PHYSICAL REVIEW D **94,** 055019 (2016)

τ- and μ-physics in a general two Higgs doublet model with $\mu-\tau$ flavor violation

Yuji Omura,[1] Eibun Senaha,[2] and Kazuhiro Tobe[3,1]

[1] *Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan*

[2] *Center for Mathematics and Theoretical Physics and Department of Physics, National Central University, Taoyuan, Taiwan 32001, Republic of China*

[3] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

(Received 18 January 2016; revised manuscript received 17 June 2016; published 15 September 2016)

Motivated by the recent CMS excess in a flavor violating Higgs decay $h \to \mu\tau$ as well as the anomaly of muon anomalous magnetic moment (muon $g-2$), we consider a scenario where both the excess in $h \to \mu\tau$ and the anomaly of muon $g-2$ are explained by the $\mu-\tau$ flavor violation in a general two Higgs doublet model. We study various processes involving μ and τ, and then discuss the typical predictions and constraints in this scenario. Especially, we find that the prediction of $\tau \to \mu\gamma$ can be within the reach of the Belle II experiment. We also show that the lepton nonuniversality between $\tau \to \mu\nu\bar{\nu}$ and $\tau \to e\nu\bar{\nu}$ can be sizable, and hence the analysis of the current Belle data and the future experimental improvement would have an impact on this model. Besides, processes such as $\tau \to \mu l^+ l^-$ $(l = e, \mu)$, $\tau \to \mu\eta$, $\mu \to e\gamma$, $\mu \to 3e$, and the muon electric dipole moment can be accessible, depending on the unknown Yukawa couplings. On the other hand, the processes like $\tau \to e\gamma$ and $\tau \to el^+l^-$ $(l = e, \mu)$ could not be sizable to observe because of the current strong constraints on the $e-\mu$ and $e-\tau$ flavor violations. Then we also conclude that contrary to the $h \to \mu\tau$ decay mode, the lepton flavor violating Higgs boson decay modes $h \to e\mu$ and $h \to e\tau$ are strongly suppressed, and hence it will be difficult to observe these modes at the LHC experiment.

DOI: 10.1103/PhysRevD.94.055019

I. INTRODUCTION

The Standard Model (SM) describes particle physics remarkably well up to the electroweak scale. In addition, the recent discovery of a Higgs boson at the LHC [1,2] strengthens the success of the SM. On the other hand, the detailed measurements of the Higgs boson properties have just started, and the whole structure of the Higgs sector may not have been unveiled. Therefore, theoretical and experimental studies of the extended Higgs sector would be important to understand the nature of the Higgs sector.

One of the simple extensions of the Higgs sector in the SM is a two Higgs doublet model (2HDM) where an additional Higgs doublet is introduced and both Higgs doublets can couple to all fermions. As a result, flavor violating phenomena mediated by the Higgs bosons are predicted [3]. In most cases, such a flavor violation has been considered to be avoided because of lack of the experimental support for the anomalous flavor violating phenomena [4–7].

However, the CMS Collaboration has recently reported an excess in a flavor violating Higgs decay $h \to \mu\tau$ [8], and it suggests that the best fit value of the branching ratio is

$$\mathrm{BR}(h \to \mu\tau) = (0.84^{+0.39}_{-0.37})\%, \tag{1}$$

where the final state is a sum of $\mu^+\tau^-$ and $\mu^-\tau^+$, and the deviation from the SM prediction is 2.4σ. In addition, the result of the ATLAS experiment has also appeared recently [9], and it is shown as

$$\mathrm{BR}(h \to \mu\tau) = (0.77 \pm 0.62)\%, \tag{2}$$

which is consistent with the CMS result within 1σ. Although these results have not been conclusive yet, they become strong motivations to study the flavor violating phenomena predicted by the beyond Standard Models [10–36].[1]

In Ref. [16], we pointed out a possibility that the $\mu-\tau$ flavor violation in general 2HDM can explain not only the CMS excess in the Higgs decay $h \to \mu\tau$, but also the anomaly of muon anomalous magnetic moment (muon $g-2$) [48]. This possibility is interesting because two unexplained phenomena can be accommodated in the 2HDM, and hence it is worth further investigating this possibility. In this paper, we study phenomena related to μ and τ lepton physics in the scenario to see whether there are any interesting predictions and constraints caused by the $\mu-\tau$ flavor violation.

[1] The lepton flavor violating Higgs decays have been investigated before the CMS excess was reported [37–47].

The paper is organized as follows. In Sec. II, we present a general 2HDM where both Higgs doublets couple to all fermions. We introduce the Yukawa interactions and Higgs mass spectrum in the model. In Sec. III, we consider a solution where the CMS excess in $h \to \mu\tau$ decay as well as the muon $g-2$ anomaly can be explained by the $\mu-\tau$ flavor violating Yukawa interactions in the model. We show the typical parameter space where both anomalies can be achieved. In Sec. IV, we discuss τ- and μ-physics in the interesting region studied in the previous section. Especially, we study $\tau \to \mu\gamma$, $\mu \to e\gamma$, muon electric dipole moment (muon EDM), $\tau \to \mu\nu\bar{\nu}$, $\tau^- \to \mu^- l^+ l^-$ $(l = e, \mu)$, $\mu^+ \to e^+ e^- e^+$, and $\tau \to \mu\eta$. The prediction of $\tau \to \mu\gamma$ can be within the reach of the Belle II experiment, which will start in the near future. The extra Higgs boson correction to $\tau \to \mu\nu\bar{\nu}$ can be as large as 10^{-3}–10^{-4}, but it is not so large in the $\tau \to e\nu\bar{\nu}$ mode. The future improvement of measurement on lepton flavor universality in $\tau \to \mu(e)\nu\bar{\nu}$ decay will be important in the scenario. We also find that unlike the $\mu-\tau$ flavor violation suggested by the CMS result, the $e-\tau$ and $e-\mu$ flavor violations are severely constrained by the $\mu \to e\gamma$ process. Then, the processes involving $e-\tau$ and $e-\mu$ flavor violations become suppressed. In Sec. V, we also discuss the implication to Higgs physics. Since $e-\tau$ and $e-\mu$ flavor violating Yukawa couplings should be small, $h \to e\tau$ and $h \to e\mu$ Higgs decay modes will not be observed in this scenario, contrary to the $h \to \mu\tau$ mode. In Sec. VI, we summarize our results.

II. GENERAL TWO HIGGS DOUBLET MODEL

In a general two Higgs doublet model, there are no symmetries to distinguish the two different Higgs doublets. Thus, both the Higgs doublets can couple to all fermions, and hence there are flavor violating interactions in the Higgs sector. In general, when the Higgs potential is minimized in the SM-like vacuum, both neutral components of Higgs doublets develop nonzero vacuum expectation values (VEVs). Note that all parameters in the Higgs potential are assumed to be real in our analysis, and then CP is not spontaneously broken by the VEVs. Taking a certain linear combination, we can define the basis where only one Higgs doublet obtains the nonzero VEV as follows:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG}{\sqrt{2}} \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA}{\sqrt{2}} \end{pmatrix}, \tag{3}$$

where G^+ and G are Nambu-Goldstone bosons, and H^+ and A are a charged Higgs boson and a CP-odd Higgs boson, respectively.[2] Then H^+ and A are in the mass eigenstates. The CP-even neutral Higgs bosons ϕ_1 and ϕ_2 can mix and form mass eigenstates, h and H $(m_H > m_h)$; in general,

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\beta\alpha} & \sin\theta_{\beta\alpha} \\ -\sin\theta_{\beta\alpha} & \cos\theta_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \tag{4}$$

Here $\theta_{\beta\alpha}$ is the mixing angle and is fixed by the Higgs potential analysis. Note that when $\cos\theta_{\beta\alpha} \to 0$ $(\sin\theta_{\beta\alpha} \to 1)$, the interactions of ϕ_1 approach those of the SM Higgs boson.

A. Yukawa interactions

In mass eigenbasis for the fermions, the Yukawa interactions are expressed by [55]

$$\begin{aligned} \mathcal{L} = &-\bar{Q}_L^i H_1 y_d^i d_R^i - \bar{Q}_L^i H_2 \rho_d^{ij} d_R^j - \bar{Q}_L^i (V_{\rm CKM}^\dagger)^{ij} \tilde{H}_1 y_u^j u_R^j \\ &- \bar{Q}_L^i (V_{\rm CKM}^\dagger)^{ij} \tilde{H}_2 \rho_u^{jk} u_R^k - \bar{L}_L^i H_1 y_e^i e_R^i - \bar{L}_L^i H_2 \rho_e^{ij} e_R^j, \end{aligned} \tag{5}$$

where i and j represent flavor indices, and $Q = (V_{\rm CKM}^\dagger u_L, d_L)^T$ and $L = (V_{\rm MNS}\nu_L, e_L)^T$ are defined. $V_{\rm CKM}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix and $V_{\rm MNS}$ is the Maki-Nakagawa-Sakata (MNS) matrix. Fermions (f_L, f_R) $(f = u, d, e, \nu)$ are mass eigenstates, and $y_f^j = \sqrt{2} m_f^i / v$, where m_f^i denote the fermion masses, are defined. Here we have assumed that the tiny neutrino masses are achieved by the seesaw mechanism introducing superheavy right-handed neutrinos, so that in the low-energy effective theory, the left-handed neutrinos have a 3×3 Majorana mass matrix. The Yukawa coupling constants ρ_f^{ij} are general 3×3 complex matrices and can be sources of the Higgs-mediated flavor changing processes.

In mass eigenstates of Higgs bosons, the Yukawa interactions are given by

$$\begin{aligned} \mathcal{L} = &-\sum_{f=u,d,e} \sum_{\phi=h,H,A} y_{\phi ij}^f \bar{f}_{Li} \phi f_{Rj} + \text{H.c.} \\ &- \bar{\nu}_{Li} (V_{\rm MNS}^\dagger \rho_e)^{ij} H^+ e_{Rj} \\ &- \bar{u}_i (V_{\rm CKM} \rho_d P_R - \rho_u^\dagger V_{\rm CKM} P_L)^{ij} H^+ d_j + \text{H.c.}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} y_{hij}^f &= \frac{m_f^i}{v} s_{\beta\alpha} \delta_{ij} + \frac{\rho_f^{ij}}{\sqrt{2}} c_{\beta\alpha}, \\ y_{Hij}^f &= \frac{m_f^i}{v} c_{\beta\alpha} \delta_{ij} - \frac{\rho_f^{ij}}{\sqrt{2}} s_{\beta\alpha}, \\ y_{Aij}^f &= \begin{cases} -\frac{i\rho_f^{ij}}{\sqrt{2}} & (\text{for } f = u), \\ \frac{i\rho_f^{ij}}{\sqrt{2}} & (\text{for } f = d, e) \end{cases} \end{aligned} \tag{7}$$

are defined with $c_{\beta\alpha} \equiv \cos\theta_{\beta\alpha}$ and $s_{\beta\alpha} \equiv \sin\theta_{\beta\alpha}$ [55]. Note that when $c_{\beta\alpha}$ is small, the Yukawa interactions of h are

[2] This base is the so-called Higgs (Georgi) basis [49–55].

almost equal to those of the SM Higgs boson; however, there are small flavor violating interactions ρ_f^{ij} which are suppressed by $c_{\beta\alpha}$. On the other hand, the Yukawa interactions of heavy Higgs bosons (H, A, and H^+) mainly come from the ρ_f^{ij} couplings.

