ENDOGENEOUS PRODUCT BOUNDARY*

Takanori Adachi
School of Economics, Nagoya University

Takeshi Ebina
Faculty of Economics, Shinshu University

Makoto Hanazono
School of Economics, Nagoya University

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Abstract

This paper analyzes the determinants of product boundary choice as discriminatory pricing. Specifically, we consider a model where a monopolist sells a base product and an add-on, which is valuable only if it is consumed together with the base product. An important feature of our model is that this additional value is allowed to be contingent on the valuation of the base product. We show that separation, in which a base product alone is also sold, yields a higher profit than integration, in which only a bundled package is sold, if and only if the range of the add-on valuation exceeds a threshold value, and that separation is more likely to be optimal as the degree of positive contingency increases. As for welfare, it is shown that in the case of separation, consumer surplus is always lower than if a seller is restricted to sell a bundled package.

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1 Introduction

In the modern economy, the boundary of a product or service is often a result from a seller's choice. Specifically, many products and services provide supplementary elements. For example, Apple's iPad installs FaceTime (a videotelephony) as an initial setting. In effect, Apple sells the basic iPad component (i.e., its operating system, iOS[1]) *and* additional ones (such as FaceTime) by naming iPad, which is essentially a bundled package. Apple also sells other separate applications for iPad with charge. They include iPhoto (an extended device for digital photograph manipulation) and iMovie (for video editing).[2] One would imagine that an iPad plus iPhoto could be actually sold as a single package called iPad, or an iPad without FaceTime could be sold as the iPad.

Similarly, more and more of digital and nondigital products are now supplemented by online materials. One such example is economics textbooks for college students. Some supplements are provided free, and others are sold with charge. For instance, while the Book Companion Site for Krugman and Wells' *Economics*[3] is free for use as long as the reader is registered to the site, Acemoglu, Laibson, and List's *Economics* does not provide its readers with free supplements. Instead, the publisher sells online supporting materials with charge (MyEconLab®).[4] Interestingly, in the examples above, additional products and services are considered as information goods, and as Shapiro and Varian (1998, p.3) emphasize, "the cost of producing (or reproducing) additional copies" of an information good "is negligible." If supplementary products are included with no charge, the product boundary necessarily reaches such additional features. While this "free of charge" is seemingly attractive to any consumers, a second thought would suggest that one cannot obtain discounts by refusing the use of supplementary materials. In another situation, a producer may sell a base product, with a supplement as an option for interested consumers. On average, is it really socially better if consumers can choose either a bundled package

[1] More precisely, it should be interpreted as the operating system with some basic applications (such as an internet browser) because the operating system alone would be useless unless it is accompanied with a minimum level of functionality.

[2] Apple calls initially installed apps "built-in apps" (see, e.g., http://www.apple.com/ios/what-is/ (retrieved: July 2015)).

[3] Its URL is: http://bcs.worthpublishers.com/krugmanwellsecon3/default.asp#t_768072____ (retrieved: July 2015).

[4] See http://www.pearsonhighered.com/acemoglu-econ/order-info/index.html (retrieved: July 2015).

or a base product than if only a package is sold?

It is thus important to understand the mechanism of product boundary choice and to evaluate its consequences. In this study, we provide a simple model of product boundary choice to focus on its role as a method of sorting consumers with different willingness to pay.[5] For this purpose, our model contains an optional good (i.e., an add-on), that is valuable *only if* the buyer consumes it along with a certain base good. We consider the situation in which a monopolistic seller produces a base good and an add-on, with constant marginal costs (both zero for simplicity in the main analysis; see Section 6 for discussions on non-zero marginal costs). Each consumer demands zero or one unit of each good, and the (innate) value of each good distributes uniformly on a rectangular area from the origin. In this setup, *integration*, in which only a bundled package is sold, corresponds to the add-on price being *zero*, whereas *separation*, in which both a base product alone and a package are sold, corresponds to a *positive* add-on price. The seller thus decides to sell a bundled package only, or to sell a base product and its add-on separately. In this way, we analyze product boundary as an *endogenous* choice by a firm.

One distinct feature of this study is that we obtain clear analytical results for the seller's problem by considering a model beyond a standard setting in which the reservation value of joint consumption equals the sum of the innate values of each good (see the next section for a literature review). Consumers may value an add-on more if they value the base product more. Because our model entails "structural complementarity" in the sense that an add-on is never consumed alone, the value of joint consumption is possibly greater than the sum of the innate values of both goods (*positive contingency*). Considering such contingency is not just out of curiosity. In the examples above, it is not unnatural that a consumer's valuation of iPhoto would increase with his or her valuation of iPad. Similarly, a college student who gains greater utility from using a textbook would probably feel more satisfaction from its accompanying workbook. However, as Venkatesh and Kamakura (2003, p.212) suggest, if a textbook and its supporting material "offer (some) overlapping

[5] Needless to say, cost reduction in aggregate production would also be a concern for a seller's choice of product boundary. However, this supply-side reason alone would not fully answer the question in many cases, especially of information goods as in the examples above: the demand-side would figure strongly in determining a product or service boundary as a seller's choice. See, e.g., Evans and Salinger (2008) for an analysis that studies the supply-side.

benefits", *negative*, rather than positive, *contingency* exists. In our model, we allow this nonzero contingency in a tractable way. Under this setting, we investigate the properties of the optimal boundary choice, and verify the following claims (Proposition 2): Given the level of contingency, separation is more likely to be optimal as the relative range of add-on valuation to base-good valuation becomes larger. Similarly, given the distribution of innate values, separation is more likely to be optimal as the degree of contingency rises. A naive thinking might mislead one to guess that strong complementarity provides the seller with a strong incentive to sell the bundled package only. Our Proposition 2 shows that the opposite is true. We provide intuitive arguments for this result in the second paragraph after Proposition 2. We then conduct a welfare analysis of the boundary choice, and show that integration is desirable from the social welfare viewpoint, provided that the monopolist retains the freedom in price choice, and thus under separation, *consumer welfare is necessarily lower than under integration* (Proposition 3). The reason is that with separation, the monopolist can exercise its market power less restrictively, resulting in inefficiency due to discriminatory pricing.

In relation to our results, Anderson (2009, pp.241-243) presents ten principles that he calls "Free Rules." He first notes that the marginal costs of such technologies as processing, bandwidth, and storage have become increasingly closer to zero, and "[b]its wants to be free" (Anderson 2009, p.241). He then goes on to state that "[if] the cost of something is heading to zero, Free is just a matter of when, not if" (Anderson 2009, p.242). He concludes that zero marginal cost is important factors for a good to be free. Our main result supports his assertion: if integration is optimal, an add-on's marginal cost must be zero (See Section 6). Our result also finds that the range of add-on valuation is another key factor in determining whether a good is offered for free.

This paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model with interdependent valuations. We then derive the optimal bundle prices for the basic model in Section 4. In Section 5, we provide a welfare analysis. It presents a clear-cut result in the case of no contingency—a result that is not shown in Chen and Nalebuff (2007). For the case of non-zero contingency, we present numerical examples by which we conjecture that the main implication still holds. Section 6 then argues the robustness of our assumption that the (constant) marginal costs are zero for both goods. Importantly, if the marginal cost of an add-on is greater than the lowest valuation for the add-on, integration is never optimal. This makes clear the condition for our main results to hold if a non-negligible marginal cost of an add-on is allowed. We also discuss how our results would be affected if competing add-ons are considered. Section 7 concludes.

2 Related Literature

In this paper, we study the case of one-way complements: one product (which we call a base product) can be consumed independently while it is essential to the consumption of another product (which we call an add-on). There are several papers that also study this case under a variety of settings of competition. First, Cheng and Nahm (2007) study product boundary in the case where the base product is produced by a monopolist and the add-on is produced by another monopolist. The quality of the base product is enhanced with the use of the complement, and consumers are one-dimensional heterogenous in evaluating the quality. In this setting, Cheng and Nahm (2007) show that if the two products are virtually symmetric complements (i.e., if the quality of independent consumption of the base product is sufficiently low), consumers purchase either both products or nothing; those who evaluate the quality high choose the former and vice versa. In terms of the bundling/tying literature (see below), this situation is similar to pure bundling. However, this results from consumers' choice, not from the sellers' choice. In particular, firms in Cheng and Nahm's (2007) model choose only prices, without deciding whether to (jointly) provide a bundled package.[6] Because of our recognition that product boundary is an important choice for a seller, this paper instead studies product boundary resulting from consideration by the supply side.

