Entrepreneurship, Financial Intermediation, and Inequality

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This paper provides a simple dynamic framework for examining the long-run relationship between financial intermediation and wealth inequality. By considering two types of entrepreneurial financing (self-financing and intermediated financing), this paper shows that wealth inequality is more severe in an economy in which all financing is intermediated than in an economy where some entrepreneurs rely on self-financing. This result is consistent with the augmented Kuznets hypothesis that large-scale production operations and financial intermediary development are associated with higher inequality.

Keywords: Entrepreneurship; Financial Intermediation; Inequality.

I. Introduction

What is the relationship between financial intermediation and inequality? Consider the case in which one needs to raise outside capital for production. Under capital market imperfections, one's borrowing opportunities would be associated with one's wealth level. On the one hand, financial intermediation might ease inequality by making less wealthy people capable of borrowing. On the other hand, one might also expect that financial intermediation would help wealthier people borrow progressively more, thus accelerating inequality. The same argument also holds for the situation where opportunities of financial access become broadened by financial liberalization such as deregulation or capital account liberalization (CAL).

Though it is difficult to gauge whether inequality *per se* is beneficial or harmful, it is important to understand how the degree of inequality in society is determined (see, e.g., Piketty (2014)). One may argue that inequality significantly affects aspects of social infrastructure such as political stability (see, e.g., Alesina and Perotti (1996)) and public safety (see, e.g., Fajnzylber, Lederman, and Loayza (1998)). Therefore, policymakers should be concerned about what determines the degree of inequality, especially when examining the balance of economic organizations and non-economic arrangements in the society. Traditionally, policymakers are expected to engage in redstribution—that is *ex-post public* transfers—in order to alleviate inequality. However, policymakers may also want to consider *ex-ante private* factors that engender generate inequality.

Considering inequality *per se* seriously, this paper provides a simple dynamic framework to study the long-term relationship between financial intermediation and wealth inequality. Specifically, it considers a deterministic dynamic model following Matsuyama (2000), who analyzes how inequality arises in an economy with an imperfect capital market. In contrast to Matsuyama (2000), the present model allows for different types of finance while eliminating the steady-state equilibria with perfect equality. It is assumed that some positive fixed amount of capital is necessary for production (i.e., non-convex technology). Then, in the presence of the capital market imperfection (due to imperfect enforcement), households (potential entrepreneurs) with limited wealth cannot borrow funds from anywhere even if they want to start production, whereas funds may be borrowed from a financial intermediary whey they own a sufficient amount of wealth. Borrowers always have the option of simply defaulting, and consequently, repayment is enforceable only with some inevitable cost. Hence, the role of financial intermediaries (FIs) is to alleviate this enforcement issue through some costly monitoring activities. The following model assumes that when the borrower defaults (which never occurs in equilibrium),[1] FIs can retrieve a fraction of the money. In this case, one can think of FIs in the model as local banks, investment banks, securities agencies, etc. However, intermediated borrowing might not be beneficial for entrepreneurs with more wealth, as it is costly in nature. Therefore, this paper examines this feature through a formal model. Though uncertainty and asymmetric information are not considered to maintain analytical tractability, a full characterization of the steady-state equilibria is provided to obtain interesting insights regarding the relationship between financial intermediation and inequality.

Specifically, this provides a theoretical basis for the augmented Kuznets hypothesis (see the next section for the related literature); (i) *inequality persists in a country and varies across countries*, and (ii) *large scale production operations and financial intermediary development are associated with higher inequality*. In addition, this paper considers two types of financing, self-financing and intermediated financing, and assumes that there is imperfect enforcement in the capital market, which allows FIs endowed with monitoring technologies to play a role in the economy. Other economic agents include households (potential entrepreneurs) that are heterogeneous only in their level of wealth. In the following analysis, the interest rate is endogenously determined. The steady-state characterization is also provided to determine the type of financial pattern prevailing in the economy as well as the characteristics of the wealth distribution. Although there is no heterogeneity such as talent, perfect equality never arises in any steady-state equilibria (i.e., inequality persists) unless the initial wealth distribution is too skewed toward the rich or poor. It is also found that wealth inequality is severe for a lower equilibrium interest rate, and that wealth inequality is more severe in an economy where all financing is intermediated than in an economy in which some entrepreneurs rely on self-financing. Furthermore, for a wide range of parameters (concerning the benefit and cost of monitoring), there are two continua of the steady-state equilibria: one is where all entrepreneurs rely on financial intermediation and the other is where some of the (richer) borrowing entrepreneurs self-finance. The multiplicity derives from the mutually reinforcing effects: in the former type of equilibria, the equilibrium interest rate is low and the supply of capital (the number of poorer agents) is large. These two effects are mutually dependent, and inequality is severe. However, in the second type of equilibrira, the equilibrium interest rate is high and the supply of capital (the number of poorer agents) is small. Thus, inequality is less severe. These features are consistent with the findings of Clarke, Xu, and Zou (2013) (see the next section). The following formal model in the present paper suggests that even if two economies that have similar values of parameters, they may end up as two different types of economies in terms of financial structure.

The remainder of the paper is organized as follows. The next section briefly reviews the related literature, and Section Ⅲ presents the dynamic model. Section Ⅳ provides the steady-state analysis, followed by Section Ⅴ in which the effects of CAL are discussed. Finally, Section Ⅵ concludes the paper.

Ⅱ. Related Literature

This section focuses on the related literature that investigates macroeconomic or developmental implications in the presence of capital market imperfections, beginning with a seminal article by Galor and Zeira (1993).[2] These papers discuss the long-term effects of an imperfect capital market on the wealth distribution. Though they are essentially silent on the *differences in finance*, one exception is a paper by Chakraborty and Ray (2006), who investigate the issue of bank-based versus market-based lending in an *Ak*-type endogenous growth model. Specifically, they extend Holmström and Tirole's (1997) incentive model (moral hazard with respect to project choice) of financial intermediation into a dynamic context. As in the present paper, the role of bank monitoring is to mitigate the agency problem, and each entrepreneur chooses how he/she borrows the required working capital. Chakraborty and Ray (2006) focus on balanced growth paths, and compare the growth rate of per capita GDP (and other macro variables) in the market-based system with that in the bank-based system. However, their model does not allow for a mixed structure of different finance types. In the long run, all entrepreneurs (except for the ones who cannot borrow) in the economy borrow either from banks or from markets, depending on the exogenous parameter values concerning monitoring effects and costs. In other words, direct and intermediated lending *cannot coexist* in an economy in Chakraborty and Ray's (2006) model. Conversely, Chakraborty and Ray (2007) allow three types of households to emerge in the steady state; (i) those that cannot borrow, (ii) those that borrow some amount from a bank, and (iii) those that rely only on the credit market. Further, they focus on two features of a financial system: depth and structure. Fnancial depth refers to how large the proportion of unconstrained borrowers is, wherereas financial structure captures the fraction of borrowers who rely only on the market among them. Basically, the initial inequality entirely determines the financial system to which the economy converges; the more unequaly during the initial stage, the less developed the economy's financial system remains. This is because Chakraborty and Ray (2006, 2007) consider a small open economy. Specifically, they assume that the interest rate is exogenously given. In the following model, the interest rate is endogenously determined as a component of equilibrium. As such, the initial distribution is just one of the factors determining the characteristics of the steady state.

The formal model presented in this paper is also motivated by empirical findings on inequality.[3] Based on the available time series data from England, Germany, and the United States, Kuznets (1955) offers a broad hypothetical view, known as he *Kuznets inverted U-shaped hypothesis*, on the relationship between economic development and income inequality. The hypothesis states that as an economy develops, income inequality rises, but in the later stage of development inequality mitigates. Kuznets (1955) attributes this change to the migration shift from the traditional agricultural sector to the modern industrial sector, where the wage dispersion is large. The congestion in the modern sector eventually makes the traditional sector attractive again, which eases inequality.

In contrast to Kuznets' (1955) original reasoning, Greenwood and Jovanovic (1990) offer a theoretical model that endogenously derives the inverted U-shaped curve by focusing on the role of *financial intermediation* (see also, e.g., Greenwood and Smith (1997) and Greenwood, Sanchez and Wang (2010)). In their model, individuals can invest in a risky but profitable project only when they pay a fixed membership fee to join a financial intermediary coalition. This fixed cost first prevents poorer individuals from accumulating wealth, which then exacerbates inequality, However, the more those rich individuals join these coalitions, the lower the entry fee becomes (since the average cost of the coalition declines as the number of members increases), which eventually eliminates inequality.

Conversely, a number of empirical studies (such as Deininger and Squire (1998), Li, Squire, and Zou (1998), and Clarke, Xu, and Zou (2013)) find little support for the Kuznets inverted U-shaped relationship between income inequality and the level of income per capita. As Li, Squire, and Zou (1998) show, although the degree of inequality seems to *persist* within an economy, it varies across economies. Based on their cross-country empirical study, Clarke, Xu, and Zou (2013) also question the role of financial intermediation in Greenwood and Jovanovic's (1990) study. Based on these points, the present paper shows inequality as a perpetuating phenomenon even in the long run. Specifically, in this model, equality never arises in any steady state equilibria. This is in sharp contrast to the models that derive wealth distribution but allow perfect equality to arise as one of the equilibria.[4] Moreover, this paper interprets institutional differences in finance as the main causes of generating these international varieties in equality, if other possibly related factors are controlled.

Though Kuznets' (1955) original inverted U-shaped hypothesis has gained little empirical support, Kuznets' (1955) analysis can still be insightful. If the modern technology that entrepreneurs adopt requires high leverage, financial intermediation will help rich people borrow more, thus preventing poorer households, which remains suppliers of capital, from starting a project. In this manner, inequality might be associated with the prevalence of the modern technology via financial intermediation. Indeed, Clarke, Xu, and Zou (2013) identify this effect in their empirical study, and term it an *augmented Kuznets hypothesis*, or as Kuznets (1955) suggests, sectorial structure matters to inequality. In particular, large-scale production operations (measured by the added value of non-agricultural sectors divided by the GDP) and financial intermediary development (measured by the amount of bank assets or private credits divided by the GDP) are associated with higher inequality (measured by the Gini index of income). To the best of this author's knowledge, there is no theoretical model that formalizes this effect of financial intermediation on inequality. Though the simple model in this paper cannot replicate all of the empirical results, it may prove useful in terms of investigating the role of financial intermediation from various perspectives.