B. Higgs mass spectrum

Let us comment on the relation between the Higgs masses and the parameters in the Higgs potential. The renormalizable Higgs potential in the general 2HDM is given by

$$\begin{aligned} V = {} & M_{11}^2 H_1^\dagger H_1 + M_{22}^2 H_2^\dagger H_2 - (M_{12}^2 H_1^\dagger H_2 + \text{H.c.}) \\ & + \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) \\ & + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_5}{2}(H_1^\dagger H_2)^2 \\ & + \{\lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)\}(H_1^\dagger H_2) + \text{H.c.} \end{aligned} \tag{8}$$

M_{12}^2, λ_5, λ_6, and λ_7 can be complex, but they are assumed to be real parameters to avoid the CP breaking. In the basis shown in Eq. (3), the Higgs boson masses can be described by the dimensionless parameters and M_{22} using the stationary conditions for the Higgs doublets [55],

$$\begin{aligned} m_{H^+}^2 &= M_{22}^2 + \frac{v^2}{2}\lambda_3, \\ m_A^2 - m_{H^+}^2 &= -\frac{v^2}{2}(\lambda_5 - \lambda_4), \\ (m_H^2 - m_h^2)^2 &= \{m_A^2 + (\lambda_5 - \lambda_1)v^2\}^2 + 4\lambda_6^2 v^4, \\ \sin 2\theta_{\beta\alpha} &= -\frac{2\lambda_6 v^2}{m_H^2 - m_h^2}. \end{aligned} \tag{9}$$

Especially, when $c_{\beta\alpha}$ is close to zero (that is, $\lambda_6 \sim 0$), we obtain the following simple expressions for the Higgs boson masses:

$$\begin{aligned} m_h^2 &\simeq \lambda_1 v^2, \\ m_H^2 &\simeq m_A^2 + \lambda_5 v^2, \\ m_{H^+}^2 &= m_A^2 - \frac{\lambda_4 - \lambda_5}{2} v^2, \\ m_A^2 &= M_{22}^2 + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2} v^2. \end{aligned} \tag{10}$$

Note that fixing the couplings, λ_3, λ_4, and λ_5, the heavy Higgs boson masses are expressed by the CP-odd Higgs boson mass m_A, which we treat as a free parameter of the model. The contribution to the Peskin-Takeuchi T-parameter [56] should be taken into account, so that we assume that it is suppressed by the degeneracy between m_A and m_{H^+} as well as the small Higgs mixing parameter $c_{\beta\alpha}$. Therefore, we set $\lambda_4 = \lambda_5$ in our analysis, which corresponds to $m_A = m_{H^+}$. The S-parameter may be relevant to our model depending on the mass spectrum. It is approximately evaluated as $S \approx \lambda_5 v^2/(24\pi m_A^2)$ in the limit that $s_{\beta\alpha} \to 1$, $m_A = m_{H^+}$, and $m_A \gg \lambda_5 v^2$. The deviation of S is small enough to evade the experimental constraint ($|\Delta S| \lesssim 0.1$) as long as λ_5 is not extremely large.

III. SOLUTION TO THE CMS EXCESS IN $h \to \mu\tau$ AND THE MUON $g-2$ ANOMALY

The CMS Collaboration has reported an excess in a Higgs boson decay mode $h \to \mu\tau$. Furthermore, it is known that there is a discrepancy between the measured value and the SM prediction of the muon anomalous magnetic moment (muon $g-2$). Both anomalies cannot be explained by the SM, and hence they might be an indication of physics beyond the SM. In this section, we discuss whether the general 2HDM can accommodate both anomalies simultaneously, and we investigate the parameter space where both anomalies can be achieved simultaneously.

A. $h \to \mu\tau$

An excess in $h \to \mu\tau$ decay mode has been reported by the CMS Collaboration: the best fit value of the branching ratio is $\text{BR}(h \to \mu\tau) = (0.84^{+0.39}_{-0.37})\%$ [8]. As discussed in the Introduction, the ATLAS Collaboration has also shown the result, $\text{BR}(h \to \mu\tau) = (0.77 \pm 0.62)\%$ [9], which is consistent with the CMS one. If the excesses are confirmed with better significance, it would indicate new physics because the SM cannot accommodate the flavor violation. Since in the general 2HDM the SM-like Higgs boson has flavor violating Yukawa interactions as discussed in the previous section, the excess can easily be explained. The expression of the branching ratio of the $h \to \mu\tau$ process is given by

$$\begin{aligned} \text{BR}(h \to \mu\tau) &= \frac{\Gamma(h \to \mu^+\tau^-) + \Gamma(h \to \mu^-\tau^+)}{\Gamma_h} \\ &= \frac{c_{\beta\alpha}^2 (|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2) m_h}{16\pi\Gamma_h}, \end{aligned} \tag{11}$$

where Γ_h is a total decay width of Higgs boson h and we adopt $\Gamma_h = 4.1$ MeV in this paper. To accommodate the CMS excess, the $\mu - \tau$ flavor violating Yukawa couplings need to satisfy the following condition:

$$\begin{aligned} \bar{\rho}^{\mu\tau} &\equiv \sqrt{\frac{|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2}{2}} \\ &\simeq 0.26 \left(\frac{0.01}{|c_{\beta\alpha}|}\right) \sqrt{\frac{\text{BR}(h \to \mu\tau)}{0.84 \times 10^{-2}}}. \end{aligned} \tag{12}$$

It is interesting to note that even in the small Higgs mixing ($|c_{\beta\alpha}| \simeq 0.01$), the $\mu - \tau$ flavor violating Yukawa couplings with the order of 0.1 can achieve the CMS excess.

B. The muon anomalous magnetic moment (muon $g-2$)

We have shown that the $\mu-\tau$ flavor violating Yukawa couplings in the general 2HDM explain the CMS excess in the $h\to\mu\tau$ decay mode. Here we consider the contribution of the $\mu-\tau$ flavor violating interaction to the muon anomalous magnetic moment (muon $g-2$).

The discrepancy between the measured value ($a_\mu^{\rm Exp}$) and the standard model prediction ($a_\mu^{\rm SM}$) of the muon $g-2$ has been reported in Ref. [48]. For example, Ref. [57] suggests the following result:

$$a_\mu^{\rm Exp}-a_\mu^{\rm SM}=(26.1\pm 8.0)\times 10^{-10}. \tag{13}$$

Here we consider whether the extra contributions induced by the $\mu-\tau$ flavor violating interactions can accommodate this muon $g-2$ anomaly. The effective operator for the muon $g-2$ is expressed by

$$\mathcal{L}=\frac{v}{\Lambda^2}\bar{\mu}_L\sigma^{\mu\nu}\mu_R F_{\mu\nu}+\mathrm{H.c.} \tag{14}$$

We note that the chirality of muon is flipped in the operator. Therefore, if there is a large chirality flip induced by the new physics, it can enhance the extra contributions to the muon $g-2$ [58]. Feynman diagrams for the one-loop corrections involving the neutral Higgs bosons and the $\mu-\tau$ flavor violating Yukawa couplings are described in Fig. 1. As shown in Fig. 1, the chirality is flipped in the internal line of τ lepton in the diagram. Therefore, it induces the $O(m_\tau/m_\mu)$ enhancement in the extra contributions to the muon $g-2$, compared with the one generated by the flavor-diagonal Yukawa coupling. We stress that the $\mu-\tau$ flavor violating interaction is essential to obtain such an enhancement. Note that both couplings $\rho_e^{\mu\tau}$ and $\rho_e^{\tau\mu}$ should be nonzero to flip the chirality in the internal τ lepton line. Finally, the expression of the enhanced extra contribution δa_μ is given by

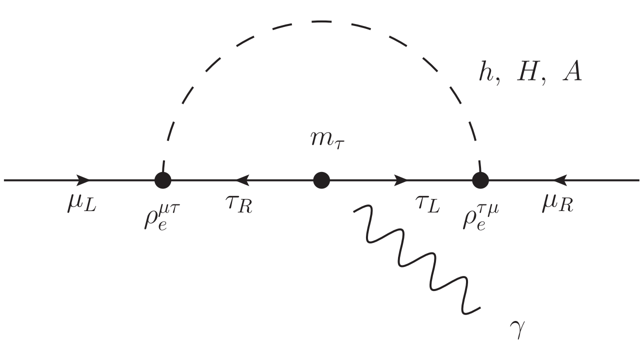

FIG. 1. A Feynman diagram for neutral Higgs boson contributions to the muon $g-2$. A photon is attached somewhere in the charged lepton line.

$$\delta a_\mu=\frac{m_\mu m_\tau\rho_e^{\mu\tau}\rho_e^{\tau\mu}}{16\pi^2}\times\left[\frac{c_{\beta\alpha}^2(\log\frac{m_h^2}{m_\tau^2}-\frac{3}{2})}{m_h^2}+\frac{s_{\beta\alpha}^2(\log\frac{m_H^2}{m_\tau^2}-\frac{3}{2})}{m_H^2}-\frac{\log\frac{m_A^2}{m_\tau^2}-\frac{3}{2}}{m_A^2}\right], \tag{15}$$

where we have assumed that $\rho_e^{\mu\tau}\rho_e^{\tau\mu}$ is real, for simplicity. We will discuss the effect of an imaginary part of these Yukawa couplings later. We note that a degeneracy of all neutral Higgs bosons suppresses the extra contribution to the muon $g-2$, as seen in Eq. (15).

The so-called Barr-Zee–type two-loop diagrams can contribute to the muon $g-2$ if diagonal elements of ρ_f are nonzero. In the parameter space we are considering here, such contributions are always subdominant and numerically unimportant. In the cases of the flavor-changing muon or tau decays, however, they can compete with the one-loop contributions and can play a significant role. Details will be discussed below.

In Fig. 2, we show numerical results for the extra contribution to muon $g-2$ (δa_μ) as a function of $c_{\beta\alpha}$

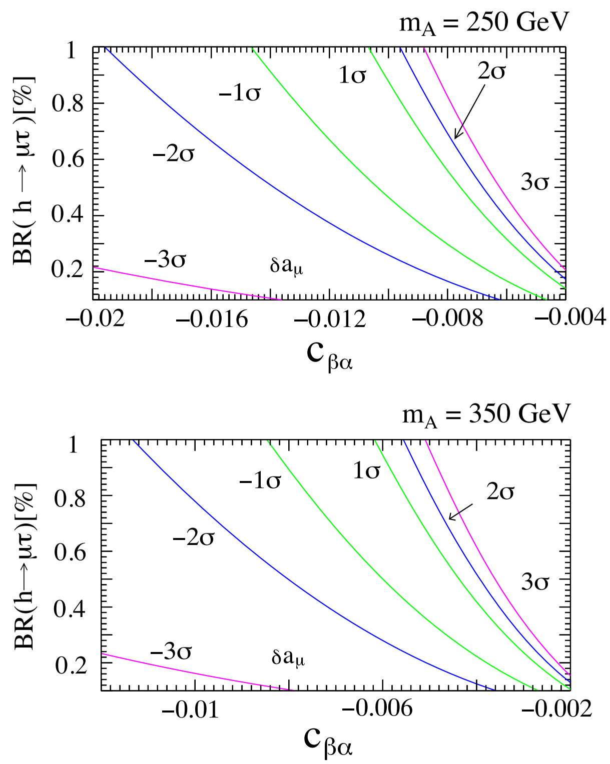

FIG. 2. Numerical result for δa_μ as a function of $c_{\beta\alpha}$ and BR($h\to\mu\tau$) for $m_A=250$ GeV (upper figure) and 350 GeV (lower figure). Regions where the muon $g-2$ anomaly in Eq. (13) is explained within $\pm1\sigma$, $\pm2\sigma$, and $\pm3\sigma$ are shown. Here we determine the mass spectrum of heavy Higgs bosons assuming $\lambda_4=\lambda_5=0.5$ in Eq. (10). We have assumed $\rho_e^{\mu\tau}\rho_e^{\tau\mu}<0$ with $\rho_e^{\mu\tau}=-\rho_e^{\tau\mu}$ to obtain the positive contribution to δa_μ.

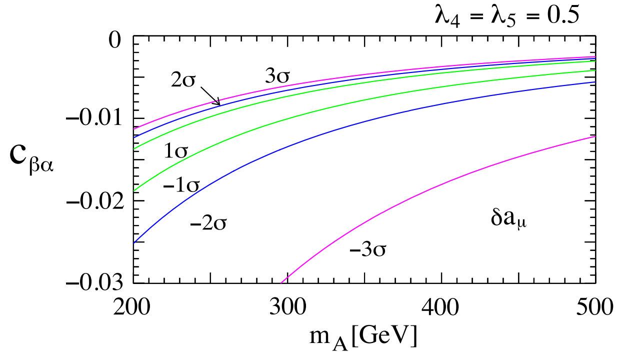

FIG. 3. Numerical result for δa_μ as a function of m_A and $c_{\beta\alpha}$ assuming $\mathrm{BR}(h \to \mu\tau) = 0.84\%$. Regions that explain the muon $g-2$ anomaly in Eq. (13) within $\pm 1\sigma$, $\pm 2\sigma$, and $\pm 3\sigma$ are shown. Here we determine the mass spectrum of heavy Higgs bosons as a function of m_A assuming $\lambda_4 = \lambda_5 = 0.5$ in Eq. (10). We have assumed $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ to obtain the positive contribution to δa_μ, and the Yukawa couplings $\rho_e^{\mu\tau(\tau\mu)}$ are fixed to realize $\mathrm{BR}(h \to \mu\tau) = 0.84\%$.