Chen and Nalebuff (2007) also study multiple scenarios of unidirectional complements.

[6] In a similar vein, Adachi and Ebina (2014) propose a simplified model of Cheng and Nahm (2007) to study the effects of a merger of two firms. Tarola and Vergari (2015) consider the situation where a monopolistic firm produces a base product whereas its add-ons are produced by the monopolist and another firm specializing in an add-on. Assuming add-ons are vertically differentiated (the quality of the base product monopolist's add-on is lower), Tarola and Vergari (2015) show that the monopolist always has an incentive to integrate the rival. See also Plotnikova, Sarangi, and Swaminathan (2015) for a related analysis.

One such scenario entails one monopolist of a base product and another monopolist producing its add-on. Both firms simultaneously and independently decide the prices. Assuming the uniform distribution, Chen and Nalebuff (2007) find that for large values of the upper bound for the add-on valuation, the add-on monopolist's profits are larger than those of the base product monopolist. In the present paper, we focus on one of the scenarios that Chen and Nalebuff (2007) consider, namely the situation where there exists a single monopoly that produces both a base product and its add-on. However, this paper complements Chen and Nalebuff (2007) by examining two important issues that Chen and Nalebuff (2007) ignore: contingency and welfare. Specifically, following Venkatesh and Kamakura (2003), we introduce the concept of contingency into our general model to allow one's valuation of the base good to affect his or her value by having an additional option.[7] This is an important generalization of Chen and Nalebuff (2007) because positive contingency would be especially relevant in the unidirectional relationship. We show that this nature affects the level of threshold whether pure bundling or mixed bundling is optimal. In addition, while Chen and Nalebuff (2007) do not consider social welfare, we examine the effects of product boundary choice on social welfare.[8]

This paper is also related to a large literature on bundling under a monopoly, initiated by Stigler (1963) and Adams and Yellen (1976).[9,10] Conceptually, pure bundling (two

[7] Venkatesh and Kamakura (2003) compare pure bundling with mixed bundling by setting up a monopolistic model where a consumer's reservation value is not merely the sum of the component prices when considering complements (superadditive) or substitutes (subadditive). They argue that pure bundling should be employed if two goods are strong complements. An important difference between this work and our model is that the two goods are asymmetric in our model. Moreover, while Venkatesh and Kamakura (2003) rely on a numerical analysis of the relative profitability of mixed bundling compared to pure bundling, we provide distinct results for the optimal bundling strategy of a monopolistic seller.

[8] The present paper is also a generalization of Pierce and Winter (1996) in the sense that we consider continuous valuation. While Pierce and Winter (1996) conclude that pure bundling is optimal when one type of consumer's valuation for the optional good is similar to the other's, as long as a monopolist serves both types (i.e., no exclusion), we obtain qualitatively similar results without the qualification of exclusion.

[9] Among others, Schmalensee (1984), Lewbel (1985), McAfee, McMillan, and Whinston (1989), Salinger (1995), and Eckalbar (2010) study two-goods, continuum-type cases. An important common idea in this body of work is that bundling reduces the dispersion of the buyers' average willingness to pay for the goods, unless the buyer's valuations for bundled goods are perfectly positively correlated. In this situation, a monopolist can extract more profits through bundling. Recent surveys on bundling include Armstrong (2006), Stole (2007), Shy (2008, Chapter 4), and Choi (2012). For other related issues, see, e.g., Prasad, Venkatesh, and Mahajan (2010) for an investigation of optimal bundling strategy in the presence of network externality, Chao and Derdenger (2013) for an analysis of price implications of mixed bundling in two-sided markets, and Derdenger and Kumar (2013) for a dynamic analysis of bundling strategy.

[10] For studies of bundling and strategic interaction, see, e.g., Nalebuff (2004), Thanassoulis (2007), Casadesus-Masanell, Nalebuff, and Yoffie (2009), Armstrong and Vickers (2010), Armstrong (2013), Jeon

goods are sold only as a bundle) corresponds to integration in the present paper, and mixed bundling (they are sold separately) to separation. Our basic model differs from these previous bundling models in that we focus on the unidirectional nature of the goods by considering one of the goods as an add-on.[11]

3 Model

To consider a multi-product monopolist's profit maximization problem, we borrow the setup from McAfee, McMillan and Whinston (1989) to model the problem, with one important departure: there is a *base* good and an *add-on* in the following sense. While a base good has its own value irrespective of the consumption of other goods, an associated optional add-on can only have value if it is consumed together with the base good.[12]

More specifically, suppose that a monopolist produces two types of goods, good 1 (base) and good 2 (add-on). Each consumer consumes up to one unit of each good. Let $v_1 \geq 0$ denote valuation of good 1 for a consumer. If the consumer purchases good 2 *in addition to* good 1, his or her valuation of the "composite" good is denoted as v_B. We now introduce valuation for good 2, v_2. It is the value for a consumer as if it is independently consumed, though it is not possible that good 2 is consumed without good 1. Thus, v_2 should be interpreted as the *conditional* utility that is realized *only if* good 1 is consumed together with good 2.[13]

We assume, as in Chen and Nalebuff (2007), that consumers are heterogenous in the sense that (v_1, v_2) is distributed uniformly on $[0, 1] \times [0, \tau]$, where $0 < \tau \leq 1$, with independence between v_1 and v_2 (the density is thus $1/\tau$). Two remarks are in order. First, the reason for employing a uniform distribution and statistical independence is that it enables us to derive an explicit solution and to provide an intuitive argument (Chen and Nalebuff (2007) and Prasad, Venkatesh, and Mahajan (2010), among others, also

and Menicucci (2012), and Chung, Lin, and Hu (2013).

[11] A similar unidirectional relationship and discount for add-ons are also studied by Adachi and Ebina (2012), based on a different setting.

[12] In this paper, we make an *a priori* distinction between a base good and an add-on. In many cases such as the example of iPad and a rubber cover, the distinction would be obvious. More broadly, the distinction could be less obvious. See, e.g., Lee, Kim, and Allenby (2013) for this issue.

[13] As usual, we assume the quasi-linear utility, and thus, income effects are not of concern.

employ the same assumption). Second, it appears natural that the highest valuation of the optional good is no greater than that of the base good.[14] The parameter τ is thus a measure of the *diversity* of consumers' valuations of the optional good.

A consumer with (v_1, v_2) then obtains utility of v_1 if good 1 alone is consumed, and, unlike Chen and Nalebuff (2007), $v_B \equiv v_1+v_2+\alpha v_1 v_2$ if both goods are consumed together, where we call α the *degree of contingency* (following Venkatesh and Kamakura (2003)).[15] We assume that α is common for all consumers. Note that $v_B - v_1 = (1 + \alpha v_1)v_2$, which implies that if $\alpha = 0$, the incremental value by consuming the composite product for (v_1, v_2) consumer is v_2, *regardless of* his or her valuation of the base product. To allow the dependency of the increment value on v_1, we allow α to be nonzero. If $\alpha > 0$, it means that the higher v_1, the higher an incremental value of v_2 for v_B, and if $\alpha < 0$, vice versa. To ensure that $v_B \geq v_1$, we assume that $\alpha > -1$.

Now, let p_B be the price that the monopolist charges for a unit of the "composite" or "packaged" product and p_1 be the price of good 1. For expositional convenience, we define "price" for good 2 as $p_2 \equiv p_B - p_1$. The costs of producing each good are constant and normalized to zero. It is thus clear that in this setting, welfare maximization requires everyone in $[0, 1] \times [0, \tau]$ consumes both goods 1 and 2. These assumptions concerning distribution support and the zero marginal costs are relaxed in Section 6.

A consumer prefers purchasing a bundled package to consuming only a base product, if and only if

$$v_B - p_B \geq v_1 - p_1 \Leftrightarrow v_2 \geq \frac{p_2}{1 + \alpha v_1} \ (\equiv v_2^{B1}(v_1; \alpha)).$$

[14] The restriction on $\tau \leq 1$ should not be important: in the case of $\alpha = 0$, the main results hold as long as $\tau \leq 3/2$ (because the optimal pure bundling price does not exceed the upper bound for v_1).

