Springer

KSME

Journal of Mechanical Science and Technology 00 (2010) 0000~0000
www.springerlink.com/content/1738-494x
submitted manuscript under review

# Sensitivity Analysis of Thermal Deformation of CFRP Laminate Reflector due to Fiber Orientation Error

Shun Tanaka[1*], Tadashige Ikeda[1] and Atsuhiko Senba[2]

[1] *Department of Aerospace Engineering, Nagoya University, Nagoya, Japan*
[2] *Department of Vehicle and Mechanical Engineering, Meijo University, Nagoya, Japan*

(Manuscript Received 000 0, 2009; Revised 000 0, 2009; Accepted 000 0, 2009)

**Abstract**

Carbon Fiber Reinforced Plastic (CFRP) laminates are sometimes adopted as a material of space observation systems because their specific stiffness is high and thermal deformation can be controlled by stacking sequence and fiber orientation. However, it is reported that unexpected out-of-plane thermal deformation is generated on the CFRP laminate plates by existence of fiber orientation error. In this research, first a CFRP reflector model was actually built and its thermal deformation was measured to know what kinds of deformation were generated on the reflector. In the experiment it was found that non-axisymmetric deformation was generated even for uniform heating. Then, a finite-element analysis (FEA) on thermal deformation of the reflector model was performed to explain the non-axisymmetric deformation. The result of the FEA showed that the non-axisymmetric deformation was generated by fiber orientation error. The thermal deformation of the CFRP reflector model was strongly affected by the fiber orientation error and accordingly fiber orientation angle of CFRP reflectors should be controlled strictly depending on desired accuracy.

*Keywords*: CFRP laminates; Reflector; Sensitivity analysis; Thermal deformation

**Nomenclature**

1-direction: Fiber direction
2-direction: Transverse direction of fiber
3-direction: Direction perpendicular to 12-direction
$E$ : Elasticity modulus of $\theta$ direction
$E_{11}$ : Elasticity modulus of 1-direction
$E_{22}$ : Elasticity modulus of 2-direction
$G$ : Shear modulus of $\theta$ direction
$G_{12}$ : Shear modulus of 12-direction
$\alpha$ : Coefficient of thermal expansion of $\theta$ direction
$\alpha_1$ : Coefficient of thermal expansion of 1-direction
$\alpha_2$ : Coefficient of thermal expansion of 2-direction
$\beta$ : Coefficient of thermal expansion of transverse direction of $\theta$ direction
$\theta$ : The angle formed by arbitrary direction and 1-direction
$\nu_{12}$ : Poisson's ratio of 12-direction
$\nu_{23}$ : Poisson's ratio of 23-direction

## 1. Introduction

In order to realize high resolution space observation, high precision space observation systems are necessary. Space observation is often performed in space and the space observation systems are exposed to severe temperature variation. Accordingly, the space observation systems are required to have high shape stability against the temperature variation.

Carbon Fiber Reinforced Plastics (CFRPs) are widely accepted as a material with high specific stiffness and small coefficient of thermal expansion. Hence they were adopted as structural elements of the space observation systems to reduce thermal deformation [1]. They were also utilized as a material of a 7.2m diameter ground based reflector (mirror) of a science mission [2].

Abusafieh et al. [3] investigated the effects of moisture absorption and microcrack on dimensional stability of CFRPs for space based reflectors and it was reported that CFRPs can be successfully used for precision structures. Yoon et al. investigated the effects of outgassing deformation and thermal deformation of CFRP reflectors on optical performance of space telescopes [4] and it was concluded that the thermal deformation of the structure degrades the optical performance significantly more than the outgassing deformation. In terms of the thermal deformation of CFRPs, it was reported that unexpected out-of-plane thermal deformation occurs on CFRP laminate plates due to existence of fiber orientation error [5, 6]. It was also reported that fiber orientation error of 0.4 degrees in standard deviation is inevitable [5]. However, a tendency of distribution of thermal deformation and its cause have yet to be sufficiently discussed.