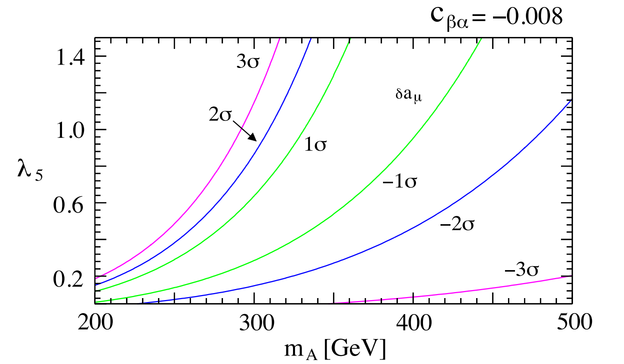

FIG. 4. Numerical result for δa_μ as a function of m_A and λ_5 assuming $\mathrm{BR}(h \to \mu\tau) = 0.84\%$ with the fixed $c_{\beta\alpha}(= -0.008)$. Here we have assumed $\lambda_4 = \lambda_5$ and $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$.

and $\mathrm{BR}(h \to \mu\tau)$ for $m_A = 250$ GeV (upper figure) and 350 GeV (lower figure). Regions where the muon $g-2$ anomaly in Eq. (13) is explained within $\pm 1\sigma$, $\pm 2\sigma$, and $\pm 3\sigma$ are shown. Here we have fixed the mass spectrum of heavy Higgs bosons assuming $\lambda_4 = \lambda_5 = 0.5$ in Eq. (10). We have assumed that $\rho_e^{\mu\tau}\rho_e^{\tau\mu} < 0$ with $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ to obtain the positive contribution to δa_μ. We only discuss the case with $c_{\beta\alpha} < 0$; however, the predictions of δa_μ and $\mathrm{BR}(h \to \mu\tau)$ do not change even if the sign of $c_{\beta\alpha}$ is flipped ($c_{\beta\alpha} \to -c_{\beta\alpha}$). One can see that there are regions where both anomalies of the muon $g-2$ and $h \to \mu\tau$ can be explained in the 2HDM. Although larger $\mathrm{BR}(h \to \mu\tau)$ is preferred by the muon $g-2$ anomaly, the regions where $\mathrm{BR}(h \to \mu\tau)$ is smaller than the CMS result are also allowed by the muon $g-2$ anomaly as long as $|c_{\beta\alpha}|$ is small.

In Fig. 3, the numerical result for the δa_μ is depicted as a function of m_A and $c_{\beta\alpha}$ fixing $\mathrm{BR}(h \to \mu\tau) = 0.84\%$. Regions that explain the muon $g-2$ anomaly in Eq. (13) within $\pm 1\sigma$, $\pm 2\sigma$, and $\pm 3\sigma$ are shown. In this plot, we take $\lambda_4 = \lambda_5 = 0.5$ to determine the mass spectrum of heavy Higgs bosons as a function of m_A [see Eq. (10)]. We assume that the Yukawa couplings $\rho_e^{\tau\mu}$ and $\rho_e^{\mu\tau}$ are chosen to realize $\mathrm{BR}(h \to \mu\tau) = 0.84\%$ with $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$. When $|c_{\beta\alpha}|$ gets smaller, δa_μ increases because the Yukawa couplings $\rho_e^{\mu\tau(\tau\mu)}$ become larger with the fixed $\mathrm{BR}(h \to \mu\tau)$. It is interesting to see that the 2HDM can explain both anomalies of the muon $g-2$ and $h \to \mu\tau$ when $|c_{\beta\alpha}|$ is small ($|c_{\beta\alpha}| \sim 0.01$) and m_A is $m_A = 200$–500 GeV. We note that the small mixing $|c_{\beta\alpha}|$ is consistent with the current results of the Higgs coupling measurements as well as the constraints from the electroweak observables. In Fig. 4, similar to Fig. 3, the numerical result for the δa_μ is shown as a function of m_A and λ_5 fixing that $\mathrm{BR}(h \to \mu\tau) = 0.84\%$ and $c_{\beta\alpha} = -0.008$. We have assumed that $\rho_e^{\tau\mu} = -\rho_e^{\mu\tau}$ and $\lambda_4 = \lambda_5$. As λ_5 gets larger, the δa_μ becomes larger because the nondegeneracy between H and A increases and it enhances the δa_μ. Figures 2, 3, and 4 show the typical interesting regions which explain both anomalies of the muon $g-2$ and $h \to \mu\tau$.

In the general 2HDM, the small mixing $|c_{\beta\alpha}|$ and small Yukawa couplings, ρ_e^{ij}, could be achieved by the small breaking of an extra symmetry such as Z_2 symmetry in type-I 2HDM. However, the typical interesting regions for both anomalies we study here require the small mixing but relatively large $\mu - \tau$ flavor violating Yukawa couplings. Therefore, the realization of the parameter regions may not be easy to understand without an understanding of the more fundamental theory. The theoretical understanding of the parameter regions we study here is beyond the scope of this paper. We think that our study together with the experimental studies would be important to unveil the more fundamental theory.

IV. τ- AND μ-PHYSICS IN THIS SCENARIO

We have seen that the general 2HDM with the $\mu - \tau$ flavor violation accommodates the muon $g-2$ anomaly and the CMS excess in $h \to \mu\tau$ decay, simultaneously. The parameter regions with $|c_{\beta\alpha}| \sim 0.01$ and $m_A \sim O(100)$ GeV are typically interesting. In this section, we investigate what kinds of predictions and/or constraints in τ- and μ-physics are given in this scenario.

A. $\tau \to \mu\gamma$

First, we discuss the process, $\tau \to \mu\gamma$. The $\mu - \tau$ flavor violating Yukawa couplings induce the flavor violating phenomena $\tau \to \mu\gamma$, as shown, for example, in Fig. 5. We parametrize the decay amplitude ($T_{\tau\to\mu\gamma}$) as follows:

$$T_{\tau\to\mu\gamma} = e\epsilon^{\alpha *}(q)\bar{u}_\mu(p-q)m_\tau i\sigma_{\alpha\beta}q^\beta(A_L P_L + A_R P_R)u_\tau(p), \tag{16}$$

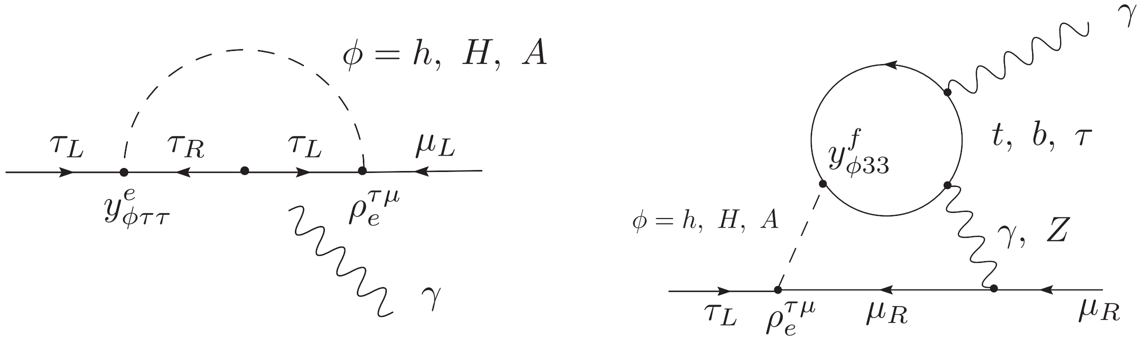

FIG. 5. Some of the Feynman diagrams that contribute to $\tau \to \mu\gamma$ processes at the one-loop level (left figure) which are induced by $y^e_{\phi\tau\tau}(\phi = h, H, A)$ Yukawa couplings and at the two-loop level (right figure) which are Barr-Zee–type contributions and induced by the third generation fermions via $y^f_{\phi 33}(f = u, d, e)$ Yukawa couplings. Diagrams where the fermion chiralities are flipped also contribute. We also have a Barr-Zee–type two-loop contribution involving the W-loop, which is not shown here.

where $P_{R,L}(= (1 \pm \gamma_5)/2)$ are chirality projection operators, and e, ϵ^α, q, p, and u_f are the electric charge, a photon polarization vector, a photon momentum, a τ momentum, and a spinor of the fermion f, respectively. The branching ratio is given by

$$\frac{\mathrm{BR}(\tau \to \mu\gamma)}{\mathrm{BR}(\tau \to \mu\bar{\nu}_\mu \nu_\tau)} = \frac{48\pi^3 \alpha(|A_L|^2 + |A_R|^2)}{G_F^2}, \tag{17}$$

where α and G_F are the fine structure constant and the Fermi constant, respectively. The lepton flavor violating Higgs contributions to A_L and A_R via $y^e_{\phi\tau\tau}(\phi = h, H, A)$ Yukawa interactions at the one-loop level (left figure in Fig. 5) are given by[3]

$$A_{L,R} = \sum_{\phi = h,H,A,H^-} A^\phi_{L,R}, \tag{18}$$

$$A^\phi_L = \frac{y^{e*}_{\phi\tau\mu}}{16\pi^2 m_\phi^2}\left[y^{e*}_{\phi\tau\tau}\left(\log\frac{m_\phi^2}{m_\tau^2} - \frac{3}{2}\right) + \frac{y^e_{\phi\tau\tau}}{6}\right] (\phi = h, H, A),$$
$$A^\phi_R = A^\phi_L|_{y^{e*}_{\phi\tau\mu} \to y^e_{\phi\mu\tau}, y^e_{\phi\tau\tau} \leftrightarrow y^{e*}_{\phi\tau\tau}} (\phi = h, H, A),$$
$$A^{H^-}_L = -\frac{(\rho_e^\dagger \rho_e)^{\mu\tau}}{192\pi^2 m_{H^-}^2}, \qquad A^{H^-}_R = 0, \tag{19}$$

where $A^\phi_{L,R}(\phi = h, H, A, H^-)$ are the ϕ contributions at the one-loop level. The Yukawa couplings $y^e_{\phi\tau\tau}(\phi = h, H, A)$ are given in Eq. (7). Here we have neglected the $O(m_\mu/m_\tau)$ contributions.

We also find that the Barr-Zee–type contributions ($A^{\mathrm{BZ}}_{L,R}$) at the two-loop level are important and dominant in the most of the cases. The third generation fermion contributions via $y^f_{\phi 33}(f = u, d, e)$ Yukawa couplings[4] (shown in the right figure in Fig. 5) and the W-boson contribution (not shown in Fig. 5) are given by[5]

$$\begin{aligned}
A^{\mathrm{BZ}}_L = &-\sum_{\phi = h,H,A; f = u,d,e} \frac{N_C Q_f \alpha}{8\pi^3} \frac{y^{e*}_{\phi\tau\mu}}{m_\tau m_{f_3}}\Big[Q_f\{\mathrm{Re}(y^f_{\phi 33}) F_H(x_{f\phi}) - i\mathrm{Im}(y^f_{\phi 33}) F_A(x_{f\phi})\} \\
&+ \frac{(1 - 4s_W^2)(2T_{3f} - 4Q_f s_W^2)}{16 s_W^2 c_W^2}\{\mathrm{Re}(y^f_{\phi 33})\tilde{F}_H(x_{f\phi}, x_{fZ}) - i\mathrm{Im}(y^f_{\phi 33})\tilde{F}_A(x_{f\phi}, x_{fZ})\}\Big] \\
&+ \sum_{\phi = h,H} \frac{\alpha}{16\pi^3} \frac{g_{\phi WW} y^{e*}_{\phi\tau\mu}}{m_\tau v}\Big[3F_H(x_{W\phi}) + \frac{23}{4}F_A(x_{W\phi}) + \frac{3}{4}G(x_{W\phi}) + \frac{m_\phi^2}{2m_W^2}\{F_H(x_{W\phi}) - F_A(x_{W\phi})\} \\
&+ \frac{1 - 4s_W^2}{8 s_W^2}\Big\{\left(5 - t_W^2 + \frac{1 - t_W^2}{2x_{W\phi}}\right)\tilde{F}_H(x_{W\phi}, x_{WZ}) + \left(7 - 3t_W^2 - \frac{1 - t_W^2}{2x_{W\phi}}\right)\tilde{F}_A(x_{W\phi}, x_{WZ}) + \frac{3}{2}\{F_A(x_{W\phi}) + G(x_{W\phi})\}\Big\}\Big],
\end{aligned} \tag{20}$$

[3]Yukawa couplings $y^e_{\phi\mu\mu}$ also contribute to $\tau \to \mu\gamma$. However, the SM part of $y^e_{\phi\mu\mu}$ is smaller than the one of $y^e_{\phi\tau\tau}$, and $\rho_e^{\mu\mu}$ is strongly constrained by the $\tau \to 3\mu$ process as discussed later. Therefore, we have neglected the contributions from $y^e_{\phi\mu\mu}$.