[15] The parameter α can be said as expressing complementarity/substitutability between the two goods *conditional on* the consumption of the add-on. Because the relationship between the main and the optional goods in our case inherits structural (one-way) complementarity (as already mentioned in Introduction), we instead use positive (resp. negative) contingency to mean $\alpha > 0$ (resp. $\alpha < 0$). One of the most important findings in the literature since Adams and Yellen (1976) is that a negative correlation between two goods favors (pure) bundling because the joint valuation of the two goods is more centered, which enables a seller to capture the homogeneous willingness to pay. Typically, the joint valuation is assumed as the simple sum of the two goods, while correlation between the goods is allowed. In contrast, the present paper, while assuming the independence of the valuation distributions, allows the interdependency at the utility function. Our result is similar to Adams and Yellen (1976) and others in the sense that a lower α favors integration. We conjecture that the two ways of modeling correlation share the same logic. We leave this question for future research.

Similarly, a consumer prefers a bundled package to nothing if and only if

$$v_B - p_B \geq 0 \Leftrightarrow v_2 \geq \frac{p_2}{1+\alpha v_1} - \frac{v_1 - p_1}{1+\alpha v_1} \ (\equiv v_2^{B0}(v_1;\alpha)).$$

Given good 2 has a non-negative value for any consumer, $p_2 = 0$ implies that a consumer buys both goods or nothing. Also, if $p_1 = 1$ and $p_2 > 0$, a generic consumer buys either both or nothing. We call these pricing strategies as *integration* (see Figure 1). On the other hand, if $p_2 > 0$, $p_2 < \tau(1+\alpha)$ and $p_1 < 1$, some consumers buy only good 1 and others consumer goods 1 and 2 together. We refer to this pricing strategy as *separation*. In this sense, *integration is nested in separation*. Clearly, a consumer whose (v_1, v_2) satisfies $0 > v_1 - p_1 \Leftrightarrow v_1 < p_1$ and $v_2 < v_2^{B0}(v_1;\alpha)$ buys nothing.

[Figures 1, 2 and 3 around here]

Given the prices and nature of the goods, the demand for the goods is depicted in Figures 2 (positive contingency: $\alpha > 0$) and 3 (negative contingency: $\alpha < 0$).[16] It is easy to see that $v_2^{B1}(v_1)$ is convex, with the slope being negative in the case of $\alpha > 0$, and positive in the case of $\alpha < 0$. It is also seen that $v_2^{B0}(v_1)$ is convex if $\alpha > 0$, and concave if $\alpha < 0$. Finally, it is verified that the two curves intersect at $(p_1, p_2/(1+\alpha p_1))$. Intuitively, the shapes for $v_1 < p_1$ is derived from the fact that as v_1 and v_2 move closer, a consumer obtains more utility from joint consumption in the case of $\alpha > 0$, and vice versa in the case of $\alpha < 0$. The shape for $v_1 \geq p_1$ is explained as follows: for a fixed value of v_2, as v_1 increases, joint consumption becomes (resp. less) attractive for $\alpha > 0$ (resp. $\alpha < 0$).

4 Integration vs. Separation

We now analyze an optimal strategy for product boundary and pricing. Let $p_1 \in [0,1]$ and $p_2 \in [0,\tau]$ be the prices of goods 1 and 2 in the regime of separation, respectively. As discussed, $p_1 = 1$ or $p_2 = 0$ is interpreted as integration. Let b denote the price of an integrated good in the regime of integration (that is, b is defined as p_B with $p_2 = 0$).

[16] In these figures, the case of $p_1 + p_2 < \tau$ is depicted. If $p_1 + p_2 \geq \tau$, the curve $v_2 = v_2^{B0}(v_1;\alpha)$ intersects the boundary $v_2 = \tau$ at $v_1 = (p_1 + p_2 - \tau)/(1+\alpha\tau)$.

4.1 The Case of No Contingency ($\alpha = 0$)

First, we present the result that comes from Chen and Nalebuff (2007). To obtain the heuristics for an optimal solution with nonzero contingency, we reconstruct their argument as it provides a framework for generalization to the case of $\alpha \neq 0$.

Proposition 1 (Chen and Nalebuff's (2007) Lemma 2)**.** *If the optimal price of good 1 (the base product) is less than or equal to* $2/3$*, then the optimal price of good 2 (the add-on) is equal to zero (i.e., integration is optimal).*

We provide heuristic arguments below to understand the essence underlying this proposiion: why the seller prefers integration to separatio if $p_1 \leq 2/3$. For a given price (p_1, p_2), where $p_2 > 0$, let the add-on price be increased by a small amount, $\varepsilon > 0$, and the base product price be decreased by the same amount ε, *keeping the price of the bundled package constant.* The price pair is now moved from (p_1, p_2) to (p_1', p_2') in Figure 4. Notice that the inframarginal consumers purchasing both goods have no effects on the profits because the package price remains the same.

[Figure 4 around here]

In Figure 4, we identify three first-order effects that the seller takes into account. First, there are new consumers who start buying a base product alone ("*Newcomer Gain*" in Figure 4). This effect is evaluated by $\varepsilon p_1 p_2/\tau$ (the selling price, p_1, multiplied by the density, $\varepsilon p_2/\tau$, in the first-order change). Second, those who continue to buy a base product alone now pay less ("*Reduced Price Loss*" in Figure 4). This is evaluated by $-\varepsilon(1-p_1)p_2/\tau$ (the loss in the price per customer, $-\varepsilon$, multiplied by the density, $(1 - p_1)p_2/\tau$). Lastly, there are consumers who switch from buying both goods to a base product alone ("*Switching Loss*" in Figure 4). This is also evaluated by $-\varepsilon p_2(1 - p_1)/\tau$ (the loss in the revenue per customer, $-(p_B - p_1) = -p_2$, multiplied by the density, $(1 - p_1)\varepsilon/\tau$, ignoring the second-order change). Hence, the gain (strictly) exceeds the losses if and only if:

$$p_1 p_2 > 2(1 - p_1)p_2 \Longleftrightarrow p_1 > 2/3 \text{ and } p_2 > 0.$$

This shows that as long as $p_2 > 0$, this screening (starting from $p_1 = 1$) should be continued until $p_1 = 2/3$ in order to increase the profits for the given sum of the two prices. We then need to check what to do for $p_2 = 0$ (see Figure 5). To see this, it is instructive to observe how strongly the seller is motivated to screen consumers by putting a base product on sale, starting from integration, in which only a bundled package is sold. The discussion for this case is also relevant in subsections 4.2 and 6.1 below. Specifically, we, starting from $(p_1, 0)$, consider a small change in p_2 by ε, keeping the price of the bundled product constant. In this case, the screening process yields only the newcomer gain (there are consumers who once purchased nothing, but now buy a base product) and the switching loss (there are also consumers who once purchased a bundled package, but now switch to purchasing a base product only), and these are merely the second-order change.

[Figure 5 around here]

The newcomer gain from this change is evaluated as the shaded area in Figure 5, multiplied by the selling price, p_1 (ignoring any higher order changes). This is equal to $\varepsilon^2 p_1 / (2\tau)$. Likewise, the switching loss is equal to (the negative of) $(1-p_1)\varepsilon^2/\tau$, as above (now, the density is $(1-p_1)\varepsilon/\tau$). Here, this marginal loss decreases as the price of the base product p_1 increases. This is because each of th switchers causes a slight decrease in revenue for the seller, and the number of the switchers, who have a valuation of the base product exceeding the base product price, decreases as the base product price increases. Now, the gain is greater if and only if $p_1/2 > 1 - p_1 \Leftrightarrow p_1 > 2/3$. Therefore, the optimal add-on price $p_2 = 0$ should be increased if the base product price is greater than 2/3.

Combining the previous arguments, the optimal prices are either $(p_1, 0)$ or $(2/3, p_2 > 0)$. We can find the optimal prices as follows: First, we solve for the optimal price under integration. Then, if it is more than 2/3, we set $p_1 = 2/3$ and find $p_2 > 0$ that maximizes the seller's profits. If it is less than 2/3, $p_2 = 0$ is indeed optimal.[17] It can be shown that *integration is suboptimal if and only if* $\tau > 2/3$ because the optimal price of an integrated product is greater than 2/3 (a formal argument is available upon request). An intuitive explanation is as follows. It is straightforward to discern that the newcomer gain is increasing in p_1, and the reduced price loss and the switching losses are decreasing in

[17] We can argue that, in this case, profit is decreasing in p_2 at $p_1 = 2/3$, so that $(2/3, p_2 > 0)$ will never be opimal.

p_1. The gain from screening is thus more likely to exceed the losses if p_1 is higher. When τ is large, the bundle price $b = p_1 + p_2$ is likely to be high because the average willingness to pay for joint consumption is large.[18] In this case, the seller finds it profitable to provide the base product alone at a price slightly lower than the bundle price, to invite consumers who are willing to buy a base product alone at a high price for the cost of losing the switching consumers as well as the price reduction, both of which only slightly affect the seller's profits for a small p_2.

