Technical Material

Statistical Error Estimation of the Feynman-α Method using the Bootstrap Method

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Applicability of the bootstrap method is investigated to estimate the statistical error of the Feynman-α method, which is one of the subcritical measurement techniques on the basis of reactor noise analysis. In the Feynman-α method, the statistical error can be simply estimated from multiple measurements of reactor noise, however it requires additional measurement time to repeat the multiple times of measurements. Using the resampling technique such as the bootstrap method, the statistical error (standard deviation and confidence interval) of measurement results obtained by the Feynman-α method can be estimated from a single measurement of reactor noise. In order to validate our proposed technique, we carried out a passive measurement of reactor noise without any external source, i.e. with only inherent

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neutron source by spontaneous fission and (α,n) reactions in nuclear fuels at the Kyoto University Criticality Assembly. Through the actual measurement, it is confirmed that the bootstrap method is applicable to approximately estimate the statistical error of measurement results obtained by the Feynman-α method.

Keywords; Feynman-α method; reactor noise; subcriticality; bootstrap method; KUCA; measurement; statistical error

1. Introduction

In the present paper, statistical error estimation for the Feynman-α method [1], or the variance-to-mean ratio method, is proposed using the bootstrap method [2]. Our proposed method is validated by an actual reactor noise measurement, which was carried out at the Kyoto University Criticality Assembly (KUCA) [3].

Subcriticality monitoring contributes safe and efficient operation and management in nuclear fuel related-facilities. The subcriticality monitoring is also important for the accelerator driven subcritical reactor [4-6], since the reactor must be kept in the subcritical state in operation. Furthermore, in the retrieval of fuel debris from Fukushima Dai-ichi units 1-3 with the submersion condition, there are potentials to bring a positive reactivity due to the change of moderation ratio, thus the subcriticality monitoring to prevent the recriticality is one of the important issues [7].

The Feynman-α method is one of the practical subcriticality measurement techniques on the basis of reactor noise analysis. In the Feynman-α method, firstly, time-series data of neutron counts (so-called "reactor noise") are measured for a steady-state subcritical system. The measured time-series data are utilized to evaluate the variation of variance-to-mean ratio $Y(T)$ for various counting gate width T. Here, Y value is an index to investigate the neutron-correlation due to fission reaction. By analyzing the variation of $Y(T)$, the prompt neutron decay constant α can be primarily obtained. Finally, the measurement value of α is converted to the subcriticality $(-\rho)$. Here, the quantification of measurement error is also important to ensure the margin of subcriticality to the critical state. In the present paper, we focus on the statistical error of the Feynman-α method as one of the measurement errors. A simple way to evaluate the statistical error is multiple measurements of reactor noise, however it requires additional measurement time to repeat the multiple times of measurements. In addition, the statistical theory for probability distributions of the $Y(T)$ and α has not yet been sufficiently established to evaluate these statistical errors only from a single measurement of reactor noise.

In order to address this problem, the purpose of the present paper is to investigate the applicability of the bootstrap method [2] to evaluate the statistical error of the Feynman-α method without multiple measurements. The bootstrap method enables to practically estimate variance and confidence interval for a sample estimate (e.g. mean, variance, median) by a large number of resamples obtained from an original data. In the case of present study, sample estimates correspond to $Y(T)$ and α, and these statistical error (standard deviation and confidence interval) are estimated using the bootstrap method for a single measurement of reactor noise. Assuming a passive measurement at a deep subcritical system (e.g. effective neutron multiplication factor $k_{\mathrm{eff}} \lesssim 0.95$), our aim is the order-of-magnitude estimate of statistical error in 1 significant figure, from a single measurement under a condition of low neutron count rate without any external neutron source, i.e. with only inherent neutron source by spontaneous fission and (α,n) reactions in nuclear fuels. In order to validate our proposed technique, an actual reactor noise measurement was carried out at the KUCA.

The contents of the present paper are as follows: In section 2, methodology of the statistical error estimation using the bootstrap methods are described. Section 3 shows validation through the actual reactor noise experiment at the KUCA. In order to confirm reliability of the proposed method, a short-time single noise measurement is repeatedly carried out and statistical errors of $Y(T)$ and α are estimated by the bootstrap method for each measurement. Then, each of the estimated statistical error is compared to the reference value obtained through repetition short-time measurements, to confirm the confidence interval estimated by the bootstrap method. Finally, concluding remarks are summarized in section 4.

2. Methodology

Let us assume a steady-state subcritical system with some kind of neutron source (e.g. spontaneous fission and (α, n) reactions in nuclear fuel, or external neutron source such as an

Am-Be source). In this subcritical system, successive time-series data of neutron counts $C_i(T_0)$ $(i = 1 \sim N)$ are measured, where T_0 is a basic counting gate width and N is the number of count data. Then, Y value is evaluated using the ratio of variance to mean [1]:

$$Y(T_0) = \frac{\sigma^2(T_0)}{\mu(T_0)} - 1, \tag{1}$$

where $\mu(T_0)$ and $\sigma^2(T_0)$ are the sample mean and the unbiased variance of neutron counts $C_i(T_0)$, respectively. If neutron counts follow the Poisson distribution, the Y value is zero, since the variance is equal to the mean in the case of Poisson distribution. However, in a subcritical neutron multiplication system, measured neutron counts do not follow the Poisson distribution, i.e. the variance is larger than the mean. This phenomena results from the neutron-correlation due to fission chain reaction.

