論文の要約

論文題目　**Bergman kernel and its boundary asymptotics**
(ベルグマン核とその境界漸近性)

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The Bergman kernel on each complex manifold is a canonical volume-form determined by the complex structure. This thesis studies the Bergman kernel and consists mainly of the following two parts. Part I is on the relations between Bergman kernels and potentials (Arakelov-Green function, Evans-Selberg potential, etc.). Part II is on the variation (in particular its asymptotic behaviors) of Bergman kernels at degeneration. After the introduction and the preliminaries among the total five chapters, Chapter 3 and Chapters 4 & 5 correspond to Part I and Part II, respectively. Asymptotic behaviors of the Bergman kernel's variation at the limiting case can imply (by convexity and plurisubharmonicity) quantitative relations between Bergman kernels and potentials, which is a direct link from Part II to Part I.

Part I

It is interesting to generalize the Suita type results to compact Riemann surfaces, whose Green functions in the usual sense do not exist. However, in arithmetic algebraic geometry it is known that on a compact manifold the Arakelov-Green function and the Arakelov metric play important roles (with applications to string theory), similarly as the Green function and the logarithmic capacity do for a bounded domain. We first dealt with a complex torus, whose Bergman kernel, Arakelov-Green function and Arakelov

metric are given by elliptic functions. We numerically found a universal constant independent of the complex structure. On the other hand, we aim to generalize the Suita type results to the potential-theoretically parabolic case. For a once-punctured complex torus, we compared the Bergman kernel and the fundamental metric by constructing the Evans-Selberg potential, deriving the fundamental metric, and discussing their asymptotic behaviors. Moreover, as the torus degenerates to a singular complex algebraic curve, we know that the Gaussian curvature of the fundamental metric can be arbitrarily close to 0, i.e., cannot be bounded from above by a negative constant, which is different from the potential-theoretically hyperbolic case (with an upper bound -4). We provided explicit formulas for Evans-Selberg potentials on unbounded planar domains and discussed their growth orders near boundaries.

Part II

A possibly more interesting question is to study the variations (in particular their asymptotic behaviors) of Bergman kernels at degeneration. Such study is much related to the variation of Hodge structures, especially the nilpotent orbit theorem. In general, the curvature semi-positivities characterize certain convexities and are associated with L^2 estimates and extensions. Our research aims to relate to these abstract objects in a quantitative way and at least three approaches work for this problem: elliptic function, Taylor expansion and pinching coordinate.

0. The so-called Legendre family of elliptic curves gives a general description of genus one compact Riemann surfaces, whose moduli space is C-{0, 1}. We found that the Poincar\'e metric there has hyperbolic growth at 0 where the curve degenerates to a singular one with a node. For the case of other

boundary points, 1 and ∞, the asymptotics are also achieved. The proofs highly depend on special properties of the Weierstrass-$\wp$ function and the elliptic modular lambda function. For other families of elliptic curves (degenerating to a singular one at 0 with a node or a cusp) that the special elliptic function method cannot be applied to, we can accurately determine the leading and subleading terms by a method based on the Taylor expansions of Abelian differentials. Information on both the singularity and the complex structure contributes to the determination of various boundary behaviors of Bergman kernels. We also found an interesting example, namely a cuspidal family of elliptic curves with non-constant periods, which is reducible to the case of a Legendre family. Such a connection between the nodal and cuspidal cases strengthens the importance of a Legendre family, since we can change coordinates holomorphically to make the reduction. Finally, (probably due to the uniformization theorem) the situations for higher genus curves are quite different.

1. Hyperelliptic & general curves with nodes. For a (non-separating) nodal family of genus two curves {y^2=x(x-t)(x-1)(x-a)(x-b)}, with distinct a, b, t in C-{0, 1}, we can determine asymptotic behaviors of Bergman kernels with precise coefficients. Previously by Habermann & Jost, the pinching-coordinate method was used to study the Bergman kernels and their induced L^2 metrics on Teichmuller spaces of general curves with separating or non-separating nodes. Nevertheless, our different approach to hyperelliptic curves has an advantage (especially if we want to know what role the given complex structure plays) that we can explicitly write down the coefficients, which usually indicate the geometry of the base varieties and their singularities.

2. Hyperelliptic curves with cusps, Case I. Let p(x) be a

polynomial of degree at least 2 with roots of distinct absolute values different from |t| and 0. In the local coordinate z=\sqrt{x}(non-zero) on a cuspidal family of hyperelliptic curves {y^2=x(x^2-t)p(x)}, write its Bergman kernel as k_t(z) dz\wedge d\bar z. Then, as t tends to 0, log k_t(z)=constant+O(t^{1/4}). Also, the second term is harmonic in t and doesn't necessarily possess a positive coefficient. Moreover, the Jacobian varieties remain being manifolds (i.e., non-degenerate), as t tends to 0.

3. Hyperelliptic curves with cusps, Case II. For distinct a, b, t in C-{0}, we consider another family of genus two curves {y^2=x(x-t)(x-t^2)(x-a)(x-b)}. Then, as t tends to 0, both coefficients of the first two terms depend only on the information away from the cusp, which is not the case for Case I. For the Jacobian varieties, the curvature form of the relative Bergman kernel has hyperbolic growth again.