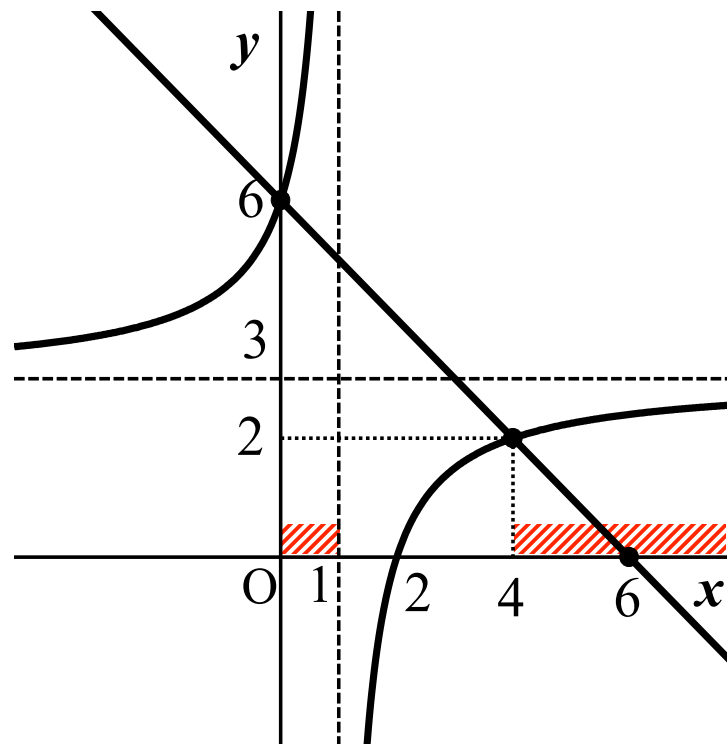
Ans.

We utilize the result of Example 1.

The cross points of $y = \frac{3x - 6}{x - 1}$ and $y = -x + 6$ are

$$\begin{aligned} \frac{3x - 6}{x - 1} = -x + 6 &\quad \therefore 3x - 6 = (x - 1)(-x + 6) \\ \therefore x(x - 4) = 0 &\quad \therefore x = 0, x = 4 \end{aligned}$$

From the figure and the cross points.
we have $0 \leq x < 1, 4 \leq x$



Exercise

Exercise 1. Solve the inequality $\frac{x+1}{x-1} > x+1$

Ans.

Pause the video and solve the problem.

Answer to the Exercise

Exercise 1. Solve the inequality $\frac{x+1}{x-1} > x+1$

Ans. • We consider two functions $y = \frac{x+1}{x-1} = \frac{2}{x-1} + 1$ and $y = x+1$

• The horizontal asymptote and vertical asymptote of the former are $y = 1$ and $x = 1$, respectively.

• The x- and y-intercepts of the former are $x = -1$ and $y = -1$, respectively.

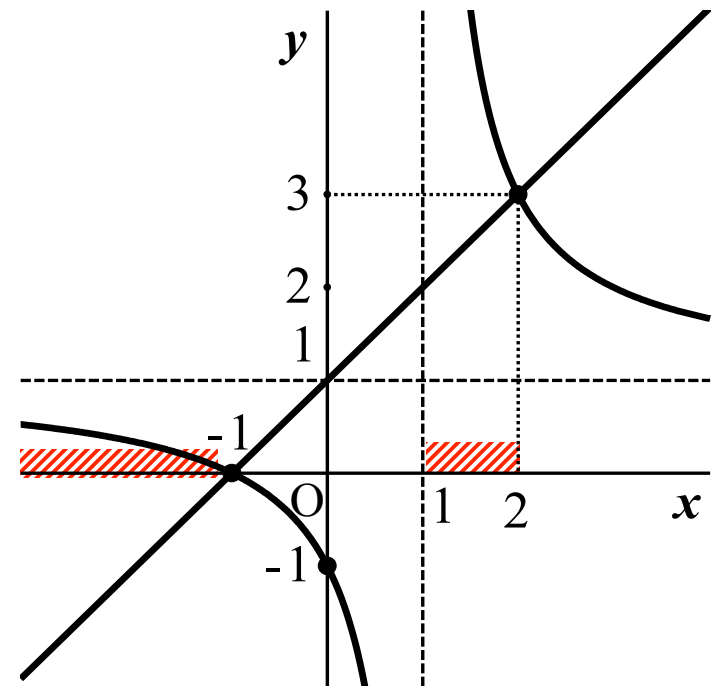
• The x- and the y-intercepts of the latter are $x = -1$ and $y = 1$, respectively.

