Course III

NAGOYA UNIVERSITY

Lesson 03
Vector Equation

3A

- Vector Equation

Vector Equation of a Line

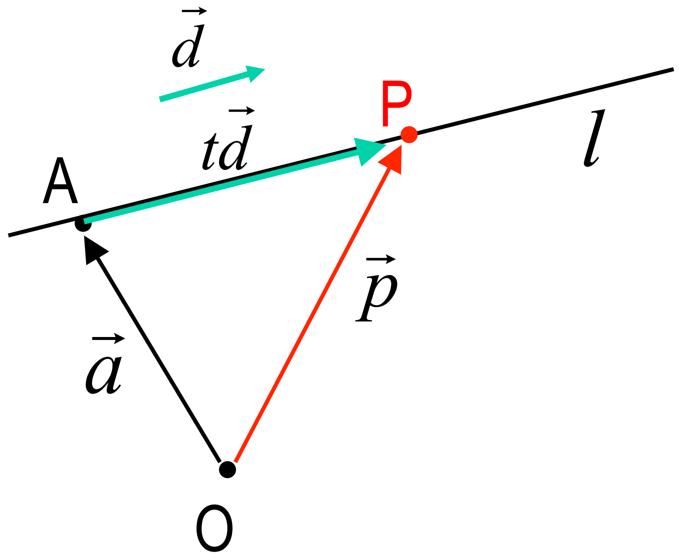
Vector equation of a line

Line passing point A($\vec{a}$)
parallel to the vector $\vec{d}$

$$\vec{p} = \vec{a} + t\vec{d}$$

t : parameter

$\vec{d}$: direction vector



Expression by components

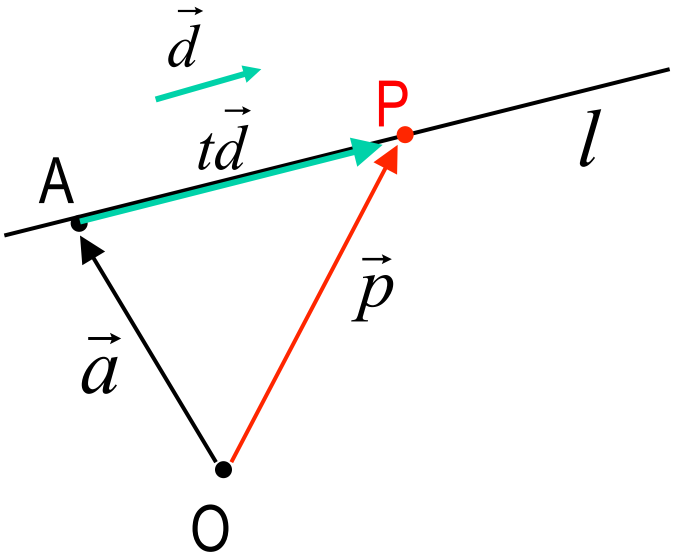
When $\vec{a} = (x_0, y_0)$, $\vec{p} = (x, y)$, and $\vec{d} = (d_x, d_y)$

$$(x, y) = (x_0, y_0) + t(d_x, d_y)$$

Parametric Equation of a Line

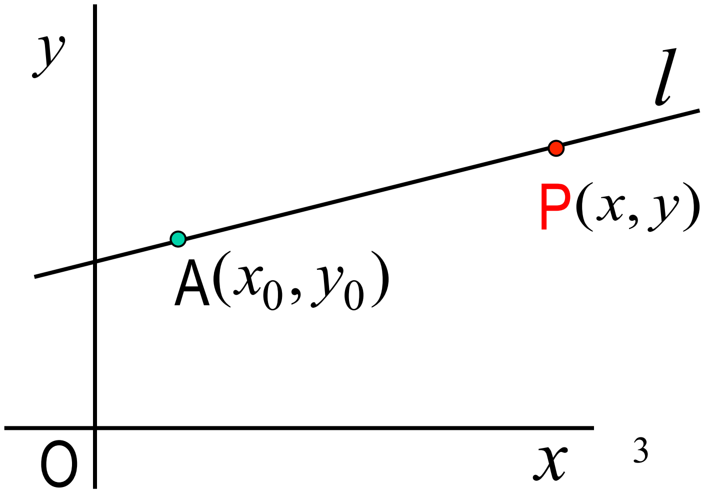
Vector equation

$$(x, y) = (x_0, y_0) + t(d_x, d_y)$$



Parametric equation

$$\left.\begin{array}{l} x = x_0 + t d_x \\ y = y_0 + t d_y \end{array}\right\}$$



One more time ?