[4]In our notation, $y^u_{\phi 33} = y^u_{\phi tt}$, $y^d_{\phi 33} = y^d_{\phi bb}$, $y^e_{\phi 33} = y^e_{\phi\tau\tau}$.

[5]The Barr-Zee contributions to $\mu \to e\gamma$ have been studied in Ref. [59]. The application to $\tau \to \mu\gamma$ is apparent, and we adopt their results for $\tau \to \mu\gamma$.

$$A_R^{\rm BZ} = A_L^{\rm BZ}(y^{e*}_{\phi\tau\mu} \to y^e_{\phi\mu\tau}, i \to -i), \tag{21}$$

where $x_{f\phi} = m^2_{f_3}/m^2_\phi$, $x_{fZ} = m^2_{f_3}/m^2_Z (f_3 = t, b, \tau$ for $f = u, d, e)$, $x_{W\phi} = m^2_W/m^2_\phi$ and $x_{WZ} = m^2_W/m^2_Z$, and $s^2_W = \sin\theta^2_W$, $c^2_W = \cos\theta^2_W$, and $t^2_W = \tan\theta^2_W$. T_{3f} denotes the isospin of the fermion. Here the couplings $g_{\phi WW} = s_{\beta\alpha}(c_{\beta\alpha})$ for $\phi = h$ $(\phi = H)$. Functions $F_{H,A}$, G, and $\tilde{F}_{H,A}$ are defined by

$$F_H(z) = \frac{z}{2}\int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-z} \log\frac{x(1-x)}{z}, \tag{22}$$

$$F_A(z) = \frac{z}{2}\int_0^1 dx \frac{1}{x(1-x)-z} \log\frac{x(1-x)}{z}, \tag{23}$$

$$G(z) = -\frac{z}{2}\int_0^1 dx \frac{1}{x(1-x)-z} \times \left[1 - \frac{z}{x(1-x)-z}\log\frac{x(1-x)}{z}\right], \tag{24}$$

$$\tilde{F}_H(x,y) = \frac{xF_H(y) - yF_H(x)}{x-y}, \tag{25}$$

$$\tilde{F}_A(x,y) = \frac{xF_A(y) - yF_A(x)}{x-y}. \tag{26}$$

Note ${\rm Im}(y^f_{\phi 33}) = 0$ for $f = h$ and H and ${\rm Re}(y^f_{\phi 33}) = 0$ for $f = A$ are satisfied, if the Yukawa couplings ρ^{ij}_f are real, as shown in Eq. (7). For simplicity, we assume that all ρ^{ij}_f are real in the calculation of $\tau \to \mu\gamma$. The contribution in the first line (the second line) of Eq. (20) comes from the effective $\phi\gamma\gamma$ vertex ($\phi Z\gamma$ vertex) induced by the third generation fermion loop, and the one in the third and the fourth lines (the fifth and the sixth lines) originates from the effective $\phi\gamma\gamma$ vertex ($\phi Z\gamma$ vertex) generated by the W-boson loop. In the analysis of $\tau \to \mu\gamma$ in Ref. [16], we have not included the Barr-Zee–type contributions induced by the effective $\phi\gamma Z$ vertex since they are subdominant contributions. Here we include them and find they change the results by about 10%.

The total amplitude $A_{L,R}$ is a sum of all contributions,

$$A_{L,R} = \sum_{\phi=h,H,A,H^-} A^\phi_{L,R} + A^{\rm BZ}_{L,R}. \tag{27}$$

In Fig. 6, numerical results for ${\rm BR}(\tau \to \mu\gamma)$ as a function of ${\rm BR}(h \to \mu\tau)$ are shown. The red and green lines correspond to the cases with $c_{\beta\alpha} = -0.012$ ($m_A = 250$ GeV) and $c_{\beta\alpha} = -0.007$ ($m_A = 350$ GeV), respectively. As one can see, if ${\rm BR}(h \to \mu\tau) = 0.84\%$ as suggested by the CMS experiment, the branching ratio for $\tau \to \mu\gamma$ can be larger than 10^{-9}, which might be within the reach of the future

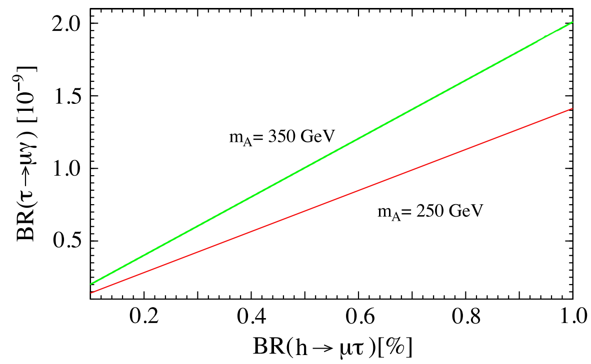

FIG. 6. ${\rm BR}(h \to \mu\tau)$ vs ${\rm BR}(\tau \to \mu\gamma)$. The red and green lines correspond to the cases with $c_{\beta\alpha} = -0.012$ ($m_A = 250$ GeV) and $c_{\beta\alpha} = -0.007$ ($m_A = 350$ GeV), respectively.

B-factory experiment, the Belle II. Note that the branching ratio ${\rm BR}(\tau \to \mu\gamma)$ does not strongly depend on the Higgs mixing parameter $c_{\beta\alpha}$ when $c_{\beta\alpha}$ is small. We note that the cancellation between the one-loop and two-loop Barr-Zee–type contributions happens, and hence the branching ratio ${\rm BR}(\tau \to \mu\gamma)$ is not simply suppressed by the heavy Higgs boson masses.

If the extra Yukawa couplings other than $\rho^{\mu\tau(\tau\mu)}_e$ are not negligible, the branching ratio ${\rm BR}(\tau \to \mu\gamma)$ could be further enhanced. For example, the extra Yukawa coupling $\rho^{\tau\tau}_e$ can contribute at the one-loop level, and on the other hand, ρ^{tt}_u can affect the branching ratio via the Barr-Zee–type two-loop contribution. In Fig. 7, numerical results for ${\rm BR}(\tau \to \mu\gamma)$ are shown as a function of $\rho^{\tau\tau}_e$ and ρ^{tt}_u. Lines for ${\rm BR}(\tau \to \mu\gamma)/10^{-8} = 0.1$ and 4.4 (current experimental limit) are shown. Here we have assumed $m_A = 350$ GeV with $\lambda_4 = \lambda_5 = 0.5$, $c_{\beta\alpha} = -0.007$, and ${\rm BR}(h \to \mu\tau) = 0.84\%$ with $\rho^{\mu\tau}_e = -\rho^{\tau\mu}_e$. This parameter

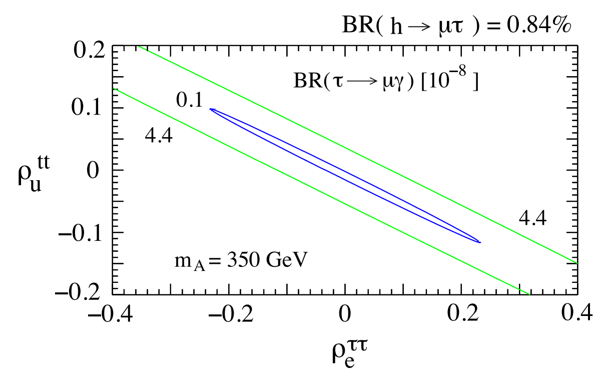

FIG. 7. Numerical result for ${\rm BR}(\tau \to \mu\gamma)$ as a function of $\rho^{\tau\tau}_e$ and ρ^{tt}_u. Lines for ${\rm BR}(\tau \to \mu\gamma)/10^{-8} = 0.1$ and 4.4 (current experimental limit) are shown. Here we have assumed $m_A = 350$ GeV with $\lambda_4 = \lambda_5 = 0.5$, $c_{\beta\alpha} = -0.007$, and ${\rm BR}(h \to \mu\tau) = 0.84\%$ with $\rho^{\mu\tau}_e = -\rho^{\tau\mu}_e$. We note that for this parameter set, $\delta a_\mu = 2.2 \times 10^{-9}$, which explains the muon $g-2$ anomaly within the 1σ.

set can enhance the muon $g-2$ as $\delta a_\mu = 2.2\times 10^{-9}$ which is within the 1σ. At present, the extra Yukawa couplings $\rho_e^{\tau\tau}$ and ρ_u^{tt} can still be larger than, for example, $O(0.1)$ with some correlation; however, the future experimental constraint would be significant for this scenario. Therefore, the $\tau\to\mu\gamma$ process would be important to probe the scenario.

B. $\mu\to e\gamma$, $\tau\to e\gamma$, and electron $g-2$

The process $\mu\to e\gamma$ cannot be induced by the only $\mu-\tau$ flavor violating Yukawa couplings. However, if $e-\mu$ or $e-\tau$ flavor-violation Yukawa couplings are not vanishing, $\mu\to e\gamma$ is easily enhanced. Since there is a strong constraint from this process, $e-\mu$ and $e-\tau$ flavor violating couplings are strongly constrained.

Similar to $\tau\to\mu\gamma$, we parametrize the decay amplitude $(T_{\mu\to e\gamma})$ as

$$T_{\mu\to e\gamma} = e\epsilon^{\alpha *}\bar{u}_e m_\mu i\sigma_{\alpha\beta}q^\beta (A_L P_L + A_R P_R) u_\mu, \quad (28)$$

and the branching ratio is given by

$$\mathrm{BR}(\mu\to e\gamma) = \frac{48\pi^3\alpha(|A_L|^2+|A_R|^2)}{G_F^2}. \quad (29)$$

The neutral Higgs contributions $A_{L,R}^\phi$ ($\phi = h, H, A$) to $A_{L,R}$ at the one loop are given by

$$A_L^\phi = \frac{1}{16\pi^2}\sum_{i=\mu,\tau}\frac{y_{\phi ie}^{e*}}{m_\phi^2}\left[\frac{m_i}{m_\mu}y_{\phi\mu i}^{e*}\left(\log\frac{m_\phi^2}{m_i^2}-\frac{3}{2}\right)+\frac{y_{\phi i\mu}^{e}}{6}\right], \quad (30)$$

$$A_R^\phi = \frac{1}{16\pi^2}\sum_{i=\mu,\tau}\frac{y_{\phi ei}^{e}}{m_\phi^2}\left[\frac{m_i}{m_\mu}y_{\phi i\mu}^{e}\left(\log\frac{m_\phi^2}{m_i^2}-\frac{3}{2}\right)+\frac{y_{\phi\mu i}^{e*}}{6}\right], \quad (31)$$

where the Yukawa couplings $y_{\phi ij}^e$ are defined in Eq. (7). Here we neglect an electron mass, and we assume that the Yukawa coupling $y_{\phi ee}^e$ is negligible.[6] The charged Higgs contribution to $A_{L,R}$ is

$$A_L^{H^-} = -\frac{(\bar{\rho}_e^\dagger \rho_e)_{e\mu}}{192\pi^2 m_{H^-}^2}, \qquad A_R^{H^-} = 0. \quad (32)$$

For nonzero $y_{\phi\mu e(e\mu)}^e$, the Barr-Zee–type contributions $(A_{L,R}^{\mathrm{BZ}})$ at the two-loop level are significant. The expression of $A_{L,R}^{\mathrm{BZ}}$ is the same as the one for the $\tau\to\mu\gamma$ case shown in Eq. (20) except that the flavor violating Yukawa couplings $y_{\phi\tau\mu(\mu\tau)}^{e(*)}$ should be replaced by $y_{\phi\mu e(e\mu)}^{e(*)}$ and the τ mass (m_τ) should be replaced by the μ mass (m_μ). The total $A_{L,R}$ is a sum of all contributions,

[6]The Yukawa coupling ρ_e^{ee} is strongly constrained by the $\tau\to\mu e^+e^-$ process, as studied later. Therefore, our assumption will be justified.

