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H-Infinity Control Design Considering Packet Loss as a Disturbance for Networked Control Systems

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SUMMARY This paper studies H_∞ control for networked control systems with packet loss. In networked control systems, packet loss is one of major weakness because the control performance deteriorates due to packet loss. H_∞ control, which is one of robust control, can design a controller to reduce the influence of disturbances acting on the controlled object. This paper proposes an H_∞ control design that considers packet loss as a disturbance. Numerical examples show that the proposed H_∞ control design can more effectively reduce control performance deterioration due to packet loss than the conventional H_∞ control design. In addition, this paper provides control performance comparisons of H_∞ control and Linear Quadratic (LQ) control. Numerical examples show that the control performance of the proposed H_∞ control design is better than that of the LQ control design.

key words: *networked control, packet loss, control design, robust control, H-infinity control, LQ control*

1. Introduction

In networked control systems, controllers and controlled objects (plants) exchange control and state information through a network, such as Wi-Fi, Zigbee, Ethernet and Internet. There are several advantages including low installation and maintenance cost, increase of system flexibility and decrease of wiring (especially in the case of wireless networks). Because of the above advantages, networked control systems are expected to be applied in many industry fields [1], [2]. However, the network may cause several problems including data packet loss, transmission delay and measurement quantization, which deteriorate control system performances [3]–[5]. Therefore, it is important for industrial application to overcome these problems and to realize a reliable control system. This paper focuses on the influence of data packet loss in networked control systems.

The existing researches can be classified into two approaches. One is "communication layer approach considering control layer" and the other is "control layer approach considering communication layer". The former communication approach focuses on reduction of the unreliability of communication networks in control systems. In this approach, a relay scheme [6], a diversity scheme [7], an adaptive modulation scheme [8], and error correction schemes [9], [10] have been proposed to reduce packet loss in networked control systems. On the other hand, the latter control approach focuses on controlling plants across an unreliable network. In this approach, linear quadratic (LQ) optimal control schemes [11], [12], predictive control schemes [13]–[16], and H_∞ control schemes [17]–[35] have been proposed. The approach of this paper belongs to the latter approach.

This paper considers H_∞ control for networked control systems. H_∞ control can design a controller to reduce the influence of disturbances acting on the controlled object. This is a notable characteristic of H_∞ control. In existing works, packet loss is regarded as long time delay in control design [17], [18]. In [19]–[22], packet loss is treated as a Markov process to use Markovian system for the control design. In [23]–[32], packet loss is considered as system stochastic parameter uncertainties. In [33]–[35], packet loss is included in control design by using switch system. These existing works do not consider packet loss as a disturbance, in other word, for packet loss, these existing works cannot sufficiently utilize the characteristic that reduce the influence of disturbances. The contribution of this paper is to make clear the effectiveness of an H_∞ control design that considers packet loss as a disturbance. The proposed control design can effectively reduce the influence of packet loss and improve the control performance. In addition, the above existing control approaches provided only the new control designs but no comparison with other control designs. Therefore, it is an interesting issue which control design leads to high control performances. This paper shows which of the proposed H_∞ control design and the LQ control design [36] can get higher control performance for reliable networked control. Our results demonstrate that the proposed H_∞ control design can get higher performance than the LQ control design.

The reminder of the paper is organized as follows. The networked control system model is described in Sect. 2. The conventional H_∞ control design based on mixed-sensitivity loop-shaping problem and the LQ control design are explained in Sect. 3. The proposed H_∞ control design is explained in detail in Sect. 4. Numerical examples of performance comparisons of the proposed H_∞ control design, the conventional H_∞ control design and the LQ control design are provided in Sect. 5. The conclusion of this study is presented in Sect. 6.

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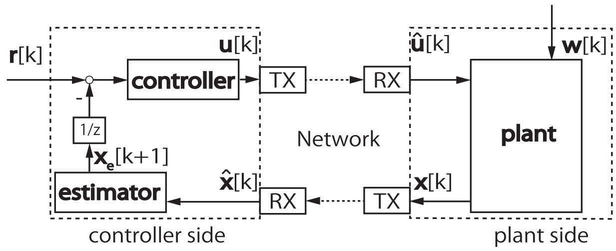

Fig. 1 Networked control system.