It is also interesting to see the effects of CAL on inequality. Based on the data of 11 emerging markets in which equity market liberalization occurred from 1986 to 1995, Das and Mohapatra (2003) find that the average middle-class income share *not* the absolute value) decreased whereas the average income share of the highest class increased and that of the lowest class showed little change. After analyzing the steady-state, Section V incorporates CAL into the dynamics of the present model, and considers its effects on wealth distribution.

Ⅲ. The Model

This section introduces a formal dynamic model of household behavior and financial intermediation. In particular, it incorporates financial intermediation into a dynamic model, following Matsuyama (2000). First, the production technology, imperfect enforceability, and the role of financial intermediation are explained. Then, the equilibrium interest rate in each period is determined, and the equilibrium dynamics of the wealth distribution and interest rate is illustrated.

1. Economic Environment

The economy is closed with an infinite, discrete time horizon $t = 0,1,2,...$. The word "closed" implies that the interest rate is *endogenously* determined in the model. Section V considers the effects of CAL on the dynamics. In this economy, there is a continuum of dynastic families that live forever. It is assumed that the total mass is normalized to be one and there is no population growth. Each agent in a dynastic family is risk-neutral and only lives for one period (reproducing one offspring). Moreover, there is a competing financial sector comprising homogeous FIs. In this paper, it is assumed that intermediaries and households are different agents.

In this economy, there is only one type of good, which can be consumed or be made for bequest (to be explained shortly). In each period t, an identical household owns the following deterministic production technology,[5] which is non-convex (due to discontinuity):

$$f(k_t) = \begin{cases} k_t & \text{if} \quad k_t \geq q, \\ 0 & \text{if} \quad 0 \leq k_t < q, \end{cases}$$

where $k_t \geq 0$ is the investment level and the unit revenue is normalized to one. This value accrues when the investment is over the normalized fixed level, which is not too large but not too small, either ($k_t = q \in [1,2)$).[6] This fixed cost can be considered as physical capital, or alternatively, entrepreneurial human capital. Note that the output is linearly increasing after the fixed threshold level of capital. This paper assumes that each household has access to an alternative "backyard" storage technology, with a per-unit return of $\rho \geq 0$ for any input level (i.e., no fixed cost is necessary to generate a return). This technology may be a traditional technology such as small-scale agriculture. Restrictions on ρ will be introduced later in this paper. Furthermore, it is assumed that each household earns exogenous non-random revenue that is normalized to one and common to all households and is non-pecuniary (so households cannot borrow or lend a part of this income). This is a technical assumption to yield steady-state results in this non-growth model.[7]

Let $a_t \geq 0$ denote the wealth of a household in generation t (which is inherited from his/her parent at $t-1$; to be explained in Subsection Ⅲ.6). The wealth level is the only source of household heterogeneity. The distribution of wealth across households is denoted by the measure $G_t(a)$ defined on the Borel subsets of $[0, \infty)$. The initial wealth distribution $G_0(a)$ is given.

Given the inherited wealth a_t (and under the constraints explained later), a household maximizes its income, after which he/she can consume it or bequeath it to his/her child at $t+1$ (to be explained in Subsection Ⅲ.6). In this paper, a household is referred to as an *entrepreneur* when the revenue is earned using the production technology. For simplicity, it is assumed that the capital for production fully depreciates in one period. This assumption would be particularly appropriate when the capital is interpreted as human capital. In this economy there is another revenue-generating opportunity for households (other than the backyard storage technology)—namely, a one-period competitive capital market (to be explained in the next paragraph) where a per-unit gross interest rate $r_t \geq 0$ accrues when a household has saved some of his/her wealth through an FI. Thus, the opportunity cost of using capital k_t for production is $\max\{\rho, r_t\} k_t$.

The timing of decision-making in period t is as follows. In the beginning, a new agent in a household inherits wealth from the parent, which is divided into savings in backyard storage (s_t) and the remainder ($a_t - s_t$). Then, he/she decides whether to become an entrepreneur, and divides ($a_t - s_t$) into the part for production (only after becoming an entrepreneur) and the part for savings in the savings market.

2. Financial Arrangement

Although a household can save any amount of money through an FI, he/she cannot borrow any amount from an FI. In addition, he/she cannot borrow directly from other households; rather, he/she can borrow only from an FI. Otherwise, self-financing is required. These features are based on the enforcement problems and the role of FIs, which is explained later in this subsection. First, let us examine the role of FIs adopted in the present paper.

Following Holmström and Tirole (1997), the present paper assumes that the role of FIs is to *monitor* an entrepreneur in order to mitigate opportunistic behaviors. One may assume that only FIs are endowed with monitoring technology, or that households have a prohibitively costly monitoring activity due to, possibly, the lack of specialization. Monitoring is a broad concept, and there exist various types of monitoring. In the present model, monitoring is undertaken when lending occurs, and determines what the lender should do in the case of default. As such, monitoring may also include the cost of writing a contract regarding the legal measures in case of default, and/or the cost of FIs' service (via renegotiation) in such a situation. This may include sending experts to the company boards. FIs offer a fixed interest rate r_t on savings deposits, and earn revenue by utilizing the deposits to provide loans to entrepreneurs. The rate charged for a loan given by an FI is the (gross) lending rate i_t andthe lending-deposit rate spread is the return to the FI for providing the financial service. An FI will choose whether to monitor or not, as there is no requirement for such monitoring. For simplicity, it is assumed that there is no fixed cost for monitoring and that the marginal cost of monitoring (per the amount of lending) is $\gamma > 0$, which is exogenous[8] and constant over time (i.e., the monitoring technology includes constant returns to scale). This can be understood as FI's disutility of labor for monitoring, or their human capital value (both of which are assumed to be non-tradeable). It is also assumed that the monitoring cost is sunk when lending occurs (in this sense monitoring is an *ex-ante* cost so that free riding is not an issue). In addition, it is assumed that there is a large number of FIs. Perfect competition in the financial sector implies that the deposits that each FI receives are equal to the loans issued by that FI and

$$i_t = r_t + \gamma \tag{1}$$

so that FIs make zero profit in equilibrium.[9] Note that it is assumed that the possibility of FI's incentive problems do not exist, and one may think that an FI can make a credible commitment, while caring about its reputation.

However, if there were no enforcement problems, then no household would want to borrow from an FI because it would be more expensive than borrowing directly from other households.[10] To validate the existence of FIs, this paper assumes that *enforcement in the capital market is imperfect* and entrepreneurs always have the option of simply defaulting.[11] More specifically, suppose that he/she borrows b_t. When the borrower does not honor repayment $i_t b_t$, the lender cannot seize all of the entrepreneur's revenue due to imperfect enforcement. In other words, an entrepreneur can pledge only up to a fraction of his/her revenue. This amount is called *pledgeable* revenue, and the following assumption is made.

Assumption 1. *Ex-ante monitoring by an FI is necessary for the borrower's pledgeable revenue to be positive.*

Specifically, it is assumed that when an entrepreneur attempts to circumvent intermediated borrowing, the pledgeable revenue is zero[12] However, it becomes λk_t when he/she relies on intermediated borrowing. Then, Assumption 1 is expressed by $0 \leq \lambda < 1$.[13] Note that λ is less than one, indicating that FIs can improve, but not perfectly correct, the enforcement problem. Parameter λ can be understood as capturing the effectiveness of monitoring and may also be strengthened by the efficiency of the economy's legal system for protecting investors (even though the financial sector is required for the same).[14] This can be interpreted as the situation in which, upon default, the fraction $1-\lambda$ of the production revenue perishes due to the costly renegotiation process. One may also interpret this as reminiscent of the costly state verification (CSV) problem: verifiability of the project is not without cost.[15]

From the borrower's perspective, these pledgeable revenues include the cost of default, which is normally seized by the lender in such a situation.[16] Thus, if an entrepreneur circumvents intermediated borrowing, he/she cannot borrow k_t and save s_t in the backyard storage technology over the wealth level a_t. That is, he/she can borrow money within the wealth level that has been saved. In this sense, this situation is referred to *self-financing*, which is expressed by

$$r_t(k_t-a_t+s_t) \leq 0. \tag{2}$$

It is assumed that households are allowed to either self-finance or borrow all the capital from an FI. Then, the borrowing constraint under intermediated borrowing becomes

$$(r_t+\gamma)(k_t-a_t+s_t) \leq \lambda k_t. \tag{3}$$

Note that in equilibrium default *never* occurs since this inequality must hold.[17] Finally, limited liability is assumed: the borrower's payment cannot exceed his/her total income.

3. Optimal Investment Decisions

Now let us focus on the revenue structure of households. Consider a household with a_t and suppose that he/she saves $s_t \leq a_t$ in the backyard storage technology, which does not occur in equilibrium in the following analysis. If he/she invests k_t, and when self-financing occurs (that is, $k_t \leq a_t-s_t$), the total income becomes $1+\rho s_t+F(k_t)+r_t(a_t-s_t)-\max\{\rho, r_t\}k_t$, Thus, this can be expressed as

$$\begin{cases} 1-(r_t-\rho)s_t+\min\{1-\rho, 1-r_t\}k_t+r_t a_t & \text{if } k_t \geq q, \\ 1-(r_t-\rho)s_t+r_t a_t-\max\{\rho, r_t\}k_t & \text{if } 0 \leq k_t < q. \end{cases}$$

Conversely, when he/she borrows from an FI, it becomes $1+\rho s_t+F(k_t)+r_t(a_t-s_t)-\max\{\rho, i_t\}k_t$ (with k_t satisfying constraint (3)). Thus, this can be expressed as

$$\begin{cases} 1-(r_t-\rho)s_t+\min\{1-\rho, 1-i_t\}k_t+r_t a_t & \text{if } k_t \geq q, \\ 1-(r_t-\rho)s_t+r_t a_t-\max\{\rho, i_t\}k_t & \text{if } 0 \leq k_t < q. \end{cases}$$

At this point, it is implicitly assumed that a household saves a_t-s_t on deposits in an FI, after which he/she self-finances or borrows from an FI. It has been shown that given the investment level k_t only households with wealth satisfying $a_t-s_t \geq k_t$ can self-finance an investment to become an entrepreneur. Similarly, only a household with wealth in the storage technology a_t-s_t that is greater than or equal to $[1-\lambda/(r_t+\gamma)]k_t$ can borrow capital from an FI. Since the minimum investment level for becoming an entrepreneur is $k_t = q$, only a household with $a_t-s_t \geq q$ can become an entrepreneur by self-financing. Similarly, a household with net wealth a_t-s_t that is greater than or equal to $[1-\lambda/(r_t+\gamma)]q$ can borrow capital from an FI.