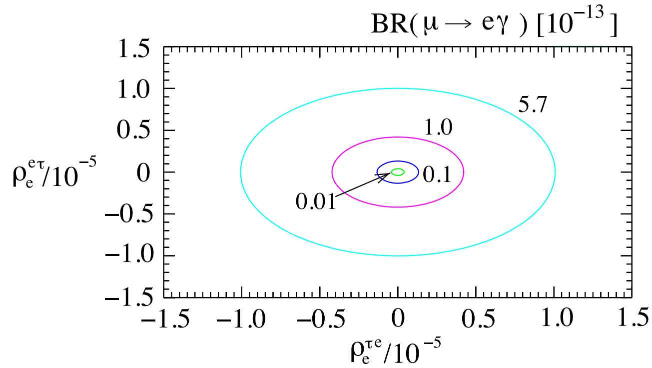

FIG. 8. Numerical result for $\mathrm{BR}(\mu\to e\gamma)$ as a function of $\rho_e^{\tau e}$ and $\rho_e^{e\tau}$. Lines for $\mathrm{BR}(\mu\to e\gamma)/10^{-13} = 5.7$ (current limit), 1.0, 0.1, and 0.01 are shown. Here we have assumed that $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$, $c_{\beta\alpha}=-0.007$, and $\mathrm{BR}(h\to\mu\tau) = 0.84\%$ with $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$, and extra Yukawa couplings ρ_f other than $\rho_e^{\mu\tau(\tau\mu)}$ and $\rho_e^{\tau e(e\tau)}$ are negligible. We note that for this parameter set, $\delta a_\mu = 2.2\times 10^{-9}$.

$$A_{L,R} = \sum_{\phi=h,H,A,H^-} A_{L,R}^\phi + A_{L,R}^{\mathrm{BZ}}. \quad (33)$$

Similar to the muon $g-2$, the contributions from the $\mu-\tau$ flavor violating Yukawa interactions together with the $e-\tau$ flavor violation have $O(m_\tau/m_\mu)$ enhancement and induce significant contributions to $\mu\to e\gamma$. In Fig. 8, we show numerical results for $\mathrm{BR}(\mu\to e\gamma)$ as a function of $\rho_e^{\tau e}$ and $\rho_e^{e\tau}$. Here we have taken $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$, $c_{\beta\alpha}=-0.07$, and $\mathrm{BR}(h\to\mu\tau)=0.84\%$ with $\rho_e^{\mu\tau}=-\rho_e^{\tau\mu}$. We have assumed that extra Yukawa couplings ρ_f^{ij} other than $\rho_e^{\mu\tau(\tau\mu)}$ and $\rho_e^{\tau e(e\tau)}$ are negligible. This parameter set corresponds to $\delta a_\mu = 2.2\times 10^{-9}$. One can see that the current limit on $\mathrm{BR}(\mu\to e\gamma)$ strongly constrains the $e-\tau$ flavor violating couplings $\rho_e^{\tau e(e\tau)}$ if the CMS excess of $\mathrm{BR}(h\to\mu\tau)$ is true. If we change the value of $\mathrm{BR}(h\to\mu\tau)$ in Fig. 8, the experimental bound of $\rho_e^{\tau e(e\tau)}$ is relaxed by the factor $\sqrt{\frac{0.84\%}{\mathrm{BR}(h\to\mu\tau)}}$ when $\rho_e^{\tau e}=\rho_e^{e\tau}$ is assumed.

If Yukawa couplings $\rho_e^{\tau e(e\tau)}$ are negligible but $\rho_e^{\mu e(e\mu)}$ are not, the Barr-Zee–type two-loop contributions are dominant.[7] We show numerical results for $\mathrm{BR}(\mu\to e\gamma)$ as a function of $\rho_e^{\mu e}$ and $\rho_e^{e\mu}$. Here we have assumed that $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$, $c_{\beta\alpha}=-0.007$, and extra Yukawa couplings ρ_f^{ij} other than $\rho_e^{\mu\tau(\tau\mu)}$ and $\rho_e^{\mu e(e\mu)}$ are negligible. As one can see from Fig. 9, $\rho_e^{\mu e(e\mu)}$ couplings are

[7]If $\rho_e^{\mu\mu}$ is also nonzero, there are also one-loop contributions as shown in Eq. (31). However, the coupling $\rho_e^{\mu\mu}$ is strongly constrained by the $\tau\to 3\mu$ bound, as discussed later. Therefore, the effect from $\rho_e^{\mu\mu}$ is negligible, and we neglect it in our numerical analysis.

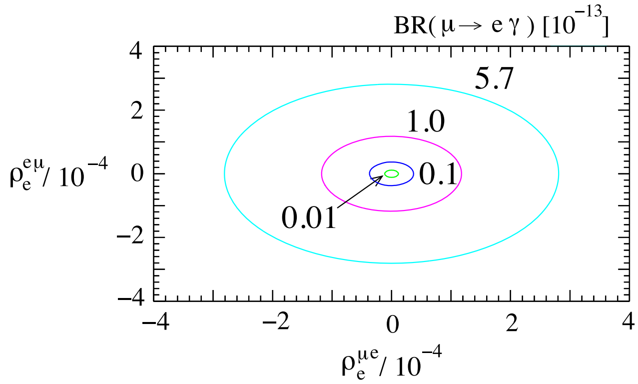

FIG. 9. Numerical result for BR($\mu \to e\gamma$) as a function of $\rho_e^{\mu e}$ and $\rho_e^{e\mu}$. Lines for BR$(\mu \to e\gamma)/10^{-13} = 5.7$ (current limit), 1.0, 0.1, and 0.01 are shown. Here we have assumed that $m_A = 350$ GeV with $\lambda_4 = \lambda_5 = 0.5$, $c_{\beta\alpha} = -0.007$, and extra Yukawa couplings ρ_f other than $\rho_e^{\mu\tau(\tau\mu)}$ and $\rho_e^{\mu e(e\mu)}$ are negligible.

also severely constrained by the $\mu \to e\gamma$ bound. Note that the prediction of $\mu \to e\gamma$ for this case does not depend on the value of BR($h \to \mu\tau$). The future improvement of BR($\mu \to e\gamma$) at the level of 10^{-14} as proposed by the MEG II experiment [60] would significantly probe the flavor structure of this scenario.

The effective operator for $\mu \to e\gamma$ also generates the $\mu - e$ conversion process in nuclei. Besides, the extra Yukawa couplings, $\rho_e^{\mu e}$ and $\rho_e^{e\mu}$, may enhance the $\mu - e$ conversion through the tree-level Higgs exchanging. The contribution depends on the extra Yukawa couplings in the quark sector as well, and then our model may also be tested by the experiments [61–64], although our prediction is vague because of the ambiguity of the Yukawa couplings.[8]

We comment on the consequence of the strong constraints on the $e - \tau$ and $e - \mu$ flavor violations. Unlike the $\mu - \tau$ flavor violation, the $e - \tau$ flavor violating Yukawa couplings in this scenario are strongly constrained as we have seen above. Therefore, the prediction of BR($\tau \to e\gamma$) is expected to be small. Similarly, because of the smallness of the $e - \tau$ and $e - \mu$ flavor violation, we also expect that the new physics contributions to the anomalous magnetic moment of electron (electron $g - 2$) should be small.

C. Muon electric dipole moment (muon EDM)

When we discussed the muon $g - 2$, we assumed that the $\mu - \tau$ flavor violating Yukawa couplings are real. If the $\mu - \tau$ Yukawa couplings are complex, the couplings $\rho_e^{\mu\tau}\rho_e^{\tau\mu}$ in Eq. (15) should be replaced by Re$(\rho_e^{\mu\tau}\rho_e^{\tau\mu})$. In addition, the imaginary parts of the Yukawa couplings generate an EDM of muon. Since the muon $g - 2$ and the muon EDM are induced by the same Feynman diagram shown in Fig. 1, these quantities are correlated via the unknown *CP*-violating phase. The effective operators for the muon $g - 2$ (δa_μ) and the muon EDM (δd_μ) are expressed by

$$\mathcal{L} = \bar{\mu}\sigma^{\mu\nu}\left(\frac{e}{4m_\mu}\delta a_\mu - \frac{i}{2}\delta d_\mu \gamma_5\right)\mu F_{\mu\nu}. \tag{34}$$

If we parametrize the complex Yukawa couplings as follows:

$$\rho_e^{\mu\tau}\rho_e^{\tau\mu} = |\rho_e^{\mu\tau}\rho_e^{\tau\mu}|e^{i\phi}, \tag{35}$$

the relation between the muon $g - 2$ (δa_μ) and the muon EDM (δd_μ) induced by the $\mu - \tau$ flavor violating Yukawa couplings is given by[9]

$$\frac{\delta d_\mu}{\delta a_\mu} = -\frac{e \tan\phi}{2m_\mu}. \tag{36}$$

Therefore, the predicted muon EDM is

$$\delta d_\mu = -3 \times 10^{-22} e \cdot \mathrm{cm} \times \left(\frac{\tan\phi}{1.0}\right)\left(\frac{\delta a_\mu}{3 \times 10^{-9}}\right). \tag{37}$$

The current limit [65] is

$$|d_\mu| < 1.9 \times 10^{-19} e \cdot \mathrm{cm} (95\%\mathrm{C.L.}), \tag{38}$$

and hence it is not sensitive to this scenario at present. However, the future improvement at the level of $10^{-24} e \cdot$ cm [66] would be significant to probe the scenario.

D. $\tau \to \mu\nu\bar{\nu}$

The Yukawa couplings $\rho_e^{\mu\tau(\tau\mu)}$ induce a correction to $\tau \to \mu\nu\bar{\nu}$ via a charged Higgs mediation, where the flavor of final neutrino and antineutrino states is summed up since it is not detected.[10]

The correction δ is given as follows:

$$\Gamma(\tau \to \mu\nu\bar{\nu}) = \frac{m_\tau^5 G_F^2}{192\pi^3}(1 + \delta),$$
$$\delta = \frac{|\rho_e^{\mu\tau}|^2 |\rho_e^{\tau\mu}|^2}{32 G_F^2 m_{H^+}^4}. \tag{39}$$

In Fig. 10, numerical results for the correction δ given above are shown as a function of $c_{\beta\alpha}$ and BR($h \to \mu\tau$) in

[8]The study on the tree-level flavor changing couplings of quarks is beyond our scope.

[9]Here, we assume that Higgs potential is *CP* conserving.

[10]In general, the unknown Yukawa couplings $\rho_e^{i\tau}$ and $\rho_e^{i\mu}$ $(i = e, \mu, \tau)$ generate the extra corrections to δ. However, the Yukawa couplings $\rho_e^{e\tau(e\mu)}$ and $\rho_e^{\mu\mu}$ are strongly constrained by $\mu \to e\gamma$ and $\tau^- \to \mu^-\mu^+\mu^-$, respectively. Therefore, the contributions from these couplings are negligible. The unknown Yukawa coupling $\rho_e^{\tau\tau}$ can be sizable, and hence it can increase the prediction of the δ. Thus our result of δ induced from $\rho_e^{\mu\tau(\tau\mu)}$ is viewed as a conservative estimate.