2. Networked Control System

The considered networked control system is shown in Fig. 1. The system is discretized at the sampling interval T_s. At time $t = kT_s (k = 0, 1, 2, \cdots)$, the state information and the control information are expressed as $\mathbf{x}[k]$ and $\mathbf{u}[k]$, respectively. The plant is assumed to be linear time invariant system, so that at the next time index $k + 1$, the resulting state information of the plant is given as follows:

$$\mathbf{x}[k+1] \quad = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\hat{\mathbf{u}}[k] + \mathbf{w}[k], \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are coefficient matrices and represent the state space model of the controlled plant. $\hat{\mathbf{u}}[k]$ is received control information at the plant side. $\mathbf{w}[k]$ is a random vector that represents system disturbances added to the controlled plant's state. The mean vector and covariance matric of $\mathbf{w}[k]$ are assumed to be $\mathbf{0}$ and $\mathbf{W}$, respectively.

The controller generates control information $\mathbf{u}[k]$ according to a target value vector $\mathbf{r}[k]$, which is inputted every T_s seconds, and the estimated state information $\mathbf{x}_e[k]$. The estimated state information is given as follows:

$$\mathbf{x}_e[k+1] \quad = \mathbf{A}\hat{\mathbf{x}}[k] + \mathbf{B}\mathbf{u}[k], \tag{2}$$

where $\hat{\mathbf{x}}[k]$ is received state information at the controller side.

Each of the state information and the control information is transmitted via the network as a packet. Packet losses are assumed to occur randomly with probability p in the network and to be surely detectable. The number of transmitted packets of state information (or control information) per second is defined as the packet transmission rate $1/T_s$.

If the plant detects packet loss of control information $\mathbf{u}[k]$, the plant side proceeds with zero input as follows:

$$\hat{\mathbf{u}}[k] = \begin{cases} \mathbf{u}[k] & \text{if received successfully,} \\ \mathbf{0} & \text{otherwise.} \end{cases} \tag{3}$$

If the controller detects packet loss of state information $\mathbf{x}[k]$, the controller side defines received state information $\hat{\mathbf{x}}[k]$ as follows:

$$\hat{\mathbf{x}}[k] = \begin{cases} \mathbf{x}[k] & \text{if received successfully,} \\ \mathbf{x}_e[k] & \text{otherwise.} \end{cases} \tag{4}$$

3. Conventional Control Design

This section explains how to design the controller and calculate the control information in the case of the conventional H_∞ control and LQ control.

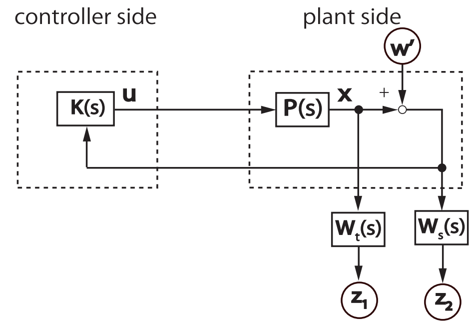

Fig. 2 Design block diagram of the conventional H_∞ control.

3.1 H_∞ Control (Mixed-Sensitivity Loop-Shaping Problem)

The conventional H_∞ control design is based on mixed-sensitivity loop-shaping problem [37]. Figure 2 shows the design block diagram of the conventional H_∞ control. In control design, $\mathbf{z}_1$ and $\mathbf{z}_2$ are named controlled outputs, and $\mathbf{w}'$ is exogenous input for evaluating the influence of system disturbances. The conventional H_∞ control design uses the H_∞ norm of the closed loop transfer function $\mathbf{G}_{\mathbf{Zw}'}(s)$ that is given by

$$\mathbf{G}_{\mathbf{Zw}'}(s) = \begin{bmatrix} \mathbf{W}_t(s)\mathbf{T}(s) \\ \mathbf{W}_s(s)\mathbf{S}(s) \end{bmatrix}, \tag{5}$$

where $\mathbf{Z} = [\mathbf{z}_1\ \mathbf{z}_2]^T$. $\mathbf{W}_t(s)$, $\mathbf{W}_s(s)$ are tuning parameters. $\mathbf{T}(s)$, $\mathbf{S}(s)$ are defined as follows:

$$\mathbf{T}(s) = \mathbf{P}(s)\mathbf{K}(s)[\mathbf{I} - \mathbf{P}(s)\mathbf{K}(s)]^{-1}, \tag{6}$$

$$\mathbf{S}(s) = [\mathbf{I} - \mathbf{P}(s)\mathbf{K}(s)]^{-1}. \tag{7}$$

$\mathbf{P}(s)$ is the plant transfer function that corresponds to $\mathbf{A}, \mathbf{B}$ of Eq. (1). To evaluate the H_∞ norm of the closed loop transfer function from $\mathbf{w}'$ to $\mathbf{z}_1$ is related with the tracking performance. On the other hand, to evaluate that from $\mathbf{w}'$ to $\mathbf{z}_2$ is related with the disturbance reduction. The controller transfer function $\mathbf{K}(s)$ is designed to minimize the H_∞ norm of $\mathbf{G}_{\mathbf{Zw}'}(s)$.

The control information $\mathbf{u}[k]$ is given as follows:

$$\begin{aligned} \mathbf{x}_c[k+1] &= \mathbf{A}_c\mathbf{x}_c[k] + \mathbf{B}_c(\mathbf{r}[k] - \mathbf{x}_e[k]), \\ \mathbf{u}[k] &= \mathbf{C}_c\mathbf{x}_c[k] + \mathbf{D}_c(\mathbf{r}[k] - \mathbf{x}_e[k]), \end{aligned} \tag{8}$$

where $\mathbf{x}_c[k]$ is a controller state. $\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c$ are coefficient matrices of the controller. These coefficient matrices correspond to discretized $\mathbf{K}(s)$ at the sampling interval T_s.

3.2 LQ Control

LQ control is one of optimal control and can design a controller to minimize the cost for tracking performance and the amount of the control input. This paper employs a linear quadratic Gaussian (LQG) control with packet loss under UDP-like protocols [36]. In this case, the designed controller gain $\mathbf{K}_{\mathrm{lq}}$ minimizes the cost function defined in Eq. (9).

$$J = \lim_{N\to\infty} \mathrm{E}\Big[\mathbf{x}[N]^{\mathrm{T}}\mathbf{Q}\mathbf{x}[N] + \sum_{k=0}^{N-1}(\mathbf{x}[k]^{\mathrm{T}}\mathbf{Q}\mathbf{x}[k] + P_k\, \mathbf{u}[k]^{\mathrm{T}}\mathbf{R}\mathbf{u}[k])\Big], \tag{9}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are arbitrary weight matrices related with the cost for tracking performance and that for the amount of the control input, respectively. P_k is Bernoulli random variables with $\mathrm{Prob}(P_k = 0) = p$. The LQ control considers packet loss as motion errors in the system. In this control design, the control information $\mathbf{u}[k]$ is given as follows:

$$\mathbf{u}[k] = \mathbf{K}_{\mathrm{lq}}(\mathbf{r}[k] - \mathbf{x}_{\mathrm{e}}[k]). \tag{10}$$

4. Proposed H$_\infty$ Control

Figure 3 shows the design block diagram of the proposed H$_\infty$ control. The structure of Fig. 3 is not special in H$_\infty$ control theory; however, the application of Fig. 3 to the networked control system, where packet loss occurs randomly, is new approach. The proposed design considers packet loss as a disturbance. Compared with the conventional H$_\infty$ design, the proposed design utilizes $\mathbf{w_u}$, $\mathbf{w_x}$ and evaluates $\mathbf{u}$ weighted by $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$ instead of $\mathbf{x}$ weighted by $\mathbf{W}_{\mathrm{t}}(s)$.

In the control design, $\mathbf{w_u}$ and $\mathbf{w_x}$ are exogenous inputs for evaluating the influence of packet loss on control information and state information as a disturbance. By evaluating the H$_\infty$ norm of the closed loop transfer function from $\mathbf{w_u}$ and $\mathbf{w_x}$ to $\mathbf{z}_2$, the designed controller can compensate the tracking performance even if disturbance act on $\mathbf{u}$ and $\mathbf{x}$. In addition, $\mathbf{z}_1$ is evaluated with weighting $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$, which is a function of packet loss rate p. This means that the frequency characteristic of the designed controller output $\mathbf{u}$ is shaped by $[\mathbf{W}_{\mathrm{e}}^{(p)}(s)]^{-1}$. If $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$ is set to be a high gain at the frequency where the influence of packet loss is large, the influence of packet loss can be reduced as a disturbance.