In this research, an experiment on the thermal deformation

[†] This paper was recommended for publication in revised form by Associate Editor 000 000-
[*]Corresponding author. Tel.: +81 (0)52-789-3284
E-mail address: tanaka.shun@g.mbox.nagoya-u.ac.jp

of a CFRP reflector model was performed to examine what extent the thermal deformation is generated on the CFRP reflectors. Additionally, a finite-element analysis (FEA) on the thermal deformation of the CFRP reflector model was performed to discuss the mechanisms of the out-of-plane thermal deformation observed in the experiment, focusing on error of fiber orientation angle. The cause of the out-of-plane displacement was also discussed analytically.

## 2. Experiment

### *2.1 Experimental setup*

A reflector model used in this research is shown in Fig. 1. It was composed of a quasi-isotropic CFRP laminate (TMP Inc., XN60/NM31, $[0/\text{-}45/90/45]_s$) and cup-shaped. The radius of curvature was 1000mm and the diameter was 300mm. It was fixed on experimental setup with a screw attached at the center.

The thermal deformation of the reflector model was measured by the experimental setup shown in Fig. 2. The reflector model was fixed on a jig by a screw. The reflector model was covered by a thermostatic chamber with a heat-resisting glass [SANMEC Inc., special-ordered] shown in Fig. 2 which can uniformly heat up to 200°C inside. The shape of the reflector model was measured by a laser displacement sensor [Keyence, LK-H080] fixed on a two-axis linear slider system [THK, KRF] through the glass. The measurement was performed at discrete points in interval of 10mm for *x* and *y*-directions in 300mm × 300mm square domain before and after heating up the reflector model, where *x*-axis corresponded to zero degree direction of the laminate of the reflector model. The out-of-plane displacement (*z*-direction) of the reflector model was obtained by subtracting the shape before heating from the shape after heating. In this process, the effect of thermal expansion of jig was removed by assuming that the displacement of center of the reflector model was equal to zero.

### *2.2 Experimental result*

Figure 3 shows the experimentally observed out-of-plane displacement at the temperature difference of 50°C. The displacement was not uniformly. Positive large deformation occurs at approximately -45 degrees and 135 degrees from *x*-axis around the edge of the reflector model and negative large deformation occurred at 45 degrees and -135 degrees around the edge. The maximum displacement was about 0.03mm.

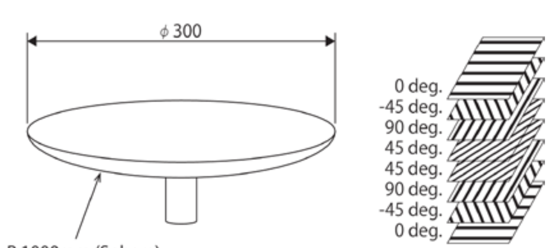


Fig. 1. The schematic view and lamination configuration of CFRP reflector model used in this research.

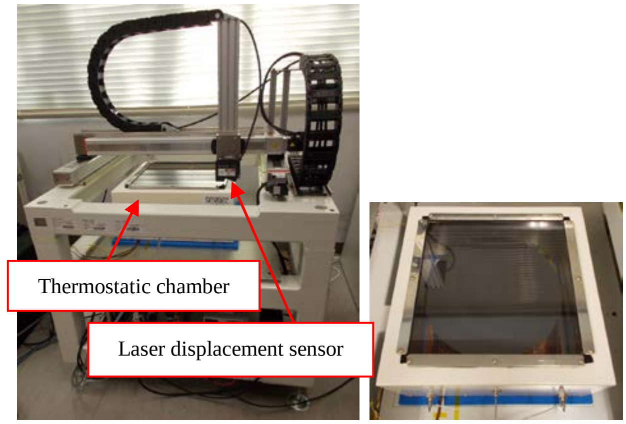


Fig. 2. Experimental setup (left) and thermostatic chamber (right) used in this research.