Now, in order to analyze the effects of the enforcement problem, the following assumption is made.

Assumption 2. $\lambda > \gamma+\rho$.

This assumption states that the monitoring cost is not very high, and hence, monitoring is socially desirable given the enforcement problem. It can also be stated that the monitoring effect is high enough, and that the backyard storage technology is not that productive. Here, this assumption implies $\beta\rho < 1$.

Note that if there were no enforcement problems, then no households would be under borrowing constraints, and there would be no financial sector. Consequently, the equilibrium interest rate would be $r_t = 1$, which is the only possible case. This is because if $r_t > 1$, all households, irrespective of wealth a_t, would want to become a lender, whereas, if $r_t < 1$, all households would want to become an entrepreneur, both of which imply that the capital market would not clear. Furthermore, note that no households would want to use the backyard storage technology since $\rho < 1$. In this case, all households are indifferent between borrowing and lending, and obtain $1+a_t$. In addition, the division of wealth in the economy between lending and borrowing is indeterminate. Therefore, the GDP in this economy is one (the identical revenue times the population) for any period.[18] These arguments can be summarized in the following proposition.

Proposition 1. *There is no income inequality across households if there are no enforcement problems (the Gini index for earnings inequality is zero). That is, irrespective of the wealth level, all households earn a net inflow of one in any period.*

It is important to note that there can be *income* inequality in period t. In this regard, given the wealth level a_t, a household obtains $1+a_t$. However, as shown in the next section, this income inequality and wealth inequality in the perfect world disappears in the long run. This is also a consequence of the assumption that there is no talent or endowment heterogeneity.

On the contrary, if there were no financial intermediary sector in this imperfect world, income inequality would arise. It is easy to see that a household cannot use the backyard storage technology if his/her wealth level a_t is less than q, with the per-unit income ρ (total revenue is $1+\rho a_t$), whereas the household will earn one if the wealth level is $a_t \geq q$ (total revenue is $1+a_t$). In this case, the aggregate earnings in period t is $\rho G_t(q)+(1-G_t(q))=1-(1-\rho)G_t(q)$. The Gini index for income inequality is calculated as

$$\begin{aligned} IncomeGINI_t^{NFI} &= 1-\frac{\rho[G_t(q)]^2}{1-(1-\rho)G_t(q)} \\ &\quad -\left(\frac{\rho G_t(q)}{1-(1-\rho)G_t(q)}+1\right)[1-G_t(q)] \\ &= \frac{(1-\rho)G_t(q)(1-G_t(q))}{1-(1-\rho)G_t(q)}, \end{aligned}$$

from which we have $\partial IncomeGINI_t^{NFI}/\partial\rho < 0$. Here, the more efficient the storage technology, the less severe the income inequality. In addition, due to an increase in $G_t(q)$, the sign of $\partial IncomeGINI_t^{NFI}/\partial G_t(q)$ is ambiguous, which can imply either an upward or downward shift. In the next section, wealth inequality in this case will be considered.

Let us now return to the world with imperfect enforcement and financial intermediation to investigate the effects of financial intermediation on equality. First, the following lemma is obtained regarding the lower bound of the equilibrium interest rate.

Lemma 1. *In equilibrium, it must be the case that* $r_t > \lambda-\gamma$.

Proof. Suppose that $r_t \leq \lambda-\gamma$ occurs. Then, borrowing constraint (3) is no longer binding for any household so that every household with $a_t - s_t < [1-\lambda/r_t]q$ demands infinite capital, thus meaning that there is excess demand of capital in this economy. *QED*

Base on Lemman 1, the following lemma is obtained, which states that savings in the backyard storage technology does not occur in equilibrium.

Lemma 2. *In equilibrium, no households use the storage technology. That is, for any household,* $s_t = 0$ *for any* t.

Proof. Suppose $s_t > 0$ for some t. Then, by reducing some amount of s_t and by placing that amount in the deposits with an FI, a household obtains, by Lemma 1, a per-unit gain $r_t > \lambda-\gamma$, which is greater than the backyard storage technology's return ρ by Assumption 2. *QED*

Since a household does not choose $s_t = 0$, it has been shown that given the investment level k_t, only a household with $a_t \geq q$ can become an entrepreneur by self-financing (as explained later, he/she may optimally borrow from an FI). Similarly, a household with wealth a_t that is greater than or equal to $[1-\lambda/(r_t+\gamma)]q$ $(\equiv \underline{a}(r_t))$ can borrow capital from an FI (as also explained later, he/she may optimally self-finances when the wealth is sufficiently high). Note that $\underline{a}(r_t) < q$ for any $r_t > \lambda-\gamma$.

Since r_t is endogenously determined, the threshold $\underline{a}(r_t)$ is also endogenously determined as a function of r_t, whereas q is not. Simple algebra shows that

$$\frac{\partial \underline{a}(r_t)}{\partial r_t} = \frac{\lambda q}{(r_t+\lambda)^2} > 0, \quad \frac{\partial^2 \underline{a}(r_t)}{\partial r_t^2} = \frac{-2\lambda q}{(r_t+\lambda)^3} < 0,$$

which means that $\underline{a}(\cdot)$ is strictly increasing and concave. This fact will be utilized in the next section. The first inequality shows that the higher the interest rate, the tighter the borrowing constraint (3). The latter relationship implies that $\underline{a}(r_t)$ is concave with respect to r_t. Then, the following lemma is obtained:

Lemma 3. *For all $r_t > \lambda-\gamma$, $\underline{a}(r_t) > 0$. That is, if $G_t(\underline{a}(r_t)) > 0$, then households (with $a_t < \underline{a}(r_t)$) that cannot obtain any external finance become net lenders.*

Proof. It is immediate from $\lim_{r_t \downarrow \lambda-\gamma} \underline{a}(r_t) = 0$ and $\partial \underline{a}(r_t)/\partial r_t > 0$. *QED*

It is also verified that

$$\frac{\partial \underline{a}(r_t)}{\partial q} = 1-\frac{\lambda}{r_t+\gamma} > 0, \quad \frac{\partial \underline{a}(r_t)}{\partial \lambda} = \frac{-q}{r_t+\gamma} < 0,$$

$$\frac{\partial \underline{a}(r_t)}{\partial \lambda} = \frac{\lambda q}{(r_t+\gamma)} > 0,$$

which means that the more the necessary amount of the fixed capital, the less effective the monitoring, and the more costly the monitoring, the more severe the threshold $\underline{a}(r_t)$.

If the interest rate is exogenous from the initial period, then $\underline{a}(r_t)$ is always a constant. Thus, the initial wealth distribution G_0 completely determines who can be a borrower and who remains a lender. In addition, there is no social mobility across entrepreneurs and non-entrepreneurs since there is no jumping process such as uncertainty in the present model. However, if the interest rate is endogenous, the initial wealth distribution is not the sole determinant. Furthermore, if CAL occurs in some period t (which means that all agents in this economy consider the interest rate as exogenously given), the threshold becomes a constant from that period on. Conversely, suppose that the government stops regulation of the interest rate in some period t. Before that period, the interest rate is exogenous, and it now becomes endogenous.

The following lemma determines the upper bound of the equilibrium interest rate.

Lemma 4. *In equilibrium, it must be the case that $r_t \leq 1-\gamma$.*

Proof. Suppose that $r_t > 1-\gamma$ holds. Then, a household with $a_t < q$ does not become an entrepreneur. This is because he/she can earn $1+r_t a_t$ by choosing $k_t = 0$, whereas the income is $1+(1-\gamma-r_t)k_t + r_t a_t$, which is less than $1+r_t a_t$, when he/she becomes an entrepreneur. As intermediated borrowing is relatively costly compared to the production benefit, there are no households which borrow from an FI, thus meaning that there is excess supply of capital in this economy. *QED*

Then, the following lemma is obtained regarding the optimal amount of capital, given the financial decision (self-finance or intermediated finance).

Lemma 5. *In equilibrium, an entrepreneurs chooses $k_t = a_t$ when he/she self-finances, and $k_t = \bar{k}(a_t, r_t) \equiv a_t/(1-\lambda/[r_t+\gamma]) > a_t$ when he/she borrows from an FI.*

Proof. If $r_t < 1-\gamma$, the entrepreneur's income is strictly increasing in k_t irrespective of whether he/she self-finances or borrows from an FI. Thus, he/she wants to borrow up to the level where the borrowing constraints (2) or (3) are binding. If $r_t = 1-\gamma$, households with $a_t \in [\underline{a}(1-\gamma), q)$ are indifferent between becoming an entrepreneur and choosing $k_t = 0$. *QED*

Note in this case that if the optimal investment level is allowed to be less than the wealth, the entrepreneur may not want to save money if he/she is qualified to borrow only from a bank. In this paper, formulation on the production technology excludes this situation.

4. Optimal Financial Decisions and Income Inequality

First, it is apparent that when an entrepreneur has no access to capital, his/her income is strictly smaller than when he/she can borrow from an FI. This is because the total income when he/she self-finances is $1+a_t$, whereas the total income when he/she borrows from an FI is

$$1+\frac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)}a_t,$$

and for $r_t < 1-\gamma$, we have $r_t < [1-\lambda r_t/(r_t+\gamma)-\gamma]/[1-\lambda/(r_t+\gamma)]$. Note here that an entrepreneur who self-finances earns the same amount of income that he/she could earn in the perfect world.

Now, it can be verified that $[1-\lambda r_t/(r_t+\gamma)]/[1-\lambda/(r_t+\gamma)] > r_t \Leftrightarrow r_t < 1-\gamma$, which is assured by Assumption 2. Thus, both incomes are strictly increasing functions of a_t. By considering the two slopes 1 and $\{[1-\lambda r_t/(r_t+\gamma)]-\gamma\}/[1-\lambda/(r_t+\gamma)]$, the following lemma can be obtained.