In the proposed design, the H$_\infty$ norm of the closed loop transfer function is given as follows:

$$\left\| \begin{matrix} \mathbf{G}_{\mathbf{Zw'}}(s) \\ \mathbf{G}_{\mathbf{Zw_u}}(s) \\ \mathbf{G}_{\mathbf{Zw_x}}(s) \end{matrix} \right\|_\infty, \tag{11}$$

and the controller transfer function $\mathbf{K}_{\mathrm{P}}(s)$ is designed to minimize this norm. $\mathbf{G}_{\mathbf{Zw'}}(s)$, $\mathbf{G}_{\mathbf{Zw_u}}(s)$ and $\mathbf{G}_{\mathbf{Zw_x}}(s)$ are defined as follows:

$$\mathbf{G}_{\mathbf{Zw'}}(s) = \begin{bmatrix} \mathbf{W}_{\mathrm{e}}^{(p)}(s)\mathbf{K}_{\mathrm{P}}(s)\mathbf{S}(s) \\ \mathbf{W}_{\mathrm{s}}(s)\mathbf{S}(s) \end{bmatrix}, \tag{12}$$

$$\mathbf{G}_{\mathbf{Zw_u}}(s) = \begin{bmatrix} \mathbf{W}_{\mathrm{e}}^{(p)}(s)\mathbf{P}(s)\mathbf{S}(s) \\ \mathbf{W}_{\mathrm{s}}(s)\mathbf{T}(s) \end{bmatrix}, \tag{13}$$

$$\mathbf{G}_{\mathbf{Zw_x}}(s) = \begin{bmatrix} \mathbf{W}_{\mathrm{e}}^{(p)}(s)\mathbf{K}_{\mathbf{P}}(s)\mathbf{S}(s) \\ \mathbf{W}_{\mathrm{s}}(s)\mathbf{T}(s) \end{bmatrix}. \tag{14}$$

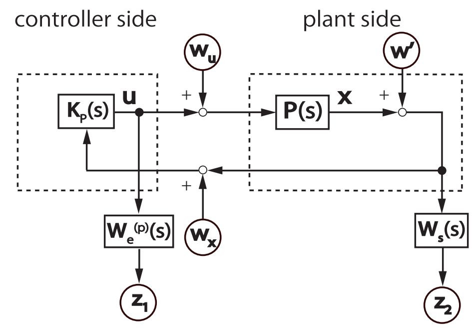

Fig. 3 Design block diagram of the proposed H$_\infty$ control.

The control information $\mathbf{u}[k]$ is similarly given by Eq. (8), where $\mathbf{A}_{\mathrm{c}}$,$\mathbf{B}_{\mathrm{c}}$,$\mathbf{C}_{\mathrm{c}}$,$\mathbf{D}_{\mathrm{c}}$ are coefficient matrices of the discretized controller $\mathbf{K}_{\mathrm{P}}(s)$.

To adopt Fig. 3 to the networked control system, an unique process is required to determine the weighting function $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$. First, time series of the influence of packet loss, which is defined as Eqs. (15) and (16), are recorded to determine $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$.

$$\mathbf{e_u}[k] = \mathbf{u}[k] - \hat{\mathbf{u}}[k], \tag{15}$$

$$\mathbf{e_x}[k] = \mathbf{x}[k] - \mathbf{x}_{\mathrm{e}}[k]. \tag{16}$$

The autocorrelation function is calculated from these time series, and then, the power density spectrum is derived by Fourier transform of the autocorrelation function. After that, the weighting function is determined based on the power density spectrum. This unique process is performed by computer simulation because it is difficult to analytically determine $\mathbf{W}_{\mathrm{e}}^{(p)}(s)$ due to the feedback loop.