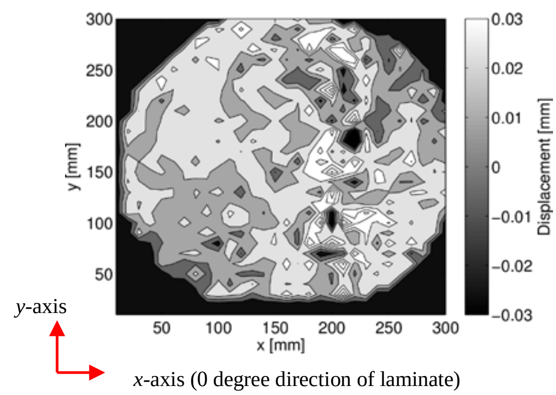


Fig. 3. Measured out-of-plane displacement of the reflector model.

## 3. Finite-element analysis

### *3.1 Analytical Model*

To discuss the mechanisms of the out-of-plane thermal deformation, first FEA was performed. In this paper, fiber orientation error was focused on as one of its causes. An engineering simulation software "ANSYS" was used in the FEA. The analytical model of the reflector model was composed of a shell element (SHELL181, 29,453nodes) and assumed to be fixed at the center. Material constants assumed in the analysis are shown in Table 1. $\nu_{23}$ was an assumed value, but other material constants were measured by tensile test and thermo mechanical analysis on the same material of the reflector model. Fiber orientation error was assumed to be ±1 degree, according to [5].

Table 1. Material constants used in the analysis.

| | |
|---|---|
| $E_{11}$ [GPa] | 340 |
| $E_{22}$ [GPa] | 5.2 |
| $G_{12}$ [GPa] | 3.9 |
| $\nu_{12}$ | 0.35 |
| $\nu_{23}$ | 0.3 |
| $\alpha_1$ [1/K] | $-0.7\times10^{-6}$ |
| $\alpha_2$ [1/K] | $35\times10^{-6}$ |

### *3.2 Effect of fiber orientation error*

Figure 4 shows the out-of-plane displacement generated on the reflector model without the fiber orientation error. It was cup-shaped and the maximum value was $1.04\times10^{-5}$ mm. This value is significantly smaller compared to the experimental result and it was made clear that this displacement was a result of the uniform expansion of the reflector model by considering the in-plane displacement. Accordingly, it can be considered that no out-of-plane thermal deformation was generated on the reflector model without fiber orientation error.

Figure 5 shows the displacement of the reflector model with the fiber orientation error of 1 degree on the bottom layer. It was saddle-shaped. The distribution of positive and negative displacement showed agreement with experimental result. The maximum value of the calculated result was $4.92\times10^{-2}$ mm and this value also shows agreement with experimental result in the order of magnitude, although the fiber orientation error of the reflector model used in the experiment is unknown.

The thermal deformations of the CFRP reflector model with fiber orientation error of +1 or -1 degree in one layer through eight layers were calculated. The out-of-plane deformations were similar to the calculated result shown in Fig. 5, but rotated in accordance with the fiber orientation angle of each layer. As an example, the distribution of positive and negative displacement of the reflector model with fiber orientation error of 1 degree in the second layer from the bottom was rotated by about -45 degrees from Fig. 5, which is equal to the difference in the original fiber orientation angle between the first and second layers from the bottom. Table 2 shows the summary of the maximum and minimum displacements of the reflector model with 1-degree fiber orientation error in each layer. This result shows that the maximum and minimum displacements occur at 45 degrees from the fiber direction with the error and that the largest thermal deformation occurs when the fiber orientation error exists in the most surficial layer. According to these results, it can be considered that even if the fiber orientation errors exist in all layers, the out-of-plane deformation when the orientation error exists in the top or bottom layers is dominant and the maximum and minimum displacements appear at ±45 degrees.