4.2 The Case of Nonzero Contingency ($\alpha \neq 0$)

We now characterize the optimal boundary choice of the monopolist with positive and negative contingency ($\alpha \neq 0$). In this subsection, we observe that even in the presence of nonzero contingency, the above argument of screening applies. First, Proposition 2 below shows that the optimal scheme is such that the monopolist adopts separation if and only if τ exceeds a threshold value. It also shows that a lower α favors integration.

Proposition 2. *For a fixed value of $\alpha > -1$, there exists a threshold of $\hat{\tau}(\alpha) \in (0, 1]$ such that integration is optimal if and only if $\tau < \hat{\tau}(\alpha)$. Conversely, for a fixed value of $\tau \in (0, 1)$, there exists a threshold $\widetilde{\alpha}(\tau) > -1$ such that integration is optimal if and only if $\alpha < \widetilde{\alpha}(\tau)$.*

[Figure 6 around here]

The proof of this proposition is displayed in Appendix. Figure 6 is a graphical presentation of this proposition. Now, we illustrate the intuitions based on the associated screening; consider the monopolist initially adopts integration with the optimal price, and now introduces the price of the base product slightly below the bundled package price while keeping the latter price constant. We argue how changes in the two exogenous parameters, τ and α, affect the marginal effects by the newcomers and the switchers in the screening. It is easy to see that the effects of τ for $\alpha = 0$ remain valid for a non-zero α, i.e., the gain from newcomers is increasing in the initial price of the bundled package

[18] Admittedly, under the assumption of the uniform distribution, not only the average but the variance is also higher. We come back to this point in Concluding Remarks.

while the loss from switchers is decreasing in it. The first part of Proposition 2 follows immediately because the optimal price of the bundled package is monotonically increasing with the high end value of add-on valuation.

To understand the second part of Proposition 2, recall that a positive contingency implies that, as the value of a base product gets higher, the associated value of a bundled package increases faster than that of a base product. This actually implies that both the newcomer gain and the switcher loss are indeed *smaller* in the case of positive contingency, since in the screening described above, more consumers continue to purchase a bundled package. If a figure such as Figures 2 and 3 is drawn, it can be verified that as α increases, the region of the newcomers that of the switchers are both smaller. However, the impact of higher contingency on these effects differs; the switcher loss decreases more than the newcomer gain. To understand this, note first that a change in α impacts more on those consumers who have a higher value of v_1 and that in the screening described above, the newcomers have relatively lower value of v_1 than the switchers. Thus, as α rises, the seller is more tempted for the screening from a given price of the bundled package, because the marginal gain from newcomers decreases *less* than the marginal loss from switchers do. To complete the argument, we need to verify that, as α rises, the optimal bundled package price actually increases. The last claim holds since the marginal consumers under integration have relatively lower value of v_1 than the inframarginal consumers, and therefore the effect of α impacts more on the latter agents.

Considering real world examples, Proposition 2 well captures the determinants of product boundary. This claim states that Apple sells iPad apps such as iPhoto and iMovie separately from the sales of iPad because the range of willingness to pay for them is sufficiently wide: they would have a high value for interested customers. However, as long as its range is sufficiently narrow, an add-on is incorporated in a product, and sold "free" (as pointed by Anderson (2009)). The same argument is also possible to the case of textbooks: whether supplementary materials are provided free or with charge is determined by the range of valuation for them. Furthermore, they are more likely to be separately sold as an option if students who gain greater utility from using a textbook also obtain greater utility from using its accompanying material as well. Thus, separate options with charge (iPhoto and iMovie, and MyEconLab) would be associated not only with a wider range of the add-on valuation but possibly with a higher contingency. We believe that this point provides an important empirical implication.

5 Social Welfare

Now, we investigate the welfare consequences of the boundary choice. In particular, we show that whenever the monopolist sells an add-on with a positive price, consumer surplus is lower than if a seller is restricted to sell only a bundled package. For $\alpha = 0$, we can derive an analytical result, as will be subsequently illustrated. Unfortunately, it is not possible to use the same argument for $\alpha \neq 0$, and thus we conduct a numerical analysis.

5.1 The Case of No Contingency ($\alpha = 0$)

[Figure 7 around here]

If separation is not possible, there are welfare gains and losses (see Figure 7). The gains come from (i) consumers who switch from no consumption to joint consumption, as the joint price is reduced, and (ii) those who switch from consuming a base product alone to joint purchase. The losses are from the consumers who switch from buying a base product alone to no good. The gains from (ii) are closely related to the efficiency distortion in the standard monopoly (or vertical differentiation) model, as these consumers are screened through the price of an add-on, p_2. However, there are some changes that affect welfare that are absent in the standard monopoly model, namely the changes in p_1 (reduced) and in b (increased). As integration is optimal for $\tau \leq 2/3$, this restriction on separation has bite only when $\tau > 2/3$. We now obtain the following proposition. The proof is relegated to Appendix.

Proposition 3. *When the monopolist chooses separation, integration is desirable from the social welfare viewpoint.*

Now, as τ increases, does the welfare loss become larger or smaller? The above proposition does not answer this question. Based on the explicit forms for social welfare (available upon request), we verify that the welfare loss becomes *larger* as τ increases, as depicted in Figure 8. As opposed to a naive belief, separation, rather than integration, is more apparent as a result from the exercise of market power.

[Figure 8 around here]

The policy implications from the result that integration is socially desirable for $\tau > 2/3$ are limited to our setup and raise cautions in application. To understand this, remember that our model is a monopolistic model with zero marginal cost, and hence separation is used only for inefficient, discriminatory pricing. If the marginal cost of add-on is positive, integration necessarily entails efficiency loss associated with the consumers with a low value of add-on. However, competition would affect the consequences of integration, which might weaken the applicability of the above result on social welfare (see also Subsection 6.3 below). Arguably, the intervention of antitrust authorities that typically prohibits bundling (integration) would reflect more of the concerns of exclusion or entry barrier in competition than price discrimination (for example, in 2007 the EU ordered Microsoft to detach Windows Media Player from its operating system for personal computers (Microsoft Windows)). Our analysis highlights the price discrimination aspects in the product boundary choice, and suggests that there should be more factors for consideration (in addition to entry and competition) in antitrust intervention.

5.2 Numerical Analysis in the Case of Nonzero Contingency ($\alpha \neq 0$)

In the case of $\alpha \neq 0$, we cannot directly apply the previous argument. Figure 9 is a representative case with $\alpha = 0.3$ (Table 3 below provides the corresponding numerical values). It is observed that social welfare under integration is higher than that under separation in this case (this is also true for $\alpha = -0.3,\ 0.1,\ -0.1$; see Tables 1, 2 and 4 below). We conjecture that Proposition 3 (possibly with some modifications) holds in the presence of contingent valuations.

[Figure 9 and Tables 1-4 around here]

6 Robustness

We now examine the robustness of the results obtained thus far in the case of $\alpha = 0$. A special feature of the basic model is that the lowest valuation for each good equals the marginal cost. It would be natural for the lowest valuation to be higher or lower than the marginal cost. Below, we consider the relevant cases, with the maintained assumption of the uniform distribution for valuations.[19] Explicit solutions would be no longer available for such modifications. We will, however, consider several cases and examine the robustness of the above results. It is verified that the main result (that separation outperforms integration if and only if the range of add-on valuation exceeds a certain threshold) holds, *except when the (constant) marginal cost of an add-on is higher than its lowest valuation.* In this case, separation is *always* optimal as the add-on is priced at least as high as the marginal cost.

6.1 Allowing the Positive Marginal Cost of the Base Good

First, suppose that the marginal cost of a base product is positive, while maintaining the other assumptions of the basic model with $\alpha = 0$. A positive marginal cost may create an extra incentive for the monopolist to adopt separation, especially for a large value of τ.

A positive marginal cost creates two effects on optimal pricing: first, it induces a higher integrated product price under integration. Recall that the optimal integrated product price is characterized by the marginal consumer gain being equal to the inframarginal consumer loss by lowering the price. The optimal integrated product price increases in the marginal cost of base product increases, since a positive marginal cost negatively affects the profit margin of the marginal consumers while it is neutral on the inframarginal consumer loss. Note that a higher integrated product price implies a higher incentive to screen consumers through separation.