In order to measure the gate-width dependence of Y value, the bunching method is conventionally employed [8]. Using the bunching method, Y values can be estimated for a bunching gate width kT_0:

$$Y(kT_0) = \frac{\sigma^2(kT_0)}{\mu(kT_0)} - 1, \tag{2}$$

where k is the number of bunching (see later, Figs.1-(a) and 1-(b)). As a result, the variation of Y is measured with respect to counting gate width $T = kT_0$. After that, the prompt neutron decay constant α can be obtained by fitting the following practical formula to measured Y values:

$$Y(T) \approx Y_\infty \left(1 - \frac{1 - \exp(-\alpha T)}{\alpha T}\right) + A\,T + B, \tag{3}$$

where Y_∞ is the saturation value which is inversely proportional to α^2, and A and B are supplemental fitting coefficients to correct the delayed neutron and the dead-time effects, respectively [9].

Now, let us consider the statistical errors of Y value and prompt neutron decay constant α. One of the difficulties lies in a fact that the statistical distribution of Y value is theoretically unknown. Furthermore, the fitting error of α does not necessarily correspond to the statistical error of α even if the statistical distribution of Y is approximated as the Gaussian distribution, since it is difficult to estimate true covariance, or experimental uncertainty and correlation, of Y among different gate widths. A simple alternative solution is multiple measurements of Y and α, thereby the statistical errors can be estimated by standard errors of them. Note, however, that it inevitably requires longer measurement time for multiple measurements.

In the present study, the bootstrap method [2] is applied to estimate the statistical errors of Y and α for a single measurement. The bootstrap method is one of the resampling techniques. In the bootstrap method, a histogram of original neutron count data is utilized as an experimentally-based probability distribution in the resampling to evaluate the statistical errors of Y and α. Detail procedures are described as follows:

1. Original time-series data of neutron counts $C_i(T_0)$ are provided by a single measurement of reactor noise. This histogram of neutron count data is utilized as a probability distribution in the bootstrap resampling procedure.
2. Set k be an arbitrary number of bunching. Note that $1 \leq k < N$.
3. The "resampling position r" is determined as $r = \xi$, where ξ means a uniform random integer number $\xi \in [1, (N - k + 1)]$. Then, neutron count $C^*(kT_0)$ is resampled by bunching the successive count data as follows:

$$C^*(kT_0) = \sum_{i=r}^{r+k-1} C_i(T_0). \tag{4}$$

4. By repeating $K(= \lfloor N/k \rfloor)$ times of random-resampling described in step 3, then "bootstrap sample" of count data is newly generated as follows:

$$\vec{C}^*(kT_0) \equiv \{C_1^*(kT_0), C_2^*(kT_0), \cdots, C_K^*(kT_0)\}. \tag{5}$$

5. Using Eq. (2) for $\vec{C}^*(kT_0)$, "bootstrap replicate $Y^*(kT_0)$" is evaluated for the bunching gate width kT_0.
6. Repeat steps 2 through 5 by varying k to obtain the variation of Y^* with respect to counting gate width.

7. In order to estimate standard deviation of the bootstrap replicate Y^*, repeat steps 2 through 6 several times. Consequently, many number of bootstrap replicates Y^{*b} are obtained for $b = 1,2,\cdots,B$. Here, B is the number of bootstrap replicates. To estimate the bootstrap confidence interval, B is typically set to $B \approx 10^3$.
8. Using dataset of Y^{*b} in step 7, standard deviation of Y^* (denoted as σ_{Y^*}) is calculated for each counting gate width kT_0:

$$\sigma_{Y^*}(kT_0) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(Y^{*b}(kT_0) - \frac{1}{B}\sum_{b'=1}^{B} Y^{*b'}(kT_0)\right)^2}\ . \qquad (6)$$

   Figure 1-(c) shows an example of the bootstrap method to estimate the bootstrap standard deviation $\sigma_{Y^*}(3T_0)$ as the statistical error for $Y(3T_0)$. As will be discussed later in Section 3.3, estimated σ_{Y^*} is a good approximation of the statistical error of Y for a single measurement.
9. Using an inverse of the estimated σ_{Y^*}, i.e. $1/\sigma_{Y^*}$, as the weight in the least square fitting process, the prompt neutron decay constant α^{*b} is evaluated by fitting Eq. (3) to each of Y^{*b}. Here, it is important to take account of the weight $1/\sigma_{Y^*}$, since the statistical error of Y differs depending on the counting gate width kT_0. Consequently, bootstrap replicates α^{*b} are obtained for $b = 1,2,\cdots,B$.
10. As the result of step 9, a frequency distribution of α^* is obtained. Based on this "bootstrap frequency distribution", the statistical error of α can be estimated as the standard deviation and the confidence interval. Namely, the "bootstrap standard deviation σ_{α^*}" is estimated as follows:

$$\sigma_{\alpha^*} = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\alpha^{*b} - \frac{1}{B}\sum_{b'=1}^{B} \alpha^{*b'}\right)^2}\ . \qquad (7)$$

   In the present paper, the "bootstrap confidence interval CI_{α^*}" is simply estimated by the percentile method. Firstly, the B bootstrap replicates α^{*b} are sorted in ascending order. Let p be a positive real number within the range of $0 < p < 1$. Then the pBth smallest values in the sorted α^{*b} are denoted as $\alpha^{*(pB)}$. Finally, in the case of 95% confidence interval level, the lower and upper limits of CI_{α^*} are estimated as $\alpha^{*(0.025B)}$ and $\alpha^{*(0.975B)}$, respectively. It is noted that, if the value of pB is not integer, $\alpha^{*(pB)}$ is numerically calculated by the linear interpolation of two adjacent neighboring points in the sorted α^{*b}.