Lemma 6. *For any $\lambda \in [0,1)$ and any $\lambda \in (0, \lambda-\rho)$ there exists a unique $\tilde{r} \in (\lambda-\gamma, 1-\gamma)$ such that for $r_t \in (\lambda-\gamma, \tilde{r}]$,*

$$[1-\lambda r_t/(r_t+\gamma)-\gamma]/[1-\lambda/(r_t+\gamma)] \geq 1$$

holds, where equality holds if and only if $r_t = \tilde{r}$. For $r_t \in (\tilde{r}, 1-\gamma)$, it is

$$[1-\lambda r_t/(r_t+\gamma)-\gamma]/[1-\lambda/(r_t+\gamma)] < 1.$$

Proof. The slope of the entrepreneur's income when he/she borrows from an FI is continuous, differentiable and strictly decreasing in $r_t \in (\lambda-\gamma, 1-\gamma)$ because

$$\frac{\partial}{\partial r_t}\left(\frac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)}\right) = -\frac{\lambda\gamma}{(r_t+\gamma)^2}\left(1-\frac{\lambda}{r_t+\gamma}\right) - \frac{\lambda}{(r_t+\gamma)^2}\left(1-\frac{\lambda r_t}{r_t+\gamma}-\gamma\right) < 0.$$

Now, it is verified that

$$1 > 1-\gamma = \lim_{r_t \uparrow 1-\gamma}\frac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)}$$

and

$$\lim_{r_t \downarrow \lambda-\gamma}\frac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)} = \infty > 1.$$

Thus, the statement in the lemma holds as per the intermediate value theorem. *QED*

Indeed, the following explicit solution for $\tilde{r}$ is obtained: $\tilde{r} = \tilde{r}(\lambda,\gamma) = (\lambda-\gamma^2)/(\lambda+\gamma)$. This lemma shows that for a sufficiently low interest rate, there are no self-financing entrepreneurs in the economy. This result might at first seem odd, but the reason is clear: an entrepreneur can borrow more under intermediated lending than direct lending, and this effect becomes greater as the interest rate becomes lower. Summarizing the argument thus far, the following proposition is obtained:

Proposition 2. *Suppose that (a) $\lambda-\gamma < r_t \leq \tilde{r}$. Then, a household does not become an entrepreneur if his/her wealth is $a_t \in [0, \underline{a}(r_t))$, but he/she borrows from an FI to become an entrepreneur if the wealth is $a_t \geq \underline{a}(r_t)$. Next, suppose that (b) $\tilde{r} < r_t \leq 1-\gamma$. Then, a household does not become an entrepreneur if his/her wealth is $a_t \in [0, \underline{a}(r_t))$, but he/she borrows from an FI if the wealth is $a_t \in [\underline{a}(r_t), q)$ or self-fi-nances if $a_t \geq q$.*

Based on this proposition, one can derive what type of income inequality arises in this economy, which is summarized in the following corollary.

Corollary 1. *In equilibrium, a household with $a_t < \underline{a}(r_t)$ earns the interest proceeds r_t from his/*

her savings, whereas one with $a_t \geq \underline{a}(r_t)$ earns the return one, which is greater than r_t from the project.

Note that the total revenue of an entrepreneur depends on whether he/she has self-financed or borrowed from an FI, and his/her inherited wealth level is a_t. As in the case of no financial intermediation, the Gini index for income inequality can be calculated as

$$IncomeGINI_t(r_t) = \frac{(1-r_t)G_t(\underline{a}(r_t))[1-G_t(\underline{a}(r_t))]}{1-(1-r_t)G_t(\underline{a}(r_t))}.$$

If $G_t(q) > G_t(\underline{a}(r_t))$ and $G_t(q)(1-G_t(q)) > G_t(\underline{a}(r_t))[1-G_t(\underline{a}(r_t))]$, then $Income\ GINI_t^{NFI} > Income\ GINI_t(r_t)$.

5. Existence and Uniqueness of the Equilibrium Interest Rate for a Fixed Wealth Distribution

Let us now verify the existence of the equilibrium interest rate r_t in period t, given the wealth distribution G_t. First, note the demand side of capital (i.e. the economy's total investment). In the case of $\tilde{r} < r_t < 1-\gamma$, the aggregate demand for intermediated capital is

$$D^I(r_t) = \frac{1}{1-\lambda/(r_t+\gamma)} \int_{\underline{a}(r_t)}^{q} adG_t(a),$$

whereas the aggregate amount of self-financing is

$$D^S(r_t) = \int_q^\infty adG_t(a)$$

so that the aggregate demand for capital is $D(r_t) = D^I(r_t) + D^S(r_t)$ for $r_t \in (\tilde{r}, 1-\gamma)$. Note that for a fixed G_t, $D^S(r_t)$ is indeed a constant. For this range, $D(r_t)$ is continuous, and decreasing in r_t. However, it is not strictly decreasing since a positive mass can exist on $\underline{a}(r_t)$ or on q. These properties also hold for the case of $\lambda-\gamma < r_t < \tilde{r}$, where

$$D(r_t) = \frac{1}{1-\lambda/(r_t+\gamma)} \int_{\underline{a}(r_t)}^{\infty} adG_t(a)$$

since there is no self-financing.

Now consider the case of $r_t = \tilde{r}$. In this case, households with $a_t \geq \underline{a}(\tilde{r})$ are indifferent between self-financing and borrowing from an FI. Thus, the demand for intermediated capital is between zero and

$$\frac{1}{1-\lambda/(\tilde{r}+\gamma)} \int_{\underline{a}(\tilde{r})}^{\infty} adG_t(a)$$

so that the aggregate demand for capital $D(r_t)$ is continuous at $r_t = \tilde{r}$.

Now let us consider the case of $r_t = 1-\gamma$. As explained earlier, households with $a_t \in [\underline{a}(1-\gamma), q)$ are indifferent between becoming an entrepreneur (by borrowing from an FI) and choosing $k_t = 0$. Due to the borrowing constraint (3), household $a_t \in [\underline{a}(1-\gamma), q)$ can borrow up to $k_t = a_t$. Thus, the demand for intermediated capital when $r_t = 1-\gamma$ is a correspondence:

$$D(1-\gamma) = \left[0, \frac{1}{1-\lambda} \int_{1-\lambda}^{q} adG_t(a)\right].$$

It can be verified that $D(r_t)$ is continuous at $r_t = 1-\gamma$. Overall, the aggregate demand is a continuous function of r_t in the relevant range.

Finally. let us focus on the supply side of capital. For any $r_t \in (\lambda-\gamma, 1-\gamma]$, the aggregate supply of capital (i.e., the economy's total savings) is

$$K_t = \int_0^\infty adG_t(a),$$

where $\int$ denotes the Lebesgue integral. Note that it does not depend on r_t. However, it is endogenously determined as it depends on G_t. The equilibrium interest rate in period t is determined by the usual market clearing condition: $K_t = D(r_t)$. Thus, it is apprarent that the following proposition holds.

Proposition 3. *The equilibrium interest rate $r_t \in (\lambda-\gamma, 1-\gamma]$ exists and is unique.*

6. Equilibrium Dynamics of Wealth Distribution and the Interest Rate

Given the initial wealth distribution G_0, the market clearing condition also has a role to recursively determine the dynamics of the equilibrium interest rate $\{r_t\}_{t=0}^\infty$ together with the wealth dynamics $\{G_t\}_{t=0}^\infty$ (caused by the dynastic motivation explained later). It is assumed that expectations are fully rational or players have perfect foresight, as there is no uncertainty. First, take any period t, and suppose that $\tilde{r} < r_t < 1-\gamma$. Then, the wealth dynamics in this case is described by

$$a_{t+1} = a_{t+1}(a_t; r_t) = \begin{cases} \beta(1+a_t) & \text{for } a_t \geq q, \\ \beta\left[1+\dfrac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)}a_t\right] & \text{for } a_t \in [\underline{a}(r_t), q), \\ \beta(1+r_t a_t) & \text{for } a_t \in [0, \underline{a}(r_t)), \end{cases}$$

where $\beta \in (0,1)$ is the parameter that shows how a household cares about the next generation. For example, a household consumes a fraction $(1-\beta)$ of income, and bequeaths fraction β to his/her child.[19] One may interpret this as an exogenous parameter of the saving rate, as in the Solow growth model.

Similarly, for $r_t \in (\lambda-\gamma, \tilde{r}]$ the wealth dynamics is described by

$$a_{t+1} = a_{t+1}(a_t; r_t) = \begin{cases} \beta\left[1+\dfrac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)}a_t\right] & \text{for } a_t \geq \underline{a}(r_t), \\ \beta(1+r_t a_t) & \text{for } a_t \in [0, \underline{a}(r_t)). \end{cases}$$

Thus, for any equilibrium interest rate r_t, the wealth transition a_t to a_{t+1} is obtained. The wealth distribution dynamics $\{G_t\}_{t=0}^{\infty}$ is determined by the following law of motion:

$$D_t = \int a_t(a_{t-1}; r_{t-1})\, dG_{t-1}.$$

Instead of investigating the dynamics per se, let us focus on the steady-state in order to study the financial pattern that emerges as well as the wealth distribution in the economy. Note that the limit wealth distribution $G_\infty(a)$ will have positive mass on the fixed points of mapping $a_{t+1} = a_{t+1}(a_t; r_\infty)$, where r_∞ is the limit interest rate. Thus, the goal here is to analyze the properties of these fixed points. As shown in the next section, the limit wealth distribution G_∞ includes a finite number of mass points. Though it does not resemble any conceivable wealth distribution in the real world, one can obtain interesting insight into the relationship between finance and inequality. For example, the steady-state Gini index of wealth can be analytically computed.

Ⅳ. Steady-State Analysis

This section examines the relationship between inequality and financial intermediation in the steady state. First, it is apparent that if there were no enforcement problems, one would have $r_\infty = 1$ from the beginning, and in the steady state all households would have the same wealth level $a_\infty = \beta/(1-\beta)$, provided that $\beta < 1$. In each period, the income of all households is $1+\beta/(1-\beta) = 1/(1-\beta)$. This is because for any period t, the wealth dynamics is described by $a_{t+1} = \beta(1+a_t)$, and the steady-state wealth level is the fixed point of this mapping. Since the households are not heterogenous with respect to production ability or consumption preferences, the difference in the wealth level eventually vanishes. Furthermore, there would be no room for FIs from the beginning, and there would be no income or wealth inequality in the long run. These arguments are summarized in the following proposition.

