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主　論　文　の　要　旨

論文題目　Bergman kernel and its boundary asymptotics
（ベルグマン核とその境界漸近性）

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論　文　内　容　の　要　旨

The Bergman kernel on each complex manifold is a canonical volume-form determined by the complex structure. This thesis studies the Bergman kernel and consists mainly of the following two parts. Part I is on the relations between Bergman kernels and potentials (Arakelov-Green function, Evans-Selberg potential, etc.). Part II is on the variation (in particular its asymptotic behaviors) of Bergman kernels at degeneration. After the introduction and the preliminaries among the total five chapters, Chapter 3 and Chapters 4 & 5 correspond to Part I and Part II, respectively. Asymptotic behaviors of the Bergman kernel's variation at the limiting case can imply (by convexity and plurisubharmonicity) quantitative relations between Bergman kernels and potentials, which is a direct link from Part II to Part I.

Part I

For any complex torus, we compute via elliptic functions the quotient of the Bergman kernel and the Arakelov metric, and obtain a sharp positive lower bound. For general compact Riemann surfaces, we further derive upper bounds of the Arakelov metric's Gaussian curveture by a method due to Berndtsson & Lempert. For a once-punctured complex torus, we compare the Bergman kernel and the fundamental metric, by constructing explicitly the Evans-Selberg potential and discussing its asymptotic behaviors. These works aim to generalize the Suita type results to potential-theoretically non-hyperbolic Riemann surfaces.

Part II

For a Legendre family of elliptic curves, using two methods (depending on special elliptic functions or not) we show that the horizontal curvature form of the relative Bergman kernel metric is strictly positive inside the moduli space and defines a Kaehler metric on $\mathbf{C}$-{0, 1}. In particular, this metric blows up and has hyperbolic growth near the node 0. For other boundary points 1 and ∞, the asymptotics are also achieved. For other families of elliptic curves degenerating to singular ones with a node or a cusp, we observe that it is more or less trivial.

For a holomorphic family of hyperelliptic curves and their Jacobians, we estimate asymptotic behaviors of the horizontal curvature forms of the relative Bergman kernel metrics near the degenerate boundaries with nodes or cusps. Specifically, the curvature form tends near a node to an incomplete metric on the parameter space, but tends near a cusp to 0. These results are different from the elliptic curve case where hyperbolic growth exists, and the type of singularities surely determines various boundary asymptotics. For the genus-two case particularly, asymptotic formulas with precise coefficients involving the complex structure information are written down explicitly.