<Figure 1>

3. Validation

3.1. Experimental Procedure

At the KUCA [3,4,10], the reactor noise experiments were carried out in the A-core (A3/8"p36EU-NU) shown in Fig. 2. Configuration of fuel assembly is shown in Fig. 3. The detail information about size and nuclide density is reported in the reference [10]. The core-average ^{235}U enrichment was 5.4wt%. The moderation ratio of H/^{235}U is approximately 270. As shown in Fig. 2, four ^{3}He detectors (#1~#4) were placed at the axially center positions of excore reflector assemblies, which have holes of 3cm in diameter to insert detectors. Using these detectors, the time-series data of neutron counts were successively measured. The measurement core was just shutdown state, i.e. all safety and control rods were fully inserted, and 3×3 assemblies were withdrawn as shown in Fig. 2. It is noted that the reactor noise was measured without any external neutron source (e.g. Am-Be or Cf source), namely, using only inherent neutron source which mainly consists of spontaneous fission of ^{238}U and (α,n) reactions of ^{27}Al due to α-decay of uranium isotopes [11]. Neutron counts rate of each detector is about 5 [count/sec] and detector dead-time is about or less than 10 [μsec], thus the dead-time effect is negligibly small.

In order to measure the reference values of Y and prompt neutron decay constant α, 93 times of reactor noise measurements were performed. In each of reactor noise measurement, the measurement time (length of time-series data) was 10 minutes and $T_0 = 10^{-4}$ [sec]. Using the conventional bunching method [8], the variation of Y was individually evaluated for each measurement, after that the corresponding α value was also estimated by fitting Eq. (3). Namely, $Y_m(kT_0)$ and α_m are obtained for $1 \leq m \leq 93$, where m indicates the trial number. Using 93 sets of $Y_m(kT_0)$ and α_m, their sample mean and standard deviation, i.e. square root of unbiased variance, of Y and α were calculated:

$$\bar{Y}(kT_0) = \frac{1}{93}\sum_{m=1}^{93} Y_m(kT_0), \tag{8}$$

$$\sigma_{Y,\mathrm{ref}}(kT_0) = \sqrt{\frac{1}{92}\sum_{m=1}^{93}\left(Y_m(kT_0) - \bar{Y}(kT_0)\right)^2}, \tag{9}$$

$$\bar{\alpha} = \frac{1}{93}\sum_{m=1}^{93} \alpha_m, \tag{10}$$

$$\sigma_{\alpha,\mathrm{ref}} = \sqrt{\frac{1}{92}\sum_{m=1}^{93}(\alpha_m - \bar{\alpha})^2}. \tag{11}$$

It is reasonable to regard these calculated values as the reference values of their mean and standard deviation, because the number of multiple measurements is sufficiently large.

The bootstrap method was applied for each of single 10-minutes measurements. In the estimation of statistical errors of Y^* and α^*, the number of bootstrap replicates was set to $B = 1000$. In order to check the effectiveness of bootstrap method, following values were evaluated for each of 10-minutes measurements: (1) standard deviation of Y^* (denoted as $\sigma_{Y^*,m}$), and (2) standard deviation of α^* (denoted as $\sigma_{\alpha^*,m}$) and 95% confidence interval of α^* (denoted as $CI_{\alpha^*,m}$), where m is the trial number ($1 \leq m \leq 93$). These values of $\sigma_{Y^*,m}$, $\sigma_{\alpha^*,m}$, and $CI_{\alpha^*,m}$ were calculated using only mth trial of single 10-minutes measurement.

<Figure 2>

<Figure 3>

3.2. Numerical Analysis

Prompt neutron decay constant α in the fundamental mode is expressed as

$$\alpha \approx \frac{\beta - \rho}{\Lambda}, \tag{12}$$

$$-\rho \approx \frac{1 - k_{\mathrm{eff}}}{k_{\mathrm{eff}}}, \tag{13}$$

where $-\rho$ means the subcriticality, k_{eff} the effective neutron multiplication factor, β the effective delayed neutron fraction, and Λ the neutron generation time. Based on the Eq. (12), measured α value can be converted to the subcriticality $-\rho$. In order to evaluate β and Λ in this experimental core, numerical analysis was performed by the continuous energy Monte Carlo code MCNP6.1 [12] with the JENDL-4 library [13]. The number of history per cycle is 40000, and total number of cycle is 4100 where the number of skip cycle is 100.

Furthermore, the uncertainty quantification of k_{eff} due to the nuclear data, or covariance of evaluated cross-section data, was conducted by the SCALE6.1.3/TSUNAMI-3D with the use of the covariance data (44groupcov) [14].

3.3. Results and Discussion

The numerical results of β and Λ are $\beta = 0.00751 \pm 0.00024$, and $\Lambda = 41.97 \pm 0.16$ [μsec], respectively. The effective neutron multiplication factor k_{eff} is evaluated as 0.94100 ± 0.00012, i.e. the subcriticality is 6.270 ± 0.014 [%$\Delta k/k$]. It is noted that these statistical errors are 2σ. Based on the numerical results of TSUNAMI-3D, the uncertainty of k_{eff} due to the nuclear data is 0.6474 ± 0.0001 [%$\Delta k/k$]. Although there are differences between MCNP6.1 and TSUNAMI-3D, the uncertainty of subcriticality due to the nuclear data is estimated to be about 600 [pcm].