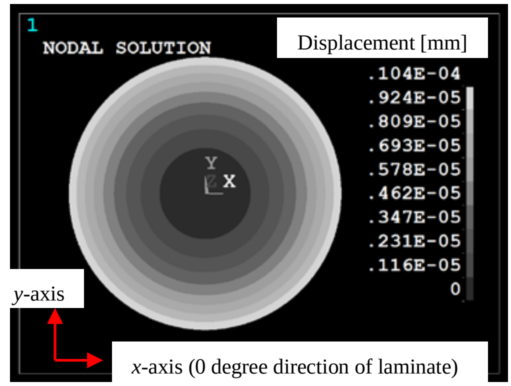


Fig. 4. Calculated out-of-plane displacement of the reflector model without fiber orientation error.

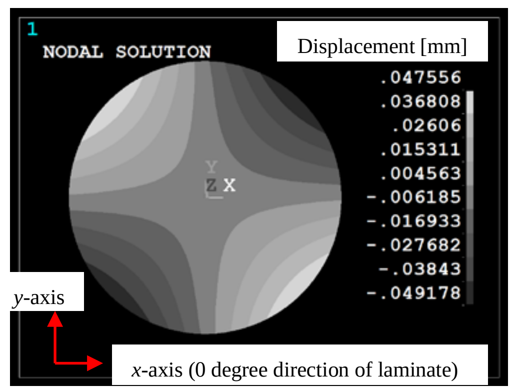


Fig. 5. Calculated out-of-plane displacement of the reflector model with fiber orientation error of 1 deg. on the lowest layer.

## 4. Analytical discussion

Next, the mechanism was analytically discussed. A virtual triangular region in a plate with unit thickness as shown in Fig. 6 is considered. The plate is supposed to be rigidly fixed at all edges and stress does not exist inside the plate at the initial state. When temperature variation of $\Delta T$ is applied, by considering the equilibrium of the force, we can obtain,

Table 2. Value of the maximum and minimum displacement and its direction with fiber orientation error in one layer.

$\times10^{-3}$ mm

| Layer | Original fiber orientation angle | Error: +1deg. | | | | Error: -1deg. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Maximum | | Minimum | | Maximum | | Minimum | |
| ⇑ | 0deg. | 49.187 | +45deg. | -47.523 | -45deg. | 47.526 | -45deg. | -49.148 | +45deg. |
| Upper | -45deg. | 16.896 | 0deg. | -17.135 | 90deg. | 17.421 | 90deg. | -17.016 | 0deg. |
| | 90deg. | 20.417 | -45deg. | -21.140 | +45deg. | 20.983 | +45deg. | -20.299 | -45deg. |
| | 45deg. | 3.429 | 90deg. | -3.359 | 0deg. | 3.413 | 0deg. | -3.468 | 90deg. |
| | 45deg. | 3.385 | 0deg. | -3.417 | 90deg. | 3.498 | 90deg. | -3.398 | 0deg. |
| | 90deg. | 21.173 | +45deg. | -20.411 | -45deg. | 20.338 | -45deg. | -20.974 | +45deg. |
| Lower | -45deg. | 17.164 | 90deg. | -16.880 | 0deg. | 17.042 | 0deg. | -17.410 | 90deg. |
| ⇓ | 0deg. | 47.556 | -45deg. | -49.178 | +45deg. | 49.182 | +45deg. | -47.523 | -45deg. |

$$E\alpha = \frac{1}{2}\left[\left(E_{11}\alpha_1 - E_{22}\alpha_2\right)\cos 2\theta + \left(E_{11}\alpha_1 + E_{22}\alpha_2\right)\right]. \quad (1)$$

By applying the material constants listed in Table 1, it is seen from Eq. (1) that the thermal stress $E\alpha\Delta T$ takes the minimum value at 0 degrees and 180 degrees and the maximum value at ±90 degrees.