• The cross-points are

$$\frac{x+1}{x-1} = x+1 \quad \therefore (x+1)(x-2) = 0 \quad \therefore x = -1, 2$$

• From the figure, the solution is

$$x < -1, 1 < x < 2$$



Lesson 9

Exponential Functions

9B

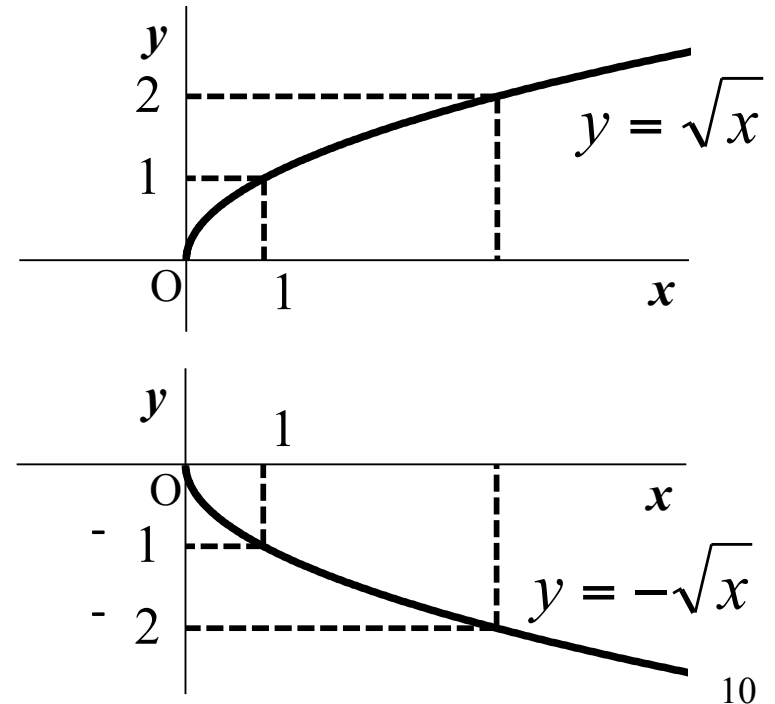
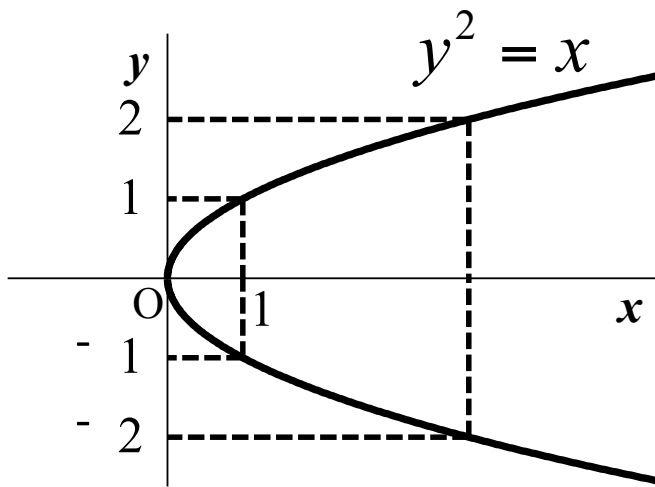
- Irrational Functions
- Standard Form
- Inequality