Proposition 4. *There is no income or wealth inequality across households in the long run if there are no enforcement problems (the Gini indices for income and wealth inequalities are zero in the long run). That is, in the long run, the wealth level of any household converges to $\beta/(1-\beta)$, and the income level of any household converges to $1/(1-\beta)$.*

1. Steady-State without FIs

Before analyzing the roles of financial intermediation, let us examine the steady state without FIs. Recall that in each t households with $a_t < q$ cannot use the backyard storage technology, whereas those with $a_t \geq q$ can invest in the project. Thus, the wealth dynamics is described by

$$a_{t+1} = \begin{cases} \beta(1+a_t) & \text{for } a_t \geq q, \\ \beta(1+\rho a_t) & \text{for } a_t \in [0, q). \end{cases}$$

It is assumed that the fixed cost for production and the the return of the storage technology are not large. More specifically, let us make the following assumption to ensure the existence of the steady state in which both rich households (entrepreneurs) and poor households (lenders) coexist in the long run. Then the wealth dynamics can be described as seen in Figure 1.

Assumption 3. $2\beta \geq q > \beta(1+\rho)$.

Note that as $q < 2$ (see Subsection Ⅲ.1), the first part of the aforementioned assumption imposes a restriction on the range of the warm-glow parameter: $\beta \in [q/2, 1)$. Also note that as $q \geq 1$ it must be the case that $\beta \geq 1/2$. Together with $\beta\rho < 1$ (from Assumption 2), $0 \leq \rho < \min\{\lambda-\gamma, 1/\beta, q/\beta-1\}$, and indeed $\sup_{\beta,q}\rho = \lambda-\gamma$.

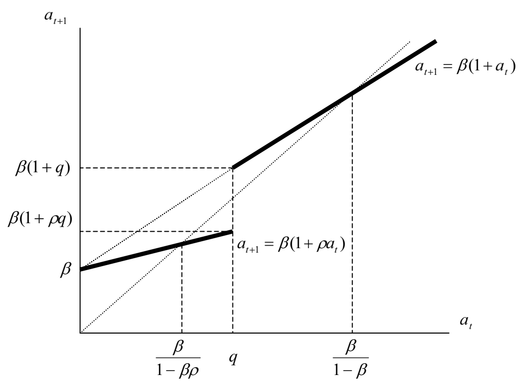

Figure 1: Wealth dynamics when there are no FIs.

Two fixed points of the mapping are observed: $\beta/(1-\beta)$ for richer households, and $\beta/(1-\beta\rho)$ for poorer households. Notice that the initial wealth distribution G_0 completely determines the future of a household: the wealth of those with $a_0 < q$ (the fraction of which is $G_0(q)$) converges to $\beta/(1-\beta\rho)$, and that of those with $a_0 \geq q$ (the fraction of which is $1-G_0(q)$) converges to to $\beta/(1-\beta)$. Unless $G_0(q) = 0$ or $G_0(q) = 1$, inequality necessarily arises.

Let the aggregate national wealth and the Gini index for wealth inequality in the steady state by denoted by NW_∞^{NFI} and by $Wealth\,GINI_\infty^{NFI}$, respectively. Then, it is verified that (note that $\beta\rho < 1$ from Assumption 2):

$$NW_\infty^{NFI} = \beta\frac{(1-\beta)G_0(q)+(1-\beta\rho)[1-G_0(q)]}{(1-\beta)(1-\beta\rho)}$$

and

$$WealthGINI_\infty^{NFI} = \frac{\beta(1-\rho)G_0(q)[1-G_0(q)]}{(1-\beta)G_0(q)+(1-\beta\rho)[1-G_0(q)]} = \frac{\beta^2(1-\rho)G_0(q)[1-G_0(q)]}{(1-\beta)(1-\beta\rho)NW_\infty^{NFI}}.$$

As expected, it is shown that $\partial NW_\infty^{NFI}/\partial\rho > 0$. Also shown is $\partial NW_\infty^{NFI}/\partial G_0(q) < 0$, although the sign of $\partial WealthGINI_\infty^{NFI}/\partial G_0(q)$ is indeterminate. If $G_0(q) > 1/2$, then $\partial WealthGINI_\infty^{NFI}/\partial G_0(q) > 0$, and for $\partial WealthGINI_\infty^{NFI}/\partial G_0(q) < 0$ to hold, it is necessary that $G_0(q) < 1/2$. This means that if there are many agents whose initial wealth is smaller than q (so that $G_0(q) > 1/2$), then an increase in $G_0(q)$ worsens the inequality. In order to examine the operation of this benchmark case, consider the following numerical example.

Example 1. Suppose that $\beta = 3/4$, $\rho = 4/15$, and $q = 1$. Then, we have $NW_\infty^{NFI}(\beta = 3/4) = 3-33G_0(q)/16$ and $WealthGINI_\infty^{NFI}(\beta = 3/4) = 33G_0(q)[1-G_0(q)]/16NW_\infty^{NFI}(\beta=3/4)$. Now, suppose that $\beta = 7/8$ (and $\rho = 4/15$, $q = 1$). In this case, $NW_\infty^{NFI}(\beta = 7/8) = 7-77G_0(q)/120$ and $Wealth\,GINI_\infty^{NFI}(\beta = 7/8) = 539G_0(q)[1-G_0(q)]/92NW_\infty^{NFI}(\beta = 7/8)$. We can verify that $7-77\,G_0(q)/120 > 3-33G_0(q)/16$ and $Wealth\,GINI_\infty^{NFI}(\beta = 3/4) > Wealth\,GINI_\infty^{NFI}(\beta = 7/8)$ for any $G_0(q)\epsilon(0,1)$. Thus, in this case, for higher β, the national wealth and the less severe the inequality is higher.

2. Financial Intermediation and Inequality

Now, let us consider the role of financial intermediation in the wealth dynamics. First, recall that there are five parameters: (i) $\lambda \in [0,1)$, the effect of monitoring; (ii) $\gamma > 0$, the unit cost of monitoring; (iii) β, the "warm-glow" parameter; (iv) q, the fixed cost for production, and (v) ρ, the per-unit return of the backyard storage technology. As shown later,

ρ does not appear in the case of active FIs due to Assumption 2. It is verified that the original Kuznets hypothesis does not hold as inequality persists in any steady state equilibrium for any initial distribution G_0. This is an important feature, because many existing models cannot exclude perfect equality from the equilibria.[20] For comparison with the case with no FIs, let us maintain the first part of Assumption 3 (i.e., $2\beta \geq q$) as well as Assumption 2 (i.e., $\lambda > \gamma + \rho$).

For notational convenience, let us define the following function:

$$H(r_\infty) = 1 - \frac{\lambda r_\infty}{r_\infty + \gamma} - \gamma,$$

which is equal to $(1-\gamma-r_\infty) + r_\infty \underline{a}(r_\infty)/q$ so that $H(r_\infty) > 0$ because $1-\gamma > r_\infty$.

There are three possible fixed points of the mapping $a_{t+1} = a_{t+1}(a_t; r_\infty)$,

$$\begin{cases} a^*(r_\infty) = \dfrac{\beta}{1-\beta r_\infty}, \\ a^{**}(r_\infty) = \left(1 - \beta q \dfrac{H(r_\infty)}{\underline{a}(r_\infty)}\right)^{-1} \beta, \text{ and} \\ a^{***}_\infty = \dfrac{\beta}{1-\beta}, \end{cases}$$

where all the households that cannot borrow in the steady-state have wealth $a^*(r_\infty)$, those that borrow from an FI have wealth $a^{**}(r_\infty)$, and those that self-finance have wealth a^{***}_∞.

Now consider the relationship among $a^*(r_\infty)$, $a^{**}(r_\infty)$, and a^{***}_∞. Note that $a^{***}_\infty > a^*(r_\infty)$ for any $r_\infty \in (\lambda-\gamma, 1-\gamma]$, $a^*(r_\infty) \gtreqless a^{**}(r_\infty) \Leftrightarrow r_\infty \gtreqless 1-\gamma$, and $a^{***}_\infty \gtreqless a^{**}(r_\infty) \Leftrightarrow r_\infty \gtreqless \tilde{r}(\lambda, \gamma)$. Also note that the domain of these functions is extended to $(\lambda-\gamma, 1]$ for the purpose of drawing the lines in the figure, although r_∞ is indeed no greater than $1-\gamma$. For all $r_\infty \in (\lambda-\gamma,\ 1-\gamma]$, it is verified that

$$\begin{cases} \dfrac{\partial a^*(r_\infty)}{\partial r_\infty} = \dfrac{\beta^2}{(1-\beta r_\infty)^2} > 0 \\ \dfrac{\partial^2 a^*(r_\infty)}{\partial r_\infty^2} = \dfrac{2\beta^3}{(1-\beta r_\infty)^3} > 0, \end{cases}$$

so that $a^*(r_\infty)$ is strictly increasing and convex. Note that $a^*(r_\infty)$ is bounded above zero since $\lim_{r_\infty \downarrow \lambda-\gamma} a^*(r_\infty) = \beta/[1-\beta(\lambda-\gamma)]$ and $\lambda-\gamma<1$. It is also verified that

$$\frac{\partial a^{**}(r_\infty)}{\partial r_\infty} = -\beta^2 q \lambda \frac{qH(r_\infty) + \gamma \underline{a}(r_\infty)}{(r_\infty+\gamma)^2 [\underline{a}(r_\infty) - \beta q H(r_\infty)]^2} < 0$$

so that $a^{**}(r_\infty)$ is strictly decreasing. It is apparent that $\lim_{r_\infty \downarrow \lambda-\gamma} a^{**}(r_\infty) = \infty$, which is based on the fact that for any $a_t > 0$, $\bar{k}(a_t, r_t) \to \infty$ as $r_\infty \downarrow \lambda-\gamma$. In this case, entrepreneurs want to borrow infinitely if there is no borrowing limit due to the linearity of the production technology. This also implies that the limit interest rate is never equal to $\lambda-\gamma$ because that would be inconsistent with the definition of steady state. As we have $\lambda-\gamma < r_t \leq 1-\gamma$ in any period t, we know that $\lambda-\gamma < r_\infty \leq 1-\gamma$ in any steady state. Figure 2, a graphical summary of the arguments thus far, is useful for investigating the existence of steady-state equilibria in the following analysis. From this figure, the following proposition is obtained.

Proposition 5. *Perfect equality never arises and financial intermediation never disappears in any steady state unless $G_0(q) = 1$ or $G_0(q) = 0$.*

This result stands in contrast to the existing literature that allows perfect equality to arise as one of the possible equilibria. In order to understand the reason why this result holds in our model, note that perfect equality requires $r_\infty = 1$. In this case, the wealth dynamics should be $a_{t+1} = \beta(1+a_t)$ for all households, which means that provided that $\beta/(1-\beta) \geq q$, all households become an entrepreneur and possess the same amount of wealth, $\beta/(1-\beta)$. In this case, however, all entrepreneurs should rely on self-financing so that no FIs survive, which means that the capital market does not clear.