As an example, the experimental results of ^{3}He detector #2 are shown and discussed. The reference mean and standard deviation of α are estimated as $\bar{\alpha} = 1753$ [1/sec] and $\sigma_{\alpha,\mathrm{ref}} = 299$ [1/sec], respectively. In order to evaluate the statistical error for the reference sample mean $\bar{\alpha}$, the central limit theorem was employed. According to the central limit theorem, the 95% confidence interval of the mean is estimated as ranging between 1693 and 1814 [1/sec], i.e. the Standard Error of the Mean (SEM) of α is $\sigma_{\alpha,\mathrm{ref}}/\sqrt{93} = 31.0$ [1/sec]. Using Eq. (12), the measurement value of subcriticality is estimated as 6.61 ± 0.26 [$\%\Delta k/k$], where the propagated error is 2σ. There is a bias of about 0.3 [$\%\Delta k/k$] $= 300$ [pcm] between the experimental and numerical results. Considering the statistical error of measurement and the numerical uncertainty due to the nuclear data, it is confirmed that the measured subcriticality agrees well with the numerical results.

Before discussing results of α, the bootstrap standard deviation $\sigma_{Y^*,m}$ is firstly investigated and compared with the reference standard deviation $\sigma_{Y,\mathrm{ref}}$, in order to check whether the each trial of $\sigma_{Y^*,m}$ can be utilized as the order estimation for the reference standard deviation $\sigma_{Y,\mathrm{ref}}$. Figure 4 shows the variation of standard deviation of Y with respect to the counting gate width T. In Fig. 4, black circles indicate reference value $\sigma_{Y,\mathrm{ref}}$, and other plots show representative examples of $\sigma_{Y^*,m}$ obtained from different 10-minutes measurements, e.g. "bootstrap trial-30" means results of $\sigma_{Y^*,30}$ which is obtained by Eq. (6) for 30$^{\mathrm{th}}$ trial of 10-minutes measurement ($m = 30$). As the counting gate width increases, the statistical error of Y tends to become larger, because the number of count data decreases inversely proportional to the number of bunching. As shown in Fig. 4, it is confirmed that each trial of bootstrap standard deviation $\sigma_{Y^*,m}$ is nearly equal to the reference value $\sigma_{Y,\mathrm{ref}}$. It is noted that $\sigma_{Y^*,m}$ fluctuates statistically between trials, because the neutron count rate is low in the 10 minutes measurement time. In order to check the statistical fluctuation of $\sigma_{Y^*,m}$, Fig. 5 shows the two-sided 95 percentile interval of $\sigma_{Y^*,m}$, which is calculated using 93 sets of $\sigma_{Y^*,m}$. As shown in Fig. 5, even after taking into consideration of the statistical fluctuation, it is confirmed that $\sigma_{Y^*,m}$ is same order as the reference value $\sigma_{Y,\mathrm{ref}}$. Thus, it is reasonable to utilize each trial of $\sigma_{Y^*,m}$ instead of $\sigma_{Y,\mathrm{ref}}$ in the least square fitting process (see step 9 in the bootstrap procedures, as described in Section 2).

<Figure 4>

<Figure 5>

Next, Fig. 6 shows an example of the bootstrap distribution of α^* for a single 10-minutes measurement (48$^{\text{th}}$ trial, $m = 48$). As shown in Fig. 6, the bootstrap distribution of α^* is slightly skew. Based on the bootstrap distribution of α^* such as Fig. 6, the standard deviation and the confidence interval can be estimated. For example, in the case of 48$^{\text{th}}$ trial, the bootstrap standard deviation $\sigma_{\alpha^*,48}$ is estimated as 275 [1/sec] by Eq. (7). And, using the percentile method, the 95% bootstrap confidence interval $CI_{\alpha^*,48}$ is simply estimated as $(1164, 2253)$ [1/sec], as shown in Fig. 6. As in the case of $\sigma_{Y^*,m}$, each trial of $\sigma_{\alpha^*,m}$ also fluctuates statistically, and the sample mean and the standard deviation of $\sigma_{\alpha^*,m}$ are about 230 and 50 [1/sec], respectively. As a result, it is confirmed that the bootstrap standard deviation $\sigma_{\alpha^*,m}$ is same order as the reference value $\sigma_{\alpha,\mathrm{ref}}$. Furthermore, in order to quantify the coverage probability of the 95% bootstrap confidence interval $CI_{\alpha^*,m}$, Fig. 7 summarizes $CI_{\alpha^*,m}$ for each of 10-minute measurements. In Fig. 7, the straight line shows the reference value $\bar{\alpha}$, the gray error bar (marked circle) indicates that $CI_{\alpha^*,m}$ contains $\bar{\alpha}$, and the black error bar (marked ×) indicates that $\bar{\alpha}$ exists out of the $CI_{\alpha^*,m}$. In order to quantify the statistical error of subcriticality by Eq. (12), right vertical axis indicates the corresponding subcriticality value, which is estimated using the numerical results of β and Λ. As shown in Fig. 7, the reference value $\bar{\alpha}$ is within the error bar, i.e. the bootstrap confidence interval $CI_{\alpha^*,m}$, in most cases. The coverage probability of bootstrap confidence interval $CI_{\alpha^*,m}$ is estimated as $81/93 \approx 87\%$ and the probability is comparatively close to 95%. It is noted that $CI_{\alpha^*,m}$ tends to slightly underestimate the coverage probability in the present technique, thus the improvement of method for estimating $CI_{\alpha^*,m}$ is one of the future subjects. For example, the coverage probability of $CI_{\alpha^*,m}$ can be improved by other complicated method (e.g. the BCa method [15,16]).

In conclusion, it is confirmed that the bootstrap standard deviation $\sigma_{\alpha^*,m}$ and the bootstrap confidence interval $CI_{\alpha^*,m}$ can be utilized as approximately good measures of the statistical error of α.