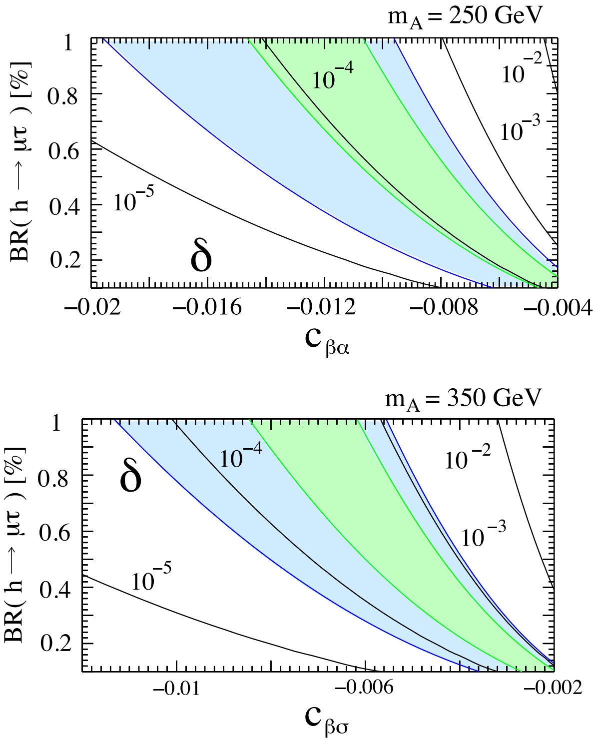

FIG. 10. Numerical result for a correction to $\tau \to \mu\nu\bar{\nu}$, δ given in Eq. (39) as a function of $c_{\beta\alpha}$ and $\mathrm{BR}(h \to \mu\tau)$ in the same parameter set of Fig. 2. Black solid lines for $\delta = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$ (from left to right) are shown for $m_A = 250$ GeV (upper figure) and $m_A = 350$ GeV (lower figure). Here we also show the region where the muon $g-2$ anomaly is explained within 1σ (light green area) and 2σ (light blue areas). Here we have assumed that the extra Yukawa couplings ρ_f other than $\rho_e^{\mu\tau(\tau\mu)}$ are negligible.

the same parameter set of Fig. 2. One can see that as the correction to the muon $g-2$ (δa_μ) gets larger, the size of δ also becomes larger, and they are correlated with each other, independent of $\mathrm{BR}(h \to \mu\tau)$. The interesting regions that explain the muon $g-2$ anomaly within 1σ predict $\delta \lesssim 10^{-4} - 10^{-3}$. The current precision of the measurement of the decay rate $\Gamma(\tau \to \mu\nu\bar{\nu})$ is at the level of 10^{-3} [48]. Therefore, the further improvement of the precision would be important for this scenario. In addition, from the τ decay, the *BABAR* Collaboration has reported a measurement of the charged current lepton universality [67], given by

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{\mathrm{BR}(\tau^- \to \mu^-\nu\bar{\nu})}{\mathrm{BR}(\tau^- \to e^-\nu\bar{\nu})} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}, \tag{40}$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$, which is a phase space factor. The universality of the gauge interaction in the SM predicts $g_e = g_\mu$, and the current experimental results are

$$\left(\frac{g_\mu}{g_e}\right) = 1.0036 \pm 0.0020(BABAR),$$
$$= 1.0018 \pm 0.0014(\text{world average}). \tag{41}$$

In our scenario, we expect the correction to $\tau \to e\nu\bar{\nu}$ would be small because of the strong constraint on the $e-\tau$ flavor violation from the $\mu \to e\gamma$ process. Therefore, the charged Higgs contribution to $\tau \to \mu\nu\bar{\nu}$ with $\mu-\tau$ flavor violating Yukawa couplings induces the significant correction to the violation of the lepton universality above,

$$\left(\frac{g_\mu}{g_e}\right)^2 = 1 + \delta. \tag{42}$$

The result from the Belle Collaboration and the further improvement of the precision of the lepton universality would have an important impact on our scenario.

E. $\tau^- \to \mu^- l^+ l^-$, $\tau^- \to e^- l^+ l^-$ ($l = e, \mu$), $\mu^+ \to e^+ e^- e^+$ and others

The nonzero Yukawa couplings $\rho_e^{\mu\tau(\tau\mu)}$ also generate processes $\tau^- \to \mu^-\mu^+\mu^-$ and $\tau^- \to \mu^- e^+ e^-$ ($\tau \to 3\mu$ and $\tau \to \mu ee$ for short, respectively) at the tree level. They are induced without unknown $\rho_e^{\mu\mu}$ and ρ_e^{ee} Yukawa couplings. The branching ratios, however, are too small to be observed. Therefore, nonzero $\rho_e^{\mu\mu}$ and ρ_e^{ee} are important for these processes.[11] The branching ratios for $\tau \to 3\mu$ and $\tau \to \mu ee$ are given by [68]

$$\frac{\mathrm{BR}(\tau \to 3\mu)}{\mathrm{BR}(\tau \to \mu\nu\bar{\nu})} = \sum_{\phi,\phi'=h,H,A} \frac{I(\phi,\phi')}{64 G_F^2},$$
$$I(\phi,\phi') = 2\left(\frac{y^e_{\phi\mu\tau} y^{e*}_{\phi\mu\mu}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\mu\tau} y^{e}_{\phi'\mu\mu}}{m_{\phi'}^2}\right) + 2\left(\frac{y^e_{\phi\tau\mu} y^{e*}_{\phi\mu\mu}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\tau\mu} y^{e}_{\phi'\mu\mu}}{m_{\phi'}^2}\right)$$
$$+ \left(\frac{y^e_{\phi\mu\tau} y^{e}_{\phi\mu\mu}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\mu\tau} y^{e*}_{\phi'\mu\mu}}{m_{\phi'}^2}\right) + \left(\frac{y^e_{\phi\tau\mu} y^{e}_{\phi\mu\mu}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\tau\mu} y^{e*}_{\phi'\mu\mu}}{m_{\phi'}^2}\right), \tag{43}$$

[11]Nonzero $\rho_e^{\tau e(e\tau)}$ and $\rho_e^{\mu e(e\mu)}$ couplings also induce the $\tau \to \mu ee$ process. However, these Yukawa couplings are strongly constrained by the $\mu \to e\gamma$ process as discussed in previous sections. Therefore, we neglect these effects.

$$\frac{\mathrm{BR}(\tau\to\mu ee)}{\mathrm{BR}(\tau\to\mu\nu\bar{\nu})} = \sum_{\phi,\phi'=h,H,A} \frac{J(\phi,\phi')}{32G_F^2},$$
$$J(\phi,\phi') = \left(\frac{y^e_{\phi\mu\tau}y^{e*}_{\phi ee}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\mu\tau}y^e_{\phi' ee}}{m_{\phi'}^2}\right) + \left(\frac{y^e_{\phi\tau\mu}y^{e*}_{\phi ee}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\tau\mu}y^e_{\phi' ee}}{m_{\phi'}^2}\right) + \left(\frac{y^e_{\phi\mu\tau}y^e_{\phi ee}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\mu\tau}y^{e*}_{\phi' ee}}{m_{\phi'}^2}\right) + \left(\frac{y^e_{\phi\tau\mu}y^e_{\phi ee}}{m_\phi^2}\right)\left(\frac{y^{e*}_{\phi'\tau\mu}y^{e*}_{\phi' ee}}{m_{\phi'}^2}\right). \quad (44)$$

Figure 11 shows $\mathrm{BR}(\tau\to 3\mu)$ (red curve) and $\mathrm{BR}(\tau\to\mu ee)$ (green curve) as a function of ρ_e^{ll} ($l=\mu$ for $\tau\to 3\mu$ and $l=e$ for $\tau\to\mu ee$). It is assumed that $c_{\beta\alpha}=-0.007$, $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$ and $\mathrm{BR}(h\to\mu\tau)=0.84\%$ with $\rho_e^{\mu\tau}=-\rho_e^{\tau\mu}$ in Fig. 11. One can see that the current experimental bounds,

$$\mathrm{BR}(\tau\to 3\mu) < 2.1\times 10^{-8}, \quad \mathrm{BR}(\tau\to\mu ee) < 1.8\times 10^{-8}, \quad (45)$$

set the strong constraints on the ρ_e^{ll} Yukawa couplings. For example, the parameter set shown in Fig. 11 requires $\rho_e^{ll} < 0.006$ ($l=\mu$, e). We note that the constraint on the $\rho_e^{\mu\mu}$ is still larger than the value of the muon Yukawa coupling in the SM ($y_\mu = \frac{\sqrt{2}m_\mu}{v} \sim 6\times 10^{-4}$).

Contrary to $\tau^-\to\mu^- l^+ l^-$, the $\tau^-\to e^- l^+ l^-$ ($l=e$, μ) process is suppressed in this scenario because the $\tau - e$ flavor violation is strongly constrained by the $\mu\to e\gamma$ process. Furthermore, since the constraints on $\rho_e^{e\mu(\mu e)}$ are stronger than those on $\rho_e^{\mu\mu(ee)}$, $\tau^-\to\mu^- e^+\mu^-$ is expected to be smaller than $\tau^-\to\mu^- l^+ l^-$. (Needless to say, $\tau^-\to e^-\mu^+ e^-$ is much suppressed.) Therefore, we expect that the $\tau^-\to e^- l^+ l^-$ and $\tau^-\to\mu^- e^+\mu^-$ processes will be small.

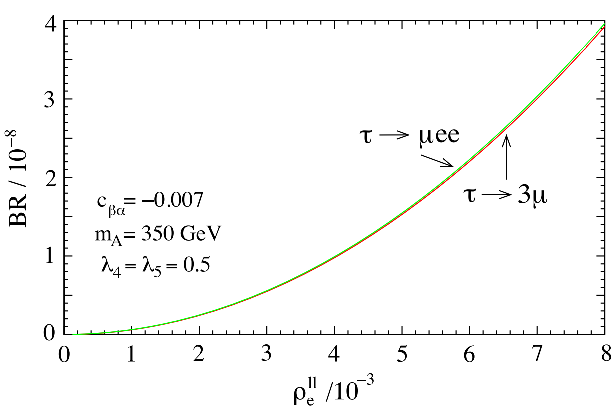

FIG. 11. $\mathrm{BR}(\tau\to 3\mu)$ (red curve) and $\mathrm{BR}(\tau\to\mu ee)$ (green curve) as a function of ρ_e^{ll} ($l=\mu$ for $\tau\to 3\mu$ and $l=e$ for $\tau\to\mu ee$). Here we have assumed that $c_{\beta\alpha}=-0.007$, $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$ and $\mathrm{BR}(h\to\mu\tau)=0.84\%$ with $\rho_e^{\mu\tau}=-\rho_e^{\tau\mu}$.

We also study $\mu^+\to e^+e^-e^+$ ($\mu\to 3e$ in short), which depends on the $\mu - e$ flavor violating Yukawa couplings $\rho_e^{e\mu(\mu e)}$ and the flavor diagonal element ρ_e^{ee}. As we have seen, the $\mu - e$ flavor violating Yukawa couplings $\rho_e^{e\mu(\mu e)}$ are constrained by the $\mu\to e\gamma$ process and the ρ_e^{ee} coupling is restricted by the $\tau\to\mu ee$ process. From Figs. 9 and 11, the current limits on $\rho_e^{\mu e(e\mu)}$ and ρ_e^{ee} are $\rho_e^{\mu e} < 2\times 10^{-4}$ for $\rho_e^{\mu e}=\rho_e^{e\mu}$ and $\rho_e^{ee} < 6\times 10^{-3}$, respectively, assuming $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$ and $c_{\beta\alpha}=-0.007$. Under these constraints, it will be interesting to see how a large branching ratio of $\mu\to 3e$ is expected. In Fig. 12, we show the $\mathrm{BR}(\mu\to 3e)$ as a function of ρ_e^{ee} and $\rho_e^{\mu e}$. In the parameter region where the constraints from $\mu\to e\gamma$ and $\tau\to\mu ee$ are satisfied, the branching ratio can be as large as about 10^{-13}. This is consistent with the current limit [48]

$$\mathrm{BR}(\mu\to 3e) < 1.0\times 10^{-12}. \quad (46)$$

The improvement of the branching ratio at the level of 10^{-16} [69] which has been proposed by the Mu3e experiment would have a significant impact on this scenario together with the improvement of $\mu\to e\gamma$ [60] and $\mu - e$ conversion in nuclei [61–64].

F. $\tau\to\mu\eta$

The $\tau\to\mu\eta$ is also generated by the extra ρ_d^{ss} Yukawa coupling via the mediation of the *CP*-odd Higgs boson at

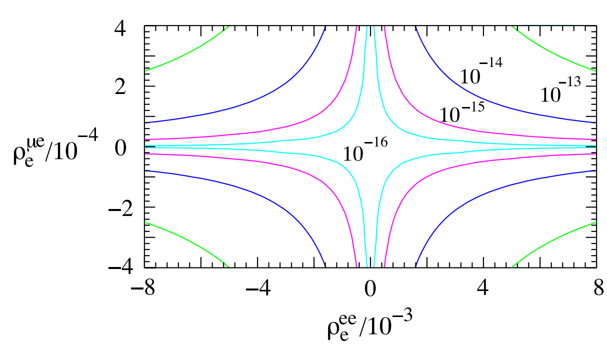

FIG. 12. $\mathrm{BR}(\mu\to 3e)$ as a function of ρ_e^{ee} and $\rho_e^{\mu e}$. Here we have assumed that $\rho_e^{\mu e}=\rho_e^{e\mu}$, $c_{\beta\alpha}=-0.007$, and $m_A = 350$ GeV with $\lambda_4=\lambda_5=0.5$.

the tree level. The expression for the branching ratio of $\tau \to \mu\eta$ is given by [70,71]

$$\mathrm{BR}(\tau \to \mu\eta) = \frac{3|\rho_d^{ss}|^2 (\bar{\rho}^{\mu\tau})^2}{32\pi} \frac{m_\tau F_\eta^2}{m_A^4 \Gamma_\tau} \left(\frac{m_\eta^2}{m_u + m_d + 4m_s} \right)^2 \times \left(1 - \frac{m_\eta^2}{m_\tau^2} \right)^2, \quad (47)$$

where m_η and F_η are the mass and the decay constant of η. For $F_\eta = 150$ MeV and $m_\eta = 548$ MeV, we obtain a constraint on ρ_d^{ss},

$$|\rho_d^{ss}| < 0.007 \left(\frac{0.3}{\bar{\rho}^{\mu\tau}} \right) \left(\frac{m_A}{350\ \mathrm{GeV}} \right)^2. \quad (48)$$

We have a strong constraint although it is still larger than the SM value of the strange quark Yukawa coupling ($y_s = \frac{\sqrt{2} m_s}{v} \sim 5 \times 10^{-4}$).