Next, let us observe how a higher interest rate benefits net lenders. This is because given that they cannot borrow from anywhere the only source of poor households' income is lending. Note also that

$$\frac{\partial a^{**}(r_\infty)}{\partial \lambda} = \frac{\beta^2 q^2 (1-\gamma-r_\infty)}{[\underline{a}(r_\infty) - \beta q H(r_\infty)]^2 (r_\infty+\gamma)} > 0,$$

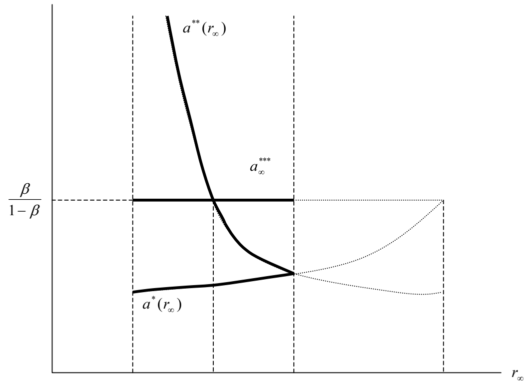

Figure 2: Relationship among $a^*(r_\infty)$, $a^{**}(r_\infty)$ and a_∞^{***}.

which means that the better the loan enforcement, the greater the entrepreneur's wealth, whereas a change in λ does not affect the lender's income. Recall that *given* the wealth distribution a change in λ alleviates inequality by mitigating the borrowing constraint since $\partial \underline{a}(r_\infty)/\partial\lambda = -1/(r_\infty+\gamma) < 0$.

The next subsection examines a steady-state equilibrium and considers the implication for inequality in the economy.

(1) Equilibria in which All Financing is Intermediated

First, let us consider the case of $\lambda-\gamma < r_\infty \leq \tilde{r}$. For the steady-state to exist, one requires

$$\beta q \frac{H(r_\infty)}{\underline{a}(r_\infty)} < 1,$$

which can be written as

$$r_\infty > \frac{\beta\gamma(1-\gamma)+\lambda-\gamma}{1-\beta+\beta(\lambda+\gamma)} \equiv \hat{r}(\lambda,\beta,\gamma).$$

Otherwise, there is no steady-state since the wealth of entrepreneurs does not converge to a certain level. If this is not the case, there are two fixed points, a_∞^* and a_∞^{**}, with there being no capital market if the following is satisfied:

$$a^*(r_\infty) < \underline{a}(r_\infty) \leq a^{**}(r_\infty).$$

Here, it can be verified that $\hat{r}(\lambda,\beta,\gamma) < \tilde{r}(\lambda,\gamma)$ as long as $\beta < 1$. To ensure the existence of steady-state equilibria with $r_\infty \in (\lambda-\gamma, \tilde{r}]$ (that is, for some $r_\infty \in (\lambda-\gamma, \tilde{r}]$ that satisfies $a^*(r_\infty) < \underline{a}(r_\infty) \leq a^{**}(r_\infty)$ to exist), the following condition is necessary:

$$\underline{a}(\hat{r}(\lambda,\beta,\gamma)) < a^{**}(\hat{r}(\lambda,\beta,\gamma)).$$

This condition always holds as long as $\beta > 0$; it can be verified by noting that this condition is equivalent to $\underline{a}(\hat{r}) - \beta q H(\hat{r}) < \beta$ and that $\lambda/(\hat{r}+\gamma) = [1-\beta+\beta(\lambda+\gamma)]/(1+\beta\gamma)$ holds.

We have two cases for $r_\infty \in (\hat{r}, \tilde{r}]$ that consist of a steady state equilibrium: (i) when $\underline{a}(\tilde{r}(\lambda,\gamma)) > a^*(\tilde{r}(\lambda,\gamma))$ holds, and (ii) when $\underline{a}(\tilde{r}) \leq a^*(\tilde{r})$ and $\hat{r} < r_L^+ \leq \tilde{r}$, where r_L^+ is the larger solution of $a^*(r_\infty) = \underline{a}(r_\infty)$ if any. Note that $a^*(r_\infty) = \underline{a}(r_\infty)$ is equivalent to $g(r_\infty) \equiv \beta r_\infty^2 - [1-\beta/q+\beta(\lambda-\gamma)]r_\infty + \lambda - \gamma(1-\beta/q) = 0$, which has at most two solutions. Note that $a^*(r_\infty) > \underline{a}(r_\infty) \Leftrightarrow g(r_\infty) > 0$.

In case (i), the equilibrium limit interest rate is $r_\infty \in (\max[\hat{r}, r_L^-], \min[r_H^+, \tilde{r}]]$, where r_L^- is the smaller solution of $g(r_\infty) = 0$ and r_H^+ is a solution of $a^{**}(r_\infty) = \underline{a}(r_\infty)$, which is unique since $a^{**}(\cdot)$ is strictly decreasing and $\underline{a}(\cdot)$ is strictly increasing. Note that if $\underline{a}(\tilde{r}) > a^*(\tilde{r})$, then such an $r_L^- \in (\lambda-\gamma, \tilde{r})$ exists as per the intermediate value theorem because and $\lim_{r_t \downarrow \lambda-\gamma} \underline{a}(r_t) = 0$, $\lim_{r_\infty \downarrow \lambda-\gamma} a^*(r_\infty)$ and the monotonicities of $\underline{a}(\cdot)$ and of $a^*(\cdot)$. It is verified that $r_H^+ \in (\tilde{r}, 1/\beta)$ exists, as $\underline{a}(\cdot)$ is bounded below q and $\lim_{r_\infty \uparrow 1/\beta} a^*(r_\infty) = \infty$. Now, $\underline{a}(\tilde{r}) > a^*(\tilde{r})$ is equivalent to $f(\lambda) \equiv (1-\beta)\lambda^2 + [\beta\gamma^2 + (1+\beta/q)\gamma + (1+1/q)\beta - 1]\lambda - \gamma[1-\beta/q+\beta\gamma(1-1/q)] < 0$.

Here, it is verified that $f(1) = \beta(1+\gamma)^2/q > 0$, $f'(1) = (1-\beta)+\gamma(\beta\gamma+\beta/q+1)+\beta/q > 0$, and $f(\gamma) = \gamma[\beta\gamma^2 + (1+2/q)\beta - 2(1-1/q)\beta\gamma - 2(1-\gamma)]$. Thus, by allowing λ^+ and λ^- be defined by the larger and smaller solution of $f(\lambda) = 0$, respectively,[21] one can verify that $f(\lambda) < 0$ if and only if $\max[\lambda^-, \gamma] < \lambda < \lambda^+$.

If case (ii) occurs, the equilibrium limit interest rate is $r_\infty \epsilon (\max[\hat{r}, r_L^-], r_L^+]$. For $r_L^+ \leq \tilde{r}$ to exist, the determinant of $g(r_\infty) = 0$ should be positive, which implies that $h(z) \equiv z^2 + 2\beta(\lambda+\gamma)z + \beta^2(\lambda-\gamma)^2 - 4\beta\lambda > 0$, where $z \equiv 1-\beta/q$ so that $1-\beta \leq z < 1-\beta/2$. Note that $h'(z) = 2z + 2\beta(\lambda+\gamma) > 0$ for $z \geq 1-\beta$. Thus, it is deduced that $h(z) > 0 \Leftrightarrow h(1-\beta) = (1-\beta)^2 + \beta^3(\lambda-\gamma)^2 - 2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$.

Summarizing the argument thus far, the following proposition is obtained. Figure 3 illustrates one case of this equilibrium:

Proposition 6. *If* $\max[\lambda^{-}, \gamma] < \lambda < \lambda^{+}$ *or if* $(1-\beta)^2+\beta^2(\lambda-\gamma)^2-2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$ *and* $\hat{r}(\lambda,\beta,\gamma) < r_L^{+}(\lambda,\beta,\gamma,q)$, *then there exists a steady-state equilibrium with* $r_\infty \in (\max[r_L^{-}, \hat{r},], \min[r_L^{+}, r_H^{+}, \tilde{r}]]$. *In this equilibrium, the wealth of entrepreneurs is* $a^{**}(r_\infty)$*, and that of lending households is* $a^{*}(r_\infty)$*.*

Bold lines on the wealth levels and the interest rate in Figure 3 depict a continuum of the steady-state equilibria, in which all entrepreneurs rely on financial intermediation and $r_\infty \in (r_L^{-}, \tilde{r}]$.

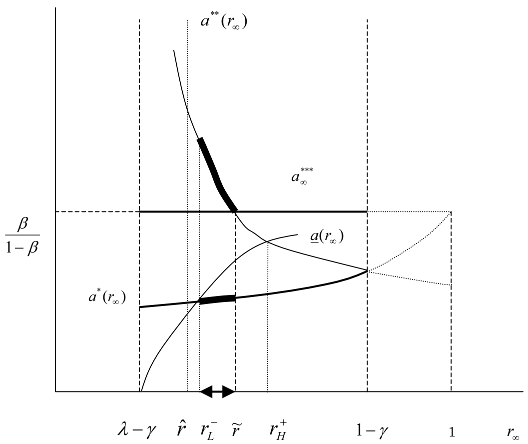

Figure 3: A case of the economy in which all financing is intermediated.

Now, let us consider the capital market clearing condition. Letting $X_L(r_\infty)$ be the fraction of net lenders in the economy "L" connotes that the interest rate is low), the following is obtained:

$$X_L(r_\infty)a^{*}(r_\infty)+[1-X_L(r_\infty)]a^{**}(r_\infty) = [1-X_L(r_\infty)]\frac{a^{**}(r_\infty)}{1-\lambda/(r_\infty+\gamma)},$$

which implies

$$X_L(r_\infty) = \frac{\lambda(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)}.$$

It can be shown that $\partial X_L(r_\infty)/\partial r_\infty = -\lambda(1+\beta\gamma)/[1-\beta(1-\gamma)](r_\infty+\gamma)^2 < 0$, which means that the greater the limit interest rate is, the less households are net lenders, which seems to be a natural consequence.