Irrational Function

Irrational Function

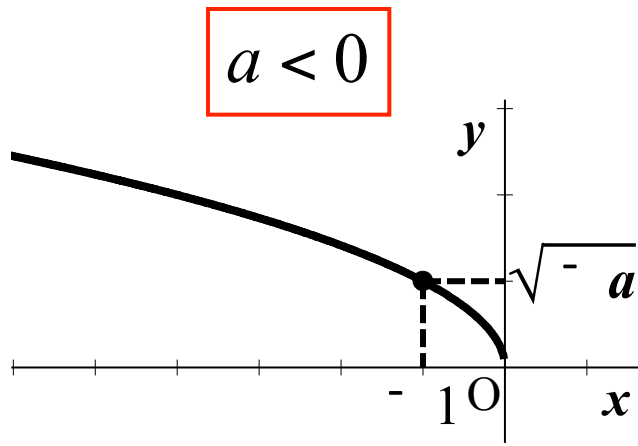
- There is no rigorous definition of irrational function.
- We can say that irrational function is the one that cannot be written as the quotient of two polynomials (but this definition is not used.).
- Customary, a function **which include variables in the root** is called an **irrational function**.

Example

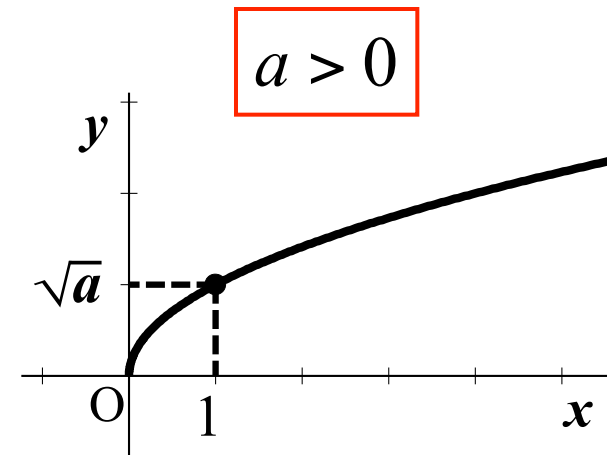


Fundamental Form

$$y = \sqrt{ax}$$



Domain $x \leq 0$
Range $y \geq 0$



Domain $x \geq 0$
Range $y \geq 0$

Function

$$y = \sqrt{ax + b} + q$$

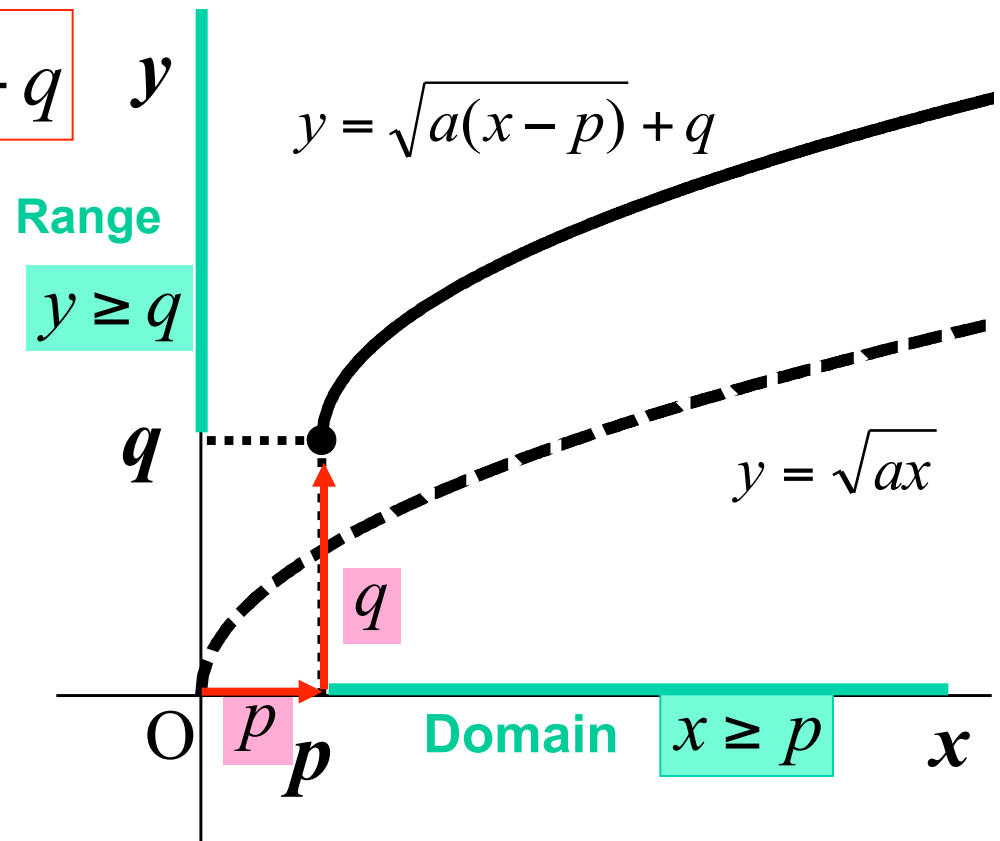
Change to the standard form

$$y = \sqrt{ax + b} + q$$

→ $y = \sqrt{a(x - p)} + q$



That wasn't so hard!



Equation and Inequality

Example 3. Answer the following questions

(1) Illustrate the graphs of the functions $y = \sqrt{2-x}$ and $y = -x + \sqrt{2}$

(2) Solve the equation $\sqrt{2-x} = -x + \sqrt{2}$

(3) Solve the inequality $\sqrt{2-x} \geq -x + \sqrt{2}$