As shown in the calculated results in the previous section, the out-of-plane deformation was not generated on the reflector model without fiber orientation error. This is because the thermal stress $E\alpha\Delta T$ was balanced due to the symmetric stacking sequence. However, when the fiber orientation error exists, the thermal stress $E\alpha\Delta T$ is asymmetric and imbalanced. The sensitivity of the difference of the thermal stress distribution by small fiber orientation error can be obtained by differentiating Eq. (1) by $\theta$ as,

$$\frac{d(E\alpha)}{d\theta} = -\left(E_{11}\alpha_1 - E_{22}\alpha_2\right)\sin 2\theta. \quad (2)$$

This value takes the maximum at 45 degrees and -135 degrees and the minimum at -45 degrees and 135 degrees. Accordingly, the thermal stress varies much at ±45 degrees and ±135 degrees inclined from the fiber direction and therefore the maximum and minimum displacements were generated on these directions, as shown in Fig. 5 and Table 2.

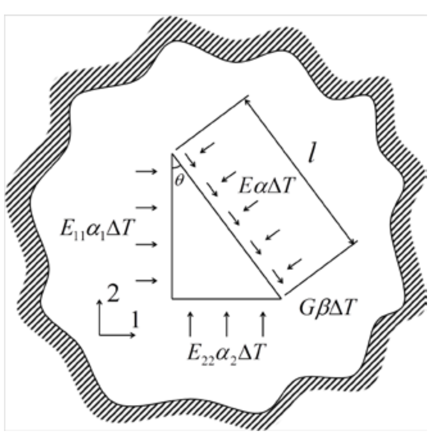


Fig. 6. Virtual triangular plate in a plate with a fixed boundary.

## 5. Conclusion

In this research, thermal deformation of a CFRP reflector model composed of quasi-isotropic CFRP laminate was measured. The result showed that the distribution of out-of-plane thermal deformation was not uniformly despite that the reflector model was heated uniformly. FEA on thermal deformation of the CFRP reflector model with and without fiber orientation error was also performed. It was found from the analysis that the thermal deformation of the CFRP reflector model was strongly affected by fiber orientation error, especially when the error existed in a surficial layer. It was also found that the maximum and minimum displacements were generated at ±45 and ±135 degrees-direction from the fiber direction. The mechanism of this finding was clarified analytically. The calculated result showed agreement with the experimental result to some extent. Accordingly, the thermal deformation observed in the experiment may be affected by fiber orientation error. Therefore, the fiber orientation angle of the CFRP reflectors should be controlled strictly depending on desired accuracy.

## Acknowledgment

This research was supported by JSPS KAKENHI Grant Number 26249131.

## References

[1] T. Ozaki, K. Naito, I. Mikami, H. Yamauchi and S. Tsuneta, High precision composite pipes for SOLAR-B optical structures, *Acta Astronautica*, 48 (2001) 321-329.

[2] A. Kawachi, Y. Hayami, J.Jimbo, S. Kamei, T. Kifune, H. Kubo, J. Kushida, S. LeBohec, K. Miyawaki, M. Mori, K. Nishijima, J. R. Patterson, R. Suzuki, T. Tanimori, S. Yanagita, T. Yoshikoshi and A. Yuki, The optical reflector system for the CANGAROO-II imaging atmospheric Cherenkov telescope, *Astroparticle Physics*, 14 (2001) 261-269.

[3] A. Abusafieh, D. Federico, S. Connell, E. J. Cohen and P. B. Willis, Dimensional stability of CFRP composites for space based reflectors, *Proc. of Optomechanical Design and Engineering 2001*, San Diego, California, USA (2001) 9-16.

[4] J. S. Yoon, H. I. Kim, J. H. Han and H. S. Yang, Effect of dimensional stability of composites on optical performances of space telescopes, *Journal of Aerospace Engineering*, 27 (2012) 40-47.

[5] Y. Arao, J. Koyanagi, S. Utsunomiya and H. Kawada, Effect of ply angle misalignment on out-of-plane deformation of symmetrical cross-ply CFRP laminates: Accuracy of the ply angle alignment, *Composite Structures*, 93 (2011) 1225-1230.

[6] Y. Arao, J. Koyanagi, S. Takeda, S. Utsunomiya and H. Kawada, Out-of-Plane Deformation due to the Ply Angle Misalignment in CFRP Laminates (The Effect of the Stacking Sequence on Thermal Deformation), *Trans. JSME*, 77 (2011) 619-628 (in Japanese).