The other hadronic τ-lepton decays have been studied in Ref. [72]. They potentially provide constraints on the other extra Yukawa couplings ρ_f in the quark sector. For details, see Ref. [72].

G. $R(D^{(*)})$ and $B \to \tau\nu$

Finally, let us briefly discuss the impact of the tree-level flavor changing neutral currents on B physics. In our model, the extra scalar exchanging may contribute to the B physics as well, if H_2 couple with the SM quarks. Many channels in the B decay have been measured by the Belle and *BABAR* experiments, and are consistent with the SM predictions, although some processes still have large uncertainties. Recently, the *BABAR* experiment has announced that there are some excesses in $B \to D^{(*)}\tau\nu$ decays: $R(D) = 0.440 \pm 0.072$ and $R(D^*) = 0.332 \pm 0.030$ [73]. $R(D)$ and $R(D^*)$ denote the ratios between $\mathrm{BR}(B \to D^{(*)}\tau\nu)$ and $\mathrm{BR}(B \to D^{(*)}l\nu)$ ($l = e, \mu$), and the deviations from the SM prediction are about 3σ [73]. The Belle and the LHCb experiments have analyzed the products, and their results are closer to the SM prediction [74,75]. Then, we still cannot conclude that the excesses really come from new physics, although it would be worthwhile to investigate the possibility of the new physics.

On the other hand, the deviation of $\mathrm{BR}(B \to \tau\nu)$ is one of the long-standing issues as well. This rare process will also strictly constrain our model. The latest results on the rare decay are given by the *BABAR* [76] and Belle [77] Collaborations. The combined result is $\mathrm{BR}(B \to \tau\nu) = 1.06(20) \times 10^{-4}$ [78]. There are still large uncertainties in both the experimental side and the theoretical side because of the ambiguity of the CKM matrix. The Belle2 experiment will measure it with higher accuracy and fix the (u, b) element of the CKM matrix.

In our model, those processes are enhanced/suppressed by the charged Higgs exchanging at the tree level. Now, let us assume that $\rho_e^{\mu\tau}$ is only sizable to enhance $\mathrm{BR}(h \to \mu\tau)$. Then, the interference between the W boson and the charged Higgs contributions is absent because the flavor of the neutrino in the final state is different in each contribution. Eventually, the deviation from the SM prediction is suppressed by $m_{H^+}^4$, and it is too small as long as $\rho_e^{\tau\tau}$ is not large.[12]

V. IMPLICATION TO HIGGS PHYSICS

We have seen that the CMS excess in $h \to \mu\tau$ is consistent with the anomaly of muon $g-2$ as well as the other experimental constraints. It will be interesting to note whether other lepton flavor violating Higgs boson decays would be possible. As we have already seen, the $e-\mu$ and $e-\tau$ flavor violating Yukawa couplings are strongly constrained mainly by the $\mu \to e\gamma$ constraint. As a consequence, the lepton flavor violating Higgs boson decays $h \to e\mu$ and $h \to e\tau$ are strongly suppressed so that the near future experiments such as the ones at the LHC could not observe these decay modes, contrary to the $h \to \mu\tau$ mode. Therefore, the nonobservation of these decays is one of the interesting predictions of this scenario.

VI. SUMMARY

The excess in $h \to \mu\tau$ has been reported by the CMS Collaboration. The discrepancy of the muon $g-2$ is also one of the long-standing issues in the particle physics. These anomalous phenomena may be a hint of physics beyond the Standard Model. At a glance, these anomalies are not related to each other. However, we have found that both anomalies are related and accommodated by the $\mu-\tau$ flavor violating Yukawa interactions in a general two Higgs doublet model, and hence this motivates further studies to see whether there are any interesting predictions and indications in the scenario. We have identified the parameter space where the CMS excess in $h \to \mu\tau$ and the muon $g-2$ anomaly are both explained, and especially we have studied τ- and μ-physics in this interesting parameter space.

One of the interesting processes in the presence of the $\mu-\tau$ flavor violation is $\tau \to \mu\gamma$. The $\mu-\tau$ flavor violation suggested by the CMS excess in $h \to \mu\tau$ and the muon $g-2$ anomaly induces the large branching ratio, and it can be as large as 10^{-9}, which is within the reach of the future experiment at the SuperKEKB. The imaginary parts of the $\mu-\tau$ flavor violating Yukawa couplings also induce the extra contributions to the muon EDM, which may also be within the planned future experiments. The necessary $\mu-\tau$ flavor violation also generates the correction to $\tau \to \mu\nu\bar{\nu}$ decay and also induces a violation of lepton universality between

[12]Even if $\rho_e^{\tau\tau}$ is sizable, large Yukawa couplings are required to explain the excesses [73].

TABLE I. Observabilities in various processes are summarized. If there is an observability in the planning experiments without introducing unknown Yukawa couplings ρ_f other than $\rho_e^{\mu\tau(\tau\mu)}$, the circle mark "$\circ$" is shown. If there is an observability, but it depends on unknown Yukawa couplings (other than $\rho_e^{\mu\tau(\tau\mu)}$), "($\circ$)" is indicated. If there is an observability when the (currently unknown) experimental improvement is achieved, the triangle mark "$\triangle$" is shown. If the event rate is expected to be too small to be observed, "$\times$" is shown.

Process	Typical value	Observability
Muon g-2	$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$	(Input)
$\tau \to \mu\gamma$	BR $\le 10^{-9}$	$\circ$
$\tau \to e\gamma$	Small	$\times$
$\tau \to \mu l^+ l^- (l = e, \mu)$	Depends on $\rho_e^{\mu\mu}$ and ρ_e^{ee}	($\circ$)
$\tau^- \to e^- l^+ l^-$, $e^-\mu^+e^-$, $\mu^-e^+\mu^-$	Small	$\times$
$\tau \to \mu\eta$	Depends on ρ_d^{ss}	($\circ$)
$\tau \to \mu\nu\bar{\nu}$	$\delta \le 10^{-3}$, lepton nonuniversality	$\triangle$
$\tau \to e\nu\bar{\nu}$	Small, lepton nonuniversality	$\triangle$
$\mu \to e\gamma$	Depends on $\rho_e^{\tau e(e\tau)}$ and $\rho_e^{\mu e(e\mu)}$	($\circ$)
$\mu - e$ conversion	Depends on $\rho_e^{\mu e(e\mu)}$ and $\rho_{d,u}^{ij}$	($\circ$)
$\mu \to 3e$	BR $\le 10^{-13}$	($\circ$)
Muon EDM	$\lvert\delta d_\mu\rvert \le 10^{-22} e \cdot \mathrm{cm}$	($\circ$)
Electron $g-2$	Small	$\times$
LFV Higgs decay mode	BR	
$h \to \mu\tau$	BR $= (0.84^{+0.39}_{-0.37})\%$	(Input)
$h \to e\tau$	Small	$\times$
$h \to e\mu$	Small	$\times$

$\tau \to \mu\nu\bar{\nu}$ and $\tau \to e\nu\bar{\nu}$. The improvement of their precisions would be interesting. The tree-level τ decays such as $\tau^- \to \mu^- l^+ l^- (l = e, \mu)$ and $\tau \to \mu\eta$ are also interesting because the extra Yukawa couplings $\rho_e^{ee(\mu\mu)}$ and ρ_d^{ss} could also induce the observable effects. On the other hand, we have found that the $e-\mu$ and $e-\tau$ flavor violating Yukawa couplings are severely constrained by mainly the $\mu \to e\gamma$ process. Because of these constraints, phenomena such as $\tau \to e\gamma$, $\tau^- \to e^- l^+ l^- (l, = e, \mu)$, $e^-\mu^+e^-$, $\mu^-e^+\mu^-$, and extra contributions to the electron $g-2$ would not be accessible in the near future experiments. Although there are many unknown Yukawa couplings in a general 2HDM, there are many interesting indications to τ- and μ-physics.

We have also commented on an implication to Higgs physics. Contrary to the $\mu - \tau$ flavor violation suggested by the CMS result, the $e-\mu$ and $e-\tau$ flavor violations in the Higgs coupling are strongly limited. Therefore, the observation of $h \to \mu\tau$ and nonobservation of $h \to e\mu$ and $h \to e\tau$ would be the important implication of the scenario.

We summarize our findings in Table I. If the CMS excess in $h \to \mu\tau$ is justified in the coming LHC run, these phenomena in τ- and μ-physics would be key to reveal the physics beyond the Standard Model.

ACKNOWLEDGMENTS

This work was supported in part by Grants-in-Aid for Scientific Research from the Japan Society for Promotion of Science (JSPS) (Grant No. 26104705 for K. T.). E. S. is supported in part by the Ministry of Science and Technology, Taiwan, under Grant No. MOST 104-2811-M-008-011.

[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).

[3] J. D. Bjorken and S. Weinberg, A Mechanism for Nonconservation of Muon Number, Phys. Rev. Lett. **38**, 622 (1977).

[4] S. L. Glashow and S. Weinberg, Natural conservation laws for neutral currents, Phys. Rev. D **15**, 1958 (1977).

[5] B. McWilliams and L. F. Li, Virtual effects of Higgs particles, Nucl. Phys. **B179**, 62 (1981).