5. Numerical Examples

5.1 Simulation Parameters

Computer simulations were performed to evaluate the performance of the proposed H$_\infty$ control system. A rotary inverted pendulum (Furuta pendulum), which is a typical underactuated object, was employed as the controlled plant. The basic structure of the rotary inverted pendulum is shown in Fig. 4 (see Appendix for its state-space model). The pendulum angle $\theta[k]$ is defined as zero when it is in its upright position and positive for counterclockwise rotation. The zero position for the arm angle $\varphi[k]$ can be arbitrarily defined and the counterclockwise rotation is defined as positive. The pendulum is controlled by applying a voltage $v_{in}[k]$ on the DC-motor for rotating the arm, i.e., $\mathbf{u}[k] = [v_{in}[k]]$. The state information $\mathbf{x}[k]$ is given as follows:

$$\mathbf{x}[k] = [\theta[k]\ \varphi[k]\ \dot{\theta}[k]\ \dot{\varphi}[k]]^T. \tag{17}$$

The pendulum's state space model of Eq. (1) is based on REALTEC RTC05 [38]. The physical parameters of the rotary inverted pendulum are shown in Table 1. The tuning parameters of the H$_\infty$ control design and the LQ control design are as follows:

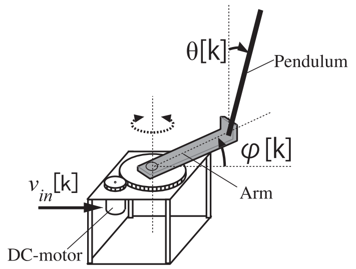

Fig. 4 Rotary inverted pendulum.

Table 1 Parameters of the rotary inverted pendulum.

Mass of pendulum rod m_p	0.016 kg
Length of the pendulum rod l_p	0.2 m
Length of the rotary arm r	0.2 m
Moment of inertia of the arm J_b	0.0048 kgm^2
DC-motor armature resistance R	8.3 Ω
DC-motor constant K_m	0.023 Nm/Amp
Gear ration (Arm:DC-motor) K_g	7.5
Gravitational constant g	9.81 m/s^2

$$\mathbf{W}_{\rm s}(s) = \tfrac{0.1s+10}{s+0.5} \cdot \mathrm{diag}\{10, 2, 0.5, 0.01\}, \tag{18}$$

$$\mathbf{W}_{\rm t}(s) = \tfrac{s+100}{0.05s+10} \cdot \mathrm{diag}\{10, 2, 0.5, 0.01\}, \tag{19}$$

$$\mathbf{Q} = \mathrm{diag}\{100, 1, 10, 0.1\}, \tag{20}$$

$$\mathbf{R} = 0.1. \tag{21}$$

These parameters are set to match the step response of the H$_\infty$ control system and the LQ control system. This setup is acceptable for comparing control performances.

The simulation starts from the initial state $\mathbf{x}[0] = [0\ 0\ 0\ 0]^T$. The rotary inverted pendulum is controlled to make its arm angle $\varphi[k]$ follow the target value $\Phi[k]$, while keeping the pendulum in an upright position, i.e.,

$$\mathbf{r}[k] = [\,0 \quad \Phi[k] \quad 0 \quad 0\,]^T. \tag{22}$$

The target value of the arm angle $\phi[k]$ is a rectangular wave:

$$\Phi[k] = \begin{cases} \pi/2 & (n-1)T \le kT_s < nT/2, \\ 0 & nT/2 \le kT_s < nT, \end{cases} \tag{23}$$

where $T = 10$[s] and $n = 1, 2, \ldots$. As the system disturbance $\mathbf{w}[k]$, Gaussian random vector is applied. The mean of $\mathbf{w}[k]$ is zero, the variance of angular elements of $\mathbf{w}[k]$ is σ^2[rad^2], and that of angular velocity elements is σ^2[(rad/s)^2]. The variance of the system disturbance is set to $\sigma^2 = 10^{-6}$. The simulation trials are repeated 1000 times and each trail lasted 1000 seconds.