Second, a positive marginal cost has a negative effect on the incentive of screening consumers through separation. The newcomer gain is negatively affected while the switcher

[19] We consider a flexible class of distribution, the bivariate beta distribution, that is sufficiently close to the uniform distribution, and verify that the main thrusts of the propositions hold (the details are available upon request).

loss is neutral. This means that for a given integrated product price, the incentive to screen consumers is weakened by a positive marginal cost of the base product.

However, the former effect dominates the latter and hence the overall effect of a positive marginal cost of the base product on screening is positive. Intuitively, the impact of a higher marginal cost is stronger on the optimal integrated price, since the ratio of the inframarginal consumer loss (unaffected) over the marginal consumer gain (affected) by decreasing the integrated product price is larger than the ratio of the switcher loss (unaffected) over the newcomer gain (affected). Hence, the optimal integrated product price increases faster than the threshold value of integrated product price at which separation (screening) dominates integration.

6.2 Allowing the Positive Marginal Cost of the Add-on

We throughout have assumed that the marginal cost and the lowest valuation of the add-on are equal to zero. As argued below, the lowest valuation should be no less than the marginal cost for integration to arise as an optimal boundary choice. Thus, it is necessary that the marginal cost of an add-on is very low and/or the valuation of those who value an add-on the least is sufficiently high,

To see this, consider that the marginal cost of producing an add-on, c_2, is positive while maintaining the other assumptions of the basic model with $\alpha = 0$. In this case, *integration is never optimal for any value of* τ.[20] To better understand this, consider integration with $p_1 > 0$ and $p_2 = 0$. Let $c_2 > 0$ be the (constant) marginal cost of the add-on. Then, it becomes clear that this price is dominated by $p_1' = p_1 - c_2$, $p_2' = c_2$: with this change, the monopolist avoids serving inefficient consumers with the add-on without incurring any loss, and attracts new consumers of the base product with $v_1 \in [p_1', p_1]$, thereby earning positive profits. Note that this argument does not depend on the assumption of the uniform distribution; therefore, the conclusion is robust for any continuous distribution. Indeed, the profit maximizing price is $p_2 > c_2$, if the distributions are uniform and the marginal cost falls in the interior of the support (this result is available

[20] The following argument is similar to Adams and Yellen's (1976) Footnote 12 in the two-regular-goods case. They (implicitly) assume that the lower bound of valuation for each good is zero, and c_1 and c_2 are positive.

upon request). This case may be applied to the real-world example of Apple's accessories for iPad, which are costly to produce and which some consumers might not appreciate enough to justify their cost. We conjecture that this argument holds in the presence of nonzero contingency.

In this sense, an add-on in our model is not necessarily to be interpreted as an information good or a product with close-to-zero marginal cost. Our results on product boundary choice also hold to the case where the marginal cost of an add-on is positive as long as it is no greater than the lowest valuation. As for welfare implications, one should note that integration necessarily entails efficiency loss associated with the consumers with a low value of an add-on. Thus, our results on welfare would be affected by introduction of a positive marginal cost of an add-on.

6.3 Competing Add-ons

Introducing competition into our model illuminates new strategic effects. Consider the case where one firm (firm 1) produces both a base product and its add-on, whereas the other firm (firm 2) produces a compatible add-on only. Assume firm 1's and 2's add-ons are horizontally differentiated and substitutable. Then, two conflicting effects of firm 1's product separation arise. First, firm 1's separation strategy invites competition in the add-on market (business stealing effect in add-ons). Second, however, firm 1 may welcome firm 2's entry into the add-on market due to the creation of new comers in the base product market. Under separation, some consumers who favor firm 2's add-on more than firm 1's now may find it desirable to purchase firm 1's base product: they are those who would not purchase the base product at all if firm 2 were excluded by firm 1's product integration. Thus, we conjecture that if the degree of horizontal differentiation is sufficiently high, firm 1 allows firm 2's entry by product separation.

7 Concluding Remarks

This paper generalizes Chen and Nalebuff (2007) to allow nonzero contingency, and develops a model of a monopolist's problem of product boundary choice with the assumption of a uniform distribution of consumers' valuations. We then study the monopolist's problem of choosing integration or separation in the context of base and add-on goods by taking into account nonzero contingency. We find that given the level of contingency, separation is more likely to be optimal as the relative range of add-on valuation to base-good valuation becomes larger. It is also shown that given the distribution of innate values, separation is more likely to be optimal as the degree of contingency increases. As for welfare implications, our analysis shows that integration is desirable from the social welfare perspective, provided that the monopolist retains the freedom in price choice, and thus under separation consumer welfare is necessarily lower than under integration.

Because of its specific assumptions, our model is certainly special. In particular, it raises the question that if there are many add-ons, as in reality, can we distinguish between bundled features and add-ons for buying by merely looking at the main goods and each feature individually? A more tractable way to ask the question is to determine, first, whether it is more profitable to bundle all add-ons as a package or to sell each of them separately, and second, whether add-ons bundled with a base good are more profitable.[21]

Obviously, there remain other interesting issues for further research. As briefly mentioned in Section 6 and as Prasad, Venkatesh, and Mahajan (2010) admit in their final remarks, uniform distributions are certainly restrictive. In particular, one would wonder how much the mean effects of increasing τ and the variance effects matter to the results. A satisfactory analysis of this awaits future research.

Appendix

Proof of Proposition 2

We prove Proposition 2 by a series of lemmata. The proof requires more than deriving the first-order conditions and applying the implicit function theorem because the optimization problem is not concave, Instead, we narrow down the candidate of solutions by the associated first-order *necessary* conditions for interior solutions (separation) and by the analogous conditions for corner solutions (integration), and then find out the global opti-

[21] For this direction of research, the framework of Armstrong (1999), Bakos and Brynjolfsson (1999), Fang and Norman (2006), Chu, Leslie and Sorensen (2011), Crawford and Yurukoglu (2012), and Chen and Riordan (2013), who analyze specific multi-good cases, would be helpful.

mum among them (Lemmata A3 and A4). Then we derive how these candidates change with respect to the parameters (Lemma A5).

First, analogous to the case of zero contingency, we look at the screening problem with $p_1 + p_2$ fixed. Consider the case of separation (i.e., $p_2 > 0$). Then, we can compute the three associated marginal effects with a small reduction in p_1 by ε (as in Figure 4 in the main text). The gain from newcomers becomes

$$\left(p_1 \times \varepsilon \frac{p_2}{1+\alpha p_1}\right)/\tau = \varepsilon \frac{p_1 p_2}{1+\alpha p_1}\frac{1}{\tau},$$

while the reduced price loss is

$$\left(\varepsilon \times \int_{p_1}^{1} \frac{p_2}{1+\alpha v_1} dv_1\right)/\tau$$

and the switching loss is

$$\left(p_2 \times \int_{p_1}^{1} \varepsilon \frac{1}{1+\alpha v_1} dv_1\right)/\tau,$$

implying the aggregate loss becomes

$$2\varepsilon \frac{[\ln(1+\alpha) - \ln(1+\alpha p_1)]p_2}{\alpha}\frac{1}{\tau}.$$

Now, let (p_1^*, p_2^*) be the pair of the optimal prices under separation. Then, the marginal gain must coincide with the marginal loss:

$$\begin{aligned} \varepsilon \frac{p_1^* p_2^*}{1+\alpha p_1^*}\frac{1}{\tau} &= 2\varepsilon \frac{[\ln(1+\alpha) - \ln(1+\alpha p_1^*)]p_2^*}{\alpha}\frac{1}{\tau} \\ \Leftrightarrow \quad \frac{p_1^*}{1+\alpha p_1^*} &= 2\frac{[\ln(1+\alpha) - \ln(1+\alpha p_1^*)]}{\alpha}, \end{aligned}$$

which shows that p_1^* is independent of τ and p_2^*.[22] We summarize the argument so far in the following lemma.

Lemma A1. *If separation is optimal, then the optimal price of the base product p_1^* is independent of $\tau \in (0, 1]$ and the optimal price of the add-on, p_2^*.*

[22] Note that if $\alpha \to 0$, then we have the same equality in the case of $\alpha = 0$ ($p_1 p_2 = 2(1 - p_1)p_2$) because

$$\lim_{\alpha \to 0} \frac{\ln(1+\alpha) - \ln(1+\alpha p_1^*)}{\alpha} = \lim_{\alpha \to 0} \frac{\dfrac{1}{1+\alpha} - \dfrac{p_1^*}{1+\alpha p_1^*}}{1} = 1 - p_1^*$$

by l'Hôpital's rule.