In addition, let the aggregate wealth in the economy when $r_\infty \in (\lambda-\gamma, \tilde{r}]$ be denoted by $NW_L(r_\infty)$. Then, it is shown that

$$NW_L(r_\infty) = \beta\frac{\lambda+r_\infty+\gamma}{[1-\beta(1-\gamma)](r_\infty+\gamma)}.$$

Similarly, the Gini index for wealth inequality in the steady state is given by

$$\begin{aligned} WealthGINI_L(r_\infty) &= 1-X_L(r_\infty)\frac{X_L(r_\infty)a^{*}(r_\infty)}{NW_L(r_\infty)} \\ &\quad -\left(\frac{X_L(r_\infty)a^{*}(r_\infty)}{NW_L(r_\infty)}+1\right)[1-X_L(r_\infty)]] \\ &= \frac{X_L(r_\infty)[1-X_L(r_\infty)][a^{**}(r_\infty)-a^{*}(r_\infty)]}{NW_L(r_\infty)}. \end{aligned}$$

(2) Equilibria in which Not All Financing is Intermediated

Now let us consider the case of $\tilde{r} < r_\infty \leq 1-\gamma$. There are three possible fixed points: $a^{*}(r_\infty)$, $a^{*}(r_\infty)$ and a_∞^{***} if the following is satisfied:

$$a^{*}(r_\infty) < \underline{a}(r_\infty) \leq a^{**}(r_\infty) < q \leq a_\infty^{***}.$$

Thus, it is important to determine whether $r_\infty \in (\tilde{r}, 1-\gamma]$ exists, which satisfies the aforementioned relationship. First, it must be the case that

$$\beta/(1-\beta) \geq q,$$

which is rewritten as $\beta \geq q/(1+q)$. Now, recall the restriction of the fixed cost $(1 \leq q < 2)$ and the first part of Assumption 3 $(\beta \geq q/2)$. It is verified that $q/2 \geq q/(1+q)$. Thus, this condition always holds under the assumptions.

Since $a^{*}(r_\infty)$ is strictly increasing, there are two cases for $r_\infty \in (\tilde{r}, 1-\gamma]$ to consist of a steady state equilibrium: (i) when $\underline{a}(1-\gamma) > a^{*}(1-\gamma) = a^{**}(1-\gamma)$ holds, and (ii) when $\underline{a}(1-\gamma) \leq a^{*}(1-\gamma)$ and $\tilde{r} < r_L^{+} \leq 1-\gamma$, where r_L^{+} is the larger solution of $g(r_\infty)=0$ if any.

In case (i), it is verified that $\underline{a}(1-\gamma) > a^{*}(1-\gamma)$ is equivalent to $\lambda < 1-\beta/\{q[1-\beta(1-\gamma)]\}$, or $\beta < (1-\lambda)q/[1+(1-\gamma)(1-\lambda)q]$. The equilibrium limit interest rate is $r_\infty \in (\max[r_H^{-}, r_L^{-}], r_H^{+}]$, where r_H^{-} is defined by

$$a^{**}(r_H^-) = q.$$

Recall that r_H^+ is defined by $a^{**}(r_H^+) = \underline{a}(r_H^+)$. Since $a^{**}(\cdot)$ is strictly decreasing, it is shown that $a^{**}(r_\infty) < q$ for $r_\infty > r_H^-$. It is also shown that $a^{**}(r_\infty) \geq \underline{a}(r_\infty)$ for $r_\infty \leq r_H^+$. Indeed, one can derive explicit forms of solutions:

$$\begin{cases} r_H^- = \dfrac{\beta\gamma(1/q-\gamma)+\lambda(1-\beta/q)-\gamma(1-\beta)}{1-(1+1/q)\beta+\beta(\lambda+\gamma)}, \text{ and} \\ r_H^+ = \dfrac{\beta\gamma(1+1/q-\gamma)+\lambda-\gamma}{1-(1+1/q)\beta+\beta(\lambda+\gamma)}, \end{cases}$$

where $r_H^- < r_H^+$ always holds.

If case (ii) occurs, the equilibrium limit interest rate is $r_\infty \in (\max[r_H^-, r_L^-], r_L^+]$. As in the previous subsection, for $r_L^+ \leq \tilde{r}$ to exist, it must be the case that $(1-\beta)^2+\beta^2(\lambda-\gamma)^2-2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$. Summarizing the arguments thus far, the following proposition is obtained.

Proposition 8. *If $q/2 \leq \beta < (1-\lambda)q/[1+(1-\gamma)(1-\lambda)q]$ or if $(1-\beta)^2+\beta^2(\lambda-\gamma)^2-2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$, then a steady-state equilibrium exists with $r_\infty \in (\max[r_L^-, r_H^-], \lim(r_L^+, r_H^-], 1-\gamma)]$. In this equilib-rium, the wealth of entrepreneurs who self-finance is a_∞^{***}, that of entrepreneurs who borrow from an FI is $a^{**}(r_\infty)$, and that of lending households is $a^*(r_\infty)$.*

Figure 4 illustrates one case of this equilibrium. Bold lines on the wealth levels and on the interest rate depict a continuum of the steady-state equilibria with $r_\infty \in (r_H^-, r_H^+)$.

Now let us consider the capital market clearing condition. Letting $X_H(r_\infty)$ and $Y_H(r_\infty)$ be the fractions of net lenders and of entrepreneurs who rely on intermediated borrowing, respectively ("H" connotes that the interest rate is high), the following is obtained:

$$\begin{aligned} X_H(r_\infty)a^*(r_\infty) + Y_H(r_\infty)a^{**}(r_\infty) \\ + [1-X_H(r_\infty)-Y_H(r_\infty)]a_\infty^{***} \\ = Y_H(r_\infty)\frac{a^{**}(r_\infty)}{1-\lambda/(r_\infty+\gamma)} + [1-X_H(r_\infty) \\ - Y_H(r_\infty)]a_\infty^{***}, \end{aligned}$$

which implies

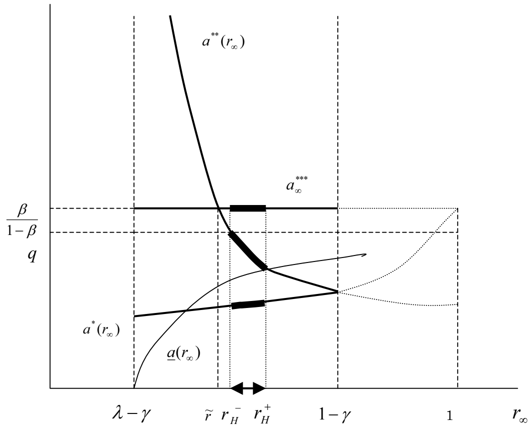

Figure 4: A case of the economy in which not all financing is intermediated.

$$\frac{\beta}{1-\beta r_\infty}X_H(r_\infty) = \frac{\beta\lambda}{[1-\beta+\beta(\lambda+\gamma)]r_\infty+[1-\beta(1-z)]\lambda-\gamma}Y_H(r_\infty).$$

Together with $X_H(r_\infty)+Y_H(r_\infty) = G_0(q)$, it is verified that

$$X_H(r_\infty) = \frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)}.$$

It can be shown that $\partial X_H(r_\infty)/\partial r_\infty = -\lambda G_0(q)(1+\beta\gamma)/[1-\beta(1-\gamma)](r_\infty+\gamma)^2 < 0$, which means that the greater the limit interest rate is, the less households are net lenders, which seems, again, to be a natural consequence.

Let the aggregate wealth in the economy when $r_\infty \in (\tilde{r}, 1-\gamma]$ be denoted by $NW_H(r_\infty)$. Then, the following is obtained:

$$\begin{aligned} NW_H(r_\infty) = & \left(1-\frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)}\right) \\ & \times\frac{\beta(r_\infty+\gamma)}{[1-\beta+\beta(\lambda+\gamma)]r_\infty+[1-\beta(1-\gamma)]\gamma-\lambda} \\ & +\frac{\beta}{1-\beta}[1-G_0(q)]. \end{aligned}$$

However, it is not easy to determine whether $NW_H(r_\infty)$ is larger than $NW_L(r_\infty)$. Furthermore, one can compare the fractions of lending households. It is verified that $X_L > X_H$ as it is equivalent to

$$\frac{\lambda(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)} > \frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)},$$

where r_∞ on the left hand side is $r_\infty \in (\lambda-\gamma, \tilde{r}]$, and the one on the right hand side is $r_\infty \in (\tilde{r}, 1-\gamma]$. Thus, more poor lending households exist in the equilibrium with $r_\infty \in (\lambda-\gamma, \tilde{r}]$ than in the equilibrium with $r_\infty \in (\tilde{r}, 1-\gamma]$. The Gini index for wealth inequality is given by

$$WealthGINI_H(r_\infty) = \frac{X_H(r_\infty)Y_H(r_\infty)[a^{**}(r_\infty)-a^*(r_\infty)]}{NW_H(r_\infty)} + [1-X_H(r_\infty)-Y_H(r_\infty)] \times \frac{X_H(r_\infty)(a_\infty^{***}-a^*(r_\infty))+Y_H(r_\infty)(a_\infty^{***}-a^{**}(r_\infty))}{NW_L(r_\infty)}.$$

Finally, note that from Propositions 6 and 8, we can see that if $\max[\lambda^-, \gamma] < \lambda < \lambda^+$ and $\beta > \max[q/2, (1-\lambda)/[\lambda+\gamma(1-\lambda)]$ (and Assumptions 2 and 3), then two continua exist: one is with $r_\infty \in (\max[\hat{r}, r_L^-], \min[\tilde{r}, r_L^+]]$ and the other is with $r_\infty \in [r_H^-, \min(r_H^+, 1-\gamma))$. Since $\beta > 1/2$, it is verified that $(2-1/\beta)$ lies between 0 and 1. This situation is depicted in Figure 5. Let us recall, for a high monitoring effect ($\lambda \geq \lambda^+$), that steady-state equilibrium does not equilbrium with $r_\infty \in (\lambda-\gamma, \tilde{r}]$ as it contradicts with the definition of steady-state. Thus, in this region, only $r_\infty \in (\max[\hat{r}, r_L^-], \min[\tilde{r}, r_L^+]]$ is permitted as a steady-state equilibrium. If the cost of monitoring is so low that $\gamma < 2-\lambda-1/\beta$, then the equilibrium with $r_\infty \in [r_H^-, \min(r_H^+, 1-\gamma))$ does not arise. In this case, all entrepreneurs want to borrow from an FI since the borrowing rate is sufficiently low.