Figure 5 shows an example of amplitude density spectrums of $\mathbf{e_u}[k]$ and $\mathbf{e_x}[k]$ (arm angles), which are recorded by simulation in the case of $p = 0.3$. This figure shows that the amplitude density spectrum of $\mathbf{e_u}[k]$ is much larger than that of $\mathbf{e_x}[k]$. In this case, it is sufficient to determine $\mathbf{W}_{\rm e}^{(p)}(s)$ only on the basis of $\mathbf{e_u}[k]$. To determine $\mathbf{W}_{\rm e}^{(p)}(s)$, an equation error method $invfreqs$ of MATLAB is used for the amplitude density spectrum from the lowest frequency (10^{-2}[rad/s]) to the second peak frequency (3 [rad/s]) of the

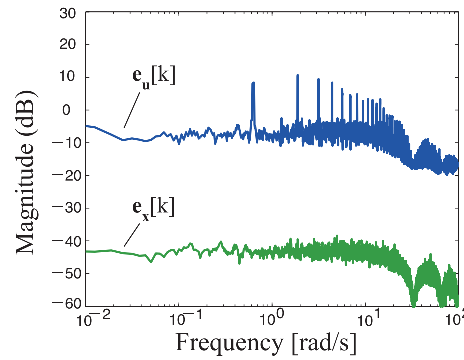

Fig. 5 Amplitude density spectrum of time series of influence of packet loss.

amplitude spectrum density of $\mathbf{e_u}[k]$. This is because the control bandwidth of this system is approximately 3 [rad/s]. Equation (24) shows the determined $\mathbf{W}_{\rm e}^{(p)}(s)$.

$$\mathbf{W}_{\rm e}^{(p=0.3)}(s) = \tfrac{0.17s^4+0.1033s^3+0.7157s^2+0.3126s+0.4157}{s^4+0.005485s^3+4.166s^2+0.01193s+2.407}. \tag{24}$$

$\mathbf{W_e}^{(p)}(s)$ is depend on packet loss rate p. In the case of $p = 0.5$, $\mathbf{W}_{\rm e}^{(p)}(s)$ is given as follows:

$$\mathbf{W}_{\rm e}^{(p=0.5)}(s) = \tfrac{0.4858s^4+0.2723s^3+2.008s^2+0.79s+1.178}{s^4+0.01841s^3+3.948s^2+0.04403s+2.162}. \tag{25}$$

The stability performance is evaluated by the pendulum fall rate and, the tracking performance is evaluated by root mean square error (RMSE) of the arm angle to that without both packet loss and system disturbances. If the angle of pendulum is $|\theta[k]| > \pi/6$, the pendulum is assumed to have fallen down. Once the pendulum has fallen down, each simulation run is terminated.

5.2 Performance Comparison of the Proposed and the Conventional H$_\infty$ Control Designs

First, to show the effectiveness of the proposed control design, performances of the conventional H$_\infty$ control and the proposed control are compared. The performance comparison of stability performance and that of tracking performance are shown in Fig. 6 and Fig. 7, respectively. In both figures, the horizontal axis is packet loss rate. Figure 6 shows that in the case of the proposed control design, the pendulum fall appears in $p = 0.6$, but in the case of the conventional control design, the pendulum fall appears in $p = 0.1$. This shows that the stability performance of the proposed H$_\infty$ control design is superior to that of the conventional H$_\infty$ control design. From Fig. 7, we can see that RMSE of the proposed H$_\infty$ control design is smaller than that of the conventional H$_\infty$ control design. Therefore, the tracking performance of the proposed H$_\infty$ control design is superior to that of the conventional H$_\infty$ control design.

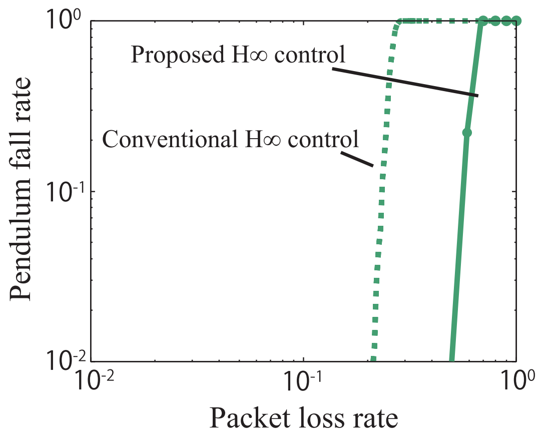

Fig. 6 Performance comparison of pendulum fall rate for the conventional and the proposed H_∞ control design with packet loss rate.