Now, let $F(p_1, \alpha)$ be defined by:

$$F(p_1, \alpha) \equiv \frac{p_1}{1 + \alpha p_1} - 2\frac{[\ln(1 + \alpha) - \ln(1 + \alpha p_1)]}{\alpha}.$$

Then, the optimal price of the base product under separation is denoted as a function of the degree of contingency solely, $p_1^* = p_1^*(\alpha)$, which is an interior solution of $F(p_1, \alpha) = 0$. The next lemma characterizes p_1^* with respect to the degree of contingency, α. Namely, a lower contingency makes the optimal price of the base good *higher* under separation. The proof is available upon request.

Lemma A2. *The optimal price of the base product under separation, $p_1^*(\alpha)$, is decreasing in α.*

As α increases, the measure of newcomer for a base product decreases since consumers with higher v_2 prefer both goods due to higher contingency. For the same reason, the measure of consumers associated with reduced price loss from purchasing a base product only also decreases in α. This implies that both the newcomer gain and the other losses decrease, noting that the two losses have exactly the same form in our setup. However, the impact of higher contingency on the losses is greater than on the newcomer gain since the newcomers have lower values of a base product. Therefore, the aggregate losses are greater than the gain at the optimal price of the base product, implying that there is a greater incentive for the seller to lower $p_1^*(\alpha)$ as α increases,

We now characterize the optimal boundary strategy. Let $b^*(\tau, \alpha)$ denote the optimal price *under integration* (it is not necessarily optimal if the choice of separation is optimal). Then, we obtain the following lemma (the proof is available upon request).

Lemma A3. *If the optimal price of an integrated product $b^*(\tau, \alpha)$ is greater than $p_1^*(\alpha)$, then separation is optimal.*

Intuitively, if the price of the bundled package exceeds the optimal price of the base product in separation, there is always a second-order gain from separation although the first-order gain is zero, as the associated screening argument shows.

Lemmata A1 and A3 imply that the optimal price pairs for separation and for integration (with $p_2 = 0$) lie on the bold lines in Figure 10.

[Figure 10 around here]

To better understand this, note first that $b^* > p_1^*$ cannot constitute an optimal strategy (Lemma A3). Second, the price pair with $p_2^* \geq \tau(1+\alpha)$ cannot be optimal because the monopolist only sells the base product. Lastly, the price pair (p_1, p_2) with $p_1 \neq p_1^*(\alpha)$ and $p_2 \in (0, \tau)$ cannot be an optimal strategy under separation (Lemma A1). Thus, the optimal pricing strategy is either "$p_2 = 0$ and $p_1 \leq p_1^*(\alpha)$" (in this case, b is used for p_1), or "$p_2 > 0$ and $p_1 = p_1^*(\alpha)$," as shown by the bold lines in Figure 10.

Combined with the next lemma (the proof is available upon request), we can determine the optimal boundary based on the comparison of $b^*(\tau, \alpha)$ and $p_1^*(\alpha)$.

Lemma A4. *If the optimal price of a bundled package $b^*(\tau, \alpha)$ is no greater than $p_1^*(\alpha)$, then integration is optimal.*

Under the assumption of this lemma, we can show that there is no local interior optimum along the line $(p_1^*(\alpha), p_2)$. Since $(p_1^*(\alpha), 0)$ is identified as an integration, we can conclude that integration with $b^*(\tau, \alpha)$ is optimal.

Finally, the positive monotonicity of the optimal price of a bundled package $b^*(\tau, \alpha)$ in both arguments is derived (the proof is available upon request).

Lemma A5. *The optimal price of a bundled package $b^*(\tau, \alpha)$ is increasing in τ and in α.*

Using all the lemmata, we can find the associated thresholds in Proposition 2. First, for a given $\alpha > -1$, $\hat{\tau}(\alpha)$ is defined by $b^*(\hat{\tau}(\alpha), \alpha) = p_1^*(\alpha)$ if such $\hat{\tau}(\alpha)$ exists, $\hat{\tau}(\alpha) = 1$ otherwise.[23] Conversely, for a given $\tau \in (0, 1]$, $\widetilde{\alpha}(\tau)$ is defined by $b^*(\tau, \widetilde{\alpha}(\tau)) = p_1^*(\widetilde{\alpha}(\tau))$

[23] We can show that for any $\alpha > -1$, there is $\tau \in (0, 1]$ such that $b^*(\tau, \alpha) < p_1^*(\alpha)$. It is easy to see that $b^*(0, \alpha) = 1/2$ since this is the same as a single good monopolist with the uniform distribution of valuation on $[0, 1]$. However, $p_1^*(\alpha) > 1/2$ for any $\alpha > -1$, because it is decreasing in α and

$$F(p_1^*(\alpha), \alpha) = 0 \Leftrightarrow \frac{p_1^*(\alpha)}{1 + \alpha p_1^*(\alpha)} = \frac{2}{\alpha} \ln\left(\frac{1+\alpha}{1 + \alpha p_1^*(\alpha)}\right)$$
$$\Leftrightarrow \frac{p_1^*(\alpha)}{2} = \left(\frac{1}{\alpha} + p_1^*(\alpha)\right) \ln\left(\frac{1/\alpha + 1}{1/\alpha + p_1^*(\alpha)}\right),$$

and by taking $\alpha \to \infty$, we have $p_1^*(\infty)/2 = p_1^*(\infty) \ln(1/p_1^*(\infty))$, so that $p_1^*(\infty) = \exp(-0.5) \cong 0.607 > 1/2$.

if such $\widetilde{\alpha}(\tau)$ exists, $\widetilde{\alpha}(\tau) = -1$ if $b^*(\tau, \alpha) > p_1^*(\alpha)$ for all $\alpha > -1$, and $\widetilde{\alpha}(\tau) = \infty$ if $b^*(\tau, \alpha) < p_1^*(\alpha)$ for all $\alpha > -1$.

To prove the first part, it suffices to show that b and τ are complementary in profit under integration, which is denoted by $\pi^I(b; \alpha, \tau)$. For a small change in the integration price b, areas B and C in Figure 11 correspond to the inframarginal consumers and the marginal consumers, respectively. Now, suppose that τ increases. First, this affects the density of consumers, but under uniform distribution, this proportional change cancels out in the first-order condition. More importantly, area B expands while area C remains the same. Therefore, in the relative sense, the gain from inframarginal consumers increases in τ, while the loss from the marginal consumers remains the same (for the case of $b > \tau$, we need to modify the argument slightly since now area C expands as τ increases. However, the main thrust remains valid). This shows that $\partial^2\pi^I/\partial\tau\partial b > 0$ and therefore $b^*(\tau, \alpha)$ is increasing in τ. Although we have assumed $\tau \leq 1$, our argument does not crucially depend on this assumption.[24] If we do not assume $\tau \leq 1$, we expect the threshold $\hat{\tau}(\alpha)$ to exist for any $\alpha > -1$.

[Figure 11 around here]

The argument for the latter half is more involved. It suffices to show $\partial^2\pi^I/\partial\alpha\partial b > 0$ to verify that $b^*(\tau, \alpha)$ is increasing in α. We examine $\partial^2\pi^I/\partial\alpha\partial b$ by looking at the changes in areas B and C in Figure 11 when α increases. As α increases, the curve that defines area B expands toward the origin with the intercepts of the vertical and horizontal axes unchanged given b. This immediately shows that the inframarginal gain increases in α. It is not immediate that area C actually shrinks in α so that the loss from the marginal consumers decreases in α. To see this, note that the loss from the marginal consumer (when $b \leq \tau$) is given by

$$b \cdot \frac{\partial}{\partial b} \int_0^b \frac{-v_1 + b}{1 + \alpha v_1} dv_1 = \frac{b}{\alpha} \ln(1 + \alpha b)$$

[24] The only concern is that the term $\ln(1 + \alpha\tau)$, which appears several times in this appendix, might not be well-defined because $1 + \alpha\tau$ can be negative if $\tau > 1$. However, the term $\ln(1 + \alpha\tau)$ appears from the restriction $\tau \leq 1$, and thus the results hold even if we allow $\tau > 1$.

and thus

$$\frac{\partial}{\partial \alpha}\left(\frac{b}{\alpha}\ln(1+\alpha b)\right) = \frac{-b}{\alpha^2(1+\alpha b)}\left((1+\alpha b)\ln(1+\alpha b)-\alpha b\right)$$
$$= \frac{-b}{\alpha^2(1+\alpha b)}\int_1^{1+\alpha b}\ln x dx \leq 0$$

because $\int_1^{1+\alpha b}\ln x dx \geq 0$ for all $\alpha > -1$ (and is equal to zero if and only if $\alpha = 0$).