Equation of a Line Passing A and B

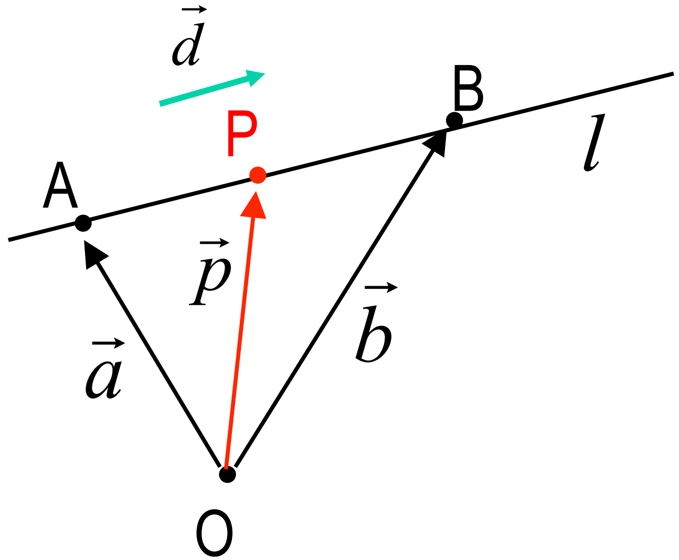
Line passing two points A $(\vec{a})$ and B $(\vec{b})$

$$\vec{d} = \vec{b} - \vec{a}$$

$$\vec{p} = \vec{a} + t(\vec{b} - \vec{a})$$

Or

$$\vec{p} = (1 - t)\vec{a} + t\vec{b}$$



[Examples 3-1] Find the vector equation which passes through the points A=(1, 2) and B=(-2, 5)

Ans.

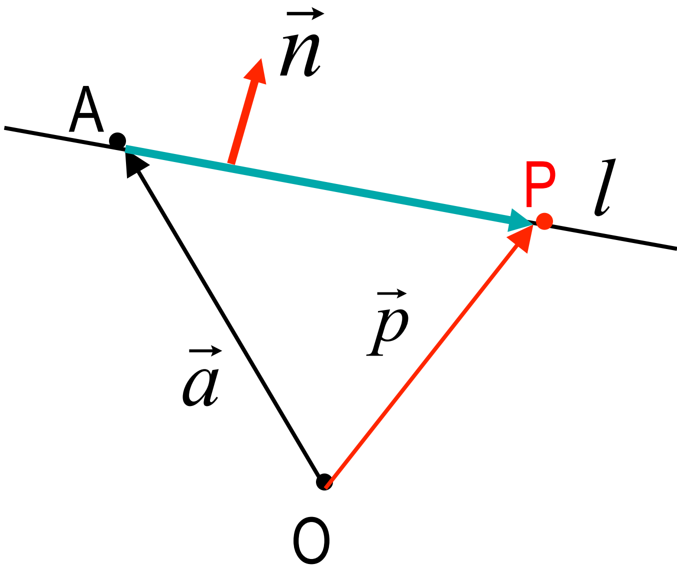
Since $\overrightarrow{AB} = (-2-1,\ 5-2) = (-3,\ 3)$

we have

$$(x, y) = (1,\ 2) + t(-3,\ 3)$$

Straight Line and Normal Vector

Line Passing through Point A($\vec{a}$) and normal to the vector $\vec{n}$



Since $\vec{AP} \perp \vec{n}$, we have

$$\vec{n} \cdot (\vec{p} - \vec{a}) = 0$$

Expression by components

When $\vec{a} = (x_0, y_0)$, $\vec{p} = (x, y)$, and $\vec{n} = (n_x, n_y)$

$$n_x(x - x_0) + n_y(y - y_0) = 0$$

Exercise

[Ex.3-1] Answer the following questions about the line which passes point A(3, 5) and is parallel to the vector

(1) Find the vector equation.