<Figure 6>

<Figure 7>

4. Conclusion

In the present paper, based on the bootstrap method, the statistical error of the Feynman-α method for a single reactor noise measurement was investigated. The bootstrap method is one of the resampling techniques and a histogram of original neutron count data is utilized as an experimentally-based probability distribution in the resampling to evaluate the statistical errors of of Y and α. As a result, it was confirmed that the bootstrap method is a simple and powerful method to approximately estimate the statistical errors of Y and α, without multiple measurements of reactor noise. As described in Sec. 3, the bootstrap method enables the order-of-magnitude estimate of these statistical errors in 1 significant figure from a single measurement of 10 [min], where the neutron count rate was about 5 [count/sec] with only inherent neutron source by spontaneous fission and (α,n) reactions in nuclear fuels. In the present technique, it is noted that the bootstrap confidence interval tends to slightly underestimate the coverage probability, thus the improvement of method for estimating the bootstrap confidence interval is one of the future subjects.

We applied our proposed technique to the experimental results of reactor noise. Our proposed technique is also applicable to the numerical simulation of reactor noise by the Monte Carlo code such as MCNP-DSP [17] and MVP [18].

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Figure captions

Figure 1. Example of bootstrap method for standard deviation of Y.

Figure 2. Top view of experimental core (A3/8"p36EU-NU).

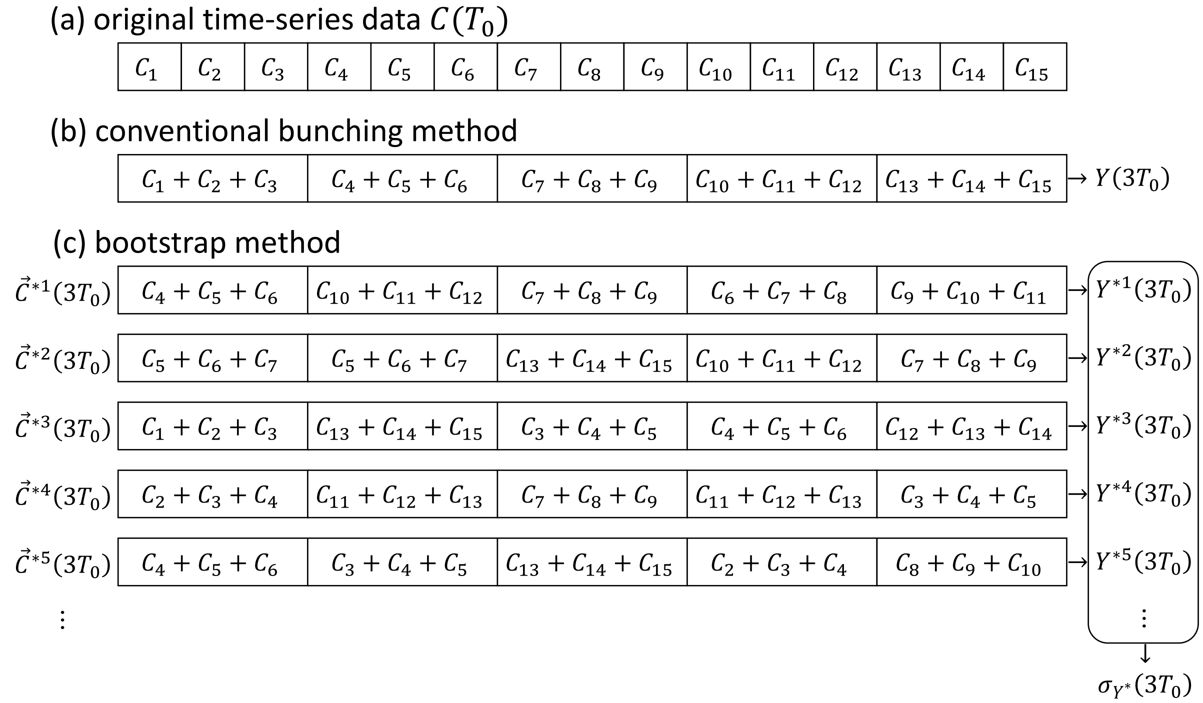
Figure 3. Description of A3/8"p36EU-NU fuel assembly.

Figure 4. Variation of standard deviation of Y with respect to counting gate width T.

Figure 5. Comparison of two-sided 95 percentile interval of bootstrap standard deviation $\sigma_{Y^*,m}$ with reference standard deviation $\sigma_{Y,\mathrm{ref}}$.

Figure 6. Example of bootstrap frequency distribution of α^* (48^{th} trial).

Figure 7. 95% bootstrap confidence interval of α^*.