Proof of Proposition 3

Let b^*, p_1^* and p_2^* denote the optimal prices. Using the optimal prices derived in Propositions AA1 and AA2 in the online appendix (available upon request), we know that for $\tau > 2/3$,

$$2/3 = p_1^*(\tau) < b^*(\tau) = \sqrt{2\tau/3} < p_1^*(\tau) + p_2^*(\tau) = \tau/2 + 1/3,$$

i.e., the price for good 1 in separation, $p_1^*(\tau)$, is smaller than the optimal price of a bundled package $b^*(\tau)$, but the the joint price in separation, $p_1^*(\tau) + p_2^*(\tau)$, is higher than $b^*(\tau)$.

Now consider the welfare gains from the consumers who switch from buying no good to joint purchase (part (i) above), whose level is denoted by A, and the welfare losses, denoted by B.

We maintain that A is greater than B for any τ, so that the total welfare gains (A plus the standard gains) must be larger through the prohibition of separation. Although calculating the exact welfare gains for A involves the integration of the consumers' willingness to pay, we only require a lower bound to support the claim.

The consumers associated with this gain are in the parallelogram, surrounded by the lines drawn by $v_1 = 0$, $v_1 = 2/3$, $v_1 + v_2 = b^*(\tau)$ and $v_1 + v_2 = p_1^*(\tau) + p_2^*(\tau)$. The area of this parallelogram is

$$\frac{2}{3}\left(\left\{\frac{2}{3}+\frac{\tau}{2}-\frac{1}{3}\right\}-\sqrt{\frac{2\tau}{3}}\right) = \frac{2}{3}\left(\frac{1}{3}+\frac{\tau}{2}-\sqrt{\frac{2\tau}{3}}\right).$$

Recall that each consumer in this parallelogram lies in the northeast of $b^*(\tau)$ line and thus its willingness to pay for joint consumption is at least as much as $b^*(\tau)$. The welfare gains A are therefore greater than $b^*(\tau)$ times the area. Now we compute an upper bound of B, the welfare losses. The consumers associated with this loss are in the triangle surrounded by the horizontal axis, the lines drawn by $v_1 = 2/3$, and $v_1 + v_2 = b^*(\tau)$. The area of this triangle is

$$\frac{1}{2}\left(\sqrt{\frac{2\tau}{3}} - \frac{2}{3}\right)^2 = \frac{1}{2}\left(\frac{2\tau}{3} + \frac{4}{9} - \frac{4}{3}\sqrt{\frac{2\tau}{3}}\right),$$

which is exactly the *same* as the parallelogram above. Each consumer in this region will switch to buying nothing when separation is prohibited. Here, each consumer's willingness to pay for good 1 alone is at most $b^*(\tau)$ such that the welfare losses B are smaller than $b^*(\tau)$ times this area. We now compute the upper bound of the losses and the lower bound of the gains,

$$A \geq \left(\frac{2}{9} + \frac{\tau}{3} - \frac{2}{3}\sqrt{\frac{2\tau}{3}}\right)\sqrt{\frac{2\tau}{3}} \geq B.$$

Hence, the welfare gains from prohibiting separation, which must be strictly larger than A, are greater than the total losses B.

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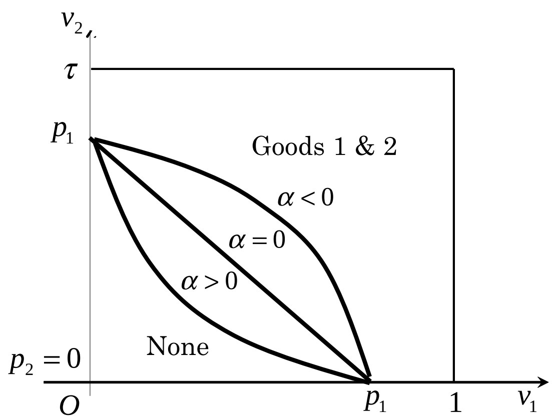

Figure 1. Demand Structure under Integration

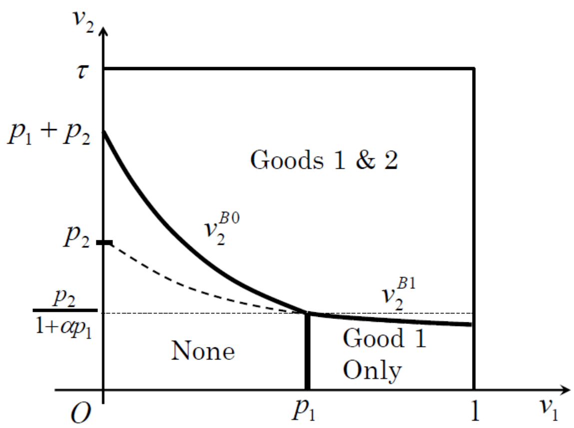

Figure 2. Demand Structure under Separation: The Case of Positive Contingency ($\alpha > 0$)

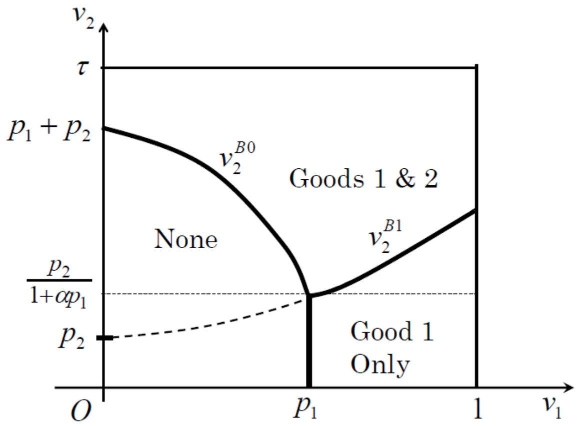

Figure 3. Demand Structure under Separation: The Case of Negative Contingency ($\alpha < 0$)

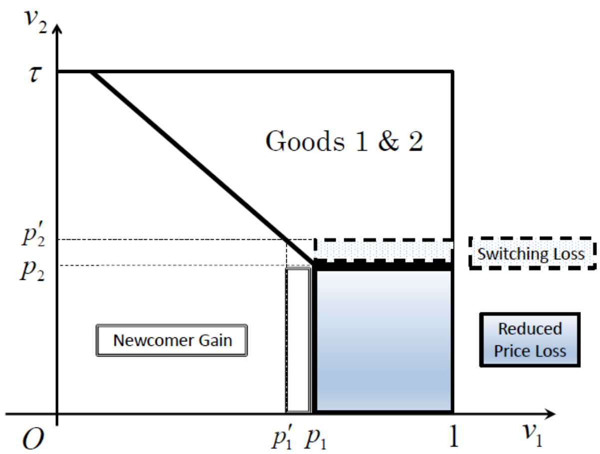

Figure 4. The Effects of Screening ($\alpha = 0$)

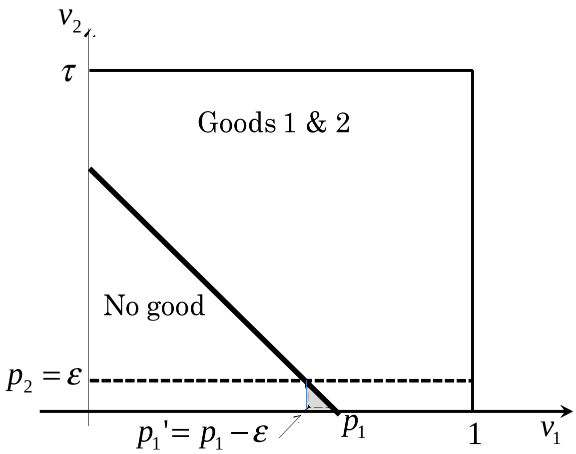

Figure 5. Suboptimality of Integration
($\alpha = 0$)