[6] O. U. Shanker, Flavor violation, scalar particles and leptoquarks, Nucl. Phys. **B206**, 253 (1982).
[7] T. P. Cheng and M. Sher, Mass matrix ansatz and flavor nonconservation in models with multiple Higgs doublets, Phys. Rev. D **35**, 3484 (1987).
[8] V. Khachatryan *et al.* (CMS Collaboration), Search for lepton-flavour-violating decays of the Higgs boson, Phys. Lett. B **749**, 337 (2015).
[9] Pierre Savard *et al.* (ATLAS Collaboration), Search for lepton-flavour-violating $H \to \mu\tau$ decays of the Higgs boson with the ATLAS detector, J. High Energy Phys. 11 (2015) 211.
[10] M. D. Campos, A. E. C. Hernández, H. Päs, and E. Schumacher, Higgs $\to \mu\tau$ as an indication for S_4 flavor symmetry, Phys. Rev. D **91**, 116011 (2015).
[11] D. Aristizabal Sierra and A. Vicente, Explaining the CMS Higgs flavor violating decay excess, Phys. Rev. D **90**, 115004 (2014).
[12] J. Heeck, M. Holthausen, W. Rodejohann, and Y. Shimizu, $H \to \mu\tau$ in Abelian and non-Abelian flavor symmetry models, Nucl. Phys. **B896**, 281 (2015).
[13] A. Crivellin, G. DAmbrosio, and J. Heeck, Explaining $h \to \mu^\pm\tau^\mp$, $B \to K^*\mu^+\mu^-$ and $B \to K\mu^+\mu^-/B \to Ke^+e^-$ in a Two-Higgs-Doublet Model with Gauged $L_\mu - L_\tau$, Phys. Rev. Lett. **114**, 151801 (2015).
[14] L. de Lima, C. S. Machado, R. D. Matheus, and L. A. F. do Prado, Higgs flavor violation as a signal to discriminate models, J. High Energy Phys. 11 (2015) 074.
[15] I. Doršner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Košnik, and I. Nišandžic, New physics models facing lepton flavor violating Higgs decays at the percent level, J. High Energy Phys. 06 (2015) 108.
[16] Y. Omura, E. Senaha, and K. Tobe, Lepton-flavor-violating Higgs decay $h \to \mu\tau$ and muon anomalous magnetic moment in a general two Higgs doublet model, J. High Energy Phys. 05 (2015) 028.
[17] S. P. Das, J. Hernández-Sánchez, A. Rosado, and R. Xoxocotzi, Flavor violating signatures of lighter and heavier Higgs bosons within two Higgs doublet model type III at the LHeC, arXiv:1503.01464.
[18] A. Crivellin, G. DAmbrosio, and J. Heeck, Addressing the LHC flavor anomalies with horizontal gauge symmetries, Phys. Rev. D **91**, 075006 (2015).
[19] D. Das and A. Kundu, Two hidden scalars around 125 GeV and h, Phys. Rev. D **92**, 015009 (2015).
[20] I. de Medeiros Varzielas, O. Fischer, and V. Maurer, $\mathbb{A}_4$ symmetry at colliders and in the universe, J. High Energy Phys. 08 (2015) 080.
[21] C. X. Yue, C. Pang, and Y. C. Guo, Lepton flavor violating Higgs couplings and single production of the Higgs boson via $e\gamma$ collision, J. Phys. G **42**, 075003 (2015).
[22] B. Bhattacherjee, S. Chakraborty, and S. Mukherjee, $H \to \tau\mu$ and excess in $t\bar{t}H$: Connecting the dots in the hope for the first glimpse of BSM Higgs signal, arXiv:1505.02688.
[23] Y. n. Mao and S. h. Zhu, On the Higgs-$\mu - \tau$ coupling at high and low energy colliders, Phys. Rev. D **93**, 035014 (2016).
[24] X. G. He, J. Tandean, and Y. J. Zheng, Higgs decay $h \to \mu\tau$ with minimal flavor violation, J. High Energy Phys. 09 (2015) 093.
[25] T. Goto, R. Kitano, and S. Mori, Lepton flavor violating Z-boson couplings from non-standard Higgs interactions, Phys. Rev. D **92**, 075021 (2015).
[26] C. W. Chiang, H. Fukuda, M. Takeuchi, and T. T. Yanagida, Flavor-changing neutral-current decays in top-specific variant axion model, J. High Energy Phys. 11 (2015) 057.
[27] A. Crivellin, J. Heeck, and P. Stoffer, A Perturbed Lepton-Specific Two-Higgs-Doublet Model Facing Experimental Hints for Physics Beyond the Standard Model, Phys. Rev. Lett. **116**, 081801 (2016).
[28] K. Cheung, W. Y. Keung, and P. Y. Tseng, Leptoquark induced rare decay amplitudes $h \to \tau^\mp\mu^\pm$ and $\tau \to \mu\gamma$, Phys. Rev. D **93**, 015010 (2016).
[29] E. Arganda, M. J. Herrero, X. Marcano, and C. Weiland, Enhancement of the LFV Higgs decay rates from SUSY loops in the inverse seesaw model, Phys. Rev. D **93**, 055010 (2016).
[30] F. J. Botella, G. C. Branco, M. Nebot, and M. N. Rebelo, Flavour changing Higgs couplings in a class of two Higgs doublet models, Eur. Phys. J. C **76**, 161 (2016).
[31] X. Liu, L. Bian, X. Q. Li, and J. Shu, $h \to \mu\tau$, muon g−2, and a possible interpretation of the Galactic Center gamma ray excess, Nucl. Phys. **B909**, 507 (2016).
[32] S. Baek and K. Nishiwaki, Leptoquark explanation of $h \to \mu\tau$ and muon $(g-2)$, Phys. Rev. D **93**, 015002 (2016).
[33] W. Huang and Y. L. Tang, Flavor anomalies at the LHC and the R-parity violating supersymmetric model extended with vectorlike particles, Phys. Rev. D **92**, 094015 (2015).
[34] S. Baek and Z. F. Kang, Naturally large radiative lepton flavor violating Higgs decay mediated by lepton-flavored dark matter, J. High Energy Phys. 03 (2016) 106.
[35] E. Arganda, M. J. Herrero, R. Morales, and A. Szynkman, Analysis of the $h, H, A \to \tau\mu$ decays induced from SUSY loops within the mass insertion approximation, J. High Energy Phys. 03 (2016) 055.
[36] D. Aloni, Y. Nir, and E. Stamou, Large BR$(h \to \tau\mu)$ in the MSSM, J. High Energy Phys. 04 (2016) 162.
[37] A. Pilaftsis, Lepton flavor nonconservation in H0 decays, Phys. Lett. B **285**, 68 (1992).
[38] J. G. Korner, A. Pilaftsis, and K. Schilcher, Leptonic *CP* asymmetries in flavor changing H0 decays, Phys. Rev. D **47**, 1080 (1993).
[39] K. A. Assamagan, A. Deandrea, and P. A. Delsart, Search for the lepton flavor violating decay $A_0/H_0 \to \tau^\pm\mu^\mp$ at hadron colliders, Phys. Rev. D **67**, 035001 (2003).
[40] A. Brignole and A. Rossi, Lepton flavor violating decays of supersymmetric Higgs bosons, Phys. Lett. B **566**, 217 (2003).
[41] S. Kanemura, K. Matsuda, T. Ota, T. Shindou, E. Takasugi, and K. Tsumura, Search for lepton flavor violation in the Higgs boson decay at a linear collider, Phys. Lett. B **599**, 83 (2004).
[42] E. Arganda, A. M. Curiel, M. J. Herrero, and D. Temes, Lepton flavor violating Higgs boson decays from massive seesaw neutrinos, Phys. Rev. D **71**, 035011 (2005).
[43] S. Kanemura, T. Ota, and K. Tsumura, Lepton flavor violation in Higgs boson decays under the rare tau decay results, Phys. Rev. D **73**, 016006 (2006).

[44] G. Blankenburg, J. Ellis, and G. Isidori, Flavour-changing decays of a 125 GeV Higgs-like particle, Phys. Lett. B **712**, 386 (2012).

[45] R. Harnik, J. Kopp, and J. Zupan, Flavor violating Higgs decays, J. High Energy Phys. 03 (2013) 026.

[46] M. Arana-Catania, E. Arganda, and M. J. Herrero, Non-decoupling SUSY in LFV Higgs decays: A window to new physics at the LHC, J. High Energy Phys. 09 (2013) 160; 10 (2015) 192].

[47] E. Arganda, M. J. Herrero, X. Marcano, and C. Weiland, Imprints of massive inverse seesaw model neutrinos in lepton flavor violating Higgs boson decays, Phys. Rev. D **91**, 015001 (2015).

[48] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[49] H. Georgi and D. V. Nanopoulos, Suppression of flavor changing effects from neutral spinless meson exchange in gauge theories, Phys. Lett. B **82**, 95 (1979).

[50] J. F. Donoghue and L. F. Li, Properties of charged Higgs bosons, Phys. Rev. D **19**, 945 (1979).

[51] L. Lavoura and J. P. Silva, Fundamental *CP* violating quantities in a $SU(2) \times U(1)$ model with many Higgs doublets, Phys. Rev. D **50**, 4619 (1994).

[52] L. Lavoura, Signatures of discrete symmetries in the scalar sector, Phys. Rev. D **50**, 7089 (1994).

[53] F. J. Botella and J. P. Silva, Jarlskog-like invariants for theories with scalars and fermions, Phys. Rev. D **51**, 3870 (1995).

[54] G. C. Branco, L. Lavoura, and J. P. Silva, *CP* violation, Int. ser. monogr. phys. **103**, 1 (1999).

[55] S. Davidson and H. E. Haber, Basis-independent methods for the two-Higgs-doublet model, Phys. Rev. D **72**, 035004 (2005); **72**, 099902(E) (2005).

[56] M. E. Peskin and T. Takeuchi, Estimation of oblique electroweak corrections, Phys. Rev. D **46**, 381 (1992).

[57] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, $(g-2)_\mu$ and $\alpha(M_Z^2)$ re-evaluated using new precise data, J. Phys. G **38**, 085003 (2011).

[58] S. Kanemitsu and K. Tobe, New physics for muon anomalous magnetic moment and its electroweak precision analysis, Phys. Rev. D **86**, 095025 (2012).

[59] D. Chang, W. S. Hou, and W. Y. Keung, Two loop contributions of flavor changing neutral Higgs bosons to $\mu \to e\gamma$, Phys. Rev. D **48**, 217 (1993).

[60] A. M. Baldini *et al.*, MEG Upgrade Proposal, arXiv:1301.7225.

[61] W. H. Bertl *et al.* (SINDRUM II Collaboration), A search for muon to electron conversion in muonic gold, Eur. Phys. J. C **47**, 337 (2006).

[62] H. Natori (DeeMe Collaboration), DeeMe experiment: An experimental search for a mu-e conversion reaction at J-PARC MLF, Nucl. Phys. B, Proc. Suppl. **248–250**, 52 (2014).

[63] Y. Kuno (COMET Collaboration), A search for muon-to-electron conversion at J-PARC: The COMET experiment, Prog. Theor. Exp. Phys. **2013**, 022C01 (2013).

[64] L. Bartoszek *et al.* (Mu2e Collaboration), Mu2e Technical Design Report, arXiv:1501.05241.

[65] G. W. Bennett *et al.* [Muon (*g-2*) Collaboration], An improved limit on the muon electric dipole moment, Phys. Rev. D **80**, 052008 (2009).

[66] Y. K. Semertzidis *et al.*, Sensitive search for a permanent muon electric dipole moment, AIP Conf. Proc. **564**, 263 (2001); R. M. Carey *et al.*, AGS letter of intent: Search for a permanent muon electric dipole moment, 2000; A. Silenko *et al.* (Muon EDM Collaboration), J-PARC letter of intent: Search for a permanent muon electric moment at the $10^{-24} e \cdot \mathrm{cm}$ level, 2003.

[67] B. Aubert *et al.* (*BABAR* Collaboration), Measurements of Charged Current Lepton Universality and $|V_{us}|$ Using Tau Lepton Decays to $e^- \bar{\nu}_e \nu_\tau$, $\mu^- \bar{\nu}_\mu \nu_\tau$, $\pi^- \nu_\tau$ and $K^- \nu_\tau$, Phys. Rev. Lett. **105**, 051602 (2010).

[68] A. Crivellin, A. Kokulu, and C. Greub, Flavor-phenomenology of two-Higgs-doublet models with generic Yukawa structure, Phys. Rev. D **87**, 094031 (2013).

[69] A. Blondel *et al.*, Research proposal for an experiment to search for the decay $\mu \to eee$, arXiv:1301.6113.

[70] M. Sher, $\vec{\tau}\mu\eta$ in supersymmetric models, Phys. Rev. D **66**, 057301 (2002).

[71] D. Black, T. Han, H. J. He, and M. Sher, $\tau - \mu$ flavor violation as a probe of the scale of new physics, Phys. Rev. D **66**, 053002 (2002).

[72] A. Celis, V. Cirigliano, and E. Passemar, Lepton flavor violation in the Higgs sector and the role of hadronic τ-lepton decays, Phys. Rev. D **89**, 013008 (2014).

[73] J. P. Lees *et al.* (*BABAR* Collaboration), Evidence for an Excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ Decays, Phys. Rev. Lett. **109**, 101802 (2012); J. P. Lees *et al.* (*BABAR* Collaboration), Measurement of an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ decays and implications for charged Higgs bosons, Phys. Rev. D **88**, 072012 (2013).

[74] M. Huschle *et al.* (Belle Collaboration), Measurement of the branching ratio of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $\bar{B} \to D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle, Phys. Rev. D **92**, 072014 (2015); A. Abdesselam *et al.* (Belle Collaboration), Measurement of the branching ratio of $\bar{B}^0 \to D^{*+} \tau^- \bar{\nu}_\tau$ relative to $\bar{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell$ decays with a semileptonic tagging method, arXiv:1603.06711.

[75] R. Aaij *et al.* (LHCb Collaboration), Measurement of the Ratio of Branching Fractions $\mathcal{B}(\bar{B}^0 \to D^{*+} \tau^- \bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{*+} \mu^- \bar{\nu}_\mu)$, Phys. Rev. Lett. **115**, 111803 (2015); Phys. Rev. Lett. **115**, 159901 (2015).

[76] J. P. Lees *et al.* (*BABAR* Collaboration), Evidence of $B^+ \to \tau^+ \nu$ decays with hadronic B tags, Phys. Rev. D **88**, 031102 (2013).

[77] I. Adachi *et al.* (Belle Collaboration), Evidence for $B^- \to \tau^- \bar{\nu}_\tau$ with a Hadronic Tagging Method Using the Full Data Sample of Belle, Phys. Rev. Lett. **110**, 131801 (2013); B. Kronenbitter *et al.* (Belle Collaboration), Measurement of the branching fraction of $B^+ \to \tau^+ \nu_\tau$ decays with the semileptonic tagging method, Phys. Rev. D **92**, 051102 (2015).

[78] F. C. Porter, Experimental Status of the CKM Matrix, arXiv:1604.04940.