(2) Find the parametric equation

(3) Derive the expression of the line by eliminating the parameter .

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex.3-1] Answer the following questions about the line which passes point A(3, 5) and is parallel to the vector $\vec{d} = (-2,4)$. (1) Find the vector equation. (2) Find the parametric equation. (3) Derive the expression of the line by eliminating the parameter .

Ans.

Let the coordinates of the moving point on the line be (x, y).

(1) $(x, y) = (3,5) + t(-2,4)$

(2) $x = 3 - 2t,\ y = 5 + 4t$

(3) $2(3 - x) = y - 5 \qquad \therefore\ y = -2x + 11$

NAGOYA UNIVERSITY

Lesson 03

Vector Equation

3B

- Vector Equation of a Circle

Vector Equation of a Circle （1）

Case : Radius r and center $\vec{c}$

Vector equation

Definition of a Circle $|\overrightarrow{CP}| = r$

Therefore

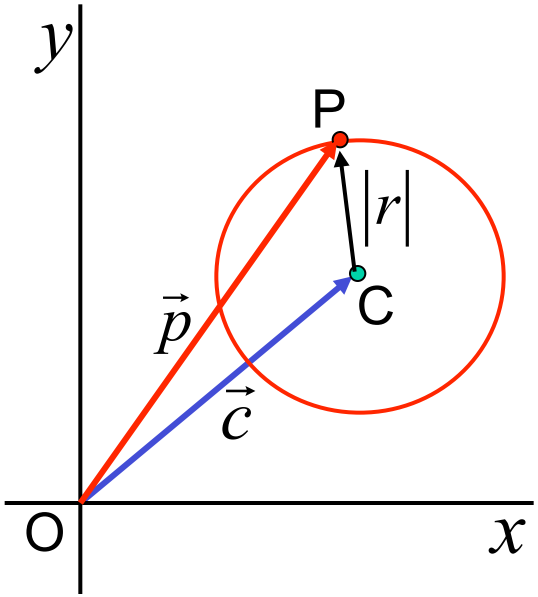
$$|\vec{p} - \vec{c}| = r$$

Making the square

$$|\vec{p} - \vec{c}|^2 = r^2$$

Therefore, from the definition of the scalar product

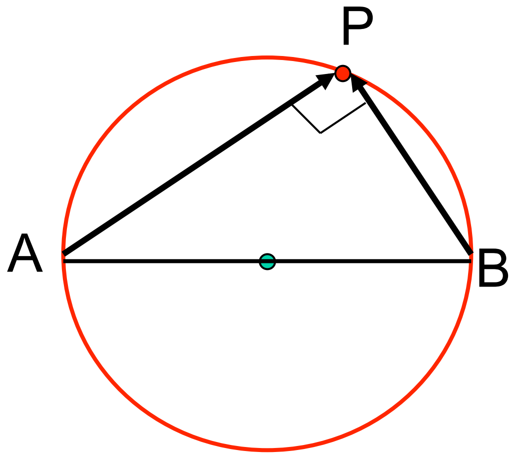
$$(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) = r^2$$



Vector Equation of a Circle (2)

Case : Diameter AB, where A($\vec{a}$) and B($\vec{b}$)

Vector equation



Circumference angle for a diameter is 90°

$$(\vec{p}-\vec{a})\cdot(\vec{p}-\vec{b})=0$$

Next, we show the radius explicitly

Since $(\vec{a}-\vec{c})\cdot(\vec{a}-\vec{c})=\left|\vec{a}-\vec{c}\right|^2=r^2$

$$(\vec{p}-\vec{a})\cdot(\vec{a}-\vec{c})=\left\{\vec{p}-\vec{c}-(\vec{a}-\vec{c})\right\}\cdot(\vec{a}-\vec{c})$$

$$=(\vec{p}-\vec{c})(\vec{a}-\vec{c})-\left|\vec{a}-\vec{c}\right|^2=(\vec{p}-\vec{c})(\vec{a}-\vec{c})-r^2=0$$