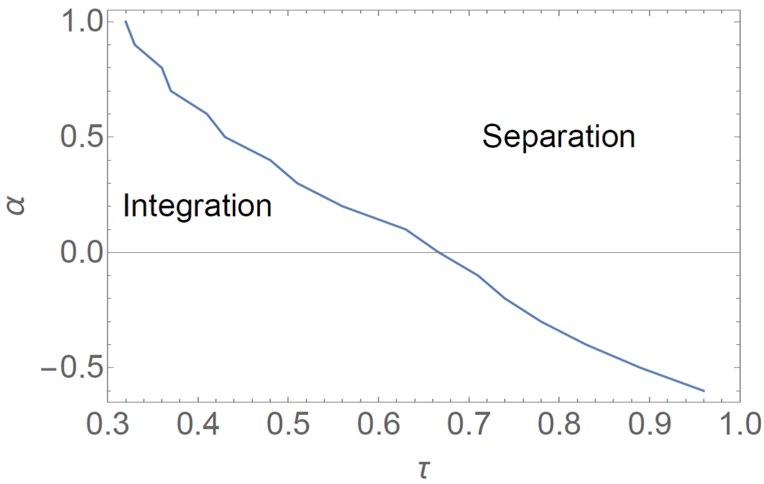

Figure 6. Integration and Separation in the Parameter Space

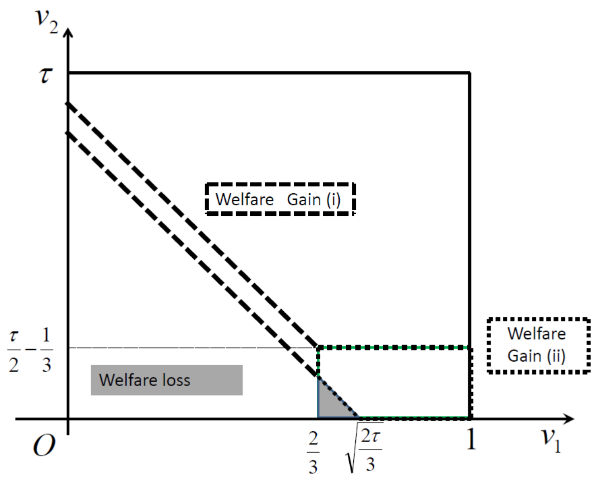

Figure 7. Effects of Prohibiting Separation

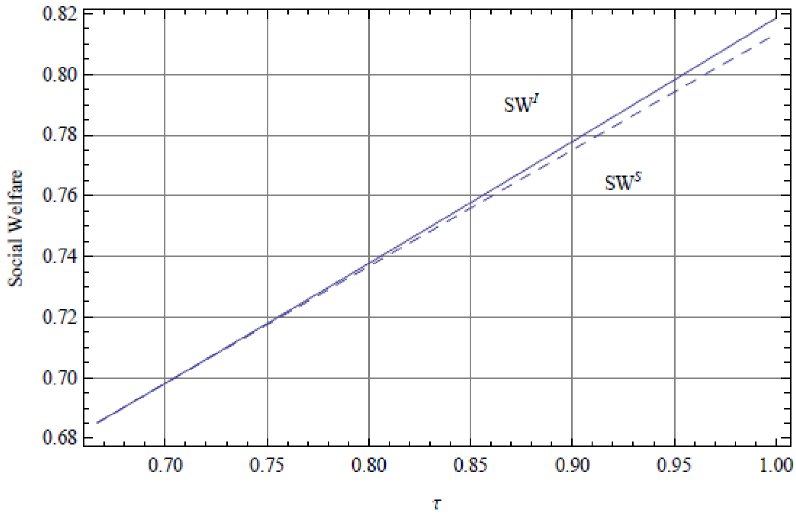

Figure 8. Social Welfare Loss by Separation (SW^I: Integration; SW^S: Separation)

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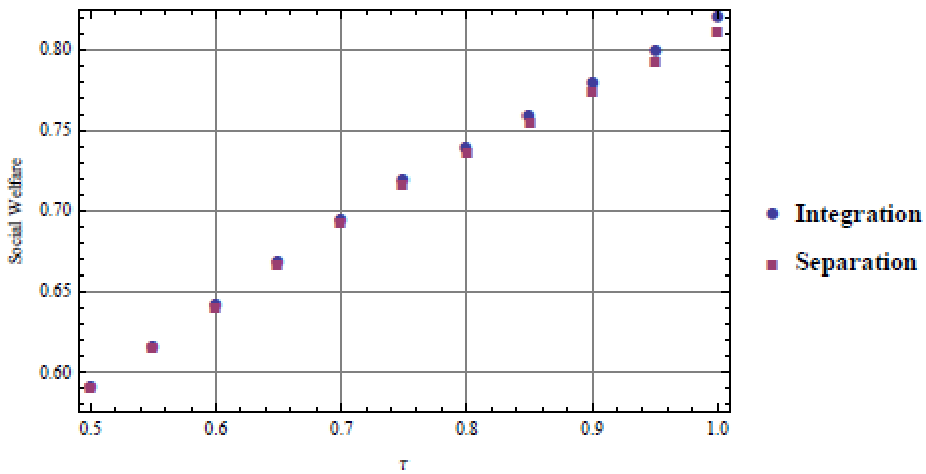

Figure 9. Welfare Comparison ($\alpha = 0.3$)

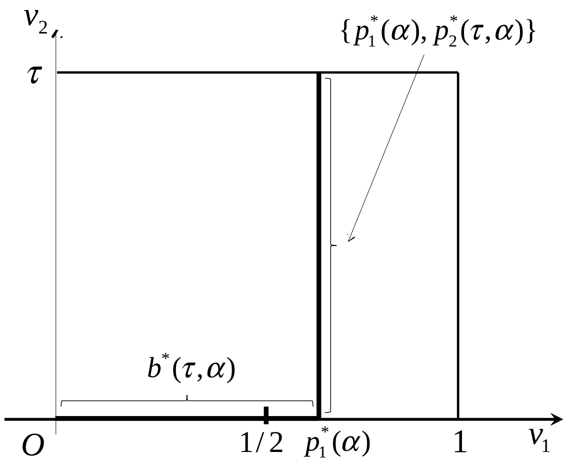

Figure 10. Possible Pairs for Optimal Prices under Separation

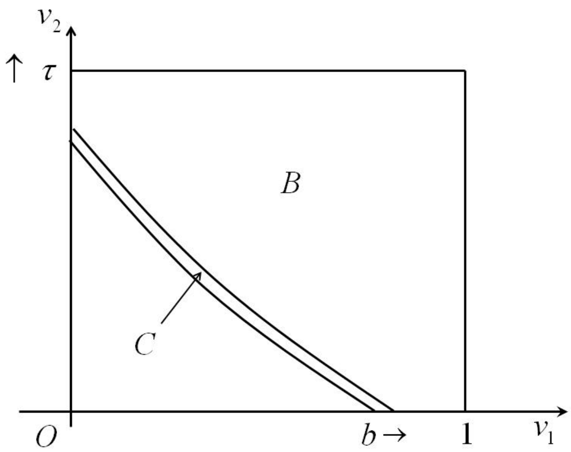

Figure 11. Marginal Effects of Integration Price

τ	Welfare	
	Integration	Separation
0.6	0.6587239	0.6587238
0.65	0.6864614	0.6863702
0.7	0.7126905	0.7124764
0.75	0.7333987	0.7326998
0.8	0.7542818	0.7528568
0.85	0.7753236	0.7729592
0.9	0.7965104	0.7930161
0.95	0.8178302	0.8130346
1	0.8392726	0.8330204

Table 1: Complementarity ($\alpha = 0.1$)

τ	Welfare	
	Integration	Separation
0.7	0.68374068	0.68373951
0.75	0.70238105	0.70221359
0.8	0.72119633	0.72060564
0.85	0.74017041	0.73893015
0.9	0.75928942	0.75719838
0.95	0.77854144	0.77541923
1	0.79791612	0.79359976

Table 2: Substitutability ($\alpha = -0.1$)

τ	Welfare Integration	Separation
0.5	0.590960336	0.590959389
0.55	0.615930270	0.615789169
0.6	0.641493938	0.640991632
0.65	0.667646519	0.666582887
0.7	0.694383140	0.692574555
0.75	0.718483483	0.717029168
0.8	0.738369256	0.735913390
0.85	0.758413503	0.754747837
0.9	0.778602489	0.773540881
0.95	0.798924264	0.792299031
1	0.819368465	0.811027511

Table 3: Complementarity ($\alpha = 0.3$)

τ	Welfare Integration	Separation
0.8	0.68834084	0.68828550
0.85	0.70526943	0.70492892
0.9	0.72234370	0.72150010
0.95	0.73955170	0.73801042
1	0.75688305	0.75446902

Table 4: Substitutability ($\alpha = -0.3$)