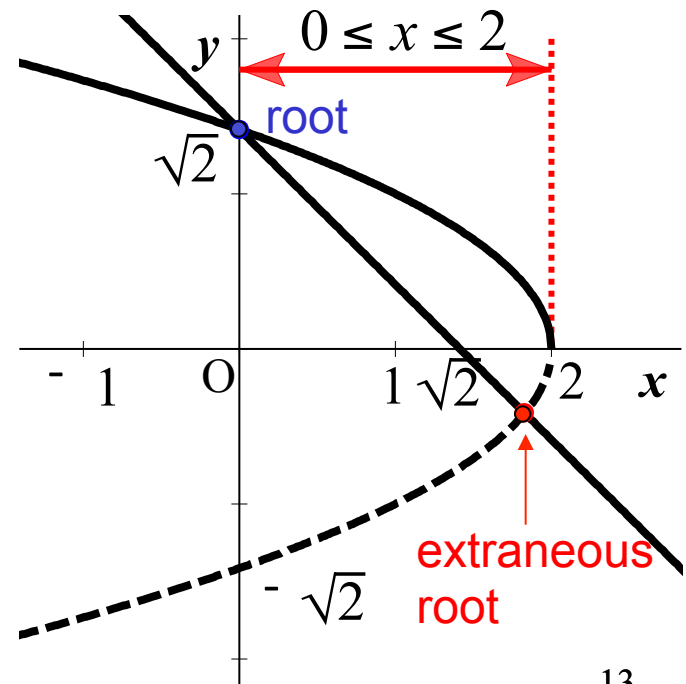
Ans. (1) See the right figure

$$(2) \sqrt{2-x} = -x + \sqrt{2} \quad \therefore 2-x = (-x + \sqrt{2})^2$$
$$\therefore x(x - 2\sqrt{2} + 1) = 0 \quad \therefore x = 0, 2\sqrt{2} - 1$$

From the figure, the root is only $x = 0$

(3) From (1) and (2), we have $0 \leq x \leq 2$

[Note] We must check that the obtained roots really satisfy the equation.



Exercise

Exercise 2. Solve the inequality $\sqrt{x+2} > x$

Ans.

Pause the video and solve the problem.

Answer to the Exercise

Exercise 2. Solve the inequality $\sqrt{x+2} > x$

Ans.

- We consider two functions $y = \sqrt{x+2}$ and $y = x$
- The x- and the y-intercepts of the former are $x = -2$ and $y = \sqrt{2}$, respectively.

- The cross-points of these functions are

$$\begin{aligned}\sqrt{x+2} &= x & \therefore x+2 &= x^2 \\ \therefore (x+1)(x-2) &= 0 & \therefore x &= -1, \quad x = 2\end{aligned}$$

Only $x = 2$ satisfies this condition.

- From the figure, the solution is
 $-2 \leq x < 2$

(Note the boundaries.)