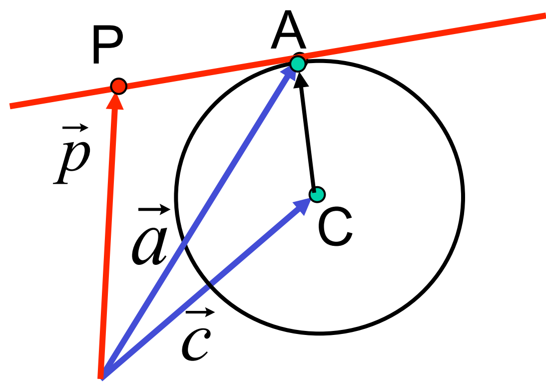
Therefore $$(\vec{p}-\vec{c})(\vec{a}-\vec{c})=r^2$$

Example

[Examples 3-2]
Find the vector equation of the tangent to the circle at A($\vec{a}$). The radius of this circle is r and the center is located at C($\vec{c}$).

Ans. Since the circumference angle for the diameter is 90°

$$(\vec{p}-\vec{a})\cdot(\vec{a}-\vec{c})=0$$



Since the radius is r

$$(\vec{a}-\vec{c})\cdot(\vec{a}-\vec{c})=\left|\vec{a}-\vec{c}\right|^2=r^2$$

Substituting this, we have

$$(\vec{p}-\vec{a})\cdot(\vec{a}-\vec{c})=\left\{\vec{p}-\vec{c}-(\vec{a}-\vec{c})\right\}\cdot(\vec{a}-\vec{c})$$

O x

$$=(\vec{p}-\vec{c})(\vec{a}-\vec{c})-\left|\vec{a}-\vec{c}\right|^2=(\vec{p}-\vec{c})(\vec{a}-\vec{c})-r^2=0$$

Therefore

$$(\vec{p}-\vec{c})(\vec{a}-\vec{c})=r^2$$

Exercise

[Ex3-2] What kind of figure does the following vector equation represent ?

$$\left|3\vec{p} - 2\vec{a}\right| = 3$$

Ans.

Pause the video and solve the problem by yourself.

Answer to the Exercise

[Ex3-2] What kind of figure does the following vector equation represent ?

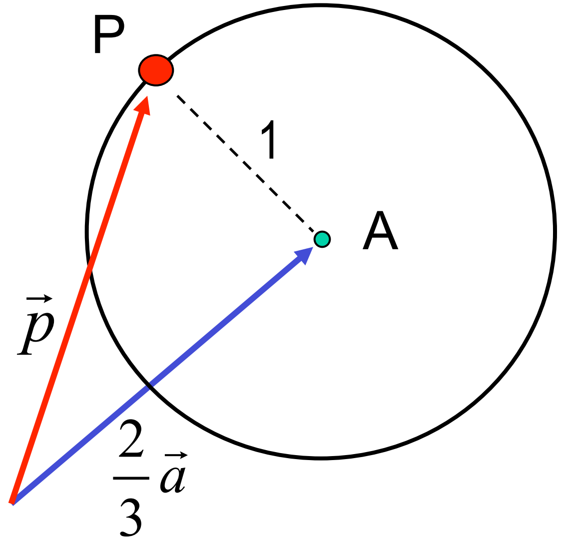
$$\left|3\vec{p}-2\vec{a}\right|=3$$

Ans.

This equation is rearranged as follows.

$$\left|3\vec{p}-2\vec{a}\right|=3$$

$$\therefore\ \left|3\left(\vec{p}-\frac{2}{3}\vec{a}\right)\right|=3$$



A circle with radius 1 and the center at the position $\frac{2}{3}\vec{a}$