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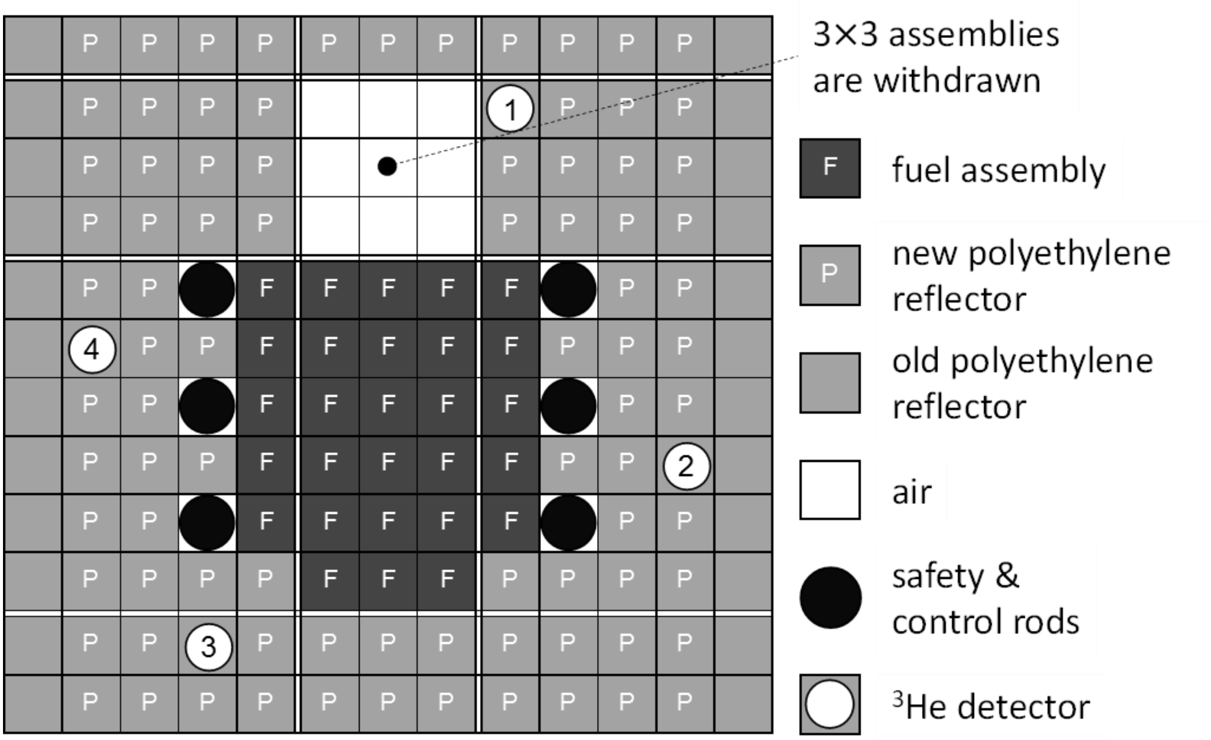

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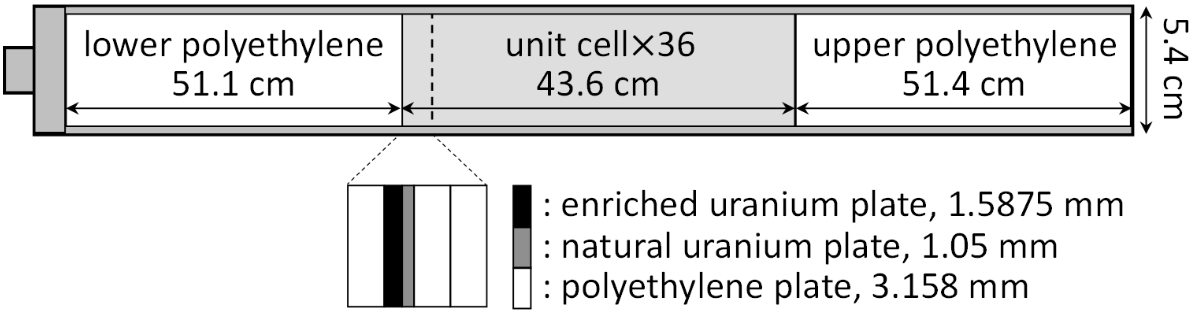

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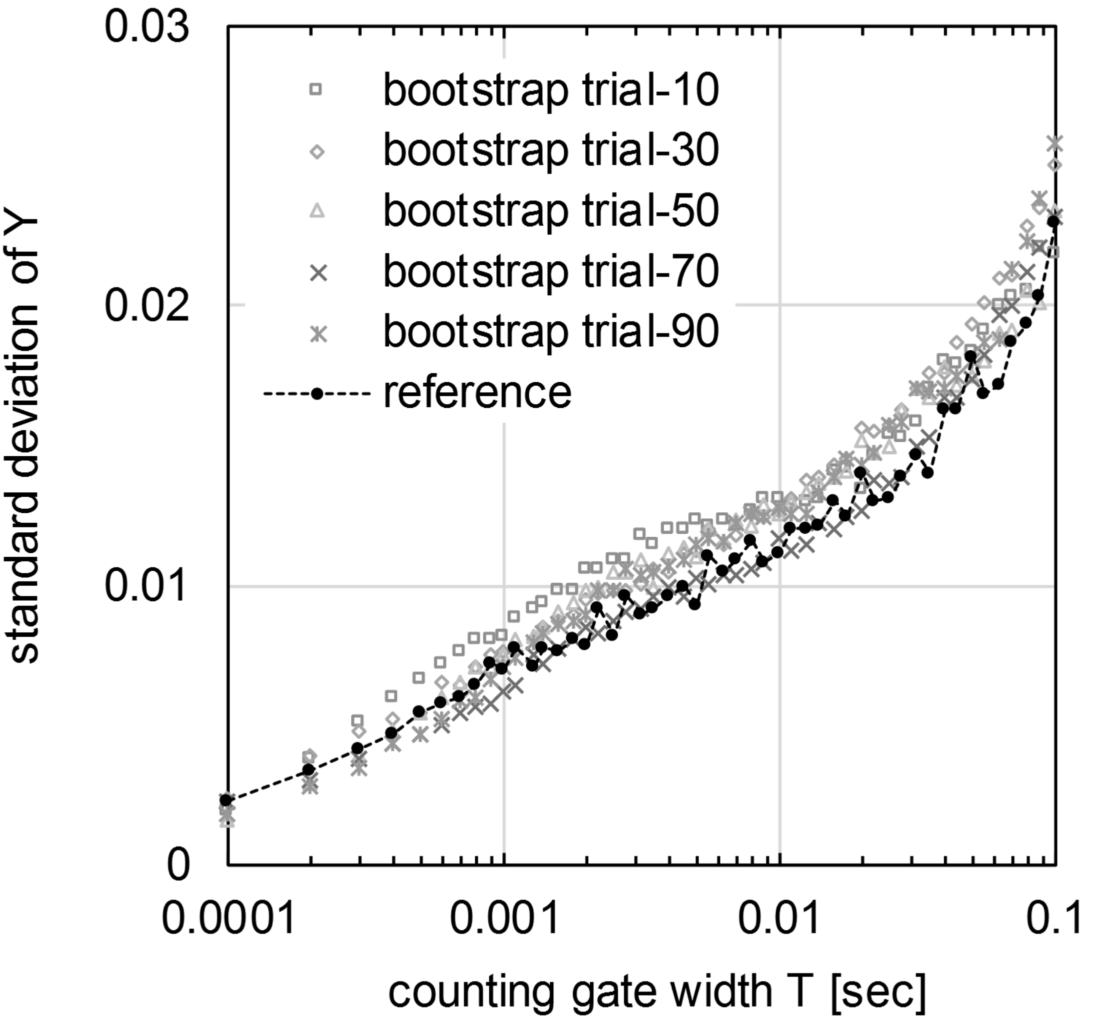