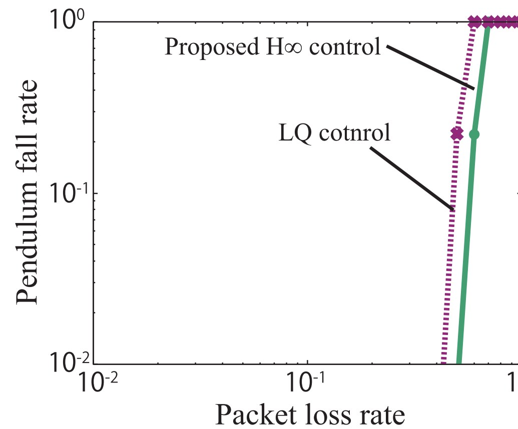

Fig. 8 Performance comparison of pendulum fall rate for the proposed H_∞ control design and LQ control design with packet loss rate.

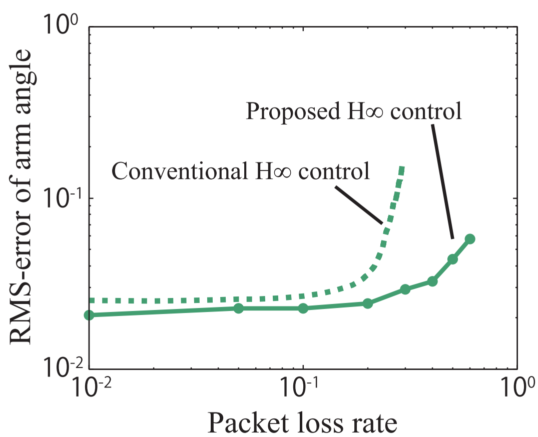

Fig. 7 Performance comparison of RMSE of arm angle for the conventional and the proposed H_∞ control design with packet loss rate.

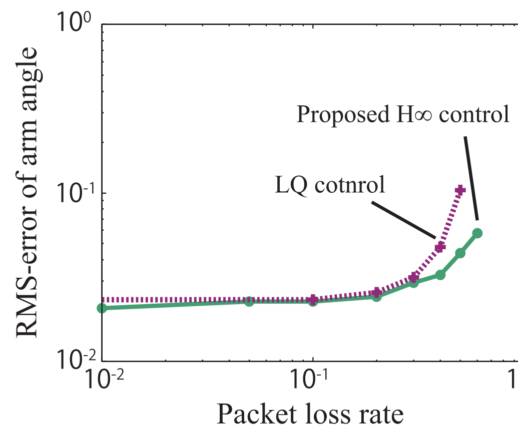

Fig. 9 Performance comparison of RMSE of arm angle for the proposed H_∞ control design and LQ control design with packet loss rate.

5.3 Performance Comparison of the Proposed H_∞ Control Design and the LQ Control Design

Next, to show whether the H_∞ control design or the LQ control design can get higher control performances against to packet loss, performance comparisons of the proposed H_∞ control design and the LQ control design are evaluated. The performance comparison of stability performance and that of tracking performance are shown in Fig. 8 and Fig. 9. In both figures, the horizontal axis is packet loss rate. Figure 6 shows that in the case of the proposed H_∞ control design, the pendulum fall appears in $p = 0.6$, but in the case of the LQ control design, the pendulum fall appears in $p = 0.4$. This shows that the stability performance of the proposed H_∞ control design is superior to that of the LQ control design. From Fig. 9, in low packet loss rate i.e., when the pendulum does not fall, there is little performance difference between the H_∞ control design and LQ control design. However, in high packet loss rate where the pendulum fall appears, the RMSE of the proposed H_∞ control design is smaller than that of the LQ control design. Therefore, the tracking performance of the proposed H_∞ control design is superior to that of the LQ control design.

6. Conclusion

This paper dealt with H_∞ control for networked control systems with packet loss. The H_∞ control design that considers packet loss as a disturbance was presented. First, the control performance of the proposed H_∞ control design and the conventional H_∞ control design was compared. The proposed control design gets better performance than the conventional control design, i.e., the proposed control design can reduce the performance deterioration due to packet loss. Next, the control performance comparison of the proposed H_∞ control design and the LQ control design was evaluated. The proposed H_∞ control design gets better performances than the LQ control design. From these results, it is shown that the proposed H_∞ control design that considers packet loss as a disturbance has superiority to realize the reliable networked control system.