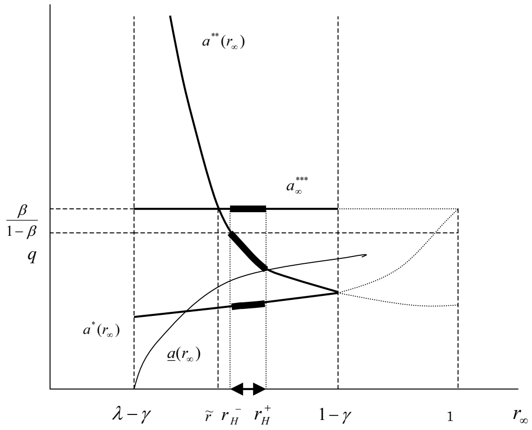

Figure 5: A case of two continua.

V. The Effects of Capital Account Liberalization on Wealth Distribution

This section considers the effects of CAL on wealth distribution. In the framework of the present analysis, CAL refers to the situation in which agents in this economy gain access to the international capital market populated with risk-neutral foreign investors whose opportunity cost is $r^W > 0$. As such, agents in this economy accept this interest rate as given. It is assumed that CAL does not change the parameters of the model; thus this paper considers the steady-state of the economy after CAL is introduced.

First, suppose that the economy is in the steady-state with $r_\infty \in (\max[\hat{r}, r_L^-], \min[r_L^+, r_L^+, \tilde{r}]]$ and that the world interest rate is $r^W \in (\hat{r}, r_\infty)$ Note that if $r^W < \hat{r}$, then after CAL, the economy does not converge to a steady-state, and borrowing entrepreneurs become infinitely wealthier. From Figure 3, it is apparent that borrowing entrepreneurs support and lending households oppose this regime change, as long as it does not make the threshold $\underline{a}(r_\infty)$ greater than $a^*(r_\infty)$. Note that it is implicitly assumed that the FI sector still behaves competitively, necessitating a reduction of the deposit rate to r^W and the lending rate to $r^W+\gamma$. If majority voting is required for the regime change and $X_L(r_\infty) > 1/2$, this economy does *not* implement CAL. However, if entrepreneurs can engage in lobbying, then CAL can be implemented. Moreover, if the world interest rate is $r^W > r_\infty$, entrepreneurs would want to oppose this regime change, whereas lending households would welcome such a change.

Next, suppose that the economy is in the steady-state with $r_\infty \in (\max[r_L^-, r_H^-], \min[r_L^+, r_H^+, 1-\gamma]]$, and that the world interest rate is $r^W \in (r_H^-, r_\infty)$. From Figure 4, it is clear that entrepreneurs borrowing from an FI welcome

CAL, whereas lending households oppose it, and self-financing entrepreneurs are neutral. However, if the world interest rate is $r^W \in (\max[\hat{r}, r_L^-], \min[r_L^+, r_L^+, \tilde{r}]]$, all self-financing entrepreneurs will support CAL since their successors will eventually become richer. Whether CAL is implemented depends on the political decision-making system as well as the steady-state fractions of lending households and entrepreneurs (self-financing or borrowing). In addition, regardless of whether they are domestic or foreign, financial sectors have a passive role in this model due to the assumed perfect competition. If the sectors gain rent due to the regulation or the increasing return to scale from monitoring, they will play a substantial role in the political decisions for CAL.

Ⅵ. Concluding Remarks

This paper presents a simple dynamic model to investigate the long-term relationship between types of financing (self-financing or intermediated financing) and wealth distribution in an economy. Specifically, it has incorporated the steady-state characterization to determine which finance pattern prevails in the economy as well as the characteristics of wealth distribution. For any steady-state, wealth inequality is higher and the interest rate is lower in the economy where intermediated capital is dominant. It is shown that for a wide range of parameters (concerning the benefit and cost of monitoring) there are two continua of the steady-state equilibria—one in which all entrepreneurs rely on financial intermediation, and the other in which some borrowing entrepreneurs self-finance. The source of multiplicity is based on the following self-fulfilling prophecy: as the interest rate is low, few richer borrowing entrepreneurs can borrow more, which makes the low interest rate is self-fulfilling due to the existence of many poor lending households. In addition, the opposite situation is also self-fulfilling. In this paper, the effects of CAL on wealth distribution were also discussed.

However, there are important questions that remained unanswered in the present paper. In particular, how do financial crises affect the change in financial systems as a whole? Moreover, how important are political processes for the evolution of and changes in financial systems? Historical experiences (see, e.g., Allen and Gale (2000, Ch.2); Allen and Gale (2004a); Bolton (2003)) suggest that characteristic features of financial systems change only if financial crises occur, regardless of whether they are due to domestic factors or to international pressure. Thus, this might suggest that financial crises have a positive effect on the workings of an economy with inevitable resultant turmoil. Furhermore, an analytical framework is necessary for investigating the issue of how political elements affect the evolution of and changes in financial systems (however, see, e.g., Bolton and Rosenthal (2002)).

Another interesting issue concerns the effect of international technological diffusion on the evolution of and changes in domestic financial systems. What is the relationship between international technological diffusion and domestic evolution of financial systems? Based on their (1999) analysis, Allen and Gale (2000, Ch.13) suggest that economies with less less innovative industries tend to adopt a bank-oriented system. However, the domestic industrial progress and financial characteristics in one economy might be a result of international technological diffusion rather than original endowment. Naturally, this issue should be investigated with a full-fledged dynamic model. Finally, extensive literature has investigated the issue of technological diffusion and economic growth. However, this literature is silent on the differences in financial systems. These and other issues concerning the evolution of and changes in financial systems should be the focus of future research.

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Notes

1) Also note that default is always voluntary. As uncertainty is not modeled in the present paper, debt overhang is not an issue.
2) The literature review is continued with reference to papers including Banerjee and Newman (1993), Aghion and Bolton (1997), Piketty (1997), Iyigun and Owen (1999), Matsuyama (2000), Mookherjee and Ray (2002), Galor and Michalopoulos (2006) and many others.
3) Note that the target of these empirical studies is *income* inequality rather than *wealth* inequality. This is because it is difficult to find an appropriate index or a good proxy for wealth inequality. Typically, the GINI index for land ownership is used. Though it is well known that wealth inequality is more severe than income inequality, the present paper does not examine this issue in detail. The benefit of considering a theoretical model is that one can systematically deal with both inequalities.
4) Mookherjee and Ray (2003) is one exception.
5) Under the fixed cost of starting production and borrowing constraints (such as the ones introduced later), households might consider joint borrowing and joint production. The present paper simply assumes that this possibility does not occur.
6) In the following analysis, this normalization will always make the equilibrium per unit interest rate less than one, which seems odd at first (since repayment is less than the amount of borrowing). However, for analytical purposes, it is more important to note that it still earns a positive amount, and this normalization does not invalidate the main thrust of the results.
7) This exogenous income should not be large. Otherwise, all households become rich enough and they can easily overcome the investment threshold caused by the non-convexity of the technology. It can be verified that normalizing the exogenous income to one satisfies this requirement.
8) Ando and Yanagawa (2004) construct a model in which monitoring technology is endogenous.
9) It is not assumed that monitoring includes increasing returns to scale, which is compatible with perfect competition. See Allen and Gale (2000, Ch. 8) and Allen and Gale (2004) for an analysis of competition in the banking sector.
10) If the interest rate is below one, then most likely, one would want to borrow an infinite amount of capital due to the linearity of the production technology. As shown later, the equilibrium interest rate under perfect enforcement should be one.
11) The present paper simply assumes away the role of intertemporal incentives such as reputational concerns. Since this author's interests are on macroeconomic issues and microeconomic ones in, say, a small community, this would not cause a serious error.
12) This might be a strong assumption, but if one allows non-zero pledgeable income when an entrepreneur escapes intermediated borrowing, then the steady-state analysis becomes complicated, thus yielding less interesting results.
13) Note that $\lambda = 1$ represents perfect enforcement. As shown later, any household with $w_t \geq 0$ does not suffer from the borrowing constraint (to be formalized shortly).
14) Seminal papers on the international differences of the legal system in protecting investors include La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997; 1998).
15) For more details regarding the formulation of this problem, see, e.g., Freixas and Rochet (2008).
16) Note that Kiyotaki and Moore's (1997) justification for the borrowing constraint can also be applied in the present context. They propose the following. Production technology is specific to the borrower, and if the lender exceeds the borrower's production, the borrower cannot produce as much as the lender. Knowing this, the lender can suggest renegotiating the initial contract to reduce the repayment, and the borrower will accept this as long as the suggested repayment is not below the level earned in production. If one assumes that it is the lender who must exceed the borrower's production, and that (unmodeled) "shadow" middlemen, who have inferior technology, are actually between lending households and

borrowing entrepreneurs, then Kiyotaki and Moore's (1997) proposal applies. Conversely, if the relationship between lending households and borrowing entrepreneurs is literally direct, then it does not hold. However, this seems less natural.

17) A similar borrowing constraint is adopted by Holmstrom and Tirole (1997), in which production is stochastic. This makes dynamic analysis less tractable. Indeed, Chakraborty and Ray (2006) incorporate Holmstrom and Tirole's (1997) incentive problem into the dynamic model. However, they assume that the most efficient technology out of the three is deterministic.

18) The following analysis ignores the exogenous revenue value when aggregate income measures such as the GDP and the Gini index are calculated. This eases computation, and enables more direct interpretation of the results.

19) This dynamics can be derived from generation t's entrepreneur's utility maximization problem if we assume the "warm-glow" utility function:

$$u=(1-\beta)\ln c+\beta \ln b,$$

where c is the amount of his/her own consumption and b is that of his/her bequest to his/her child. The indirect utility as a function of the realized income I becomes

$$u(I)=(1-\beta)^{1-\beta}\beta^{\beta}I,$$

which is linear. Therefore, this formulation is consistent with the assumption that households are risk-neutral. See, e.g., Newman (2007) for an analysis of risk-bearing and etnerepreneurship.

20) An exception is Mookherjee and Ray (2003).

21) There are always two solutions for $f(\lambda)=0$, as the determinant is

$$[\beta\gamma^2+(1+\beta/q)\gamma+(1-1/q)\beta-1]^2 + 4\gamma(1-\beta)[(1-\beta/q)+\beta\gamma(1-1/q)],$$

which is positive since $1-\beta/q>0$ and $q\geq 1$.

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