Figure 4. Variation of standard deviation of Y with respect to counting gate width T.

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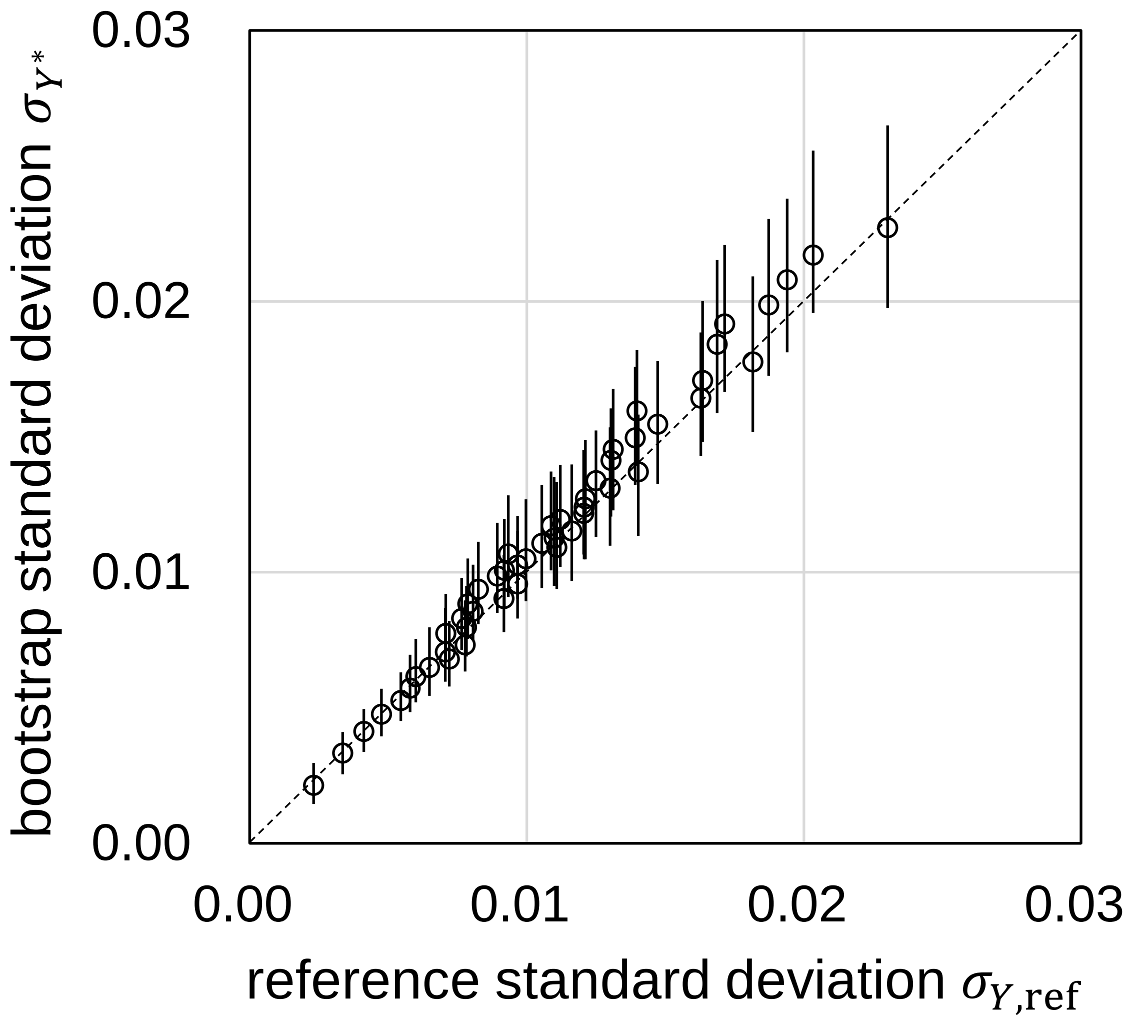

Figure 5. Comparison of two-sided 95 percentile interval of bootstrap standard deviation $\sigma_{Y^*,m}$ with reference standard deviation $\sigma_{Y,\mathrm{ref}}$.

