

論文の要約

論文題目 J-Stability, Montel's theorem, and Artin-Mazur zeta functions in non-Archimedean dynamics

(非アルキメデスの力学系における J-stability, Montel 定理, Artin-Mazur ゼータ関数)

氏名 李 正勳

In this dissertation, we consider the dynamics of iterated rational maps from the projective line over an algebraically closed, complete, and non-Archimedean field to itself.

After preparing some basic notion in Chapter 2, we present three theorems proved or re-proved by the author: J -stability (Chapter 3), Montel's theorem (Chapter 4), and Artin-Mazur zeta functions (Chapter 5) in non-Archimedean dynamical systems.

In Chapter 1, we first review the background and basic notion of non-Archimedean dynamical systems.

In Chapter 2, we review some standard facts on non-Archimedean dynamical systems.

In the first section, we recall the rigorous definitions of non-Archimedean fields, projective lines, and rational maps. Then we review some of their properties.

In the second section, we consider the Fatou-Julia theory in non-Archimedean dynamical systems with the rigorous definitions of equicontinuity, Fatou sets, and Julia sets. We also see some basic properties of Julia sets.

In Chapter 3, we prove our first main result of this dissertation: J -stability of immediately expanding rational maps.

In 1983, R. Mane, P. Sad, and D. Sullivan introduced the notion of J -stability of rational maps in complex dynamical systems, which roughly means that the dynamics on the Julia sets of two given rational maps are dynamically equivalent if those two rational maps are close enough. They also showed that a rational map is J -stable if it has a neighborhood in the set of rational maps on which the number of attracting cycles is constant.

Our first main result is a non-Archimedean analogue of Mane, Sad, and Sullivan's theorem in complex dynamical systems.

In Chapter 4, we present our second main result of this dissertation: an alternative proof of the non-Archimedean Montel theorem for rational dynamics.

Motivated by Montel's theorem in complex analysis, L-C. Hsia originally proved the non-Archimedean Montel theorem, which is also called Hsia's criterion, and applied it to non-Archimedean dynamical systems.

Our second main result is to give an alternative and simple proof of Hsia's criterion for rational dynamics by using two fundamental results on non-Archimedean Green functions introduced by J. H. Silverman and S. Kawaguchi.

In Chapter 5, we prove our final main result of this dissertation: the rationality of Artin-Mazur zeta functions of non-Archimedean dynamical systems. In 1965, as a generalization of Weil's local zeta functions, M. Artin and B. Mazur introduced the Artin-Mazur zeta functions of a topological dynamical system. Our final main result is a non-Archimedean analogue of A. Hinkkanen's result which shows the rationality of Artin-Mazur zeta functions of complex dynamical systems.