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Appendix:

A.1 State-Space of Rotary Inverted Pendulum

The rotary inverted pendulum is a typical under-actuated object. Here, the angle, angular velocity, and angular acceleration of the pendulum are denoted as $\theta(t)$, $\dot{\theta}(t)$, and $\ddot{\theta}(t)$, respectively. Those of the arm are denoted as $\varphi(t)$, $\dot{\varphi}(t)$, and $\ddot{\varphi}(t)$, respectively. The input voltage to the DC motor is denoted as $v_{\mathrm{in}}(t)$. The linear approximated continuous-time state-space model of the rotary inverted pendulum is represented as follows [38].

$$\begin{bmatrix}\dot{\theta}(t)\\ \dot{\varphi}(t)\\ \ddot{\theta}(t)\\ \ddot{\varphi}(t)\end{bmatrix} = \boldsymbol{A}_{\mathrm{c}}\begin{bmatrix}\theta(t)\\ \varphi(t)\\ \dot{\theta}(t)\\ \dot{\varphi}(t)\end{bmatrix} + \boldsymbol{B}_{\mathrm{c}} v_{\mathrm{in}}(t), \tag{A$\cdot$1}$$

$$\boldsymbol{A}_{\mathrm{c}} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ g\frac{J_{\mathrm{a}}+m_{\mathrm{p}}r_{\mathrm{a}}^2}{l_{\mathrm{p}}J_{\mathrm{a}}} & 0 & 0 & \frac{r_{\mathrm{a}}K_{\mathrm{g}}^2K_{\mathrm{m}}^2}{R_{\mathrm{m}}l_{\mathrm{p}}J_{\mathrm{a}}}\\ -\frac{m_{\mathrm{p}}r_{\mathrm{a}}g}{J_{\mathrm{a}}} & 0 & 0 & -\frac{K_{\mathrm{g}}^2K_{\mathrm{m}}^2}{R_{\mathrm{m}}J_{\mathrm{a}}}\end{bmatrix}, \tag{A$\cdot$2}$$

$$\boldsymbol{B}_{\mathrm{c}} = \begin{bmatrix} 0\\ 0\\ -\frac{r_{\mathrm{a}}K_{\mathrm{g}}K_{\mathrm{m}}}{R_{\mathrm{m}}l_{\mathrm{p}}J_{\mathrm{a}}}\\ \frac{K_{\mathrm{g}}K_{\mathrm{m}}}{R_{\mathrm{a}}J_{\mathrm{a}}}\end{bmatrix}. \tag{A$\cdot$3}$$

Here, m_{p} and l_{p} are the mass and the half length of the pendulum. r_{a} and J_{a} are the length and central moment of inertia of the arm. R_{m}, K_{m}, and K_{g} are the DC motor's resistance, motor torque constant, and gear ratio, respectively. g is the gravitational acceleration constant.

The discrete-time state-space model of the rotary inverted pendulum with sampling interval T_{s} and zero-order hold input is represented as follows [39].

$$\begin{bmatrix}\theta[k+1]\\ \varphi[k+1]\\ \dot{\theta}[k+1]\\ \dot{\varphi}[k+1]\end{bmatrix} = \boldsymbol{A}\begin{bmatrix}\theta[k]\\ \varphi[k]\\ \dot{\theta}[k]\\ \dot{\varphi}[k]\end{bmatrix} + \boldsymbol{B} v_{\mathrm{in}}[k], \tag{A$\cdot$4}$$

$$\boldsymbol{A} = e^{\boldsymbol{A}_{\mathrm{c}}T_{\mathrm{s}}}, \quad \boldsymbol{B} = \int_0^{T_{\mathrm{s}}} e^{\boldsymbol{A}_{\mathrm{c}}\tau}\boldsymbol{B}_{\mathrm{c}}\,\mathrm{d}\tau\,, \tag{A$\cdot$5}$$

where $\phi[k] = \phi(kT_{\mathrm{s}})$, $\dot{\phi}[k] = \dot{\phi}(kT_{\mathrm{s}})$, $\theta[k] = \theta(kT_{\mathrm{s}})$, and $\dot{\theta}[k] = \dot{\theta}(kT_{\mathrm{s}})$.

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