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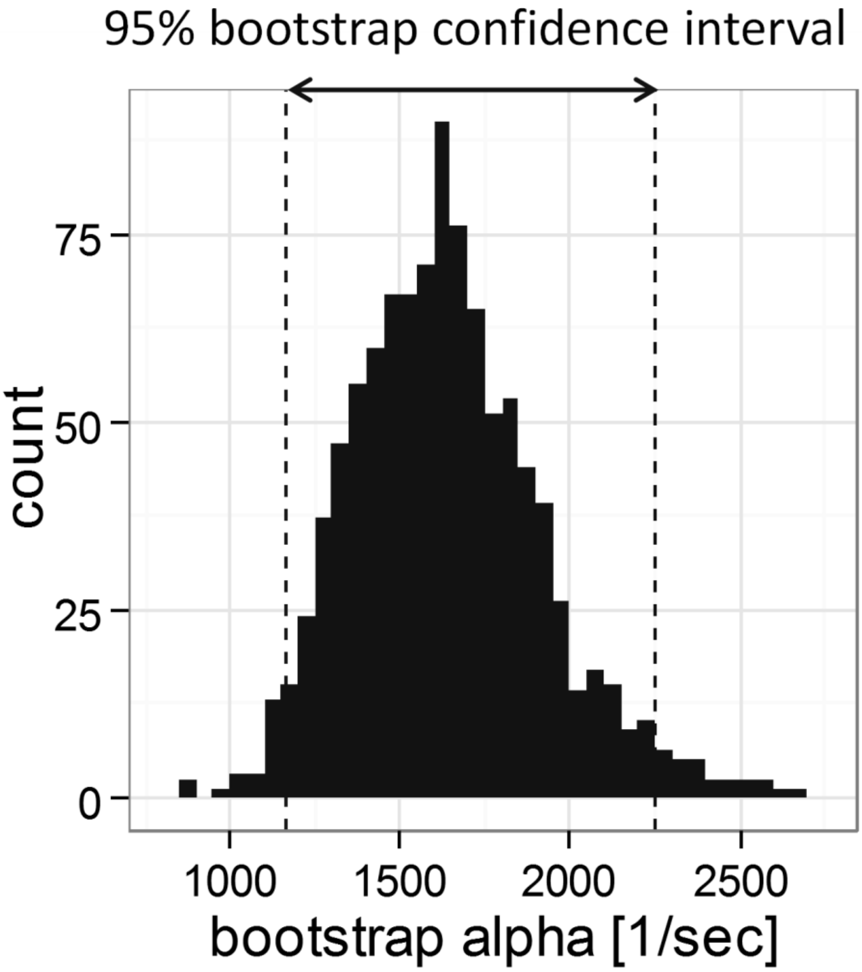

Figure 6. Example of bootstrap frequency distribution of α^* (48th trial).

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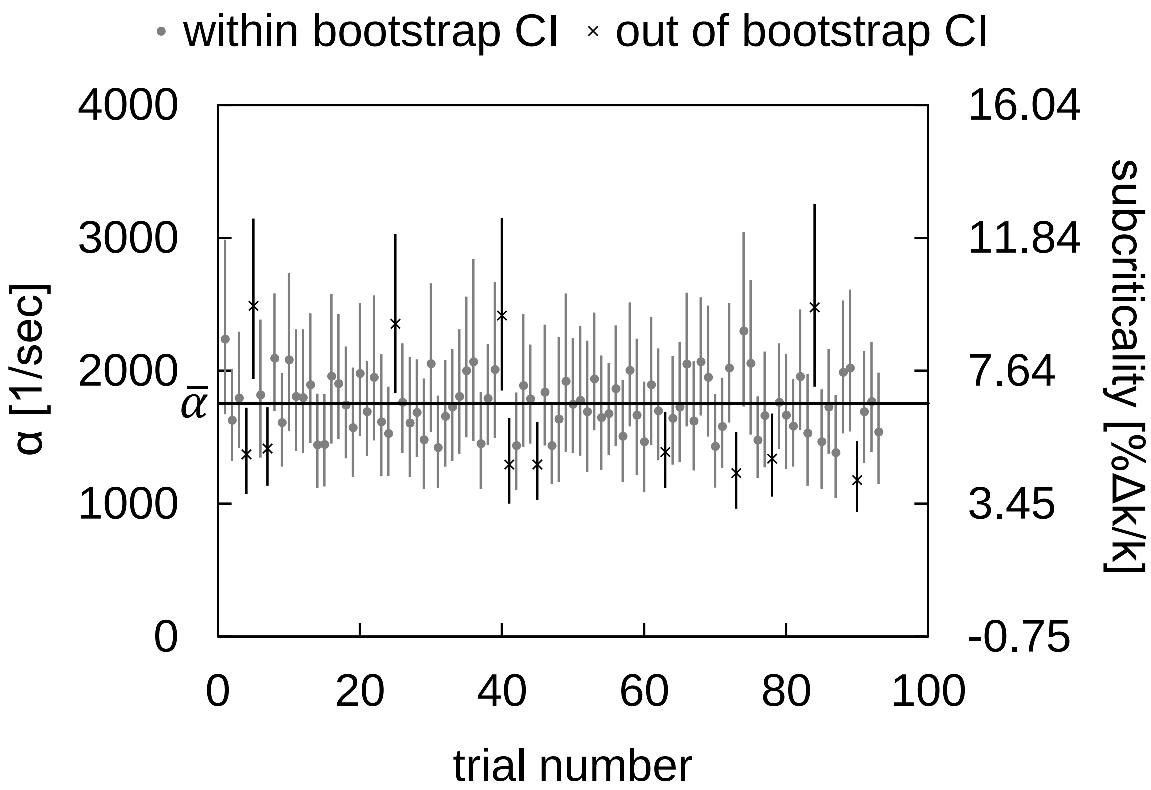

Figure 7. 95% bootstrap confidence interval of α^*.

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